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MDCCCLVI.

ADVERTISEMENT.

The Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body,

upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

The Meteorological Journal hitherto kept by the Assistant Secretary at the Apartments of the Royal Society, by order of the President and Council, and published in the Philosophical Transactions, has been discontinued. The Government, on the recommendation of the President and Council, has established at the Royal Observatory at Greenwich, under the superintendence of the Astronomer Royal, a Magnetical and Meteorological Observatory, where observations are made on an extended scale, which are regularly published. These, which correspond with the grand scheme of observations now carrying out in different parts of the globe, supersede the necessity of a continuance of the observations made at the Apartments of the Royal Society, which could not be rendered so perfect as was desirable, on account of the imperfections of the locality and the multiplied duties of the observer.

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ERRATA.

Page 264, line 1, *for* strains *read* stresses.

Page 270, line 34, *for* and minima *read* or minima.

PHILOSOPHICAL TRANSACTIONS.

I. *On the Constitution and Properties of Ozone.* *By* THOMAS ANDREWS, *M.D., F.R.S., M.R.I.A., Professor of Chemistry in Queen's College, Belfast.*

Received May 16,—Read June 21, 1855.

AMONG the many interesting bodies which the researches of modern chemists have brought to light, few are more remarkable than the substance to which the name of ozone has been given, whether we consider its many singular and anomalous properties, or its intimate relations with the most important and widely-diffused element in nature. For the first recognition of ozone and description of its properties, we are indebted to the sagacity of SCHÖNBEIN, to whom the entire merit of the discovery unquestionably belongs. His earlier experiments were, however, chiefly directed to the elucidation of its properties, and of the conditions under which it is formed; but not being accompanied by quantitative determinations, they did not throw any clear light on its actual constitution. The subject has also attracted of late years the attention of several very distinguished physical and chemical inquirers, among whom I may particularly mention MARIGNAC, DE LA RIVE, BERZELIUS, WILLIAMSON, FREMY and BECQUEREL, and BAUMERT.

SCHÖNBEIN has shown that a body having a peculiar and highly characteristic odour and very similar properties is formed under the three following conditions:—

1. When electrical sparks are passed through atmospheric air.
2. When pure water, or water holding certain acids or salts in solution, is decomposed by the voltaic current, the new substance appearing, along with the oxygen gas, at the positive pole.
3. When certain bodies, and particularly phosphorus, are slowly oxidized at common temperatures in atmospheric air.

Two distinct questions here arise for consideration. Is the same substance produced under these different conditions, or has SCHÖNBEIN included under the name of ozone substances having different compositions, although agreeing in some of their

properties? And next, what is the composition of ozone, or, if there be more ozones than one, how are they respectively constituted?

The experiments of WILLIAMSON * indicated the production of water, when ozone obtained by electrolysis was decomposed by being passed over heated copper, and BAUMERT † obtained similar results when he passed a stream of electrolytic oxygen through a tube containing anhydrous phosphoric acid, which was heated at one point to redness. These experiments were not, however, adapted to yield quantitative results, but they led to the general conclusion that this variety of ozone is an oxide of hydrogen containing more oxygen than water. But from another and very important experiment, to which I shall have occasion hereafter very fully to refer, BAUMERT has concluded that it is a teroxide of hydrogen, HO_3.

On the other hand, the experiments of DE LA RIVE and of FREMY and BECQUEREL ‡ have shown, that pure and dry oxygen gas may be converted by the electrical spark into ozone.

I am not aware of any experiments on ozone obtained by the action of phosphorus on atmospheric air, which throw any distinct light on its constitution. MARIGNAC passed a stream of this ozonized air through a solution of iodide of potassium, till the whole of the iodide was converted into iodate of potassa, and concluded that ozone produced in this way must be either oxygen in a peculiar state, or a peroxide of hydrogen.

According to the results, therefore, of the most recent investigations, it would appear,—

That the substances comprehended under the name of ozone are not identical;

That the ozone obtained by the action of the electrical spark on oxygen gas is oxygen itself in an altered or allotropic state;

That the ozone obtained by the electrolytic decomposition of water is an oxide of hydrogen, having the formula HO_3; and

That the ozone obtained by the action of phosphorus on oxygen is either oxygen itself, or a compound of oxygen and hydrogen §.

The subject of ozone has at intervals engaged my attention during the last four or five years, and I was actually occupied with a series of experiments on the production of ozone by the electrical spark, when the appearance of FREMY and BECQUEREL's able researches induced me for the moment to lay aside the inquiry. The publication of BAUMERT's memoir led me subsequently to resume it, as his results were not in accordance with those which I had previously arrived at. But the method proposed by that physicist to determine whether ozone is an oxide of hydrogen, or oxygen in an allotropic condition, appeared to be so well suited to the purpose, that

* Memoirs of the Chemical Society, vol. ii. p. 395. † POGGENDORFF's Annalen, Band lxxxix. S. 39.

‡ Annales de Chimie, 3ème série, xxxv. p. 62.

§ For a very complete account of all that is known on this subject, see the Article "Ozon" in the Handwörterbuch der Chemie, Band v. S. 835 (Braunschweig, 1853).

on resuming the inquiry, I considered it necessary in the first instance carefully to repeat his experiments. The results which I at first obtained were so far in accordance with those of BAUMERT, that they showed that the increase in weight of the apparatus was always more than the weight of the ozone, as deduced from its chemical action, but the relative proportion of these quantities was not in accordance with his results; nor, on repeating my own experiments, did they agree with one another. It was evident, therefore, that some disturbing cause existed which complicated the reaction, and, on further investigation, I not only found that such a cause did really exist, but succeeded in ascertaining its nature and the means of avoiding it. The experiments, on being now repeated, gave results very consistent with one another, and altogether at variance with the view that hydrogen is a constituent of ozone.

The apparatus which I employed was arranged as follows:—A is a vessel (Plate I. fig. 1) of about two litres capacity, containing a mixture of one measure of pure and strong sulphuric acid, and seven measures of distilled water. The cylinder B, which is filled with a similar solution, is closed below with a diaphragm of bladder, so as to prevent effectually any mixture of the gases evolved at the two poles. A platina wire, *pp*, traverses and is fused into a short glass tube, fitted by grinding into the tubulated neck *b*: this wire terminates below in a bunch of fine platina wires, which form the positive pole of a voltaic arrangement. The negative pole is a platina plate, *p'*, immersed in the liquid of the outer vessel. The vessel A was placed in a larger vessel containing cold water, to which ice was in some experiments added. This vessel has been omitted in the drawing for the sake of distinctness. CC'C'' is a continuous tube, united by fusion with the larger neck of B, and filled from C' to C'' with fragments of pumice, moistened with pure sulphuric acid. The length of the desiccating column was nearly one metre. D is a LIEBIG's apparatus, to the ends of which glass tubes were fused, which had previously been fitted by grinding, the one into the neck *c* of CC'C'', the other into a tube, which was in like manner fused to a second LIEBIG's apparatus, E. The connexions *c* and *e* were, therefore, formed by glass surfaces carefully ground. In my earlier experiments, these connexions were made by means of small and dry corks, which, on the whole, are more convenient than ground-glass joints, and are quite unobjectionable, as, when the surface is small and the cork dry, the amount of ozone destroyed by contact with the cork is wholly inappreciable. Caoutchouc connectors of any kind are altogether inadmissible; they are attacked with such energy by ozone, even when diluted with 1000 times its volume of other gases, that the tube becomes perforated in the course of a few minutes. The vessel D contained a solution of iodide of potassium, acidulated with a little hydrochloric acid, and the vessel E, concentrated sulphuric acid. The U-tube F, filled with pumice moistened with sulphuric acid, prevented any moisture from passing backwards into E. The oxygen evolved was collected in the graduated glass vessel G, inverted over water. The volume of the

oxygen gas was determined only for the purpose of ascertaining its relation to the ozone produced.

The mixture of oxygen and ozone, having been perfectly desiccated in its passage through the long tube CC′C″, enters the vessel D, where the ozone is decomposed, iodine being set free and caustic potassa formed, which latter, combining with the free hydrochloric acid, forms chloride of potassium. If a neutral solution of iodide of potassium is employed, the reaction is more complicated; for, while the greater part of the iodine is set free as before, and dissolves in the excess of iodide of potassium, iodate of potassa and caustic potassa are at the same time formed. Whether the solution be taken in an acid or neutral state, the final result is in this respect always the same, that the active oxygen enters into a chemical combination in the vessel D, and increases the weight of the liquid contained in that vessel.

The increase in weight of the vessels D and E will give the entire weight of the ozone, whether that body be allotropic oxygen, or an oxide of hydrogen. On the former supposition, the decomposition of the iodide of potassium will result in the substitution of oxygen for iodine, both remaining in D, while the sulphuric acid in E will retain the moisture which would otherwise be swept away by the current of dry gas; on the latter, ozone will become resolved into water and oxygen, both of which will be retained in the vessels D and E. Now by determining the amount of free iodine in the iodide of potassium solution at the end of the experiment, the amount of active oxygen by which it has been displaced may be easily calculated; and on comparing this with the increase in weight of the vessels D and E, it will at once be seen whether ozone be a peroxide of hydrogen yielding water in its decomposition.

Two experiments of this kind were performed by BAUMERT; in the first, the increase in weight of the apparatus amounted to 0·0133 grm., and the weight of the oxygen, as calculated from the iodine set free, to 0·0081 grm.; in the second, the same quantities were respectively 0·0149 grm. and 0·00989 grm. The iodide of potassium was employed in the state of a neutral solution, and the iodate of potassa was subsequently decomposed by the addition of a little hydrochloric acid.

It was from these results that BAUMERT inferred that the ozone which accompanies oxygen obtained by the electrolysis of water, is an oxide of hydrogen having the formula HO_3; and this conclusion, deduced from experiments which were devised with great skill and executed with care, has, in Germany at least, received very general assent.

Having, as already mentioned, found, on a repetition of these experiments, that a different expression resulted for the composition of ozone from every new trial, I instituted a diligent search into all the circumstances of the experiment, and at last succeeded in referring the irregularities to the presence of a small, but appreciable quantity of carbonic acid, which, unless very great precautions be taken, is always present in electrolytic oxygen. When baryta water was substituted for the solution of iodide of potassium in D, a precipitate of carbonate of baryta appeared in the

course of a few minutes. With caustic potassa in the same vessel, the increase in weight, for the same volume of oxygen gas, was considerably greater than with the solution of iodide of potassium, and at the end of the experiment it was found that carbonate of potassa had been formed. Now as a small quantity of free potassa is always produced during the action of ozone on a neutral solution of iodide of potassium, it appeared not improbable that this would seize upon a portion of the carbonic acid just referred to, and thus the augmentation in the weight of the apparatus would depend upon two distinct causes,—the ozone reaction, and the absorption of carbonic acid. To prevent the occurrence of the latter, it was only necessary to acidulate the solution of iodide of potassium, so as to prevent the formation of free potassa, or to boil for some time the liquid subjected to electrolysis. The acidulation of the solution alone was found to be sufficient to prevent the carbonic acid from being absorbed, for when this precaution was attended to, the results were the same, whether the electrolyte was boiled immediately before the commencement of the experiment or not. With this modification, the irregularities previously observed in different trials disappeared, and the simple and interesting result was obtained, that the increase in weight of the apparatus was exactly equal to the amount of oxygen deduced by calculation from the iodine set free.

I will now describe the chief precautions which I adopted to avoid, as far as possible, all sources of error in the following experiments, the delicacy of which will at once be apparent, if we consider that not more than 40 milligrammes of ozone are contained, under the most favourable circumstances, in 10 litres of electrolytic oxygen; and that it was necessary to have the arrangements so perfect, that this large quantity of gas (supposed to be free from ozone) should traverse the apparatus without producing any appreciable change in the united weight of the vessels D and E.

The solution of iodide of potassium employed in all the experiments had the same composition, although the quantity of ozone obtained in some cases was three times greater than in others. It consisted of 2 grms. of iodide of potassium dissolved in $22\frac{1}{2}$ grms. of a weak solution of hydrochloric acid, containing 2 per cent. of pure acid. As it is difficult to procure iodide of potassium entirely free from iodate of potassa, I always prepared, at the commencement of each experiment, two similar solutions, of which one was introduced into D, and the other preserved in a ground stoppered vial, till the experiment was finished. The amount of free iodine in both was determined at the same time, and the difference taken to represent the exact quantity of iodine due to the ozone reaction. The correction for the iodate of potassa in the original solution, when reduced, rarely represented more than 0·001 grm. oxygen, but quantities of this magnitude must not be neglected in these experiments.

Previous to weighing the vessels D and E, one litre of atmospheric air, deprived of carbonic acid and carefully desiccated, was passed through the apparatus. The object of this precaution was to bring every part of the apparatus into the same state

at the beginning of the experiment, in which it would be at the end. The same volume of dry air was passed through the apparatus at the conclusion of each experiment.

It is rarely necessary in chemical investigations to apply a correction to the direct indications of the balance for changes in the temperature and pressure of the atmosphere, during the interval between two successive weighings. By preserving the apartment at a pretty uniform temperature, the corrections for thermometric changes may be confined within very narrow limits, but the movements of the barometer are not under our control; and when, as in these experiments, a period of two and sometimes of three days elapsed between the first and second weighings, it occasionally happened that the change in the atmospheric pressure was considerable, and an appreciable error (amounting in some instances to nearly 0·002 grm.) would have occurred, if no correction had been applied*.

To ascertain how far the action of the apparatus might be relied on, one or two preliminary experiments were made, which gave very satisfactory results. The vessel D, containing pure water, E, sulphuric acid, and another LIEBIG's condenser, also containing sulphuric acid, having been interposed between E and F, 3·5 litres of oxygen gas not containing ozone, followed by 1 litre of atmospheric air, were passed through the apparatus. The time occupied in the passage of the gas was about five hours. The vessel D lost 0·0311 grm., while E gained 0·0315 grm., the third vessel not sustaining any appreciable change of weight. If, therefore, D and E had been weighed together, the change of weight would have been only 0·0004 grm. In another experiment, in which a solution of strong caustic potassa was placed in D, the loss of D was 0·0175 grm., and the gain of E 0·0172 grm., the difference being less than one-third of a milligramme. Other experiments of the same kind, with different solutions in D, gave similar results. It is evident, therefore, that at the rate at which the gas traversed the apparatus, the whole of the moisture carried off from the liquid in D was retained by the sulphuric acid in E.

To determine whether a notable quantity of iodine would be carried over by the current of the gas from D to E, a solution of iodide of potassium containing a large

* This correction was calculated as follows:—To the volume in cubic centimetres of the solution of iodide of potassium in D and of sulphuric acid in E, was added the volume of the glass of which the vessels D and E were formed. From this was deducted the volume of the weights employed Let V be the difference of the volumes so found in cubic centimetres; p and p', the atmospheric pressures in English inches at the first and second weighings; t and t', the corresponding temperatures in Centigrade degrees; x, the weight, in grammes, of a volume of air equal to V, measured under the pressure p, and at the temperature t; x', the weight of the same volume of air at p' and t'. Then, since 1 cub. cent. air, at 0°, and under a pressure of 29·92 inches, weighs 0·00129 grm.,

$$x'-x=V\left(\frac{1}{1+0\cdot00367t'}\cdot\frac{p'}{29\cdot92}-\frac{1}{1+0\cdot00367\,.\,t}\cdot\frac{p}{29\cdot92}\right).0\cdot00129.$$

The value of $x'-x$ is to be added to, or subtracted from, the increase of weight, as found by direct experiment, according as it is a positive or negative quantity.

quantity of free iodine was introduced into D, and a solution of pure iodide of potassium into E. After passing 4 litres of air through the apparatus, E was found to contain 0·0015 grm. iodine. This is equivalent to one-tenth of a milligramme of oxygen, and, from the large excess of iodine in the first solution, must be a greater quantity than could have been carried over in any of the subsequent experiments, although in some of them larger volumes of gas were passed through the apparatus.

The free iodine was determined according to the very delicate method first, I believe, proposed by BUNSEN. A dilute solution of sulphurous acid was prepared, and its strength determined, immediately before analysing the liquid in D, by ascertaining how many measures of it were required to destroy a known weight of free iodine in a solution of iodide of potassium. A corresponding experiment was made with the solution in D, from which the quantity of free iodine in it was deduced by a very simple calculation.

I. 10·2 litres of electrolytic oxygen containing ozone were passed through the apparatus at the rate of about three-quarters of a litre per hour. At the first weighing, the barometer was 29·85 in. and the thermometer, 5°·9 C.; at the second weighing, the barometer was 29·98 in. and the thermometer, 5°·3. The value of V (see preceding note) was 47 cub. cent. The gain in weight of the double apparatus D and E was 0·0375 grm., which gives, when corrected for atmospheric changes, for the true gain,

0·0379 grm.

The free iodine in the solution contained in D, was neutralized by 112·7 measures of a dilute solution of sulphurous acid. The other solution of iodide of potassium, which had been prepared at the same time as the first, and to which the same amount of acid had been added, required 0·8 measure of the same solution of sulphurous acid for neutralization. Hence the iodine eliminated by the action of the ozone was equivalent to 111·9 measures. Next, 0·5341 grm. pure iodine was added, together with 2 grms. of iodide of potassium, to a few drops of water, and when both were dissolved, the solution was diluted till it occupied exactly 100 cub. cent. From the mean of two experiments which closely agreed with one another, it appeared that 100 measures of the solution of sulphurous acid neutralized 95·96 cub. cent. of this solution, and hence 1 measure of the former corresponded to 0·00512 grm. iodine. From these data it follows, by an easy calculation, that the iodine disengaged by the ozone amounted to 0·609 grm., and the equivalent of oxygen to

0·0386 grm.

II. 2·72 litres of electrolytic oxygen were passed through the apparatus at the same rate as before. At first weighing, barometer 29·60 in., thermometer 5°·8 C.; at second weighing, barometer 29·60 in., thermometer 6°·0 C. Gain of weight of D and E 0·0107 grm., corrected,

0·0107 grm.

The free iodine in D, after deducting the iodine due to the small quantity of iodate

of potassa in the original solution, was neutralized by 30·23 measures of a solution of sulphurous acid, of which, as ascertained by direct experiment made at the time, 1 measure neutralized 0·00521 grm. free iodine. Hence the oxygen due to the displacement of iodine was

0·0100 grm.

III. 2·86 litres of the same gas as in the preceding experiments were passed through the apparatus. At first weighing, barometer 30·06 in., thermometer 6°·6 C.; at second weighing, barometer 30·20 in., thermometer 6°·1 C. Gain of weight of D and E 0·0152 grm., corrected,

0·0154 grm.

The free iodine in D, corrected as before, was neutralized by 41·52 measures of a solution of sulphurous acid, of which 1 measure neutralized 0·00525 grm. iodine; hence the weight of oxygen, as deduced from the weight of iodine set free, was

0·0138 grm.

IV. 6·45 litres of electrolytic oxygen were passed through the apparatus. At first weighing, barometer 29·96 in., thermometer 6°·8 C.; at second weighing, barometer 29·29 in., thermometer 7°·8 C. Gain of weight of D and E 0·0303 grm., corrected,

0·0288 grm.

The free iodine in D was neutralized by 100·4 measures of a solution of sulphurous acid, of which 1 measure neutralized 0·00441 grm. iodine; hence the weight of oxygen deduced in this way was

0·0281 grm.

V. 6·8 litres of electrolytic oxygen passed. At first weighing, barometer 30·53 in., thermometer 9°·8 C.; at second weighing, barometer 30·44 in., thermometer 10°·4 C. Gain of weight of D and E 0·0254, corrected,

0·0251 grm.

The free iodine in D neutralized 107·9 measures of a solution of sulphurous acid, of which 1 measure was equivalent to 0·00358 grm. iodine; hence the weight of oxygen deduced from the iodine set free was

0·0273 grm.

Collecting these results and adding them together, so as to obtain the mean of the whole, we have

	Ozone deduced from the increase in weight of the apparatus.	Ozone deduced from the iodine liberated.
I.	0·0379 grm.	0·0386 grm.
II.	0·0107 grm.	0·0100 grm.
III.	0·0154 grm.	0·0138 grm.
IV.	0·0288 grm.	0·0281 grm.
V.	0·0251 grm.	0·0273 grm.
	0·1179 grm.	0·1178 grm.

The agreement is complete, and proves unequivocally that water is not a product of the decomposition of ozone, which therefore does not contain hydrogen as a constituent. If its composition were HO_3, the apparatus would have increased 0·1841 grm. in weight, instead of 0·1179 grm.

The amount of ozone formed in these experiments was tolerably uniform. For 1 litre of oxygen the following weights of ozone were obtained:—

	I.	0·0038 grm.
	II.	0·0037 grm.
	III.	0·0046 grm.
	IV.	0·0043 grm.
	V.	0·0040 grm.
Mean . .		0·0041 grm.

In the arangement above described, the oxygen gas derived from the electrolytic decomposition of water was therefore accompanied by about $\frac{1}{350}$th of its weight of ozone.

In order to remove every possible doubt from these results, I fitted up an apparatus from every part of which organic substances were excluded. No diaphragm was used, and all the connexions were made, either by fusing the ends of the connecting tubes together, or by means of ground glass joints. The arrangement is represented in fig. 2. Two platina wires (fig. 3) were fused into the end of a glass tube, which was fitted by grinding to the tubulated neck *b* of the vessel A. The tube BB′B″ was connected at *a* with the vessel A by a ground joint, and with C by fusion. It contained pumice moistened with sulphuric acid. The vessel C was also filled with sulphuric acid, and was connected by a ground glass joint with the iodide of potassium vessel D. The vessel E contained, as before, sulphuric acid. In this experiment, both the hydrogen and the oxygen traversed the apparatus, the accuracy of which was thus exposed to a very severe test.

Twenty-two litres of the mixed gases were passed through the apparatus. The gain in weight of D and E was 0·0135 grm., the respective heights of the barometer at the first and second weighings having been 28·96 in. and 29·57 in., and the temperatures 11°·1 and 10°·0. The correction for change of pressure and temperature is therefore +0·0014 grm., and the true gain

0·0149 grm.

The free iodine in D, due to the action of the ozone, neutralized 62·65 measures of a solution of sulphurous acid, of which 1 measure corresponded to 0·00373 grm. iodine. The weight of ozone deduced from the iodine set free is therefore

0·0148 grm.

The identity of these results is very satisfactory, when it is considered that this small weight of ozone was separated from 22 litres, or nearly five gallon measures of

the mixed gases. The relative quantity of ozone to the amount of water decomposed is less than in the former experiments, arising perhaps partly from a single platina wire having been in this case employed as the positive pole. In this experiment, great care was taken to exclude both carbonic acid and nitrogen from the electrolyte.

My next object was to determine, by careful quantitative experiments, whether water is really formed, as WILLIAMSON and BAUMERT have stated, when ozone is decomposed by heat. For this purpose, the same general arrangement was employed as in the first series of these experiments; but the first LIEBIG's apparatus D, instead of being filled with a solution of iodide of potassium, was now empty, and placed in the upper part of a metallic cylinder (fig. 4 HH), where it was raised to a temperature of about 400° C., by a current of heated air from a LESLIE's burner. To the sulphuric acid apparatus E, was permanently attached and weighed along with it, a small U-tube G, containing anhydrous phosphoric acid, so as to secure the condensation of the last trace of aqueous vapour, if any were present. The oxygen gas was collected and measured as in the former experiments.

Two experiments were made. In the first, 6·8 litres of oxygen containing 0·027 grm. ozone were passed through the apparatus; in the second, 9·6 litres containing 0·038 grm. ozone. The compound sulphuric and phosphoric acid apparatus was found, all corrections having been made, to have increased, in the one case one-third, and in the other case one-half of a milligramme in weight. Such quantities can only be referred to the unavoidable errors of experiment. If ozone were a compound body having the constitution HO_3, the apparatus would have gained in the first experiment 10, and in the second 14 milligrammes.

That ozone cannot contain nitrogen will appear from the following experiment. Two platina wires were hermetically sealed into the bottom of a small flask, into which water, containing a little sulphuric acid, was introduced and made to boil rapidly for some time. While the water was in a state of ebullition, the wires were connected with the poles of a voltaic arrangement, so as to disengage the mixed gases along with the vapour of water. So long as the ebullition continued, no ozone made its appearance; but on allowing the liquid gradually to cool, without arresting the current, its presence soon became manifest from its odour and action on iodide of potassium paper. The ebullition and the current of the mixed gases must have rendered the presence of nitrogen here impossible.

One question still remains to be answered. Does ozone, besides oxygen, contain any other constituent which is not absorbable by any of the reagents employed? Although the gas which escaped from the apparatus, after the separation of the ozone, appeared to be pure oxygen, yet it would be rash to assert that it might not have contained some unknown body amounting to $\frac{1}{1000}$th of its weight, and having no very salient properties. This question appeared to me to admit of solution in another way. It will be seen, in a subsequent part of this paper, that there can be no doubt of the formation of ozone from pure and dry oxygen by the action of the electrical

spark, and nothing is easier than to convert the whole of a given volume of oxygen into ozone in presence of a solution of iodide of potassium. The next step in the inquiry was therefore to ascertain whether ozone derived from electrolysis, from the action of the electrical spark, and from the oxidation of phosphorus, exhibited a perfect identity in all its properties.

One of the most remarkable properties of ozone is its destruction by heat, or rather its conversion by heat into ordinary oxygen. To ascertain the temperature at which this change occurs, the vessel D, fig. 4, was placed in a bath of mercury, and the gas examined as it escaped, without previously passing it through the rest of the apparatus. On heating the mercurial bath, the amount of ozone, as determined by its action on iodide of potassium paper*, did not notably diminish till the temperature attained 230° C. It still continued, however, very intense till the thermometer rose to 235°. Between that point and 240° the ozone reactions entirely disappeared, when the ozone was in a very dilute state; but when more concentrated, slight traces of ozone could still be discovered, which no doubt would have also disappeared if the current of gas had been passed very slowly. Time is in fact an element in this action. Even at the temperature of 100° C. ozone is slowly destroyed. Two similar tubes were filled, at the same time, and by the same process, with ozone diluted as usual with oxygen, and afterwards hermetically sealed. One of these tubes was maintained for three hours in a vapour bath at 100° C., the other was not exposed to heat. On examining both tubes at the end of the time, it was found that the ozone in the tube which had been exposed to heat was perceptibly less than in the other. I have no doubt that, even at the common temperature of the air, ozone preserved in an hermetically sealed glass tube would gradually change into common oxygen. I made an experiment of this kind two or three years ago, which resulted in the disappearance of the ozone, but I do not remember the source from which the ozone was derived, nor what precautions were taken to dry the gas.

On the other hand, ozone brought directly into contact with the vapour of water at the boiling-point is instantly destroyed.

To obtain a continuous stream of ozone from the action of the electrical spark, a current of pure oxygen gas, obtained from the decomposition of the chlorate of potassa, and purified and dried by passing through tubes containing hydrate of potassa and sulphuric acid, was exposed to a rapid succession of electrical sparks. To obtain a sufficient stream of electricity, an electrical machine, firmly screwed down to the floor of the apartment, was connected by a belt with a heavy cast-iron wheel, 40 inches in diameter, contained in a frame which was also firmly secured to the floor. By this arrangement, the machine could be worked for any length of time

* Bibulous paper which has been dipped into a solution of iodide of potassium of moderate strength and afterwards allowed to dry, but still retaining its hygrometric moisture, is the most convenient test of ozone. If it be exposed to a continuous current of dry air, it should be removed from time to time and its hygrometric moisture restored.

continuously, the plate performing about 360 revolutions per minute. It was of course necessary to apply very frequently a hand rubber covered with amalgam to the plate, and it required the cooperation of three persons to permit the work to be easily performed. On passing the gas through the apparatus at nearly the same rate as in the experiments already described, an abundant stream of ozone was obtained, which enabled me to institute a very exact comparison between its properties and those of ozone obtained by electrolysis.

When heated in the mercurial bath, ozone prepared in this way was rapidly destroyed at the temperature of 237° C., which is the same temperature at which electrolytic ozone was also destroyed. The vapour of boiling water, in like manner, caused all the ozone reactions to disappear.

The action of water at common temperatures and of alkaline solutions upon ozone is very remarkable. It is commonly stated that caustic potassa absorbs ozone, but that pure water, and solutions of lime, baryta, and ammonia, have no action upon it. This statement is far from being accurate. Pure water does not absorb ozone, and a stream of air containing ozone may be passed for any length of time through water without producing any change in the properties of the water. I have also preserved ozone for several days in a stoppered vial containing a little distilled water, and although the vessel was agitated from time to time, the ozone did not disappear. On the other hand, pure water has the property of destroying a small quantity of ozone. If ozone, obtained by the electrolysis of water, or by the action of the electrical spark, or by means of phosphorus, be largely diluted with atmospheric air, it will entirely disappear, if an attempt be made to collect it in a jar inverted over water. The following experiment is more precise. A flask provided with a ground glass stopper, of the capacity of half a litre, was filled with equal volumes of water and atmospheric air and inverted in the pneumatic trough. The ozone in a single bubble of electrolytic oxygen, passed quietly through the water into this volume of air, could easily be detected; but on agitating the water briskly, even four or five bubbles were deprived of their ozone. The same gas, agitated with twice its volume of lime water, or one-third of its volume of baryta water, also ceased to exhibit the reactions of ozone. In like manner, the action of caustic potassa is also limited. A strong solution of that alkali in a LIEBIG's apparatus deprived one litre of electrolytic oxygen of its ozone, after which the ozone passed freely through it. These phenomena are singular and characteristic, and are the same with ozone from whatever source it is derived.

Peroxide of manganese destroys ozone, affording an interesting example of what is commonly called catalytic action. The oxide of manganese does not increase in weight, nor is water formed. Ozone from the three sources gives the same results.

The odour of ozone, from whatever source derived, is the same. The same remark applies to its property of bleaching, without producing at first an acid reaction.

Iodide of potassium is decomposed with the formation of iodate of potassa, and

oxidable substances in solution, as the protosulphate of iron, are raised to a higher state of oxidation by all the varieties of ozone.

It would not be difficult to extend this comparison, but enough has been shown, I conceive, to establish the absolute identity in properties of ozone in whatever way it may be prepared. Any difference which, on a superficial examination, may appear to exist, will be found on further inquiry to arise from the ozone being in a more or less dilute state.

That ozone is formed by the action of the electrical spark on perfectly dry oxygen, is placed beyond all doubt by the following experiment. The curved tube *a*, fig. 5, having two platina wires, *pp'*, hermetically sealed into it, was inverted over mercury and carefully filled with pure oxygen, after which a little sulphuric acid was introduced into one end (*b*). The whole was allowed to remain for twenty-four hours, when the oxygen was considered to be perfectly dry. Electrical sparks were now passed for some time between the platina wires, after which a solution of iodide of potassium was introduced into the other end of the tube. It became immediately coloured from the formation of free iodine, and the colour continued slowly to increase as the ozone was gradually absorbed.

Again, a solution of iodide of potassium may be made to absorb the whole of the oxygen in a narrow tube, by the passage of electrical sparks. This experiment has been described by FREMY and BECQUEREL, and I have myself repeatedly verified its accuracy. With a thermometer tube 2 inches long, the whole of the oxygen may be made to disappear in the course of one minute. The solution becomes always red from the decomposition of the iodide of potassium.

We have already seen that neither hydrogen nor nitrogen can be constituents of ozone, whether it be obtained from electrolysis, or from the action of the electrical spark on oxygen; and further, that all the supposed varieties of ozone exhibit in all respects identical properties.

Connecting all these facts together, it clearly follows,—

That no gaseous compound having the composition HO_3 is formed during the electrolysis of water; and

That ozone, from whatever source derived, is one and the same body, having identical properties and the same constitution, and is not a compound body, but oxygen in an altered or allotropic condition.

II. *On the Formation and some of the Properties of Cymidine, the Organic Base of the Cymole Series. By the Rev.* John Barlow, *Chaplain-in-Ordinary to Her Majesty's Household at Kensington Palace, M.A., F.R.S., Vice-President and Secretary of the Royal Institution of Great Britain, &c.*

Received June 14,—Read June 21, 1855.

By submitting the nitro-compounds of the series of hydrocarbons, of which benzole is the lowest term, to Zinin's well-known process, chemists have successively obtained the organic bases corresponding to these hydrocarbons. Aniline, toluidine, xylidine and cumidine have been prepared in this manner. Aniline was obtained by this process by Zinin * himself in 1845; toluidine by Muspratt and Hofmann †, in 1845; xylidine by Cahours ‡, in 1850; and cumidine by Nicholson, in 1847 §. Whilst the last-named chemist was engaged in the laboratory of the Royal College of Chemistry with the study of cumidine, the derivative of cumole, Dr. Noad, at the suggestion of Dr. Hofmann, pursued the same direction of research towards cymole, the only remaining hydrocarbon of this group, with a view of completing the series of the alkaloids ||. In his experiments on the action of concentrated nitric acid on cymole, Dr. Noad found that this hydrocarbon differs somewhat from the other members of the series. Instead of furnishing the nitro-substitute, which is the link of connexion between the hydrocarbon and the alkaloid, cymole was found to undergo a partial oxidation, a portion of the carbon being eliminated in the form of carbonic acid with the simultaneous formation of several acids which belong to a group of bodies standing lower in the scale of organic compounds. The study of these very interesting acids, toluylic and nitrotoluylic, appears to have detached Dr. Noad's attention from the formation of the substitution-product of cymole; and when he again returned to this inquiry he soon quitted it, in order to pursue still further a research in which he had already been successful ¶.

At the suggestion, and under the direction of Dr. Hofmann, the following investigation has been made, in the hope of filling up a gap still existing in the series of

* Journal für practische Chemie, Band xxxvi. S. 98.

† Mem. Chem. Soc. vol. ii. p. 367.

‡ Comptes Rendus, tome xxx. p. 321.

§ "On Cumidine, a new Organic Base," Reports of Royal College of Chemistry, p. 178.

|| Mem. Chem. Soc. vol. iii. p. 421.

¶ "On some of the Products of the Decomposition of Nitrotoluylic Acid," Philosophical Transactions, vol. cxliv.

bases, and the desired alkaloid has been obtained; thus completing the inquiry which was commenced in the Royal College of Chemistry seven years ago.

Preparation of Nitrocymole.—It has been already stated that cymole, when heated in the usual way with concentrated nitric acid, produces substances differing greatly from the expected substitution-compound. To prevent this result, cymole and nitric acid, having been severally kept for some minutes in a freezing mixture of ice and salt, were cautiously mixed. The cold cymole was dropped from a pipette into the cold nitric acid. The mixture was at first brown, but, on continued addition of cymole, it gradually changed to green; and when the substitution was complete, it assumed the consistency of cream. It was then projected into cold water, when the oily reddish-brown nitrocymole subsided to the bottom of the vessel. This was washed, first with water, and lastly, with a weak solution of carbonate of soda.

Properties of Nitrocymole.—In the purest state in which it has yet been obtained, nitrocymole is a reddish-brown, transparent, oily fluid, of greater specific gravity than water, in which it is insoluble. It does not appear to be readily decomposed by contact of air. When distilled with water it produces a neutral oil, which floats on water, than which it is, consequently, of less specific gravity. It was found impossible to obtain the boiling-point either of nitrocymole or of this oily distillate, or to procure the body in a state fit for analysis. But subsequent examination of substances derived from it removed all doubt that its composition is represented by the formula $C_{20} H_{13} NO_4$.

Formation of Cymidine from Nitrocymole.—The process of ZININ, although successfully adopted in obtaining the lower bases of this series, was not found equally effectual when applied to this new substitution-product. A better result, though the quantity was still very scanty, was obtained by BÉCHAMP's modification of this method *. Nitrocymole was added to iron-filings brought to the consistency of paste by mixture with acetic acid, whereupon an immediate and considerable elevation of temperature indicated the commencement of chemical action. From the distillation of this mixture at a carefully-regulated heat a complicated product resulted. A considerable proportion of the distillate was found to be insoluble in hydrochloric acid. This was set aside for separate examination, the result of which will be stated at the close of this memoir. To the portion of the distillate which was soluble in hydrochloric acid, soda was added in quantity just sufficient to neutralize the acid. The base (for which the name cymidine is suggested by obvious analogy) was then separated by ether and subsequently distilled. The same instability of composition, and tendency to oxidize, which have been noted as characterizing the hydrocarbon cymole, exist in this base, derived from it. Accordingly, it was found necessary to perform the distillation in an atmosphere of hydrogen, in order to prevent the conversion of the cymidine, at a high temperature, into a resin.

* Annales de Chimie et de Physique, tome xlii. p. 193.

Properties of Cymidine.—The specific gravity of cymidine is less than that of water. Its boiling-point is about 250° Cent.; it has a yellow colour; it is odorous; it does not affect test-paper. It is slightly soluble in water, and completely soluble in alcohol and ether. The quantity of this alkaloid available for experiment was too small to admit of its being entirely purified by distillation. It was therefore necessary to have recourse to the analysis of its platinum-salt for ascertaining its composition.

Platinum-salt of Cymidine.—When bichloride of platinum is added to an aqueous solution of the hydrochlorate of cymidine, a characteristic yellow salt is obtained, which, like cymidine itself, is slightly soluble in water, more soluble in alcohol, and most of all in ether.

I. ·4598 grm. of this substance, burnt with chromate of lead and oxide of copper, gave

·5695 grm. of Carbonic acid.
·1935 grm. of Water.
·1270 grm. of Platinum.

This, calculated for 100 parts, gives 33·777 Carbon.
4·675 Hydrogen.
27·620 Platinum.

II. ·4213 grm. of the same substance, ignited, gave ·1178 grm. of platinum, which, calculated for 100 parts, gives 27·699.

These numbers lead to the formula $C_{20}H_{15}N, HCl, PtCl_2$, as may be seen from the following comparison of the theoretical and experimental numbers:—

	Theory.		Experiment.
		Per cent.	Per cent.
20 equivs. Carbon . .	120	33·784	33·777
16 equivs. Hydrogen .	16	4·505	4·675
1 equiv. Nitrogen . .	14	3·941	
3 equivs. Chlorine . .	106·5	29·983	
1 equiv. Platinum . .	98·7	27·787	27·641 {mean of the two experiments.
1 equiv. Platinum-salt	355·2	100·000	

Hydrochlorate of Cymidine.—When strong hydrochloric acid is added to cymidine, fumes are evolved, the hydrochlorate of cymidine, in the form of an oily layer, rises to the surface of the fluid, and crystallizes on being agitated and evaporated. Like the salts of aniline, it stains firwood yellow, but, unlike these salts, it produces no reaction with the chloride of lime; it also stains the skin red.

A determination of hydrochloric acid in the hydrochlorate of cymidine was made.

Weight of hydrochlorate taken . . . ·412 grm.
Weight of chloride of silver . . . . ·319 grm.

From whence the per-centage number 19·679 hydrochloric acid is obtained. This

number agrees with the per-centage corresponding to the formula $C_{20} H_{15} N HCl$, as will appear on comparing the theoretical and experimental numbers.

	Theory.		Experiment.
		Per cent.	Per cent.
1 equivalent of Cymidine . . .	149	80·281	
1 equivalent of Hydrochloric acid .	36·5	19·719	19·679
1 equivalent of Hydrochlorate of Cymidine . .		100	

Sulphate of Cymidine.—A white crystalline salt, soluble in water.

Oxalate of Cymidine.—A white crystalline salt, soluble in water.

Gold Salt.—A yellow crystalline salt, slightly soluble in water.

Iodine has no perceptible reaction on cymidine.

Bromine produces a very feeble reaction on this base.

When chloride of cyanogen was added to cymidine, a slight action took place, and the resulting substance, after having been boiled with water, and filtered, gave a precipitate on the addition of caustic soda; thus affording evidence of the presence of a salt of a new solid base, probably analogous to melaniline.

On adding chloride of benzoyle to cymidine, a slight action ensued, and a few minute crystals appeared, probably of benzo-cymidide.

Nitric acid acts violently on cymidine; a semi-solid substance separating on addition of soda.

These are, however, merely qualitative experiments, and require further elaboration.

Examination of substance, insoluble in hydrochloric acid, produced during the formation of Cymidine.

Having been purified by repeated distillation, this substance was burnt with oxide of copper, when the following result was obtained:—

·2623 grm. gave ·8615 carbonic acid and ·2530 water.

The formula of cymole is $C_{20} H_{14}$. On comparing the per-centage numbers derived from this formula with those obtained from the substance by experiment, we find

	Theory.		Experiment.
		Per cent.	Per cent.
10 equivalents of Carbon . .	120	89·552	89·213
14 equivalents of Hydrogen .	14	10·448	10·672
	134	100·000	99·885

The chemical identity of this substance with cymole is thus established.

The boiling-point 175° C. also coincides with that of camphogene, which is recognized as the isomer of cymole. There is however an important physical distinction between cymole and this isomeric substance. It has already been stated, that when cymole is submitted to the action of fuming nitric acid at $-17\frac{7}{9}$° C. a reddish-brown

nitrocymole is produced, which is specifically heavier than water. But when this cymole-isomer (α-cymole) is similarly treated, there results a dark oily liquid, which, on being projected into water, first blackens, then becomes pale yellow, and finally floats on the surface, indicating a specific gravity less than that of water.

From this *nitro-α-cymole*, an α-cymidine was also obtained by BÉCHAMP's process, and from this a platinum-salt was formed. The platinum determination of this salt gave the following result:—

Weight of platinum-salt taken	·1840 grm.
Weight of platinum obtained	·0509 grm.

This gave a per-centage weight of 27·662 platinum, differing only by ·125 from the theoretical per-centage 27·787, calculated from the formula $C_{20} H_{15} N, HCl, PlCl_2$. Further researches are however necessary to establish the relation of this substance to cymidine.

An unsuccessful attempt to obtain the boiling-point of nitrocymole has been already noticed. The distillate from this operation was lighter than water, and had the colour and general appearance of α-cymole; when treated with fuming nitric acid in a freezing mixture, the resulting substance presented a close resemblance to nitro-α-cymole.

Nitrocymole and cymidine, the formation of which is described in the preceding memoir, complete the series of nitro-substitutions and bases corresponding to the hydrocarbons, of which benzole is the lowest homologue, as will appear from the following synoptical table:—

Hydrocarbons.		Nitro-substitutions.		Bases.	
Benzole	$C_{12} H_6$	Nitrobenzole	$C_{12} H_5 NO_4$	Aniline	$C_{12} H_7 N.$
Toluole	$C_{14} H_8$	Nitrotoluole	$C_{14} H_7 NO_4$	Toluidine	$C_{14} H_9 N.$
Xylole	$C_{16} H_{10}$	Nitroxylole	$C_{16} H_9 NO_4$	Xylidine	$C_{16} H_{11} N.$
Cumole	$C_{18} H_{12}$	Nitrocumole	$C_{18} H_{11} NO_4$	Cumidine	$C_{18} H_{13} N.$
Cymole	$C_{20} H_{14}$	*Nitrocymole*	$C_{20} H_{13} NO_4$	*Cymidine*	$C_{20} H_{15} N.$

The author of this memoir desires, in conclusion, to record his sense of the valuable assistance he received from Mr. ALFRED NOBLE, Student in the College of Chemistry.

III. *Some Observations on the Ova of the Salmon, in relation to the distribution of Species; in a letter addressed to* CHARLES DARWIN, *Esq., M.A., V.P.R.S. &c.*

By JOHN DAVY, *M.D., F.R.S., Inspector-General of Army Hospitals.*

Received March 27,—Read April 26, 1855.

MY DEAR SIR,

IN a letter with which you have favoured me, that of the 28th of January, you did me the honour to ask my aid in an inquiry in which you take an interest, in common, as you remark, with most naturalists, viz. the geographical distribution of species, especially that of fish. At the same time you expressed your opinion that some useful information might be procured by experiments on the impregnated ova of the latter, were they so conducted as to show what the ova are capable of bearing without loss of vitality, and under exposure to circumstances such as might be compatible with their being conveyed from one river or lake to another, adhering, for instance, to the plumage, beak or legs of birds. In reply, I acquainted you of my willingness, should I have an opportunity, to accede to your wishes; and, that occurring, having been so fortunate as to procure the means of making some experiments likely to be elucidatory, I have now the pleasure of communicating the results obtained.

All the experiments I have to describe have been made on the ova of the Salmon, for which I have been indebted to two gentlemen, JOHN BARKER, Esq., of Broughton Lodge in Cartmel, and WILLIAM AYRTON, Esq., of Chester. By the first, through one of his keepers, I was supplied with a considerable quantity of ova, taken from a breeding-bed in the Leven, a river that flows out of Windermere, and from a part of it near Newby Bridge, about eighteen miles distant from my house. Through the latter I obtained ova from Overton on the Dee, taken from boxes in which they had been placed in the process, as it has been called, of artificial breeding.

Both gentlemen were so good as to desire the keepers, in packing the ova, to attend to the directions I gave in writing, with the intent of commencing the inquiry even in the act of their being sent. Those from the Leven were divided into three portions; one, of 110 ova, was contained in an eight-ounce vial, two-thirds full of water, which was changed more than once on the way; another, of 75 ova, was enclosed in wet wool; and the third, of 62 ova, in dry wool. The latter two were in a small box, the lid on, which box as also the bottle were carried by hand. These ova reached me in about twenty-four hours from the time they were taken from the river, and were received on the 6th of February. They all appeared healthy and in

good progress of development, the eyes of the embryos being visible, and the blood-corpuscles distinct in the vessels of the vitelline membrane, when placed under the microscope, using a glass of one-inch focal distance. Without loss of time they were variously distributed; some in shallow earthenware pans, some in finger-glasses used at table, and with water in all little more than sufficed to cover them. No gravel was added. The water employed was well-water of considerable purity, of about 50° Fahr., and was changed once daily, and once only. The vessels were kept in a room, the temperature of which seldom exceeded 50°, and was rarely below 46°. Most of these eggs proved productive, and have yielded young and vigorous fish. The first which broke their shell appeared on the 15th of February, the last on the 17th of March: of the total number not more than three or four aborted.

The ova from the Dee were received on the 7th of February, conveyed by rail, and had been sent off the preceding day. One portion of them was in a two-ounce vial, two-thirds full of water; another, in a vial of the same size full of water; a third, in dry sand; a fourth, in wet sand; a fifth, in wet cotton-wadding; and a sixth, in dry wadding: all enclosed in a covered box. These ova on arrival exhibited no signs of organic development. They were distributed immediately much in the same manner as the preceding, and were treated in the same way, but with a different result. All of them in succession became opake from imbibing water, and not in a single instance were there any indications afforded of vital progress; leading to the inference that they were dead when they reached me. From Mr. Ayrton I have recently been informed that the ova remaining in the box from which those had been taken were doing well; and hence, necessarily, the conclusion, that the journey had been fatal to those I received. This may have been owing to their having been sent at so early a stage; and I may mention in confirmation, that a second supply which was forwarded to me later—three weeks later—sent by post in moist wool, in a more advanced stage, nearly as much advanced as those from the Leven, arrived alive and are now hatched. It may perhaps be said, that the treatment of the unsuccessful ova after I received them, especially as to the manner in which the water was supplied, was the cause of their failure: but this does not appear to me probable, having found it to succeed with the ova of the delicate Charr,—ova taken by myself from the parent-fish, and impregnated forthwith and immediately distributed in the same kind of vessels as those now used, and the water in which, of the same quality, was changed once only daily.

Having premised thus much, I shall now describe the several experiments which I have made for the purpose of testing the power of endurance of the ova. Unless otherwise specified, it is to be understood that the ova in each instance used were of those from the Leven.

I. *Of Exposure to the Atmosphere.*

1. An ovum exposed for an hour on a slip of glass to the air of a room at 64°, placed near a fire, became dry superficially without its circulation being stopped.

Returned to water, its circulation was distinct at the end of forty-eight hours. Nine days after it was in a dying state.

2. An ovum on a slip of glass was exposed to the air of a room at 52° for two hours. The shell then had become at one spot indented as from shrinking, the effect of evaporation, yet the circulation seemed unimpaired; but transferred to water, the circulation presently stopped, the egg becoming opake from the absorption of water, and of course dead.

3. An ovum exposed to the air of a room on a watch-glass from noon till 4 P.M., the thermometer rising from 49° to 51°, had become dry and shriveled. From the state of the shell the blood-vessels were indistinct under the microscope. Put into water, in one or two minutes a rupture of the shell took place and the young fish escaped. It was very languid, only the slightest indications of life being perceptible; yet the heart did not cease its feeble action till the eighth day, counting from the rupture of the shell.

4. An ovum on a support of glass was exposed to the air of a room at 49° for an hour and ten minutes; its shell was slightly indented. Returned to water, on the second day the young fish burst its shell, was vigorous, and so continued.

5. An ovum was exposed to the air of a room during the night for about ten hours; the thermometer under 50°. The following morning it was found shriveled; put into water, the shell presently burst; the young fish, excepting for a slight motion of its pectoral fins, appeared lifeless, and it soon died.

6. An ovum placed on a rock in the open air in the shade at 38°, after two hours was slightly shriveled and its circulation had become languid. The following morning its circulation had ceased, and it shortly became opake.

7. An ovum placed on snow during a thaw with occasional gentle rain, the air about 34°, and kept there from half-past nine in the morning till four in the afternoon, did not appear to be shrunk, nor was its circulation interrupted. Replaced in water, its circulation the following day was active.

II. *Of Exposure to Moist Air.*

To ascertain the effect of exposure to moist air, I have made many experiments, as by placing the wet ova in watch-glasses covered with other glasses of the same size; keeping them in moist wool, from which water had been wrung out; and in vials slightly wet within; in each instance taking the precaution to allow of the admission of air. The trials have been made at temperatures varying from 34° to 50°. The results have been so uniform that I do not think it necessary to enter into minute details. The ova in no instance appear to have materially suffered, whether the exposure has been for an hour or for several days. Thus, in one experiment, nine ova were kept in a vial, one of six-ounce capacity, eleven days; examined then under the microscope, the circulation in each of them appeared to be vigorous, as vigorous as before; and, replaced in water, they all produced healthy fish, and sooner on an

average than those constantly kept in water. These ova were from the Leven. In another experiment, with ova last received from the Dee, four were kept in a vial of the same capacity and merely moist within, fourteen days without apparently suffering; they were all hatched on being replaced in water. And, in a third trial, two ova, also from the Dee, have been kept in moist wool twelve days, also without any appearance of injury, these too having been hatched after having been put into water.

III. *Of Exposure in Air and Water to a Temperature at or below the Freezing-point.*

1. An ovum exposed on a watch-glass to the open air from 4 P.M. one day to 10 A.M. the following, the thermometer at 30° at the commencement and termination of the trial, had become slightly shriveled and its circulation was stopped; put into water with snow, so as to be gradually thawed if frozen, it did not revive; its death was denoted by its yelk becoming opake.

2. An ovum exposed to the open air at about 30° for an hour, was found adhering to the slip of glass on which it rested by a frozen drop of water, so that it could be carried inverted without falling off. Under the microscope, still attached to the glass by ice, the blood-corpuscles were seen in slow motion in the vessels; in one vessel they were moving backward and forward. Where adhering to the glass the ovum was slightly flattened. Removed to water, the following day the embryo was seen active and the circulation vigorous; thirteen days later the young fish burst its shell, and was to all appearance uninjured.

3. Another ovum, exposed to the open air of 29° for an hour and twenty minutes, was found frozen to the glass, but without loss of vitality. The result was the same as that of the preceding experiment.

4. An ovum exposed in water in a watch-glass to the open air during the night, the thermometer so low as 20°, was found in the morning included in ice and dead; the yelk had become opake and was probably frozen.

5. Exposed an egg in a wine-glass to the open air from 3 P.M. one day to 10 A.M. the next, the thermometer as low as 22°. The whole of the water was frozen; when thawed no circulation was visible in the ovum; two days after a feeble circulation was detected, which ceased the following day and the yelk became opake.

6. An ovum exposed in water to the open air, about 31°, in an hour was covered with a pellicle of ice; the circulation had become languid. An hour and a half later, the thermometer at 30°, the ovum was included in ice; the circulation much the same. The experiment was continued about eighteen hours longer, the ovum included in ice at about the same temperature; the circulation was now languid but distinct, and the ovum was nowise altered in appearance.

7. An ovum was exposed in water in a wine-glass to the open air below the freezing-point. When a pellicle of ice had formed on the water, the glass was surrounded with wool in a little box and left in the open air. During the night the thermometer fell to 9°. The ovum in the morning was found adhering to the bottom of the

glass by ice, and the inside of the glass was coated with ice, the greater portion of the egg however remaining in water. The ice thawed, the circulation was seen going on, and it soon became active. Twelve days after, the young fish burst its shell, was, and has continued vigorous.

8. An ovum was exposed in water to the open air at 28°. In about two hours the water was frozen at the surface and spicula of ice had formed round the ovum, as it were shooting from it. Thawed on the following day, the circulation was found to be vigorous, and in eleven days a young active fish was produced.

9. An ovum, one of those last received from the Dee, was exposed to the open air, placed on green moss, and left so exposed during three entire days and nights. It was then returned to water. In six hours after it was hatched. The young fish was languid and in point of size comparatively diminutive, as if prematurely produced, yet the action of the heart was vigorous, and the circulation as seen under the microscope normal. It may be right to notice the kind of weather that prevailed during the exposure of the ovum. During the first twenty-four hours the thermometer by day was between 36° and 38°; there was some rain, ·39 inch was the quantity, and partial sunshine; during the night the thermometer on the grass fell so low as 29°·5; there was a little rain, ·02 inch. During the second twenty-four hours the thermometer by day varied from 39° to 33°; the air most of the time was misty, but without rain; at night the thermometer fell to 28°, yet, as there was no frost in the morning, it was probably so low only for a very short time. During the last twenty-four hours, the state of atmosphere and the temperature differed but little from what they were in the preceding. Part of the time, especially during the latter third, the ovum was a good deal protected by the leaves of the moss, between which it had sunk. At the end of the three days it was neither dry nor shriveled, and only very slightly indented, and that on the point on which it rested. It is worthy of remark, that it was the first hatched of the ova last received from the Dee; and that the young fish, now six days old, is alive and thriving.

IV. *Of Exposure in Water to a Temperature of, or above 70°.*

In these trials ova were employed and young fish, and chiefly the latter, as better adapted to show the effect of the high temperature. In each instance the ovum or young fish was put into a thin glass vessel of the capacity of about four ounce measures, nearly full of water, and this vessel was placed in a water-bath of the temperature required. The temperature given in each following instance was that of the water in which the subject of the experiment was immersed.

1. An ovum kept two hours and a half in water at 70°, placed under the microscope, was found to have its circulation somewhat impaired, rendered more languid; kept in two hours more, the temperature rising to 80°, no further injurious effect was produced, at least that was apparent. The vessel was now withdrawn from

the bath and allowed to cool gradually. When next seen, ten hours later, a young fish had burst its shell and was vigorous.

2. An ovum and a young fish were kept in water between 68° and 72° about eight hours. The ovum, one of those from the Dee, was then found hatched, and the young fish produced was tolerably active. The following day both were exposed about nine hours to a temperature between 70° and 80°, rarely reaching 80°. At the end of this time they appeared languid, and when in motion disposed to irregular movements. Removed from the water-bath, on the following day they were active, and exhibited no peculiarity appreciable that could be attributed to the higher temperature to which they had been subjected. The ovum in its hatching in this instance preceded all the others from the Dee, with the exception of the one already mentioned, that exposed three days to the open air.

3. A young fish and an ovum were put into water which in the bath presently acquired the temperature of 82°, and in an hour rose to 85°. Now taken out and allowed to cool gradually, the circulation in the young fish was found to be very languid, the heart contracting feebly. The following day it was found dead. The ovum did not appear to suffer materially; three days after, it was hatched and a vigorous young fish was produced.

4. An ovum kept in water for two hours, at a temperature from 90° to 95°, lost its translucency, and opened under water was found to be dead.

5. An ovum, one of the last from the Dee, kept half an hour in water at 100°, afforded the same result.

6. A young fish was kept in water three hours, the temperature of which at the commencement was 70°; it rose to 85°, and when taken from the bath it had fallen to 82°. The heart then was acting with tolerable vigour, and the day following the fish appeared to be nearly in its usual state: five days later it was alive and tolerably active, but less vigorous than those which had not been so exposed.

7. A young fish kept in water an hour at 84° was found dead. No action of the heart was perceptible nor of any of the muscles when it was taken out. Another young fish was put into the water when cold without experiencing any bad effect. This trial was made to be certain that the fatal effect was not owing to want of air in the water.

8. A young fish was kept in water rising in temperature from 78° to 81° three hours and a half without any permanent bad effect that was appreciable. When taken out it appeared torpid, but the heart was acting well. Two days after the fish was as active as before.

9. A young fish, kept two hours in water between 88° and 90°, was, when taken out, dead.

10. A young fish, kept only a few minutes in water at 92°, appeared to be dying when taken out; the circulation in its tail was stopped and the heart was acting feebly;

in about a quarter of an hour it ceased to act. The following morning the fish had a sodden appearance, and its disintegration had commenced.

11. A young fish was put into water at 80°; after three hours, when the temperature had risen to 85°, it appeared to be dead; its body was bent and it had become pallid. Under the microscope the heart was seen acting feebly, and the circulation was proportionably languid. On the following day the body had become unbent; the circulation in the tail had ceased, but the heart was still acting feebly. Two days later the heart's action had ceased, and the only vestige of life was indicated by a just perceptible motion of the lower jaw, which was protracted three days longer.

12. A young fish was kept in water gradually rising from 78° to 88° for three hours. At 85°, the heart acting, no circulation was perceptible in the tail; at 88° the body had become bent and pale, and the heart's action arrested.

V. *Of the Effect of Salt and Brackish Water.*

1. An active young fish and an ovum in which the circulation was vigorous, were put into a solution of common salt of the specific gravity 1026, which it may be conjectured is nearly the degree of saltness of the sea at the estuaries of our salmon rivers. The fish immediately became restless, and the heart's action accelerated. At the end of five hours it appeared to be dying; the heart's action had become so languid as not to suffice for the circulation; notwithstanding, life was not entirely extinct, as was indicated by a feeble motion of the lower jaw, till about forty-eight hours from the commencement of the experiment. The dead fish was colourless and contracted in all its dimensions, and shortened at least one-third of its length.

The effect of the salt water on the ovum was equally fatal, but judging from the circulation, life was protracted in it a few hours longer.

2. An active young fish was put into a solution of common salt of the specific gravity 1016. It lived about four days, the heart's action gradually becoming feebler till the circulation ceased. When dead there was an accumulation of blood in the large vessels, and, as in the former instance, a diminution of the bulk of the fish, as if from contraction. The saline solution, it may be remarked, was changed daily so as to be sure that death was not owing to, or had been hastened by, deficiency of air.

3. A young fish was put into a solution of salt reduced to the specific gravity 1007, so as to be only slightly brackish. Immediately on immersion it showed great restlessness and increased activity, which continued with little abatement for several days. It has now been in the solution ten days. During the two last its activity has diminished, and at times it has appeared to be dying. It is rather more changed in form than the fish of the same age left in spring water, and the vitelline sack is decidedly more diminished, as if from increased vascular action produced by the stimulus imparted by the solution.

4. An ovum from the Dee, the circulation in which was active, was put into saline

water of the same specific gravity as the last. It was hatched at the end of about forty-eight hours. The young fish was at first languid; now, on the fourth day, it is little altered; it is seen commonly lying on its side, and is restless only by fits and starts.

Besides the experiments above detailed, I have made others, but differing so little in their results, that I do not think it necessary to describe them even in confirmation.

VI. *Concluding Remarks.*

On the conclusions which may be drawn from the experiments as bearing on the subject under consideration I shall be very brief; for the sake of order I shall advert to each section.

From the experiments detailed in the first section, it would appear that the ova of the Salmon in an advanced stage can be exposed to the open air, if dry, but a short time, at ordinary temperatures, without loss of vitality; but for a considerable time, if the temperature be low and if the air be moist; the limit in the former case not having exceeded an hour, whilst in the latter it has exceeded many hours.

From the experiments in the second section, it would appear that the vitality of the ova was as well preserved in air saturated with moisture, as it would have been had they been in water.

From the experiments in the third section, it would appear that the ova might be included in ice without losing their vitality; but that if exposed to a temperature many degrees below the freezing-point, probably effecting their congelation, they were deprived of their vitality.

From the experiments in the fourth section, it would appear that both the ova and the young fish were capable of bearing a temperature of about 80° or 82° in water for a moderate time with impunity, but not without loss of life at a higher temperature, any exceeding 84° or 85°.

From the experiments in the fifth section, it would appear that a degree of saltness of water equal, or nearly equal, to that of sea-water is pretty speedily fatal both to the ovum of the Salmon and to the young fish; that the same effect is produced on the young fish by brackish water of specific gravity 1016, but in a longer time; and that when the solution is so diluted as to be reduced to the specific gravity 1007, the advanced ovum may be hatched in it, and the life of the young fish may be sustained in it for many days, but with diminishing power.

Finally, in reference to the distribution of species, do not many of the preceding results render it probable, in the instances of fish of the salmon-kind, and by analogy of other kinds, that it may be effected in the manner you have suggested in proposing the inquiry, viz. by means of impregnated ova conveyed by animals, whether birds or quadrupeds, adhering to some part of their body, such as their feathers or hair, feet or mouth,—by the latter provided the temperature do not exceed 84° or 85°? And, during rain or snow, are we not warranted in concluding that an ovum may be taken

from one river to another without loss of vitality by an Otter or Heron or other aquatic bird, if lodged in the mouth of the one—with the proviso mentioned above, —or in the bill of the other; or during a time of frost or snow if adhering to the feet of either of the animals mentioned?

When my attention was first given to the subject, which was before I was favoured with your letter, I imagined that the impregnated ova might be conveyed in the stomach of birds, taken up from one river, and, it might be, disgorged in another, without loss of vitality, inasmuch as the ova of the Salmon found in the stomach of a trout have been known to be productive when returned to water. For an authenticated instance of the kind, I may refer to a report by Mr. HALLIDAY, the agent of Messrs. EDMUND and THOMAS ASHWORTH, on the artificial process of breeding Salmon carried on at Oughterard in Galway*. It was to test this conjecture that the experiments in the fourth section were made; and, I may add, with negative results, knowing as we do, that the temperature of the stomach of birds is usually above 100° of FAHR.

Besides the main and express object for which the preceding experiments were made, I trust the results may be of some use in aiding to solve the question as to the period, the age, at which the impregnated ova of fish are most retentive of life, and consequently, are in the state best fitted for transport without loss of life; and that those in the two last sections may help to explain the absence of the Salmonidæ in tropical seas and in those approaching to them in temperature, such as the Mediterranean; and may also throw a little light on some of the peculiar habits as well as on the localities of their migratory species.

I am, my dear Sir, yours very truly,

Lesketh How, Ambleside, JOHN DAVY.
March 21, 1855.

* The Report is attached to "A Treatise on the Propagation of Salmon and other Fish," by EDMUND and THOMAS ASHWORTH: London, SIMPKIN and MARSHALL, 1853.

IV. *On the Effect of Local Attraction upon the Plumb-line at Stations on the English Arc of the Meridian, between Dunnose and Burleigh Moor; and a Method of computing its Amount. By the Venerable* JOHN HENRY PRATT, *M.A., Archdeacon of Calcutta. Communicated by the Rev.* J. CHALLIS, *M.A., F.R.S. &c.*

Received June 5,—Read June 21, 1855.

1. IN a former communication I endeavoured to point out a method for calculating the deflection of the plumb-line at stations on the Indian arc, caused by the attraction of the Himalayas and the vast regions beyond, with a view to the correction of the astronomical amplitudes of the measured subdivisions of the arc, before they are applied to the determination of the ellipticity of the earth.

The same subject is taken up in the present paper, but in reference to one of the ENGLISH arcs, that between Dunnose and Burleigh Moor; and a different method of calculating the attraction is given.

I. *Calculation of the Ellipticity of the English Arc between Dunnose and Burleigh Moor, without taking account of Local Attraction.*

2. The data for this calculation are taken from MUDGE's 'Trigonometrical Survey of England,' vols. ii. iii., and are as follows:—

Arc between	Amplitude.	Arc in feet.	Latitude of middle point.
1. Dunnose and Greenwich	0° 51′ 31″·39	313696·0	51° 2′ 53″·316
2. Greenwich and Blenheim	0 21 47·90	132802·0	51 39 33·316
3. Blenheim and Arbury Hill	0 23 0·30	139822·0	52 1 57·831
4. Arbury Hill and Clifton	1 14 3·40	450045·3	52 50 29·580
5. Clifton and Burleigh Moor	1 6 50·11	406462·9	54 0 56·335

Captain KATER has shown, by an examination of the scale employed, that the lengths of the arcs in feet should all be corrected by multiplying by 0·00007 and adding the results; but as I shall use only the ratios of these arcs to each other, this correction need not be applied.

3. From these data I now proceed to compare these five subdivisions of the arc between Dunnose and Burleigh Moor, two and two, and thence deduce the ten values of the ellipticity which the ten combinations will give, and the arithmetic mean of them, which will be a fair representation of the mean ellipticity of the whole arc, upon the supposition that the above amplitudes are correct; that is, upon the supposition that there is no local attraction.

Let λ be the amplitude of any one of these arcs, expressed in seconds of a degree:
μ the latitude of its middle point;
α the number of feet in the arc;
a the semi-axis major of the ellipse to which the arc belongs;
ε the ellipticity of the arc.

Then, by the Integral Calculus,

$$\frac{\alpha}{a}=\lambda\left\{1-\varepsilon\left(\frac{1}{2}+\frac{3}{2}\frac{\sin\lambda}{\lambda}\cos 2\mu\right)+\frac{1}{16}\varepsilon^2\left(1+15\frac{\sin\lambda}{\lambda}\cos\lambda\cos 4\mu\right)\right\},$$

neglecting the cube and higher powers of ε;

$$\therefore\ \lambda=\frac{\alpha}{a}\left\{1+\varepsilon\left(\frac{1}{2}+\frac{3}{2}\frac{\sin\lambda}{\lambda}\cos 2\mu\right)\right.$$
$$\left.+\varepsilon^2\left(\frac{1}{2}+\frac{3}{2}\frac{\sin\lambda}{\lambda}\cos 2\mu\right)^2-\frac{1}{16}\varepsilon^2\left(1+15\frac{\sin\lambda}{\lambda}\cos\lambda\cos 4\mu\right)\right\}.$$

The coefficient of ε^2 in this may be written thus:

$$\frac{1}{4}\left\{\frac{3}{4}+\frac{15}{4}\frac{\sin\lambda}{\lambda}\cos\lambda+6\frac{\sin\lambda}{\lambda}\cos 2\mu+9\left(\frac{\sin\lambda}{\lambda}\right)^2\left(1-\frac{5}{6}\frac{\lambda}{\tan\lambda}\right)\cos^2 2\mu\right\}.$$

Since $\tan\lambda$ is always greater than λ, the coefficient of $\cos^2 2\mu$ is always positive; and therefore the greatest value which the coefficient of ε^2 can attain is when $\cos 2\mu=1$; in which case it equals

$$\frac{1}{4}\left(\frac{3}{4}+\frac{15}{4}\frac{\sin\lambda}{\lambda}\cos\lambda+6\frac{\sin\lambda}{\lambda}\right)+\frac{9}{4}\left(\frac{\sin\lambda}{\lambda}\right)^2\left(1-\frac{5}{6}\frac{\lambda}{\tan\lambda}\right),$$

or $3-\frac{3}{8}\lambda^2$, neglecting higher powers of λ, which is always small. As the ellipticity is a very small fraction, and this coefficient is not a large number, the square of ε may be neglected without any perceptible error. Hence

$$\lambda=\frac{\alpha}{a}\left\{1+\varepsilon\left(\frac{1}{2}+\frac{3}{2}\frac{\sin\lambda}{\lambda}\cos 2\mu\right)\right\};$$

or if we make $$E=\frac{1}{2}+\frac{3}{2}\frac{\sin\lambda}{\lambda}\cos 2\mu,$$

then $$\lambda=\frac{\alpha}{a}(1+\varepsilon.E).$$

Let $\lambda_1\lambda_2\lambda_3\ldots\alpha_1\alpha_2\alpha_3\ldots E_1E_2E_3\ldots$ be the values of λ, α, E for the several arcs.

Hence $$\frac{\lambda_1}{\lambda_2}.\frac{\alpha_2}{\alpha_1}=1+\varepsilon(E_1-E_2),$$

or $$\varepsilon=\frac{\frac{\lambda_1}{\lambda_2}.\frac{\alpha_2}{\alpha_1}-1}{E_1-E_2}.$$

Put $$\frac{\lambda}{\alpha}=A,$$

then $$\varepsilon=\frac{\frac{A_1}{A_2}-1}{E_1-E_2}.$$

Similar expressions for ε may be obtained by comparing all the five arcs two and two; and hence

$$\text{mean ellipticity}=\frac{1}{10}\Sigma\left(\frac{\frac{A_1}{A_2}-1}{E_1-E_2}\right).$$

4. The following Table exhibits the values of E and A for the several arcs (see Appendix):—

	Values of E.	Values of A.
1st arc	0·18569	0·00985473
2nd arc	0·15446	0·00984850
3rd arc	0·13545	0·00987184
4th arc	0·09455	0·00987324
5th arc	0·03571	0·00986587

From this, taking the arcs two and two, we obtain the following results (see Appendix):—

Arcs compared.	Ellipticity deduced therefrom.
1st and 2nd	+0·0202690
1st and 3rd	−0·0344944
1st and 4th	−0·0205622
1st and 5th	−0·0075277
2nd and 3rd	−0·1244090
2nd and 4th	−0·0418294
2nd and 5th	−0·0148295
3rd and 4th	−0·0596822
3rd and 5th	+0·0607577
4th and 5th	+0·0125964

Mean value . . $=-0·0209711=-\frac{1}{47·6846}$.

5. This mean value differs widely from the mean ellipticity of the whole earth, which is about $\frac{1}{300}$. The discrepancy must arise, either from the English arc being curved very differently to the mean meridian of the earth and belonging to an ellipse of which the polar axis is *greater* than the equatorial in the ratio of 48·6846 : 47·6846, or from the amplitudes being incorrectly determined. In this I assume that the arcs themselves are measured with such exactness as to preclude the possibility of error in the ellipticity from this source. (An error of 100 feet would not make an error of 1″ in the amplitude.)

It is evident that the latter is the true cause of the ellipticity coming out so different to the mean ellipticity of the earth; for the ellipticities deduced from the comparison of the several subdivisions of the arc, two and two, would not vary among themselves, as the last Table shows they do, if the whole arc were elliptic. It may

be concluded that the true cause is error in the amplitudes, or in the latitudes of the stations terminating the several arcs. And as these latitudes have been deduced from the most careful observations, it must be inferred that the errors arise from local attraction affecting the plumb-line.

6. An examination of the values of A deduced above will serve to show where the chief sources of attraction lie.

If the elements of the mean meridian of the earth be taken (as laid down by Mr. AIRY in his Article on the Figure of the Earth) to be

$$a=20923713 \text{ feet}, \ \varepsilon=\frac{1}{300\cdot 8},$$

then the formula for A in art. 3. leads to the following values of A for arcs with their middle latitudes the same as those of the subdivisions of the English arc under consideration [the calculation is given in the Appendix]:—

1st arc, value of A=0·00986402
2nd arc, value of A=0·00986300
3rd arc, value of A=0·00986238
4th arc, value of A=0·00986104
5th arc, value of A=0·00985911

If we take the differences between these and the values before deduced, we have

1st arc, defect of A below the mean =0·0000929
2nd arc, defect of A below the mean =0·0001450
3rd arc, excess of A above the mean =0·0000946
4th arc, excess of A above the mean =0·0001220
5th arc, excess of A above the mean =0·0000676

If these are multiplied by the lengths of the several arcs, the results will be the errors in the amplitudes; that is, on the supposition that the ellipticity of the arc is the same as the mean ellipticity of the whole earth, and that the discrepancies in the amplitudes arise from local attraction alone.

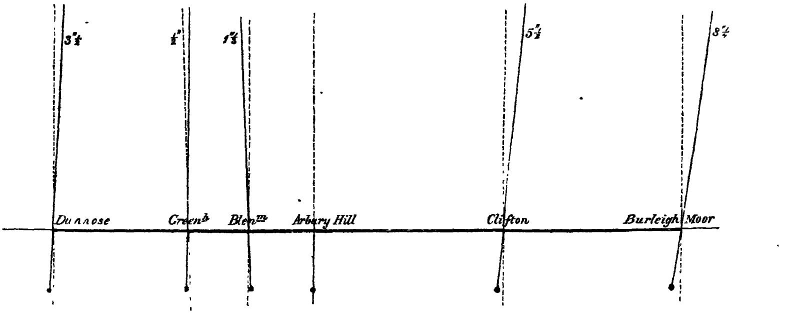
The results are as follows (see Appendix):—

1st arc, amplitude is in *defect* $2''\cdot 914$
2nd arc, amplitude is in *defect* 1·926
3rd arc, amplitude is in *excess* 1·323
4th arc, amplitude is in *excess* 5·491
5th arc, amplitude is in *excess* 2·748

From this it appears that the local attraction is so distributed as to make the observed zeniths of Dunnose and Greenwich, and also of Greenwich and Blenheim, to approach each other, and those of the extremities of the other three arcs to recede from each other; and *that* in the degree marked by the above angles.

7. In the following diagram I lay down these angles (on an exaggerated scale) in

order to represent to the eye the effects produced by local attraction. The parallel dotted lines are the positions which the verticals at the several stations would assume,



when corrected for local attraction, if the arc were bent up into a straight line at right angles to the vertical at Arbury Hill, which is the station nearest to the middle of the arc. The other lines passing through the stations of the arc show the direction and relative magnitude (the actual magnitude would be hardly discernible) of the deflection of the plumb-line at the other five stations relatively to that at Arbury Hill.

This diagram shows that the plumb-line at Arbury Hill and Greenwich is nearly equally affected, and that on the Arbury side of Blenheim there must be more attracting matter than on the Greenwich side. At Clifton the plumb-line is drawn much to the south, and still more so at Burleigh Moor, as might be expected from its proximity to the sea.

8. The chief peculiarity is in the arc between Dunnose and Greenwich. As Dunnose is on the south coast of the Isle of Wight, it might have been (*à priori*) expected that the plumb-line would have been drawn considerably northward, so as to *increase* the amplitude between that place and Greenwich, whereas it is diminished. This peculiarity must arise from the character of the immediate neighbourhood of Dunnose and the form of the coast there. That there is some peculiar arrangement of the mass in the neighbourhood of Dunnose, more lying on the south of its parallel than on the north, appears from the table of differences of amplitude between Southampton and Dunnose, Boniface Down, Week Down, and Port Valley, given at p. xl of the Introduction to Captain YOLLAND's 'Astronomical Observations made with AIRY's Zenith Sector,' published in 1852. The following is the Table alluded to:—

	Amplitudes.		G—A.
	Geodetical.	Astronomical.	
Southampton and Dunnose	0° 17′ 43″·24	0° 17′ 39″·53	+3″·71
Southampton and Boniface Down ...	0 18 37·36	0 18 36·13	+1·23
Southampton and Week Down	0 18 56·71	0 18 55·26	+1·45
Southampton and Port Valley	0 19 1·58	0 19 1·16	+0·42
Southampton and Black Down	0 13 36·69	0 13 37·79	−1·10

For the position of these places see Plate II. Three of them are in the Isle of Wight, south of the parallel of Dunnose. The Table shows that the plumb-line at Dunnose is affected in the southerly direction with reference to each of these three places, Boniface Down, Week Down, and Port Valley, to a considerable degree, viz. $2''\cdot48$, $2''\cdot26$, and $3''\cdot29$; and therefore there must be some large mass in the immediate neighbourhood of Dunnose Station between its parallel and that of Boniface Down. The Table shows that the amplitude between the parallel of Southampton and Black Down on the Dorsetshire coast is increased—as we should expect. This confirms the supposition that there is some peculiarity in the Isle of Wight south of Dunnose, which an actual geographical survey can alone determine.

9. The calculation I have gone through, in the last article but two, shows that the amplitude of the whole arc between Dunnose and Burleigh Moor is increased by local attraction to the amount of $4''\cdot722$.

10. This discussion of the arc between Dunnose and Burleigh Moor suggests the importance of obtaining, in the best way we can, the amount of local attraction at the several stations of the arc by some direct means, that the corrections may be applied to the amplitudes before they are used in the Problem of the Figure of the Earth. For although these errors in the amplitudes are rendered less injurious to the result by comparing the arc with other arcs separated from it considerably in latitude, an arc which *per se* leads to so unusual an ellipticity cannot be so safely employed in the general problem, as when it is freed from the source of error which seems to lead to that ellipticity.

11. It may in the end appear, even after the corrections for local attraction are applied, that the curvature of the English arc is different from the mean curvature, and, as I have stated in my former paper, the science of geology would tend rather to favour such a conception. But even in that case, the values of the ellipticities of the separate portions of the arc must present a much more uniform appearance than those deduced in this paper from the present data (see art. 4.).

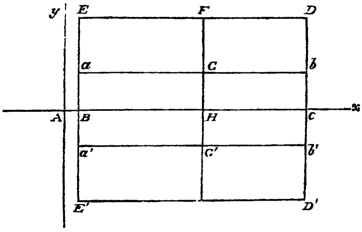
I proceed now to the second part of this communication, to obtain a formula for calculating the attraction.

II. *Investigation of a Formula for calculating the amount of Local Attraction on any Station.*

12. In the former paper I obtained a formula for calculating the attraction by supposing the superficial mass cut into lunes by great circles passing through the attracted station, and the lunes divided into compartments according to a certain law. The formula thence deduced may be applied to attraction in England. But I will now deduce another formula which in some cases may be found more convenient in practice. The former method takes account of the curvature of the earth, and determines the attraction of any masses lying anywhere above the sea-level between the given station and the antipodes. In the present instance I shall suppose

the attracting mass to lie wholly on a plane—a supposition which will lead to no error in the calculation of attraction on stations in the British Isles.

13. Let the horizontal plane through the station be taken as the plane of xy, the axes of x and y being chosen in such a position (at right angles to each other) as may be found in any particular case most convenient for the application of the resulting formula. Let z be measured vertically downwards. Suppose the attracting mass cut by vertical planes, parallel to the co-ordinate planes, into a number of masses on rectangular bases—the bases being of any size, large or small, and if necessary all different from each other, the dimensions of each being determined by the contour of the upper surface of the mass, so that a fair average height of the mass may be easily found. The smaller the parallelograms are the more accurate will be the result, but then they will be more numerous and the calculation more tedious. The nearer the parallelograms are to the station, and also the more irregular the contour of the country, the more attention will be required to make a judicious dissection of the mass. By supposing the several divisions thus made to be levelled down to their average height above the level of the sea, I conceive the whole attracting body to consist of a number of tabular masses of various dimensions.



Let A be the station on which the attraction is to be found; Ax, Ay the axes of x and y; BCDE the projection on the plane of xy of a tabular mass, lying in contact with the vertical plane zx, and very near to the plane zy; AB$=m$; X, Y coordinates to the furthest angle D; xyz coordinates to any point in the mass; H the height of the mass, its base being on the sea-level; h he height of A above the sea; both H and h I suppose to be small compared with X and Y, so that the squares of $\frac{h}{X}$ and $\frac{h}{Y}$ may be neglected; ρ the density of the mass.

Then $\rho dxdydz$ is the mass of an element,

$$\frac{\rho dxdydz}{x^2+y^2+z^2} \text{ its attraction on A,}$$

$$\frac{\rho xdxdydz}{\{x^2+y^2+z^2\}^{\frac{3}{2}}} \text{ its attraction on A parallel to } x.$$

Hence, whole attraction of the tabular mass on A parallel to x

$$=\rho\iiint\frac{xdxdydz}{\{x^2+y^2+z^2\}^{\frac{3}{2}}}, \text{ from } x=m \text{ to } x=\text{X},$$
$$y=0 \text{ to } y=\text{Y},$$
$$z=h-\text{H} \text{ to } z=h.$$

Integrating first with respect to y, then z, and then x,

$$\text{attraction } =\rho\text{Y}\iint\frac{xdxdz}{(x^2+z^2)\sqrt{x^2+\text{Y}^2+z^2}}$$

$$=\varrho\mathrm{Y}\iint\frac{xdxdz}{(x^2+z^2)\sqrt{x^2+\mathrm{Y}^2}}\left\{1+\frac{z^2}{x^2+\mathrm{Y}^2}\right\}^{-\frac{1}{2}}$$

$$=\varrho\mathrm{Y}\iint\frac{xdxdz}{(x^2+z^2)\sqrt{x^2+\mathrm{Y}^2}}\left(1-\frac{z^2}{2(x^2+\mathrm{Y}^2)}\right),\text{ neglecting } z^4,\text{ \&c.}$$

$$=\frac{\varrho\mathrm{Y}}{2}.\iint\frac{xdxdz}{(x^2+\mathrm{Y}^2)^{\frac{3}{2}}}\left(\frac{3x^2+2\mathrm{Y}^2}{x^2+z^2}-1\right)$$

$$=\frac{\varrho\mathrm{Y}}{2}\int\frac{dx}{(x^2+\mathrm{Y}^2)^{\frac{3}{2}}}\left\{(3x^2+2\mathrm{Y}^2)\tan^{-1}\frac{z}{x}-z.x+\text{const.}\right\}$$

$$=\frac{\varrho\mathrm{Y}}{2}\int\frac{dx}{(x^2+\mathrm{Y}^2)^{\frac{3}{2}}}\left\{(3x^2+2\mathrm{Y}^2)\left(\tan^{-1}\frac{h}{x}-\tan^{-1}\frac{h-\mathrm{H}}{x}\right)-\mathrm{H}.x\right\}$$

$$=\frac{1}{2}\varrho\mathrm{Y}.\int\frac{dx}{(x^2+\mathrm{Y}^2)^{\frac{3}{2}}}\left\{-\mathrm{H}.x+(3x^2+2\mathrm{Y}^2)\right.$$

$$\left.\times\left(\frac{h-(h-\mathrm{H})}{x}-\frac{h^3-(h-\mathrm{H})^3}{3x^3}+\ldots+(-1)^n\frac{h^{2n+1}-(h-\mathrm{H})^{2n+1}}{(2n+1)x^{2n+1}}+\ldots\right)\right\}$$

Put
$$\mathrm{K}_{2n+1}=\frac{h}{\mathrm{H}}\frac{1}{2n+1}\left\{1-\left(1-\frac{\mathrm{H}}{h}\right)^{2n+1}\right\}.$$

The maximum value of this coefficient is $\frac{1}{2n+1}$. This may be proved by putting $h=\mathrm{H}\sin^2\theta$, or $\mathrm{H}=h\sin^2\theta$, according as h is less or greater than H, and finding the maximum by the Differential Calculus.

Hence, attraction of whole mass parallel to x

$$=\frac{1}{2}\varrho\mathrm{Y}\frac{\mathrm{H}}{h}\int\frac{dx}{(x^2+\mathrm{Y}^2)^{\frac{3}{2}}}\left\{2xh+(2\mathrm{Y}^2-3\mathrm{K}_3h^2)\frac{h}{x}\right.$$

$$\left.-(2\mathrm{Y}^2\mathrm{K}_3-3\mathrm{K}_5h^2)\frac{h^3}{x^3}+\ldots+(-1)^n(2\mathrm{Y}^2\mathrm{K}_{2n+1}-3\mathrm{K}_{2n+3}h^2)\frac{h^{2n+1}}{x^{2n+1}}+\ldots\right\};$$

or, since the squares of $\frac{h}{\mathrm{Y}}$ may be neglected,

$$=\varrho\mathrm{Y}\frac{\mathrm{H}}{h}\int\frac{dx}{(x^2+\mathrm{Y}^2)^{\frac{3}{2}}}\left\{xh+\mathrm{Y}^2\frac{h}{x}-\mathrm{Y}^2\mathrm{K}_3\frac{h^3}{x^3}+\ldots+(-1)^n\mathrm{Y}^2\mathrm{K}_{2n+1}\frac{h^{2n+1}}{x^{2n+1}}+\ldots\right\}.$$

Now by the Integral Calculus,

$$\int_m^{\mathrm{X}}\frac{dx}{x^{2n+1}(x^2+\mathrm{Y}^2)^{\frac{3}{2}}}=-\frac{2n+1}{\mathrm{Y}^2}\int_m^{\mathrm{X}}\frac{dx}{x^{2n+1}\sqrt{x^2+\mathrm{Y}^2}}+\frac{1}{\mathrm{Y}^2}\left\{\frac{1}{\mathrm{X}^{2n}\sqrt{\mathrm{X}^2+\mathrm{Y}^2}}-\frac{1}{m^{2n}\sqrt{m^2+\mathrm{Y}^2}}\right\}$$

and
$$\int_m^{\mathrm{X}}\frac{dx}{x^{2n+1}\sqrt{x^2+\mathrm{Y}^2}}=(-1)^n\frac{1}{2}\frac{3}{4}\cdots\frac{2n-1}{2n}\frac{\mathrm{L}}{\mathrm{Y}^{2n+1}}$$

$$+\frac{1}{2n\mathrm{Y}^2m^{2n}}\left\{\sqrt{m^2+\mathrm{Y}^2}\left(1-\frac{2n-1}{2n-2}\frac{m^2}{\mathrm{Y}^2}+\ldots+(-1)^{n-1}\frac{3}{2}\cdots\frac{2n-1}{2n-2}\left(\frac{m}{\mathrm{Y}}\right)^{2n-2}+\ldots\right)\right.$$

$$\left.-\sqrt{\mathrm{X}^2+\mathrm{Y}^2}\left(1-\frac{2n-1}{2n-2}\frac{\mathrm{X}^2}{\mathrm{Y}^2}+\ldots+(-1)^{n-1}\frac{3}{2}\cdots\frac{2n-1}{2n-2}\left(\frac{\mathrm{X}}{\mathrm{Y}}\right)^{2n-2}+\ldots\right)\left(\frac{m}{\mathrm{X}}\right)^{2n}\right\}.$$

where
$$\mathrm{L}=\log_e\frac{\mathrm{X}}{m}\frac{1+\sqrt{1+\frac{m^2}{\mathrm{Y}^2}}}{1+\sqrt{1+\frac{\mathrm{X}^2}{\mathrm{Y}^2}}}.$$

I shall make m so small that the squares and higher powers of $\frac{m}{X}$ and $\frac{m}{Y}$ may be neglected. Hence these formulæ become

$$\int_m^X \frac{dx}{x^{2n+1}(x^2+Y^2)^{\frac{3}{2}}} = \frac{2n+1}{Y^2}\int_m^X \frac{dx}{x^{2n+1}\sqrt{x^2+Y^2}} - \frac{1}{Y^3 m^{2n}}$$

$$\int_m^X \frac{dx}{x^{2n+1}\sqrt{x^2+Y^2}} = (-1)^n \frac{1}{2}\,\frac{3}{4}\cdots\frac{2n-1}{2n}\,\frac{L}{Y^{2n+1}} + \frac{1}{2nYm^{2n}}$$

and
$$L = \log_e \frac{\frac{X}{m}}{1+\sqrt{1+\frac{X^2}{Y^2}}}.$$

These formulæ fail when $n=0$; but in that case

$$\int_m^X \frac{dx}{x(x^2+Y^2)^{\frac{3}{2}}} = \frac{L}{Y^3} + \frac{1}{Y^2}\left(\frac{1}{\sqrt{X^2+Y^2}} - \frac{1}{\sqrt{m^2+Y^2}}\right)$$

by direct integration,

$$= \frac{L-1}{Y^3} + \frac{1}{Y^2\sqrt{X^2+Y^2}};$$

also
$$\int_m^X \frac{x\,dx}{(x^2+Y^2)^{\frac{3}{2}}} = \frac{1}{\sqrt{m^2+Y^2}} - \frac{1}{\sqrt{X^2+Y^2}} = \frac{1}{Y} - \frac{1}{\sqrt{X^2+Y^2}}.$$

Substituting these in the expression for the attraction,

Attraction of the whole mass on A parallel to x

$$\begin{aligned}
&= \varrho Y\frac{H}{h}\Big\{\frac{h}{Y} - \frac{h}{\sqrt{X^2+Y^2}} + \frac{h}{Y}L - \frac{h}{Y} + \frac{h}{\sqrt{X^2+Y^2}} \\
&\qquad + K_3\left(\frac{3}{2}\,\frac{h^3}{Y^3}L - \frac{1}{2}\,\frac{h}{Y}\,\frac{h^2}{m^2}\right) \\
&\qquad + \cdots\cdots \\
&\qquad + K_{2m+1}\left(\frac{3}{2}\,\frac{5}{4}\cdots\frac{2n+1}{2n}\,\frac{h^{2n+1}}{Y^{2n+1}}L + (-1)^n\frac{1}{2n}\,\frac{h}{Y}\,\frac{h^{2n}}{m^{2n}}\right) \\
&\qquad + \cdots\cdots\Big\} \\
&= \varrho H\Big\{L\left(1+\tfrac{3}{2}K_3\frac{h^2}{Y^2} + \cdots + \tfrac{3}{2}\,\tfrac{5}{4}\cdots\tfrac{2n+1}{2n}\,K_{2n+1}\frac{h^{2n}}{Y^{2n}} + \cdots\right) \\
&\qquad - \left(\tfrac{1}{2}K_3\frac{h^2}{m^2} - \tfrac{1}{4}K_5\frac{h^4}{m^4} + \cdots\cdots + (-1)^n\frac{1}{2n}K_{2n+1}\frac{h^{2n}}{m^{2n}} + \cdots\right)\Big\};
\end{aligned}$$

or, neglecting the squares and higher powers of $\frac{h}{Y}$,

$$= \varrho H\left\{L - \left(\tfrac{1}{2}K_3\frac{h^2}{m^2} - \tfrac{1}{4}K_5\frac{h^4}{m^4} + \cdots + (-1)^n\frac{1}{2n}K_{2n+1}\frac{h^{2n}}{m^{2n}} + \cdots\right)\right\}.$$

It has been laid down that h and m are both small quantities. This expression shows what the limiting value of their ratio must be, that this formula may be capable of use.

In order that the series above may be convergent,

$$\frac{1}{2n}K_{2n+1}\frac{h^{2n}}{m^{2n}} \div \frac{1}{2n-2}K_{2n-1}\frac{h^{2n-2}}{m^{2n-2}}$$

must be less than unity;

$$\therefore \quad \frac{h^2}{m^2} \text{ must be less than } \frac{2n}{2n-1}\frac{K_{2n-1}}{K_{2n+1}}$$

$$\ldots\ldots\ldots\ldots \frac{2n(2n+1)}{(2n-1)(2n-2)}.$$

The least value of this is when $n=\infty$, when it $=1$;

$$\therefore \quad m \text{ must not be less than } h.$$

This then is the value we shall give to m; and the attraction of the whole mass on A parallel to x

$$=\varrho H\left\{L-\left(\frac{1}{2}K_3-\frac{1}{4}K_5+\cdots+(-1)^n\frac{1}{2n}K_{2n+1}+\cdots\right)\right\};$$

or, calling the series S, $\quad =\varrho H(L-S)$.

14. If BCD′E′ be a tabular mass lying symmetrically on the opposite side of the axis of x, the attraction of this mass on A parallel to x will obviously be the same as the attraction of the mass BCDE. This, indeed, our formula shows; for if $-Y$ be put for Y it does not change the value of L. The same is not true if we change the sign of X; the reason of which is, that negative powers of x occur in the integration, and these all become infinite as we pass across the axis of y from $x=m$ to $x=-X$. On account of this we must calculate the attraction of the masses all lying on one side of y and add them together, and then separately those on the other side and add them together, and take the difference of the results.

15. The tabular mass has been hitherto supposed to be always in contact with the axis of x, and at a small distance m from that of y, and is therefore restricted in its position. I will now, however, deduce a more general formula.

Draw ab and $a'b'$ at equal distances parallel to x on opposite sides; and let $Ba=y$.

Then the attraction of either of the masses aC or a'C on A parallel to x,

$$=\varrho H\left\{\log_e\left[\frac{\frac{X}{m}}{1+\sqrt{1+\frac{X^2}{y^2}}}\right]-S\right\},$$

S being independent of the coordinates X, y. Subtracting this from the attraction of EC, and adding it also, we have,

Attractions of tabular masses Eb and Eb'

$$=\varrho H\log_e\left[\frac{\sqrt{1+\frac{X^2}{y^2}}+1}{\sqrt{1+\frac{X^2}{Y^2}}+1}\right] \text{ and } \varrho H\left\{\log_e\left[\frac{\frac{X^2}{m^2}}{\left(\sqrt{1+\frac{X^2}{y^2}}+1\right)\left(\sqrt{1+\frac{X^2}{Y^2}}+1\right)}\right]-2S\right\}.$$

Now draw FGHG′ parallel to the axis of y, and let $AH=x$. Then the attractions of tabular masses on Fb and Fb'= those of the masses on Eb and Eb'— those of the masses on EG and EG′: hence

Attraction on A parallel to x of the tabular masses on Fb and Fb'=

$$\varrho \mathrm{H} \log_e \left[\frac{\sqrt{1+\frac{\mathrm{X}^2}{y^2}}+1}{\sqrt{1+\frac{\mathrm{X}^2}{\mathrm{Y}^2}}+1} \cdot \frac{\sqrt{1+\frac{x^2}{\mathrm{Y}^2}}+1}{\sqrt{1+\frac{x^2}{y^2}}+1}\right] \text{ and } \varrho \mathrm{H} \log_e \left[\frac{\frac{\mathrm{X}^2}{x^2}\left(\sqrt{1+\frac{x^2}{y^2}}+1\right)\left(\sqrt{1+\frac{x^2}{\mathrm{Y}^2}}+1\right)}{\left(\sqrt{1+\frac{\mathrm{X}^2}{y^2}}+1\right)\left(\sqrt{1+\frac{\mathrm{X}^2}{\mathrm{Y}^2}}\right)+1}\right]$$

These may be written in the following form:—

$$\varrho \mathrm{H} \log_e \left[\frac{\sqrt{1+\frac{y^2}{\mathrm{X}^2}}+\frac{y}{\mathrm{X}}}{\sqrt{1+\frac{\mathrm{Y}^2}{\mathrm{X}^2}}+\frac{\mathrm{Y}}{\mathrm{X}}} \cdot \frac{\sqrt{1+\frac{\mathrm{Y}^2}{x^2}}+\frac{\mathrm{Y}}{x}}{\sqrt{1+\frac{y^2}{x^2}}+\frac{y}{x}}\right] \text{ and } \varrho \mathrm{H} \log_e \left[\frac{\left(\sqrt{1+\frac{y^2}{x^2}}+\frac{y}{x}\right)\left(\sqrt{1+\frac{\mathrm{Y}^2}{x^2}}+\frac{\mathrm{Y}}{x}\right)}{\left(\sqrt{1+\frac{y^2}{\mathrm{X}^2}}+\frac{y}{\mathrm{X}}\right)\left(\sqrt{1+\frac{\mathrm{Y}^2}{\mathrm{X}^2}}+\frac{\mathrm{Y}}{\mathrm{X}}\right)}\right].$$

If the numerator and denominator of the fraction under the logarithm in the second of these be multiplied by

$$\left(\sqrt{1+\frac{y^2}{x^2}}-\frac{y}{x}\right)\left(\sqrt{1+\frac{y^2}{\mathrm{X}^2}}-\frac{y}{\mathrm{X}}\right),$$

its value will not be changed, but it will become

$$\varrho \mathrm{H} \log_e \left[\frac{\left(\sqrt{1+\frac{y^2}{\mathrm{X}^2}}-\frac{y}{\mathrm{X}}\right)\left(\sqrt{1+\frac{\mathrm{Y}^2}{x^2}}+\frac{\mathrm{Y}}{x}\right)}{\left(\sqrt{1+\frac{\mathrm{Y}^2}{\mathrm{X}^2}}+\frac{\mathrm{Y}}{\mathrm{X}}\right)\left(\sqrt{1+\frac{y^2}{x^2}}-\frac{y}{x}\right)}\right].$$

Now this is precisely the same as the first when $-y$ is put for y; and it is only in this change of sign in y that the parallelogram Fb' differs from Fb. Hence the first formula includes the second, and is applicable to all tabular masses lying on the right (or the positive side) of the axis of y, on either side of x. It will be observed, however, that the same is not true of Y in this formula, as may easily be seen by going through a process similar to this transformation for y. We must therefore remember not to let Y be negative. The way to obviate this in the use of the formula is to make the direction in which Y is measured in any particular case the direction of positive ordinates for that case, and then y will be positive or negative as it lies on the same or the opposite side of the axis of x from Y.

16. The formula can be very much simplified as follows.

Put $$\frac{\mathrm{Y}}{x}=\tan\theta_1,\ \frac{y}{\mathrm{X}}=\tan\theta_2,\ \frac{\mathrm{Y}}{\mathrm{X}}=\tan\theta_3,\ \frac{y}{x}=\tan\theta_4;$$

$$\therefore \quad \sqrt{1+\frac{\mathrm{Y}^2}{x^2}}+\frac{\mathrm{Y}}{x}=\frac{1+\sin\theta_1}{\cos\theta_1}$$

$$=\tan\left(45^\circ+\frac{1}{2}\theta_1\right),$$

and similarly of the others.

Hence, since 0·43429 is the modulus of common logarithms,

$$\text{attraction}=\frac{\varrho \mathrm{H}}{0\cdot 43429}\Big\{\log\tan\left(45^\circ+\frac{1}{2}\theta_1\right)+\log\tan\left(45^\circ+\frac{1}{2}\theta_2\right)$$
$$-\log\tan\left(45^\circ+\frac{1}{2}\theta_3\right)-\log\tan\left(45^\circ+\frac{1}{2}\theta_4\right)\Big\}.$$

17. A similar formula is true for the attraction parallel to the axis of y; and may be obtained from the above by interchanging X and Y, x and y.

18. As a kind of test of the truth of the formula deduced in the last article but two, let $\mathrm{X}=x+dx$ and $\mathrm{Y}=y+dy$. In this case the tabular mass becomes an elementary vertical prism of height H and base $dx.dy$; and by expanding in powers of dx and dy the expression comes out, as it should, attraction $=\frac{\varrho \mathrm{H}.dx.dy.x}{(x^2+y^2)^{\frac{3}{2}}}$.

19. The attraction of the earth on a point at its surface, that is, gravity $(g)=\frac{4\pi}{3}\mathrm{D}.a$, where D is the mean density of the earth, and a the radius,

$$\therefore\quad \mathrm{D}=\frac{3}{4\pi}\frac{g}{a};$$

and the coefficient of the last formula in art. 16

$$=\frac{\varrho}{\cdot 434}\mathrm{H}=\frac{3}{4\pi(\cdot 434)}\frac{\varrho}{\mathrm{D}}\frac{g}{a}\mathrm{H}.$$

Let the density of the attracting mass be taken to be that of granite*, that is, about

* Should the density differ from this in any particular case of application of the formula, especially for parts in the immediate neighbourhood of the plumb-line, then the coefficient must be altered in proportion.

Judging from the following Table, which is taken from Colonel's SABINE's volume on the Pendulum, p. 338, it would appear that pendulum experiments afford a good means of measuring the relative density of the parts of the earth's crust near the place of observation. He gives this as the result of his calculations on the subject:—

Stations.	Excess or defect of vibrations.	Scale of density [of the strata beneath].
St. Thomas	+5·58	100
Ascension	+5·04	94
Spitzbergen	+3·50	79
Jamaica	+0·28	45
New York	+0·00	43
Greenland	−0·08	43
Sierra Leone	−0·12	42
London	−0·28	41
Hammerfest	−0·52	37
Bahia	−1·80	26
Drontheim	−3·10	12
Trinidad	−4·12	2
Maranham	−4·34	1

Thus pendulum experiments seem well calculated to test the existence of local attraction, at any rate the vertical part of it. But they can be of no service in determining the part of local attraction which is effective in deflecting the plumb-line, except in determining the *density* of the attracting mass; for the part which is effective in one case is entirely inoperative in the other. If G be the force of local attraction, and this be resolved into two forces, V vertical and H horizontal, then H is the only part which has any sensible effect in

half the mean density of the earth; and put $a=20923713$ feet. Then the above coefficient $=\frac{g}{76127500}$ H, H being expressed in feet. Hence also

Tangent of angle of deflection of the plumb-line at A in the vertical plane through x caused by the tabular mass on Fb

$$= \text{attraction parallel to } x \div g$$
$$=\frac{\text{H}}{76127500}\left\{\log\tan\left(45^\circ+\frac{1}{2}\theta_1\right)+\log\tan\left(45^\circ+\frac{1}{2}\theta_2\right)\right.$$
$$\left.-\log\tan\left(45^\circ+\frac{1}{2}\theta_3\right)-\log\tan\left(45^\circ+\frac{1}{2}\theta_4\right)\right\};$$

or, angle of deflection of the plumb-line in the vertical plane through x

$$=\frac{1''}{369}\text{H}\left\{\log\tan\left(45^\circ+\frac{1}{2}\theta_1\right)+\log\tan\left(45^\circ+\frac{1}{2}\theta_2\right)\right.$$
$$\left.-\log\tan\left(45^\circ+\frac{1}{2}\theta_3\right)-\log\tan\left(45^\circ+\frac{1}{2}\theta_4\right)\right\}.$$

20. From this easily flows the following Rule for calculating the deviation caused in the plumb-line in any plane by the attraction of a tabular mass of which the height above the sea-level is H feet.

Take the origin of coordinates at the station where the plumb-line is. Let the plane of xy *be horizontal, and the axis of* x *in the vertical plane in which the amount of deflection is to be found.*

Write down the coordinates XY xy *of the furthest and nearest angles of the tabular mass from the origin;* Y *is always to be considered positive, and* y *positive or negative accordingly.*

Form four ratios, by first dividing each ordinate by the abscissa not belonging to it, and then by dividing each ordinate by its own abscissa, viz. $\frac{\text{Y}}{\text{x}}, \frac{\text{y}}{\text{X}}, \frac{\text{Y}}{\text{X}}, \frac{\text{y}}{\text{x}}$.

Look in a Table of Tangents for the four angles of which the tangents equal the above ratios.

Form four more angles by adding (subtracting if they be negative) *half of each of these angles just found to* 45°.

From the sum of the log-tangents of the first two of these angles subtract the sum of the log-tangents of the second two.

This result, multiplied by H *feet and by* $\frac{1''}{369}$, *will give the required deflection in seconds of a degree.*

deflecting the plumb-line; and V is the only part which has any influence in altering the time of vibration of the pendulum, as it is easily proved that a small constant horizontal force, though it affects the arc of vibration, has no effect on the time of vibration. Thus the determinations of local attraction by the pendulum cannot assist in determining the effect of local attraction on the plumb-line, except in as far as they assist in pointing out the relative density of the mass which deranges the normal state of things.

The simplicity of this Rule will be seen in the application I propose to make of it in the next section.

It should be mentioned that the only restriction to be attended to in the application of this Rule is, that the ratio of the height of the attracted station above the sea to each of the horizontal coordinates of the nearest angle of the attracting mass must be small, and so small that its square may be neglected.

If any part of the attracting mass is nearer to the station than this, the approximate formula must for that part of the mass be abandoned, and a direct calculation made *.

III. *Application of the Formula to obtain a rough approximation to the meridian deflection of the Plumb-line at Burleigh Moor.*

21. In the Plate attached to this paper, an outline sketch is given of the east of England, with a view to show how the land lies with reference to Burleigh Moor—the station to which I now propose to apply the formula, by way of illustration, with the scanty data which I have been able as yet to obtain. By the help of accurate survey maps no doubt a very close approximation might be obtained to the actual amount of the deflection, not only at Burleigh Moor, but at the other stations. The data used in this calculation are taken from the outline map of England in the third volume of General MUDGE's Account of the English Survey, published in 1811. A number of heights of stations are marked down on the map; and it is from these, as I have no other source of information, that I have inferred the average height of the masses into which I divide the land. But as these heights, as I conjecture, almost all appertain to *elevated* points, visible from a distance for the purposes of the survey, their average will be much greater than the average height of the country to which they appertain. I have taken the height of the masses equal to three-fifths of the average height above the sea of the various stations belonging to that mass. The result which I arrive at will therefore be only a rough approximation, for want of

* If XY are the horizontal coordinates to the middle of a vertical prism, of which the height measured from the sea-level is H, h being the height of the station, and A the area (in square feet) of the horizontal section of the prism; then, if A be small, the horizontal attraction of the prism parallel to x

$$=\frac{\rho AX}{X^2+Y^2}\left\{\frac{h}{\sqrt{X^2+Y^2+h^2}}+\frac{H-h}{\sqrt{X^2+Y^2+(H-h)^2}}\right\}.$$

By the same reasoning as in art. 19, this

$$=\frac{\rho}{D}\frac{AX}{X^2+Y^2}\left\{\frac{h}{\sqrt{X^2+Y^2+h^2}}+\frac{H-h}{\sqrt{X^2+Y^2+(H-h)^2}}\right\}\frac{g}{8764500},$$

D being the mean density of the earth, and A being expressed in square feet, XYHh in feet.

Or, the angle of deflection, in the vertical plane through the axis of x, caused by this prism

$$=\frac{\rho}{D}\frac{AX}{X^2+Y^2}\left\{\frac{h}{\sqrt{X^2+Y^2+h^2}}+\frac{H-h}{\sqrt{X^2+Y^2+(H-h)^2}}\right\}\frac{1''}{424\cdot 9}.$$

The value of this must be found for each of the vertical prisms near the station, and their sum taken.

more accurate data. But the calculation may be useful to illustrate the use of the formula.

22. If a meridian line be drawn about thirty miles west of Burleigh Moor, the resultant attraction on that place of the portion of the British Isles to the west of that line will be due west or nearly so, whatever be its amount, which is doubtless small. For the mountain region of Cumberland and Westmoreland lies due west; and those of Scotland and Wales lie at about the same bearing north and south of west, and therefore taking their mass to be about the same their resultant attraction will be west. The attraction too of the level country west of the line laid down, and of the table on which the mountain regions rest, will be about west. So that we may conclude that the part of the land which is effective in deflecting the plumb-line at Burleigh Moor in the plane of the meridian is that portion which lies east of the line.

This tract of country I divide into four portions, A, B, C, D, as marked in the Plate. The small irregular portions *a* and *b* on opposite sides of the station I suppose to counteract each other. The station itself I suppose to be in the centre of a neutral parallelogram, of which the north and east sides are the average line of sea cliff in that neighbourhood. The distances of these cliffs I put down as 3 and 10 miles. This I deduce merely from the map in the account of the Survey: it is in these assumptions regarding the parts nearest the station that the chief sources of error will lie in the present calculation, from insufficient data. The portion to the west of the station, marked *c*, will have no effect in the direction of the meridian.

The mass A will produce a deflection northward; the other masses, southward. The average heights I take to be 505, 628, 448, 394 feet above the sea-level.

23. The following Table is formed from the formula given in art. 16, according to the Rule laid down in art. 20.

Coordinates.	Ratios.	Ratios in decimals.	First angles.	Second angles.	Log-tangents.	Results.
For A (lying to the *north* of the parallel through Burleigh Moor).						
X = 23 miles	$\frac{30}{3}$ =	10·0000000	84° 17′ 20″	87° 8′ 40″	1·3020723	0·1825496
Y = 30	$\frac{10}{23}$ =	0·4347826	23 29 50	56 44 55	0·1833190	Deflection in merid^{n}. $\times \frac{505}{369} = -0''·250$
x = 3	$\frac{30}{23}$ =	1·3043478	52 31 20	71 15 40	0·4694955	
y = 10	$\frac{10}{3}$ =	3·3333333	73 18 0	81 39 0	0·8333462	
For B (lying to the *south*).						
X = 40	$\frac{30}{3}$ =	10·0000000	84 17 30	87 8 45	1·3022389	1·8638725
Y = 30	$-\frac{16}{40}$ =	−0·4000000	−21 48 2	34 5 59	$\bar{1}$·8306168	$\times \frac{628}{369} = +3''·172$
x = 3	$\frac{30}{40}$ =	0·7500000	36 52 6	63 26 3	0·3010169	
y = −16	$-\frac{16}{3}$ =	−5·3333333	−79 23 10	5 18 25	$\bar{2}$·9679663	
For C.						
X = 88	$\frac{32}{40}$ =	0·8000000	38 39 35	64 19 47	0·3181903	0·3192385
Y = 32	$-\frac{30}{88}$ =	−0·3409091	−18 49 30	35 35 15	$\bar{1}$·8546701	$\times \frac{448}{369} = +0''·388$
x = 40	$\frac{32}{88}$ =	0·3636364	19 59 0	54 59 30	0·1546388	
y = −30	$-\frac{30}{40}$ =	−0·7500000	−36 52 6	26 33 57	$\bar{1}$·6989831	
For D.						
X = 270	$\frac{80}{88}$ =	0·9090909	42 16 26	66 8 13	0·3542184	0·3278590
Y = 80	$-\frac{30}{270}$ =	−0·1111111	− 6 20 30	41 49 45	$\bar{1}$·9518325	$\times \frac{394}{369} = +0''·350$
x = 88	$\frac{80}{270}$ =	0·2962963	16 30 15	53 15 7	0·1268639	
y = −30	$-\frac{30}{88}$ =	−0·3490909	−19 14 38	35 22 41	$\bar{1}$·8513280	
						Total deflection to south = 3″·660

24. Hence, according to this rough approximation, the deflection of the plumb-line at Burleigh Moor to the south is 3″·660. It will be observed that the attraction of the space B is much larger in its amount than that of any of the other spaces; in

fact, it is 87 per cent. of the whole attraction, although it is of smaller dimensions than those to the south of it. This arises from its proximity to the station, its distance being put down as 3 miles south of Burleigh Moor—that being the supposed distance of that station from the average line of coast on the north. If I had made this 2 miles instead of 3, the attraction of B would have produced a deflection of $4''\cdot180$ instead of $3''\cdot172$, so considerable is the effect of the parts lying nearest to the attracted station. This shows the importance of an accurate survey being made of the neighbourhood of each of the terminal stations of the several portions of the arc, that the local deflections may be accurately calculated.

25. Should it prove, on a careful survey of the neighbourhood of Burleigh Moor, that the deflection above deduced is correct, then the

Deflection at Clifton is . . .	$0''\cdot912$ to the south.
Deflection at Arbury Hill is .	4·579 to the north.
Deflection at Blenheim is . .	5·902 to the north.
Deflection at Greenwich is .	3·976 to the north.
Deflection at Dunnose is . .	1·062 to the north.

And by making use of the Table from Captain Yolland's volume quoted in art. 8,—

Deflection at Southampton will be . .	$4''\cdot772$ to the north.
Deflection at Boniface Down will be .	3·542 to the north.
Deflection at Week Down will be . .	3·322 to the north.
Deflection at Port Valley will be . . .	4·352 to the north.
Deflection at Black Down will be . .	5·872 to the north.

The coast about Black Down attains an altitude of about 800 feet, and the whole of England and Scotland lies north of it. The deflection, therefore, of the plumb-line at that place deduced above is about what might have been anticipated, viz. $5''\cdot872$ northward, if the amount at Burleigh Moor be what my calculation brings it out, viz. $3''\cdot660$.

But it is only an accurate survey which can afford data fully satisfactory, upon which to base the calculation of the deflection of the plumb-line at the extremities of the several portions of the arc. When the true amounts of deflection are calculated, and the amplitudes corrected for local attraction, the process followed in art. 3 will bring out the actual ellipticity of the English arc; when it will be seen whether it is or is not more curved than the ellipticity $\frac{1}{300\cdot8}$ would indicate.

Deep River, Cape of Good Hope,
September 23, 1854.

Appendix.

Calculation of the Values of E (see art. 4).

$$E=\frac{1}{2}+\frac{3}{2}\frac{\sin\lambda}{\text{arc}\,\lambda}\cos 2\mu$$

$$=\cdot 5+\frac{1\cdot 5}{\sin 1''}\frac{\sin\lambda}{\lambda''}\cos 2\mu$$

$$=\cdot 5+\log^{-1}\left(5\cdot 4905168+\log\frac{\sin\lambda}{\lambda}+\log\cos 2\mu\right).$$

Arc 1. $2\mu=102°\ 5'\ 46''\cdot 6$

$$E_1=\cdot 5-\log^{-1}\begin{bmatrix}5\cdot 4905168\\ \bar{6}\cdot 6855586\\ \bar{1}\cdot 3212823\\ \hline \bar{1}\cdot 4973577\end{bmatrix}=0\cdot 18569.$$

Arc 2. $2\mu=103°\ 19'\ 6''\cdot 6$

$$E_2=\cdot 5-\log^{-1}\begin{bmatrix}5\cdot 4905168\\ \bar{6}\cdot 6855720\\ \bar{1}\cdot 3624091\\ \hline \bar{1}\cdot 5384979\end{bmatrix}=0\cdot 15446.$$

Arc 3. $2\mu=104°\ 3'\ 55''\cdot 7$

$$E_3=\cdot 5-\log^{-1}\begin{bmatrix}5\cdot 4905168\\ \bar{6}\cdot 6855716\\ \bar{1}\cdot 3856669\\ \hline \bar{1}\cdot 5617553\end{bmatrix}=0\cdot 13545.$$

Arc 4. $2\mu=105°\ 40'\ 59''\cdot 2$

$$E_4=\cdot 5-\log^{-1}\begin{bmatrix}5\cdot 4905168\\ \bar{6}\cdot 6855413\\ \bar{1}\cdot 4318788\\ \hline \bar{1}\cdot 6079369\end{bmatrix}=0\cdot 09455.$$

Arc 5. $2\mu=$

$$E_5=\cdot 5-\log^{-1}\begin{bmatrix}5\cdot 4905168\\ \bar{6}\cdot 6855475\\ \bar{1}\cdot 4907261\\ \hline \bar{1}\cdot 6667904\end{bmatrix}=0\cdot 03571.$$

Calculation of the Values of A (see art. 4).

$$A=\log^{-1}(\log\lambda-\log a).$$

$\lambda_1=3091''\cdot 39$	$a_1=313696$
$\lambda_2=1307\cdot 90$	$a_2=132802$
$\lambda_3=1380\cdot 30$	$a_3=139822$
$\lambda_4=4443\cdot 40$	$a_4=450045$
$\lambda_5=4010\cdot 11$	$a_5=406463$

$$A_1=\log^{-1}\begin{bmatrix}3\cdot 4901538\\ 5\cdot 4965090\\ \hline \bar{3}\cdot 9936448\end{bmatrix}=0\cdot 00985473.$$

$$A_2=\log^{-1}\begin{bmatrix}3\cdot 1165745\\ 5\cdot 1232046\\ \hline \bar{3}\cdot 9933699\end{bmatrix}=0\cdot 00984850.$$

$$A_3=\log^{-1}\begin{bmatrix}3.1399735\\ 5.1455755\\ \bar{3}.9943980\end{bmatrix}=0.00987184.$$

$$A_4=\log^{-1}\begin{bmatrix}3.6477154\\ 5.6532559\\ \bar{3}.9944595\end{bmatrix}=0.00987324.$$

$$A_5=\log^{-1}\begin{bmatrix}3.6031562\\ 5.6090210\\ \bar{3}.9941352\end{bmatrix}=0.00986587.$$

Calculation of the Values of ε (see art. 4).

Arcs compared.	Values of $\log\left(\frac{A_1}{A_2}\right)$.	$\frac{A_1}{A_2}-1$.	$\varepsilon=\frac{\frac{A_1}{A_2}-1}{E_1-E_2}$.	Values of ε.
1st and 2nd...	0.0002749	+0.000633	$+\frac{63.3}{3123}=$	$\log^{-1}\begin{bmatrix}1.8014037\\ 3.4945720\\ \bar{2}.3068317\end{bmatrix}=+0.0202690$
1st and 3rd...	$\bar{1}.9992468$	−0.001733	$-\frac{173.3}{5024}=$	$\log^{-1}\begin{bmatrix}2.2387986\\ 3.7010496\\ \bar{2}.5377490\end{bmatrix}=-0.0344944$
1st and 4th...	$\bar{1}.9991853$	−0.008874	$-\frac{187.4}{9114}=$	$\log^{-1}\begin{bmatrix}2.2727696\\ 3.9597090\\ \bar{2}.3130606\end{bmatrix}=-0.0205622$
1st and 5th...	$\bar{1}.9995096$	−0.001129	$-\frac{112.9}{14998}=$	$\log^{-1}\begin{bmatrix}2.0526939\\ 4.1760333\\ \bar{3}.8766606\end{bmatrix}=-0.0075277$
2nd and 3rd..	$\bar{1}.9989719$	−0.002365	$-\frac{236.5}{1901}=$	$\log^{-1}\begin{bmatrix}2.3738311\\ 3.2789801\\ \bar{1}.0948510\end{bmatrix}=-0.1244090$
2nd and 4th..	$\bar{1}.9989104$	−0.002506	$-\frac{250.6}{5991}=$	$\log^{-1}\begin{bmatrix}2.3989811\\ 3.7774993\\ \bar{2}.6214818\end{bmatrix}=-0.0418294$
2nd and 5th..	$\bar{1}.9992347$	−0.001761	$-\frac{176.1}{11875}=$	$\log^{-1}\begin{bmatrix}2.2457594\\ 4.0746336\\ \bar{2}.1711258\end{bmatrix}=-0.0148295$
3rd and 4th ..	$\bar{1}.9989385$	−0.002441	$-\frac{244.1}{4090}=$	$\log^{-1}\begin{bmatrix}2.3875678\\ 3.6117233\\ \bar{2}.7758445\end{bmatrix}=-0.0596822$
3rd and 5th ..	0.0002628	+0.000605	$+\frac{60.5}{9974}=$	$\log^{-1}\begin{bmatrix}1.7817554\\ 3.9988694\\ \bar{2}.7828860\end{bmatrix}=+0.0607577$
4th and 5th ..	0.0003243	+0.000747	$+\frac{74.7}{5884}=$	$\log^{-1}\begin{bmatrix}1.8733206\\ 3.7696727\\ \bar{2}.1036479\end{bmatrix}=+0.0125964$

Mean of the ten values of $\varepsilon=-0.0209711$

$$=-\frac{1}{47.6846}$$

Calculation of the Values of A *for an ellipse, in which* $a=20923713$ *feet, and* $\varepsilon=\frac{1}{300\cdot8}$ (see art. 6).

$$\frac{\lambda}{\text{arc}}=\frac{1}{a}(1+\text{E}.\varepsilon)=\frac{\varepsilon}{a\sin 1''}\left(\frac{1}{\varepsilon}+\text{E}\right).$$

$$=\log^{-1}-\left[\begin{array}{r}7\cdot3206389\\ \bar{6}\cdot6855749\\ \bar{2}\cdot4782778\\ \hline 4\cdot4844916\end{array}\right]\times\left[300\cdot8+\text{E}=\begin{array}{r}300\cdot98569\\ 300\cdot95446\\ 300\cdot93545\\ 300\cdot89455\\ 300\cdot83571\end{array}\right]$$

$$=\log^{-1}\left[\begin{array}{r}\bar{5}\cdot5155084+2\cdot4785458\\ 5007\\ 4733\\ 4144\\ 3293\end{array}\right]$$

$$=\log^{-1}\left[\begin{array}{r}\bar{3}\cdot9940542\\ \bar{3}\cdot9940091\\ \bar{3}\cdot9939817\\ \bar{3}\cdot9939228\\ \bar{3}\cdot9938377\end{array}\right]$$

	Differences between these and the former values of A.
$=0\cdot00986402$	$-0\cdot00000929$
$0\cdot00986300$	$-0\cdot00001450$
$0\cdot00986238$	$+0\cdot00000946$
$0\cdot00986104$	$+0\cdot00001220$
$0\cdot00985911$	$+0\cdot00000676$

Calculation of the Errors in amplitude (see art. 6).

Multiply these last differences by the lengths of the arcs.

1st arc, amplitude is in *defect* $\log^{-1}\left[\begin{array}{r}\bar{6}\cdot9680157\\ 5\cdot4965090\\ \hline 0\cdot4645247\end{array}\right]=2''\cdot914$

2nd arc, amplitude is in *defect* $\log^{-1}\left[\begin{array}{r}\bar{5}\cdot1613680\\ 5\cdot1232046\\ \hline 0\cdot2845726\end{array}\right]=1''\cdot926$

3rd arc, amplitude is in *excess* $\log^{-1}\left[\begin{array}{r}\bar{5}\cdot9758911\\ 5\cdot1455755\\ \hline 0\cdot1214666\end{array}\right]=1''\cdot323$

4th arc, amplitude is in *excess* $\log^{-1}\left[\begin{array}{r}\bar{5}\cdot0863598\\ 5\cdot6532559\\ \hline 0\cdot7396158\end{array}\right]=5''\cdot491$

5th arc, amplitude is in *excess* $\log^{-1}\left[\begin{array}{r}\bar{6}\cdot8299467\\ 5\cdot6090210\\ \hline 0\cdot4389677\end{array}\right]=2''\cdot748$

POSTSCRIPT.

Since the above was written, I have had the opportunity of seeing a notice of the communication of the Astronomer Royal on the density of table-lands supposed to be supported by a dense fluid or semifluid mass, and the use he makes of his suggestions to remove the discrepancy, pointed out in my first communication, between the values of the deflection of the plumb-line in India, as determined by calculating the attraction of the Himalayas, and as indicated by the results of the Great Trigonometrical Survey. The following difficulties occur to me in the way of this highly ingenious and philosophical method of removing the discrepancy:—

1. It assumes that the hard crust of the earth is sensibly lighter than the fluid or semifluid mass, imagined to be a few miles below the surface. But I know of no law, except the unique law of water and ice, which would lead us to suppose that the fluid mass in consolidating would expand and become lighter. One would rather expect it to become denser, by loss of heat and mutual approximation of its particles.

2. There is, moreover, every reason to suppose that the crust of the earth has long been so thick, that the position of its parts relatively to a mean level cannot be any longer subject to the laws of floatation. If the elevations and depressions of the earth's surface have always remained exactly what they were at the time when the laws of floatation ceased to have an uncontrolled effect, then the same reasoning would no doubt apply in our case, as if they still had their full sway. But geology shows that other laws are in constant operation (arising most probably, as Mr. BABBAGE has suggested, from the expansion and contraction of the solid materials of the crust), which change the relative levels of the various parts of the earth's surface, quite irrespectively of the laws of floatation. If Mr. HOPKINS's estimate of the thickness of the crust be correct, viz. at least 1000 miles, these laws of change in the surface must have been in operation for such an enormous interval of time, as quite to obliterate any traces of the form of surface which the simple principles of hydrostatics would occasion. Indeed, it seems to me highly probable, that the elevation of the Himalayas and the vast regions beyond may have taken place altogether from a slow upheaving force arising from this cause.

I am inclined to think that the only explanation of the discrepancy between my calculation and the results of the Indian Survey is to be found in the greater curvature of the Indian arc.

London, June 2, 1855.

Note added after the reading of the Paper.

A further difficulty arises from the peculiar law which the thickness of the crust must follow, if the present form of the surface arises altogether from hydrostatic principles. For if an *excess* of matter at the surface, in table-lands and mountains, implies a deficiency of matter below and therefore a protrusion of the light crust

down into the heavy fluid supporting it; so must a *deficiency* of matter near the surface, in deep and wide oceans, imply an excess of matter below the ocean-bed, and therefore a protrusion of the heavy fluid up into the light crust. If this were not the case, the fluid below would by its greater upward pressure burst up the crust beneath the ocean, and this would lead to a catastrophe.

Hence the thickness of the crust must follow this singular law,—that wherever its upper part is increased by rising into mountains and table-lands, its lower part is also increased by projecting downwards into the internal fluid; and wherever, on the other hand, the upper part is diminished by sinking into ocean-beds, there also the lower part is diminished by the heavy fluid protruding into it.

This law appears to be contrary to what we should expect from the process of cooling.

Lausanne, October 8, 1855.

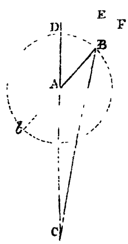
V. *Discussion of the Observed Deviations of the Compass in several Ships, Wood-built and Iron-built: with a General Table for facilitating the examination of Compass-Deviations. By* G. B. AIRY, *Esq., Astronomer Royal.*

Received September 14,—Read November 22, 1855.

IN the year 1839 I communicated to the Royal Society a paper (printed in the Philosophical Transactions of that year) containing the results of examination of the compass in two iron-built ships, and a general theory of the effect of the transient induced magnetism of iron in disturbing the direction of the compass-needle. The result of the theory of induced magnetism may be stated as follows.

First, I must premise, in explanation of the term "polar-magnet-deviation" which I shall frequently have occasion to use, the following theorem on the disturbance of the compass by a magnetized steel bar, or one which possesses independent polar magnetism, in no way referred to the influence of the existing terrestrial magnetism. Let the line CA, fig. 1, represent in magnitude and in direction the terrestrial directive force. [In the applications of the theory to ships, the terrestrial directive force is diminished in a constant ratio differing little from unity; and then it must be understood that CA represents the terrestrial force so diminished.] A is understood to be the magnetic-north end of the line. And let AB represent, in proportional magnitude and in direction, the directive force of the magnetized steel bar or "polar-magnet," B corresponding to that end of the polar-magnet which possesses boreal magnetism. Then the directive force which really acts on the compass-needle will be represented in proportional magnitude and direction by CB; and the angle ACB will be the angle of deviation of the compass. And if the polar-magnet be turned round in azimuth, so that the point B occupies successively different points in the circumference of the circle, the angle of deviation will have successively the different magnitudes and the different directions (right or left of the line CA) given by this construction for these different circumstances. This theorem is very simply founded on the ordinary "composition of forces," and is abundantly proved by experiment. The deviation ACB is what I shall call "polar-magnet-deviation." In some cases it is convenient to refer the azimuth of the polar-magnet to the *true* magnetic meridian or CA, and then the polar-magnet-deviation is given by this formula:

Fig. 1.



$$\tan ACB = \frac{AB . \sin BAD}{CA + AB . \cos BAD},$$

where BAD is the *true* azimuth of the polar-magnet. In other cases it is convenient to refer the azimuth of the polar-magnet to the *disturbed* direction of the compass-needle or BE, and then the polar-magnet-deviation is given by this formula:

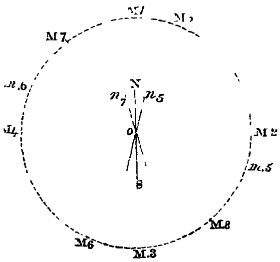
$$\sin \mathrm{ACB} = \frac{\mathrm{AB}}{\mathrm{CA}} \cdot \sin \mathrm{EBF},$$

where EBF is the *apparent* azimuth of the polar-magnet. In either case, the law which connects the polar-magnet-deviation with the azimuth (true or apparent) of the polar-magnet is what I shall call "the law of polar-magnet-deviation."

Secondly: the disturbing effect of the polar-magnet, whose power is represented by AB, may be completely neutralized by attaching to the same frame (whether it be a ship, or an experimental wood frame, &c.) which carries that polar-magnet, another polar-magnet in the opposite position, its power and direction being represented by the line A*b*.

Thirdly: if, instead of the polar magnetism of a steel bar, the disturbing force upon the compass be that of the transient induced magnetism in a nearly spherical mass of soft iron possessing no permanent magnetism, placed in the same horizontal plane as the compass; and if NOS, fig. 2, represent the position taken by the needle under the action of terrestrial magnetism only; then if the mass of soft iron be in either of the positions M_1, M_2, M_3, M_4, it will not disturb the needle NOS: if the mass of soft iron be placed either in the quadrant between M_1 and M_2 (as at M_5) or in the quadrant between M_3 and M_4 (as at M_6), it will make the point N deviate towards n_5; and if the soft iron be placed in either of the remaining quadrants (as at M_7 or M_8) it will make the point N deviate towards n_7. The amount of deviation is proportional to the sine of double the angle of azimuth of the disturbing mass, that is to the sine of double the angle M_1OM_5, or M_1OM_8, &c. If the disturbing mass be carried round the circle in the direction $M_1M_2M_3M_4M_1$, the deviation of the needle (estimated positive when the point N is moved towards the right, and negative when towards the left) will in the four quadrants have the signs $+-+-$. The deviation following this law I shall call "quadrantal deviation."

Fig. 2.



Fourthly: the deviation produced by the mass of soft iron at M_5 will *not* be corrected by placing a similar mass at M_6 (which, instead of correcting the deviation, will double it), but it *will be* corrected by placing a similar mass either at m_5 or at m_6, the angles M_5Om_5 and M_5Om_6 being supposed to be right angles. Similarly, the deviation produced by the mass at M_6 will be neutralized by a similar mass either at m_5 or at m_6, if the angles M_6Om_5, M_6Om_6 are right angles. Thus the "quadrantal deviation" may be corrected by attaching to the same frame (whether it be a ship, or a wooden experimental frame, &c.), which carries the mass that produces the "quadrantal deviation," another mass, at the same level as the compass but in an

azimuth differing 90° from that of the disturbing mass. And, if it be found that when a ship's head is in the quadrant between N. and E., or between S. and W., the needle deviates to the right, and the opposite way for the remaining quadrants; or that, in respect to the quadrants of azimuth of ship's head, the quadrantal deviation follows the law $+-+-$ which I shall call "positive quadrantal deviation;" the inference is that the deviation is of the same kind as would be produced by a mass of iron at the same level as the compass, either headward or sternward of the compass: and it may be neutralized by placing a mass of iron at the same level as the compass, either on the starboard or on the port side. But if the deviation follow the law $-+-+$ in respect of the four quadrants of azimuth, which I shall call "negative quadrantal deviation," it may be neutralized by placing a mass of iron at the same level as the compass, either headward or sternward of the compass. All these laws I have abundantly confirmed by experiment.

These being premised, the laws of the deviation of the compass produced by the transient induced magnetism of a ship, as shown by swinging the ship round in a given locality, will (according to the theory to which I have referred) be as follows:—

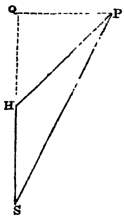
(1). There will be a force, similar to the force of a polar-magnet, and producing a polar-magnet-deviation. In northern magnetic latitudes, the nature of the effect will usually be the same as if the boreal magnetism were towards the ship's head: in southern magnetic latitudes, it will be usually the same as if the austral magnetism were towards the ship's head. The absolute magnitude of the polar-magnet-force will be a multiple of the vertical terrestrial magnetism; the proportion which it bears to the terrestrial directive force, which is the proportion of AB to AC in fig. 1 (supposing no other polar-magnet-force to act), will be a multiple of the tangent of dip, without regard to the absolute force.

(2). There will be a quadrantal deviation; and this deviation will be the same in all magnetic latitudes, and whatever be the magnitude of the earth's magnetic force. It will usually be a positive quadrantal deviation.

These are the disturbances that are produced by transient induced magnetism only. But if the iron that enters into the composition of a ship possess independent polar magnetism similar to that of a magnetized steel bar (*i. e.* not depending on the terrestrial magnetism at the present moment for its existence; and not changing its amount or quality or direction in regard to the ship's keel, while the ship is swung round in different positions), which from the slowness of its changes, though probably more variable than that of a steel bar, I propose to call "subpermanent magnetism;" it will be necessary for us to consider how the expression for the effects of this subpermanent magnetism can be most easily combined with those for the induced transient magnetism. It is readily seen that the polar-magnet-force of subpermanent magnetism must be combined with the polar-magnet-force of induced transient magnetism; and that, at a given locality, they cannot be separated. In

fig. 3, let SH represent the magnitude and direction of the polar-magnet-force of induced magnetism, directed from the ship's stern to her head (this diagram having no relation to the direction of terrestrial magnetism), and let HP represent the magnitude and direction of the subpermanent magnetism, which, inasmuch as its direction is invariable with respect to the ship, is inclined at a constant angle to SH: then the force resulting from the composition of these two will be represented in magnitude and direction by SP, which is invariable in magnitude, and inclined at a constant angle to SH. And this force will appear, in the phenomena of compass-disturbance at any one locality, as a whole, and cannot immediately be separated into the two parts SH and HP.

Fig. 3.



All that can be done is this. At a given locality we can find the direction of SP with regard to the ship's keel, if, by methods to be explained below, we can find the "neutral position of the ship in reference to the polar-magnet-force," or (which is the same thing) the azimuth of the ship's head, or of the line SH, when the polar-magnet-deviation vanishes, or when the line SP coincides with the magnetic meridian. And we can find the magnitude of SP by methods to be explained below. Therefore we can find SQ and QP. And if we assume HQ to be constant, and SH proportional to the earth's vertical magnetic force at the given locality, we shall be able, by comparison of the results at different localities where the vertical force has different magnitudes, to discover the value of SH at each place and the value of HQ. The value of QP however requires no combination of results found at different places, and is not liable to any uncertainty.

But without at present insisting on this separation of the subpermanent magnetism from the polar-magnet-force of induced magnetism, we can lay down the following rule:—

(3). The whole disturbance of the compass, whether the ship be wood-built or iron-built, will be represented by the sum of the effects of two forces, which separately would produce these two disturbances: one, a polar-magnet-deviation whose neutral point may be in any direction; the other, a quadrantal deviation, which may be expected to be a positive quadrantal deviation, following the law of signs $+-+-$ as depending on the quadrants of azimuth of the ship's head. And the whole disturbance will be very nearly (but not exactly) the algebraic sum of these two disturbances: the slight departure from that law will be the subject of examination below.

The practical problem, then, of analysing a given series of compass-deviations, is reduced to the dividing of them into two parts, of which one follows the law of polar-magnet-deviation, and the other follows the law of quadrantal deviation, subject to the trifling correction to which I have just alluded.

In the experiments with the 'Rainbow' and 'Ironsides' (which are treated in my

paper in the Philosophical Transactions, 1839), I was guided by experiments on horizontal intensity as determined by vibrations; but even then I found the computation of polar-magnet-deviation so troublesome, that I executed the calculations by graphical construction. But in other cases, where there is no determination of horizontal intensity, the computation of polar-magnet-deviation would be very much more troublesome. This consideration, together with the paucity of instances in which a comparison of the ship's magnetism in different localities was possible, prevented me from entering further into the numerical calculations of ships' magnetism. But having lately received from Captain Washington, R.N., Hydrographer to the Admiralty, the records of observations in several ships, which I desired to treat numerically; I remarked that the trouble of calculation might be much diminished, and the process might be made perfectly direct and definitive, by the previous preparation of a Table of Polar-Magnet-Deviation; and I proceeded therefore at once to compute the Table which is appended to this paper. Of this Table I will now give a short description.

The table is a Table of double-entry. One of the arguments is the "Modulus," which is the same as the proportion of AB to AC in fig. 1. It is given to every ·01 from ·00 to ·80. The other argument is the "Apparent Azimuth of the Ship's Head from the Neutral Position," which is the same as the "apparent azimuth of the polar magnet" or "azimuth of the polar magnet as measured from the disturbed position of the compass-needle," or the angle EBF in fig. 1. This is used as the argument of the Table, because, in the examination of the disturbance of ships' compasses, it is usually most convenient to fix the ship in position by means of its own compass; and in fact all the observations supplied to me have been made in positions of the ship so determined. As the observations of deviation of ships' compasses are usually made from "point" to "point" of azimuth, the division of the circle here employed is that by points and decimals of a point. The Table is carried to 8 points only, as the polar-magnet-deviations from 8 to 16 points are the same in reversed order; and those from 16 to 32 points are the same as those from 0 to 16 points with change of sign. At the bottom of each column is the "Mean of all the Polar-Magnet-Deviations for each value of the Modulus," which is necessary for enabling us to determine the value of the modulus in any given case.

In ascertaining, from a given series of observed compass-deviations, the neutral position and modulus to be used in the application of this Table, it will be necessary to recognize the existence of a deviation following very nearly the law of quadrantal deviation, and the given numbers must therefore be so combined that quadrantal deviation will be *ipso facto* eliminated. This will be done by so arranging the process that the numbers for a whole semicircle of apparent azimuth will be added together algebraically. This being understood, we may now proceed with advantage to investigate the nature of the terms produced by combining the effects of the polar-magnet-force and the quadrantal force.

Use the notation of the paper of 1839, as far as it goes, and use also the following notation:—

In fig. 1, let CA represent the terrestrial horizontal force multiplied by $(1-M)$, M being a constant peculiar to each ship.

μ the modulus, or the proportion of AB to CA. It will be remarked that SP in fig. 3 is the representative of AB in fig. 1.

α the angle QSP, fig. 3, or the starboard angle made by the compound polar-magnet-force with the ship's keel.

A the true eastern azimuth of the ship's head.

A′ the eastern azimuth of the ship's head as referred to the needle disturbed by polar-magnet-force only.

A″ the eastern azimuth of the ship's head as referred to the needle disturbed by polar-magnet-force and quadrantal force.

$\left.\begin{array}{l}A+\alpha\\ A'+\alpha\\ A''+\alpha\end{array}\right\}$ the corresponding azimuths of the compound polar-magnet-force. $A+\alpha$ is the same as the angle BAD in fig. 1.

Δ' the compass-deviation to the east produced by polar-magnet-force only $=A-A'$ $=(A+\alpha)-(A'+\alpha)$.

Δ'' the additional deviation produced by the quadrantal force $=A'-A''=(A'+\alpha)$ $-(A''+\alpha)$.

And the following double equation is accurate:—

$$\frac{\sin\Delta'}{\mu}=\sin\overline{A'+\alpha}=\frac{\sin\overline{A+\alpha}}{\sqrt{\{1+2\mu\cos\overline{A+\alpha}+\mu^2\}}}.$$

Then, neglecting MP only, the formulæ in the paper of 1839 give—

$$\text{Whole force to north}=\mathrm{I}.\cos\delta.(1-M).(1+\mu.\cos\overline{A+\alpha}+P.\cos 2A)$$

$$\text{Whole force to east}\ =\mathrm{I}.\cos\delta.(1-M).(\mu.\sin\overline{A+\alpha}+P.\sin 2A).$$

Therefore $$\tan\overline{\Delta'+\Delta''}=\frac{\mu.\sin\overline{A+\alpha}+P.\sin 2A}{1+\mu.\cos\overline{A+\alpha}+P.\cos 2A}.$$

But $$\tan\Delta'=\frac{\mu.\sin\overline{A+\alpha}}{1+\mu.\cos\overline{A+\alpha}}.$$

Therefore, retaining the complete multiplier of the first power of P, but no higher powers of P,

$$\tan\Delta''=\frac{P.\sin 2A+\mu P.\sin\overline{A-\alpha}}{1+2\mu.\cos\overline{A+\alpha}+\mu^2}.$$

This quantity, however, is not that which we shall have occasion to use, for the following reason. The polar-magnet-deviation which we shall take out from the Table is taken for an argument which is referred to the position of the compass-needle as disturbed *by all causes*; it is therefore taken out, not for argument $A'+\alpha$ (which would give us Δ exactly), but for argument $A''+\alpha$. Let the quantity

thus taken from the Table be called $\Delta_{,}$, and the correction required be called $\Delta_{,,}$. Then $\Delta_{,}+\Delta_{,,}=\Delta'+\Delta''$; and

$$\Delta_{,,}=(\Delta'-\Delta_{,})+\Delta''=\mu(\sin\overline{A'+\alpha}-\sin\overline{A''+\alpha})+\Delta'' \text{ nearly}$$

$$=\mu(\sin\overline{A''+\alpha+\Delta''}-\sin\overline{A''+\alpha})+\Delta''=\Delta''(1+\mu.\cos\overline{A''+\alpha}) \text{ nearly.}$$

If in the computation of this small quantity we reject powers of μ following the first,

$$\Delta_{,,}=(1-\mu.\cos\overline{A''+\alpha}).P.(\sin 2A+\mu.\sin\overline{A-\alpha}).$$

But $$\sin 2A=\sin 2A''+2\mu.\cos 2A''.\sin\overline{A''+\alpha} \text{ nearly.}$$

After all reductions, $\Delta_{,,}=P.\sin 2A''+\mu P.\cos 2A''.\sin\overline{A''+\alpha}$.

The first term of this expression is in the very convenient form of a quadrantal term referred to the apparent azimuth of the ship's head. The general influence of the second term is, that it produces no effect on the maxima of the quadrantal terms, that it slightly increases the polar-magnet-deviation when $A''=0$ or 180°, and slightly diminishes it when $A''=90^\circ$ or 270°; and this will be practically a sufficient description of its characteristic effects.

But as, in the aggregate of numbers, small terms become sensible which are scarcely sensible in the individual numbers, it will be desirable to ascertain the effect of this on a semicircular group. Combine the term $\mu P.\cos 2A''.\sin\overline{A''+\alpha}$ with the approximate polar-magnet-deviation $\mu.\sin\overline{A''+\alpha}$, and integrate from $A''+\alpha=0$ to $A''+\alpha=\pi$: the result is $2\mu\left(1-\frac{P}{3}\cos 2\alpha\right)$. Without the small term we should have obtained 2μ. Hence it appears that the result for modulus found from semicircular groups, which may be called the "Approximate Modulus," must be multiplied by $1+\frac{P}{3}\cos 2\alpha$ in order to obtain the "True Modulus." Again, conceive the "approximate starboard angle made by the compound polar-magnet-force with the ship's keel" to be $\alpha+\beta$: then a semicircular sum from $A''+\alpha+\beta=\frac{\pi}{2}$ to $A''+\alpha+\beta=\frac{3\pi}{2}$ ought to vanish; the integral between these limits, omitting $P\beta$ and β^2, is $2\left\{\sin\beta-\frac{2P}{3}\sin 2\alpha\right\}$; hence $\beta=\frac{2P}{3}\sin 2\alpha$; and this quantity must be subtracted from the "Approximate Starboard Angle," or $\alpha+\beta$, in order to obtain the "True Starboard Angle," or α.

Thus it appears that, in the column of Tabular Polar-Magnet-Deviations, we are not comparing the tabular deviation due to the modulus μ and the starboard angle α, but that due to the modulus $\mu\left(1-\frac{P}{3}\cos 2\alpha\right)$ and starboard angle $\alpha+\frac{2P}{3}\sin 2\alpha$; and therefore our residual numbers ought to represent

$$\mu.\sin\overline{A''+\alpha}+P.\sin 2A''+\mu P.\cos 2A''.\sin\overline{A''+\alpha}-\mu.\left(1-\frac{P}{3}\cos 2\alpha\right).\sin\left(A''+\alpha+\frac{2P}{3}\sin 2\alpha\right).$$

Expanding the last term, this quantity becomes, after all reductions,

$$P.\sin 2A''+\mu P\left(\frac{1}{2}\sin \overline{3A''+\alpha}-\frac{1}{6}\sin \overline{A''+3\alpha}\right).$$

It will scarcely be necessary to tabulate the small terms; an estimate of their general effect can very well be formed in the mind.

The entire process will therefore be the following:—

1. For the nautical terms N., N.b.E., N.N.E., &c., use the numeral reckoning of points 0, 1, 2, &c., as far as 31, which will correspond to N.b.W. And for deviation E and deviation W, use the algebraical signs deviation + and deviation —. It will always be convenient to place the + deviations and the — deviations in separate columns.

2. Clear the deviations of constant error by adding together all the + deviations, adding together all the — deviations, combining them algebraically, taking the mean of the sum, and applying this mean with sign changed to every deviation. The deviations thus corrected will be the base of all the following operations.

3. In writing down, in columns, the corrected deviations, repeat those from 0 to 15 points, in sequence to those from 0 to 31 points; so that the Table contains forty-eight lines.

4. A conjecture will easily be formed as to the approximate value of the azimuth for the "neutral position:" and then two or three neighbouring half-points are to be adopted for trial. Thus, if the azimuth for neutral position appears to be near 3^p or 4^p, the positions to be tried may be $2^p{\cdot}5$, $3^p{\cdot}5$, $4^p{\cdot}5$.

5. The trial of these azimuths will be effected by dividing the series of observed deviations, not at these azimuths, but at azimuths distant from them 8 points on each side. Thus, to make trial of the assumption $2^p{\cdot}5$, the observed deviations are to be divided at $26^p{\cdot}5$ and $10^p{\cdot}5$. And the criterion will be given by adding algebraically all the deviations from 27^p to 10^p, both included; a little accuracy will be gained if we also add in a separate sum all the deviations from 11^p to 26^p, both included, and subtract this sum from the former. It will be remarked that the quadrantal deviation is here eliminated.

6. If our assumption $2^p{\cdot}5$ for the neutral position were strictly correct, the sum or difference of sums found in the manner just stated would $=0$. As this usually will not prove to be true, we must try the next assumption $3^p{\cdot}5$ in like manner. The comparison of the sums or differences of sums will give the correction to be applied to $2^p{\cdot}5$ with very great accuracy. The azimuth thus determined is strictly an "approximate neutral position," and its supplement to 32^p is the "approximate starboard angle."

7. The approximate neutral position being thus determined, the observed deviations are to be divided into two groups, one division being at the interval in which the neutral position falls, the other at the interval distant from it by 16^p. The algebraic

sums of the deviations in the two groups are to be taken: one is to be subtracted from the other, and the remainder is to be divided by 32. The quotient is the mean deviation. This is to be compared with the Means of Polar-Magnet-Deviations at the foot of the columns of the Table. Two adjacent Tabular Means being found, one greater and one less than the mean deviation just obtained, and the values of modulus corresponding to those two tabular means being noted, there is no difficulty in finding by interpolation the value of modulus corresponding to the mean deviation just obtained. This is the "Approximate Modulus."

8. By use of the approximate neutral position, the angle of apparent azimuth from the neutral position will be formed for every observation. Using this as the argument of Azimuth in the Table, the Polar-Magnet-Deviation is to be taken out for every observation with two tabular values of Modulus, one greater and one less than the approximate modulus just found. Between these, the Polar-Magnet-Deviation will be interpolated for the approximate modulus; and thus the Tabular Polar-Magnet-Deviation corresponding to the Approximate Modulus will be obtained for every observation.

9. Subtracting this Tabular Polar-Magnet-Deviation algebraically from the Observed Deviation, the residual quantity will consist of Quadrantal Deviation, of the small correction $\Delta_{\prime\prime}$, and of errors of observation. Neglecting the two last mentioned, a pretty accurate estimate of the coefficient of quadrantal deviation may be got by omitting the values for 0^p, 8^p, 16^p, 24^p, and dividing the sum of each group of seven numbers by 5; the quotient will be the coefficient, or the Quadrantal Deviation for 4^p, 12^p, 20^p, 28^p. The conversion of this coefficient into abstract number (radius $=1$) gives the numerical coefficient P.

10. The angle $\frac{2}{3}\times$ coefficient of quadrantal deviation $\times$ sine of twice the approximate starboard angle is to be subtracted from the approximate starboard angle to give the "True Starboard Angle." And the approximate modulus is to be multiplied by $1+\frac{P}{3}\times$ cosine of twice the approximate starboard angle to give the "True Modulus."

11. The Headward Modulus $=$ True Modulus $\times$ cosine True Starboard Angle; and the Starboard Modulus $=$ True Modulus $\times$ sine True Starboard Angle. As the modulus is the proportion of the disturbing force to the terrestrial horizontal force (slightly diminished everywhere in the same proportion), we must, for the exhibition of the absolute values of the disturbing forces, multiply these quantities by the terrestrial horizontal force. Then (referring to the statements at the commencement of this paper for the results of theory) we shall have,

Headward Modulus $\times$ Terrestrial Horizontal Force $=$ H $+$ N $\times$ Terrestrial Vertical Force,
Starboard Modulus $\times$ Terrestrial Horizontal Force $=$ S,

where H and S are the forces of the ship's subpermanent magnetism in the headward and starboard directions; and N is a constant peculiar to the ship, depending

on the arrangement of the mass of iron, and having relation only to the ship's capacity for induced magnetism, but in no way related to terrestrial magnetism.

If we can reconcile the observations made in the same ship at various localities by making H and S constant, then the subpermanent magnetism is truly permanent. In any case, N must be constant for the same ship.

Perhaps the process of obtaining the various elements of a ship's magnetism will be rendered a little more intelligible by exhibiting the work in a single instance.

Iron-Steamer "Trident," examined at Greenhithe, 1852, September.

1. Deviations as registered, substituting only the 0, 1, 2, 3, &c. points for N., N.b.E., N.N.E., N.E.b.N., &c., and the signs + and − for E. and W.

Apparent azimuth of ship's head.	Deviation.		Apparent azimuth of ship's head.	Deviation.	
	+	−		+	−
p	° ′	° ′	p	° ′	° ′
0		2 55	16	2 30	
1	3 40		17	0 0	
2	9 15		18		2 30
3	12 32		19		5 50
4	15 50		20		9 10
5	17 40		21		11 52
6	19 30		22		14 35
7	19 5		23		16 47
8	18 40		24		19 0
9	17 15		25		20 22
10	15 50		26		21 45
11	14 10		27		22 10
12	12 30		28		20 0
13	11 30		29		16 30
14	9 20		30		13 0
15	5 55		31		7 32

The sum of the + deviations is $+205^\circ\ 12'$; the sum of the − deviations is $-203^\circ\ 58'$; the algebraical sum of all is $+1^\circ\ 14'$; which implies a mean error of $+0^\circ\ 2'$. Applying the correction $-0^\circ\ 2'$ to every deviation, the next table is formed.

2 & 3. Deviations as corrected for constant error.

Apparent azimuth of ship's head.	Deviation. +	Deviation. −	Apparent azimuth of ship's head.	Deviation. +	Deviation. −
p	° ′	° ′	p	° ′	° ′
0		2 57	24		19 2
1	3 38		25		20 24
2	9 13		26		21 47
3	12 30		27		22 12
4	15 48		28		20 2
5	17 38		29		16 32
6	19 28		30		13 2
7	19 3		31		7 34
8	18 38		0		2 57
9	17 13		1	3 38	
10	15 48		2	9 13	
11	14 8		3	12 30	
12	12 28		4	15 48	
13	11 28		5	17 38	
14	9 18		6	19 28	
15	5 53		7	19 3	
16	2 28		8	18 38	
17		0 2	9	17 13	
18		2 32	10	15 48	
19		5 52	11	14 8	
20		9 12	12	12 28	
21		11 54	13	11 28	
22		14 37	14	9 18	
23		16 49	15	5 53	

4. The neutral position appears to be somewhere near 1^p, and therefore trial may be made with the two assumptions $0^p{\cdot}5$ and $1^p{\cdot}5$.

5. For trial $0^p{\cdot}5$, the sums must be taken from 9^p to 24^p, and from 25^p to 8^p.

$$\begin{array}{ll} \text{Sum from } \overset{p}{9} \text{ to } \overset{p}{24} = +\ \overset{\circ}{88}\ \overset{\prime}{44} - \overset{\circ}{80}\ \overset{\prime}{0} = +\ \overset{\circ}{8}\ \overset{\prime}{44} \\ \text{Sum from } 25 \text{ to } 8 = +115\ 56 - 124\ 30 = -\ 8\ 34 \\ \qquad\qquad \text{Excess of the first} \ldots\ldots +17\ 18 \end{array}$$

For trial $1^p{\cdot}5$, the sums must be taken from 10^p to 25^p, and from 26^p to 9^p.

$$\begin{array}{ll} \text{Sum from } \overset{p}{10} \text{ to } \overset{p}{25} = +\ \overset{\circ}{71}\ \overset{\prime}{31} - \overset{\circ}{100}\ \overset{\prime}{24} = -\overset{\circ}{28}\ \overset{\prime}{53} \\ \text{Sum from } 26 \text{ to } 9 = +133\ 9 - 104\ 6 = +29\ 3 \\ \qquad\qquad \text{Excess of the first} \ldots\ldots -57\ 56 \end{array}$$

6. A change of 1^p in the assumption has changed the "excess" from $+17^\circ\ 18'$ to $-57^\circ\ 56'$, or has changed it by $75^\circ\ 14'$. Hence, the assumption which would make the "Excess" $=0$ is $0^p{\cdot}5 + 1^p \times \frac{17^\circ\ 18'}{75^\circ\ 14'} = 0^p{\cdot}5 + 1^p \times 0{\cdot}23 = 0^p{\cdot}5 + 0^p{\cdot}23 = 0^p{\cdot}73$. This is the "approximate neutral position"; and its supplement or $31^p{\cdot}27$ is the "approximate starboard angle," or the approximate value of α.

7. The neutral position being $0^p{\cdot}73$, the observed deviations which are to be grouped for ascertaining the approximate modulus are those from 1^p to 16^p in one part, and those from 17^p to 0^p in the other part. The sums are respectively $+204^\circ\ 40'$ and $-204^\circ\ 30'$; the excess of the first is $409^\circ\ 10'$; dividing by 32, the

quotient is 12° 47′. On examining the "Mean of Polar-Magnet-Deviations" at the foot of the columns of the Table, it is found that the Mean corresponding to Modulus 0·34 is 12° 35′, and that corresponding to Modulus 0·35 is 12° 58′. Hence the Modulus for mean 12° 47′ is 0·3452.

8 & 9. The neutral position being $0^p{\cdot}73$, the "Apparent Inclination from Neutral Position" for azimuth of ship's head 0^p will be $31^p{\cdot}27$; and so for the others. Then two series of numbers are interpolated from the Table of Polar-Magnet-Deviations for the fractional parts of the points; one for Modulus 0·34, and the other for Modulus 0·35. Between the pairs of corresponding numbers thus found, a third interpolation is made for the fractional part of the Modulus, to 0·3452; and thus are obtained the Tabular Polar-Magnet-Deviations required. Subtracting these from

Apparent azimuth of ship's head.	Apparent inclination from neutral position.	Tabular polar-magnet-deviation. Modulus 0·34.	Modulus 0·35.	Modulus 0·3452.	Excess of corrected observed deviation.	One-fifth of quadrantal group.
p	p	° ′	° ′	° ′	° ′	
0	31·27	− 2 47	− 2 52	− 2 50	(−0 7)	
1	0·27	+ 1 2	+ 1 4	+ 1 3	+2 35	
2	1·27	+ 4 49	+ 4 57	+ 4 53	+4 20	
3	2·27	+ 8 26	+ 8 40	+ 8 33	+3 57	
4	3·27	+11 45	+12 6	+11 56	+3 52	+3° 57′
5	4·27	+14 38	+15 5	+14 52	+2 46	
6	5·27	+16 59	+17 31	+17 16	+2 12	
7	6·27	+18 42	+19 16	+19 0	+0 3	
8	7·27	+19 40	+20 16	+19 59	(−1 21)	
9	8·27	+19 51	+20 27	+20 10	−2 57	
10	9·27	+19 14	+19 50	+19 33	−3 45	
11	10·27	+17 52	+18 24	+18 9	−4 1	
12	11·27	+15 48	+16 16	+16 3	−3 35	−3 33
13	12·27	+13 9	+13 32	+13 21	−1 53	
14	13·27	+10 0	+10 18	+10 9	−0 51	
15	14·27	+ 6 31	+ 6 42	+ 6 37	−0 44	
16	15·27	+ 2 47	+ 2 52	+ 2 50	(−0 22)	
17	16·27	− 1 2	− 1 4	− 1 3	+1 1	
18	17·27	− 4 49	− 4 57	− 4 53	+2 21	
19	18·27	− 8 26	− 8 40	− 8 33	+2 41	
20	19·27	−11 45	−12 6	−11 56	+2 44	+3 19
21	20·27	−14 38	−15 5	−14 52	+2 58	
22	21·27	−16 59	−17 31	−17 16	+2 39	
23	22·27	−18 42	−19 16	−19 0	+2 11	
24	23·27	−19 40	−20 16	−19 59	(+0 57)	
25	24·27	−19 51	−20 27	−20 10	−0 14	
26	25·27	−19 14	−19 50	−19 33	−2 14	
27	26·27	−17 52	−18 24	−18 9	−4 3	
28	27·27	−15 48	−16 16	−16 3	−3 59	−3 30
29	28·27	−13 9	−13 32	−13 21	−3 11	
30	29·27	−10 0	−10 18	−10 9	−2 53	
31	30·27	− 6 31	− 6 42	− 6 37	−0 57	

the corrected observed deviations, we have the excess, which clearly follows with considerable accuracy, a Positive Quadrantal law, perhaps slightly disturbed by the residual small terms in the manner explained above. Its Mean Coefficient is +3° 35′ or $0^p{\cdot}3185$, whose equivalent in abstract number is +0·0626=P.

10. From these numbers, $\frac{2}{3}$ quadrantal coefficient × sine of twice approximate

starboard angle $=-0^{p}\cdot06$; which subtracted from approximate starboard angle gives True Starboard Angle $=31^{p}\cdot33$. And $\frac{P}{3}\times$ cosine of twice approximate starboard angle $=0\cdot020$: adding this to unity and multiplying the Approximate Modulus, we have the True Modulus $=0\cdot352$.

11. Expressing the magnitude of the terrestrial magnetic force in the manner introduced by GAUSS for *Absolute Measure*, and adopting the English foot and English grain as the units of length and weight, the measure of terrestrial horizontal force at Greenhithe is 3·79, and that of terrestrial vertical force is 9·66. Forming the quantities "True Modulus $\times$ cosine True Starboard Angle $\times$ Terrestrial Horizontal Force," and "True Modulus $\times$ sine True Starboard Angle $\times$ Terrestrial Horizontal Force," we have for the 'Trident' at Greenhithe in 1852,

$$+1\cdot375=H+N\times9\cdot663$$
$$-0\cdot182=S;$$

and these results, with that just obtained for the Coefficient of Quadrantal Deviation, are the most advanced that can be obtained from the deviations of the compass in the 'Trident' observed at Greenhithe only.

I shall now exhibit, in a tabular form, the results of the twenty-nine series of deviations which have reached me. Nos. 1 to 13 are extracted from the work of the late Captain JOHNSON, R.N., "On the Deviations of the Compass." The signs of the compass-deviations of the 'Erebus' at St. Helena are changed, on the authority of Colonel SABINE, as conveyed to me by ARCHIBALD SMITH, Esq. Nos. 14 to 29 have been communicated to me by Captain WASHINGTON, R.N., Hydrographer to the Admiralty.

It must be remarked that the first column, in every case, is the registered deviation as given by the observer (a few numbers in brackets being supplied by interpolation), and not the deviation as cleared of mean error or index error. In some cases this mean error is large (thus with the 'Simoom' at Simon's Town it amounts to 1° 47′), and here it greatly modifies the true deviation, and even causes the original deviation to appear less on some points than the residual error. The residual error is formed by computing the polar-magnet-deviation from the approximate elements at the top of the Table, and subtracting it from the deviation corrected for constant mean error only; it therefore contains the quadrantal deviation, the small terms produced by combination of polar-magnet-deviation with quadrantal deviation, and the accidental errors of observation. The coefficients of quadrantal deviation below are formed by omitting the residual errors for 0^{p}, 8^{p}, 16^{p}, 24^{p}, and taking one-fifth part of the sums of the residual errors in the groups between them; and the mean coefficient is formed by changing the signs of the second and fourth coefficients of quadrantal deviation, and taking one-fourth of the algebraical sum. The true elements are formed by correcting the approximate elements in the manner just explained.

No.	1.		2.		3.		4.	
Ship's name	Erebus.							
Place of observation	Gillingham.		Porto Praya.		St. Helena.		Cape of Good Hope.	
Time of observation								
Index correction	+0° 16′		−0° 24′		0° 0′		+0° 19′	
Approximate modulus	·069		·033		·008		·020	
Approx. starboard angle	0^p·40		0^p·93		1^p·88		14^p·98	
	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.
Apparent azimuth of ship's head. p								
0	−0° 6′	(−0° 9′)	+0° 56′	(+0° 11′)	+0° 4′	(−0° 6′)	−0° 24′	(−0° 18′)
1	+0 49	0 0	+1 17	+0 11	+0 16	+0 1	−0 24	−0 6
2	+1 39	+0 6	+1 53	+0 27	+0 28	+0 10	−0 1	+0 31
3	+2 16	+0 5	+2 12	+0 29	+0 51	+0 29	−0 11	+0 34
4	+2 59	+0 14	+2 10	+0 14	+1 10	+0 46	−0 22	+0 35
5	+3 18	+0 7	+2 30	+0 23	+0 48	+0 23	−0 31	+0 37
6	+3 46	+0 17	+2 47	+0 34	+0 57	+0 31	−0 51	+0 25
7	+4 53	+1 13	+2 40	+0 24	+0 55	+0 29	−1 2	+0 21
8	+3 42	(+0 1)	+1 56	(−0 18)	+0 32	(+0 7)	−1 15	(+0 12)
9	+3 17	−0 16	+1 44	−0 25	+0 13	−0 9	−1 34	−0 6
10	+2 50	−0 25	+1 18	−0 40	−0 5	−0 24	−1 43	−0 16
11	+2 21	−0 30	+1 25	+0 5	−0 20	−0 35	−2 1	−0 38
12	+1 53	−0 26	+1 9	−0 19	−0 20	−0 31	−1 53	−0 37
13	+1 23	−0 17	+0 25	−0 44	−0 25	−0 31	−1 41	−0 33
14	+0 48	−0 10	+0 42	−0 5	−0 30	−0 31	−1 26	−0 29
15	+0 19	+0 7	+0 32	+0 6	−0 24	−0 19	−1 14	−0 29
16	−0 28	(+0 7)	+0 39	(+0 36)	−0 28	(−0 18)	−0 39	(−0 7)
17	−0 52	+0 29	−0 10	+0 8	−0 8	+0 7	−0 16	+0 4
18	−1 34	+0 31	+0 2	+0 40	−0 1	+0 17	+0 2	+0 8
19	−2 8	+0 35	−0 9	+0 46	+0 14	+0 36	+0 40	+0 33
20	−2 45	+0 32	−0 26	+0 42	+0 4	+0 28	+0 52	+0 33
21	−3 24	+0 19	−1 34	−0 15	+0 8	+0 33	+1 6	+0 36
22	−4 3	−0 2	−1 24	+0 1	+0 9	+0 35	+1 5	+0 27
23	−4 40	−0 28	−1 49	−0 21	−0 12	+0 14	+1 14	+0 29
24	−4 19	(−0 6)	−1 33	(−0 7)	−0 32	(−0 7)	+1 9	(+0 20)
25	−4 9	−0 4	−1 15	+0 6	−0 29	−0 7	+0 51	+0 1
26	−3 51	−0 4	−1 5	+0 5	−0 25	−0 6	+0 33	−0 16
27	−3 28	−0 5	−1 25	−0 53	−1 3	−0 48	+0 15	−0 30
28	−3 3	−0 12	−1 15	−0 35	−0 40	−0 29	+0 3	−0 35
29	−2 30	−0 18	−0 52	−0 31	−0 33	−0 27	−0 2	−0 32
30	−2 1	−0 31	−0 20	−0 21	−0 27	−0 26	−0 10	−0 29
31	−1 12	−0 28	−0 27	−0 49	−0 2	−0 7	−0 16	−0 23
Quadrantal coefficients	+0° 24′ −0 23 +0 23 −0 20		+0° 32′ −0 24 +0 20 −0 36		+0° 34′ −0 36 +0 34 −0 30		+0° 35′ −0 38 +0 34 −0 33	
Mean quadrantal coefficient	+0 23		+0 28		+0 34		+0 35	
True modulus	·069		·033		·008		·020	
True starboard angle	0^p·40		0^p·92		1^p·86		14^p·99	

No.	5.		6.		7.		8.	
Ship's name	Erebus.		Bloodhound.					
Place of observation	Kerguelen's Land.		Plymouth.		Constantinople.		Piræus.	
Time of observation			1845.		1846.			
Index correction	0° 0′		−0° 27′		−0° 6′		−0° 9′	
Approximate modulus	·067		·259		·185		·161	
Approx. starboard angle	15ᵖ·81		31ᵖ·15		30ᵖ·91		30ᵖ·70	
	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.
Apparent azimuth of ship's head. p								
0	+0° 8′	(0° 0′)	−1° 0′	(+1° 0′)	−4° 20′	(−2° 10′)	−1° 45′	(+0° 26′)
1	−0 22	+0 15	+3 20	+2 26	+0 20	+0 26	+0 15	+0 39
2	−1 1	+0 20	+7 15	+3 28	+4 0	+2 1	+2 55	+1 30
3	−1 31	+0 31	+10 50	+4 18	+7 0	+3 1	+7 15	+4 4
4	−2 9	+0 29	+13 20	+4 15	+9 0	+3 9	+8 35	+3 46
5	−2 13	+0 54	+14 40	+3 22	+10 20	+2 50	+10 20	+4 3
6	−3 1	+0 30	+15 0	+1 53	+11 20	+2 29	+11 0	+3 30
7	−3 31	+0 14	+15 0	+0 33	+11 50	+1 55	+9 55	+1 26
8	−3 48	(+0 3)	+14 0	(−1 14)	+12 50	(+2 17)	+9 15	(+0 9)
9	−3 53	−0 4	+13 20	−2 6	+10 30	−0 17	+8 5	−1 17
10	−4 12	−0 35	+11 50	−3 13	+9 0	−1 39	+6 35	−2 44
11	−3 51	−0 33	+10 20	−3 47	+6 50	−3 13	+5 35	−3 19
12	−3 20	−0 30	+9 20	−3 17	+5 30	−3 35	+4 15	−3 52
13	−2 50	−0 34	+7 50	−2 50	+3 0	−4 47	+3 15	−3 48
14	−2 4	−0 28	+6 20	−2 1	+2 50	−3 20	+2 55	−2 48
15	−1 10	−0 16	+4 20	−1 23	+2 40	−1 41	+2 35	−1 35
16	−0 21	(−0 13)	+2 40	(−0 14)	+2 20	(−0 2)	+2 45	(+0 16)
17	+0 44	+0 7	+1 20	+1 20	+1 30	+1 12	+1 15	+0 33
18	+1 53	+0 32	−0 50	+2 3	+0 40	+2 27	+1 25	+2 32
19	+2 33	+0 31	−2 40	+2 58	−0 10	+3 37	+1 35	+4 28
20	+3 23	+0 45	−5 20	+2 51	−2 40	+2 59	−0 5	+4 26
21	+3 46	+0 39	−7 10	+3 14	−4 0	+3 18	−1 55	+4 4
22	+4 5	+0 34	−9 50	+2 23	−6 0	+2 39	−3 45	+3 27
23	+3 56	+0 11	−11 10	+2 23	−7 30	+2 13	−7 5	+1 6
24	+3 40	(−0 11)	−13 0	(+1 20)	−10 0	(+0 21)	−9 5	(−0 17)
25	+3 35	−0 14	−15 50	−1 18	−11 20	−0 45	−11 45	−2 41
26	+3 11	−0 26	−16 40	−2 31	−12 30	−2 3	−13 5	−4 4
27	+2 37	−0 41	−16 40	−3 27	−11 20	−1 29	−12 25	−3 49
28	+2 1	−0 49	−15 30	−3 47	−12 30	−3 37	−10 45	−2 56
29	+2 7	−0 9	−13 30	−3 44	−11 0	−3 25	−9 30	−2 45
30	+1 9	−0 27	−11 20	−3 53	−8 30	−2 32	−8 45	−3 20
31	+0 26	−0 28	−6 0	−1 11	−6 20	−2 11	−5 0	−1 8
Quadrantal coefficients	+0° 39′ −0 36 +0 40 −0 39		+4° 3′ −3 43 +3 26 −3 58		+3° 10′ −3 42 +3 41 −3 16		+3° 48′ −3 53 +4 7 −4 9	
Mean quadrantal coefficient	+0 38		+3 48		+3 27		+3 59	
True modulus	·068		·264		·189		·164	
True starboard angle	15ᵖ·82		31ᵖ·22		30ᵖ·99		30ᵖ·82	

No.	9.		10.		11.		12.	
Ship's name	Jackal.						Trident.	
Place of observation	Plymouth.		Tagus.		Piræus.		Greenhithe.	
Time of observation	1845.		1847.		1846.		1846.	
Index correction	−0° 12′		+0° 4′		−0° 4′		−0° 37′	
Approximate modulus	·297		·216		·182		·366	
Approx. starboard angle	31ᵖ·40		31ᵖ·10		31ᵖ·58		0ᵖ·32	
	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.
Apparent azimuth of ship's head. p 0	− 2° 15′	(−0° 27′)	− 1° 0′	(+1° 14′)	− 0° 40′	(+0° 9′)	+ 3° 12′	(+1° 17′)
1	+ 2 40	+1 8	+ 3 0	+2 49	+ 3 0	+1 44	+ 8 25	+2 26
2	+ 8 58	+4 8	+ 6 40	+4 5	+ 6 30	+3 14	+13 44	+3 52
3	+13 7	+5 9	+ 9 20	+4 27	+ 8 20	+3 11	+18 37	+5 10
4	+15 18	+4 29	+11 20	+4 19	+10 40	+3 50	+21 7	+4 35
5	+17 15	+3 59	+12 20	+3 27	+11 20	+3 4	+21 45	+2 41
6	+18 7	+2 53	+12 0	+1 36	+12 20	+2 56	+22 15	+1 22
7	+17 47	+1 9	+12 0	+0 30	+10 30	+0 20	+22 26	+0 32
8	+15 35	(−1 54)	+11 40	(−0 31)	+10 0	(−0 33)	+20 18	(−1 46)
9	+14 38	−2 49	+10 20	−2 3	+ 9 0	−1 30	+18 50	−2 31
10	+13 22	−3 28	+ 9 20	−2 45	+ 8 0	−2 5	+16 30	−3 19
11	+11 40	−3 54	+ 9 0	−2 20	+ 6 15	−2 59	+13 0	−4 32
12	+ 9 55	−3 48	+ 5 40	−4 27	+ 4 0	−4 4	+11 0	−3 38
13	+ 8 5	−3 15	+ 4 40	−3 49	+ 3 15	−3 20	+ 7 30	−3 43
14	+ 5 35	−2 59	+ 3 20	−3 17	+ 3 0	−1 51	+ 6 0	−1 26
15	+ 4 12	−1 16	+ 2 40	−1 46	+ 1 20	−1 37	+ 2 20	−1 5
16	+ 2 20	(+0 8)	+ 2 0	(−0 ·6)	+ 0 40	(−0 17)	− 0 6	(+0 35)
17	− 0 12	+0 56	+ 1 20	+1 39	− 1 0	+0 8	− 2 45	+2 0
18	− 2 32	+1 54	− 1 0	+1 43	− 1 20	+1 48	− 5 58	+2 40
19	− 4 55	+2 39	− 1 0	+4 1	− 2 40	+2 21	− 8 42	+3 31
20	− 6 30	+3 55	− 3 40	+3 29	− 2 40	+4 2	−10 55	+4 23
21	− 8 45	+4 7	− 4 0	+5 1	− 4 40	+3 28	−13 25	+4 25
22	−10 52	+3 58	− 7 40	+2 52	− 5 40	+3 36	−17 10	+2 29
23	−13 50	+2 24	−10 0	+1 38	− 8 20	+1 42	−19 17	+1 23
24	−15 45	(+1 14)	−12 40	(−0 21)	− 9 40	(+0 45)	−20 35	(+0 15)
25	−17 25	−0 22	−13 20	−0 49	−11 20	−0 58	−21 22	−1 15
26	−18 25	−1 59	−15 20	−3 7	−12 0	−2 3	−21 12	−2 37
27	−18 25	−3 15	−15 0	−3 32	−12 40	−3 34	−19 40	−3 22
28	−17 15	−3 56	−14 40	−4 25	−11 5	−3 9	−18 15	−4 51
29	−15 45	−4 49	−14 40	−6 3	− 9 40	−3 13	−14 40	−4 41
30	−11 55	−3 45	− 9 20	−2 35	− 8 0	−3 17	− 9 38	−3 26
31	− 7 25	−2 21	− 5 40	−1 6	− 4 30	−1 41	− 3 50	−1 39
Quadrantal coefficients	+4° 35′ −4 18 +3 59 −4 5		+4° 15′ −4 5 +4 5 −4 19		+3° 40′ −3 29 +3 25 −3 35		+4° 8′ −4 3 +4 10 −4 22	
Mean quadrantal coefficient	+4 14		+4 11		+3 32		+4 11	
True modulus	·305		·220		·186		·375	
True starboard angle	31ᵖ·46		31ᵖ·19		31ᵖ·61		0ᵖ·29	

No.	13.		14.		15.		16.	
Ship's name	Trident.		Pandora.				Mæander.	
Place of observation	Malta.		Plymouth.		Auckland.		Sheerness.	
Time of observation	1847.		1851, February.		1853, January.		1852, September.	
Index correction	−0° 38′		+0° 47′		+0° 43′		+0° 15′	
Approximate modulus	·240		·043		·045		·021	
Approx. starboard angle	31ᵖ·64		0ᵖ·28		14ᵖ·50		2ᵖ·33	
	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.
Apparent azimuth of ship's head. p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	+ 1 19	(+1 40)	+0 10	(+0 48)	0 0	(−0 2)	+0 30	(+0 14)
1	+ 4 29	+2 8	+0 30	+0 40	−0 4	+0 24	+0 45	+0 18
2	+ 6 59	+2 0	+0 50	+0 32	−0 45	+0 13	+0 35	−0 3
3	+11 19	+3 52	+0 50	+0 7	−1 33	−0 5	+0 30	−0 16
4	+14 9	+4 29	+1 45	+0 41	−1 30	+0 21	+0 45	−0 7
5	+14 59	+3 26	+1 20	−0 1	−1 56	+0 26	+0 55	0 0
6	+15 29	+2 28	+1 50	+0 16	−1 50	+0 52	+1 0	+0 4
7	+14 29	+0 29	+1 55	+0 15	−2 35	+0 23	+1 0	+0 6
8	+13 19	(−1 9)	+2 5	(+0 24)	−2 40	(+0 31)	+0 30	(−0 19)
9	+12 9	−2 15	+2 0	+0 23	−3 27	−0 11	+0 30	−0 11
10	+10 9	−3 38	+1 20	−0 7	−3 25	−0 9	+0 30	−0 2
11	+ 9 9	−3 29	+0 50	−0 22	−3 45	−0 34	+0 30	+0 9
12	+ 7 9	−3 55	−0 25	−1 18	−3 23	−0 25	+0 20	+0 12
13	+ 6 9	−2 56	−1 10	−1 39	−3 28	−0 46	+0 25	+0 31
14	+ 4 59	−1 47	−0 20	−0 23	−2 50	−0 28	0 0	+0 19
15	+ 3 39	−0 37	−0 40	−0 15	−2 16	−0 20	0 0	+0 33
16	+ 2 29	(+0 52)	−0 45	(+0 11)	−1 35	(−0 7)	−0 10	(+0 36)
17	+ 0 19	+1 24	−1 25	−0 1	−0 36	+0 22	−0 35	+0 22
18	− 1 21	+2 22	−1 0	+0 52	+0 20	+0 48	−1 40	−0 32
19	− 2 51	+3 20	−1 50	+0 27	+0 50	+0 48	−1 40	−0 24
20	− 5 1	+3 23	−2 0	+0 38	+1 26	+1 1	−2 10	−0 48
21	− 7 21	+2 56	−2 10	+0 45	+1 0	+0 4	−2 0	−0 35
22	− 9 31	+2 14	−2 40	+0 28	+1 10	−0 6	−1 40	−0 4
23	−10 41	+2 3	−3 0	+0 14	+0 55	−0 37	−1 30	−0 6
24	−12 51	(+0 21)	−3 40	(−0 25)	+1 40	(−0 5)	−1 30	(−0 11)
25	−14 21	−1 13	−4 17	−1 6	+2 36	+0 46	−1 25	−0 14
26	−14 21	−1 50	−3 50	−0 49	+1 35	−0 15	−0 40	+0 32
27	−15 1	−3 39	−3 50	−1 4	+0 47	−0 58	−0 40	+0 21
28	−14 21	−4 33	−2 30	−0 3	+0 30	−1 2	−0 20	+0 18
29	−11 31	−3 42	−2 40	−0 37	+0 45	−0 31	−0 20	+0 4
30	− 8 41	−3 11	−1 20	+0 17	+0 5	−0 51	−0 20	−0 9
31	− 4 21	−1 21	−0 50	+0 19	+1 5	+0 40	−0 15	−0 28
Quadrantal coefficients	+3° 46′ −3 43 +3 32 −3 54		+0° 30′ −0 44 +0 41 −0 37		+0° 31′ −0 35 +0 28 −0 26		0° 0′ +0 18 −0 25 +0 5	
Mean quadrantal coefficient	+3 44		+0 38		+0 30		−0 12	
True modulus	·245		·044		·045		·021	
True starboard angle	31ᵖ·67		0ᵖ·27		14ᵖ·52		2ᵖ·32	

No.	17.		18.		19.		20.	
Ship's name	**Mæander.**		**Virago.**				**Plumper.**	
Place of observation	Simon's Bay.		Plymouth.		Valparaiso.		Portsmouth.	
Time of observation	1853, March.		1851, September.		1852, September.		1853, September.	
Index correction	+0° 23′		−0° 19′		+0° 9′		+0° 27′	
Approximate modulus	·044		·129		·012		·106	
Approx. starboard angle	15p·80		0p·58		6p·25		0p·90	
Apparent azimuth of ship's head.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.
p 0	−1° 0′	(−0° 43′)	+1° 0′	(−0° 10′)	+1° 0′	(+0° 31′)	+0° 35′	(−0° 2′)
1	−1 0	−0 13	+2 50	+0 15	+1 40	+1 9	+2 20	+0 33
2	−1 0	+0 16	+4 0	+0 5	+2 20	+1 49	+3 40	+0 50
3	−1 10	+0 32	+5 0	−0 7	+2 20	+1 50	+5 0	+1 14
4	−2 40	−0 34	+6 30	+0 23	+2 20	+1 52	+5 55	+1 23
5	−2 40	−0 15	+8 0	+1 5	+2 30	+2 6	+6 45	+1 38
6	−3 12	−0 32	+9 40	+2 13	+2 20	+2 1	+6 35	+1 5
7	−2 40	+0 10	+9 40	+1 58	+0 20	+0 8	+6 0	+0 23
8	(−2 55)	(−0 1)	+8 40	(+0 59)	−0 30	(−0 35)	+5 45	(+0 13)
9	−3 10	−0 18	+7 20	−0 3	−1 0	−0 57	+4 40	−0 33
10	−2 55	−0 11	+6 40	−0 8	−1 20	−1 9	+3 45	−0 56
11	−2 25	+0 7	+4 30	−1 29	−2 0	−1 42	+2 50	−1 6
12	−2 10	+0 4	+3 0	−1 57	−2 10	−1 43	(+1 45)	−1 17
13	−1 50	+0 2	+2 0	−1 43	−2 40	−2 7	(+0 40)	−1 20
14	−1 0	+0 26	+1 0	−1 22	−2 40	−2 1	(−0 25)	−1 17
15	−0 10	+0 48	+0 40	−0 17	−1 20	−0 37	−1 30	−1 11
16	−0 50	(−0 21)	−0 50	(−0 18)	−0 10	(+0 37)	−1 45	(−0 16)
17	0 0	−0 1	−1 20	+0 37	−0 10	+0 39	−1 50	+0 49
18	+0 50	+0 20	−2 20	+0 57	0 0	+0 49	−2 40	+1 2
19	+1 0	+0 4	−3 0	+1 29	+0 50	+1 38	−3 40	+0 58
20	+0 50	−0 30	−3 35	+1 54	+0 50	+1 36	−4 25	+0 59
21	+1 20	−0 19	−5 20	+0 57	−1 30	−0 48	−4 35	+1 24
22	+1 50	−0 4	−6 30	+0 19	+0 50	+1 27	−4 55	+1 27
23	+2 0	−0 4	−7 10	−0 6	+0 20	+0 50	−5 35	+0 54
24	+2 0	(−0 8)	−8 20	(−1 17)	−0 20	(+0 3)	−6 15	(+0 9)
25	+2 40	+0 34	−8 20	−1 35	−1 0	−0 45	−6 35	−0 30
26	+1 40	−0 18	−7 40	−1 30	−1 0	−0 53	−6 25	−0 52
27	+1 57	+0 11	−6 20	−0 59	−1 30	−1 30	−6 0	−1 12
28	+1 50	+0 22	−6 0	−1 41	−1 0	−1 9	−5 30	−1 36
29	+1 20	+0 14	−3 20	−0 15	−1 0	−1 15	(−4 20)	−1 28
30	+1 10	+0 30	−0 50	+0 54	−1 0	−1 21	−3 10	−1 26
31	0 0	−0 12	+0 40	+0 59	−0 10	−0 35	−1 5	−0 32
Quadrantal coefficients	−0° 9′ +0 10 −0 9 +0 14		+1° 10′ −1 24 +1 13 −0 49		+2° 11′ −2 3 +1 14 −1 30		+1° 25′ −1 32 +1 31 −1 31	
Mean quadrantal coefficient	−0 11		+1 9		+1 44		+1 30	
True modulus	·044		·130		·012		·106	
True starboard angle	15p·80		0p·56		6p·18		0p·87	

No.	21.		22.		23.		24.	
Ship's name	Plumper.		Spy.				Trident.	
Place of observation	St. Catherine's.		Sheerness.		St. Paul's Loando.		Greenhithe.	
Time of observation	1852, April.		1854, August.		1852, December.		1852, September.	
Index correction	−0° 3′		+1° 35′		+0° 15′		−0° 2′	
Approximate modulus	·048		·040		·077		·345	
Approx. starboard angle	1^p·55		27^p·56		31^p·93		31^p·27	
	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.
Apparent azimuth of ship's head. p 0	+1° 0′	(+0° 7′)	−5° 0′	(−1° 41′)	−0° 40′	(−0° 21′)	−2° 55′	(−0° 7′)
1	+2 30	+1 7	−5 0	−2 0	+0 20	−0 13	+3 40	+2 35
2	+3 20	+1 30	−3 0	−0 22	+1 20	−0 3	+9 15	+4 20
3	+3 20	+1 8	0 0	+2 14	+2 20	+0 10	+12 32	+3 57
4	+3 40	+1 11	+1 20	+3 7	+3 10	+0 19	+15 50	+3 52
5	+4 0	+1 18	+0 15	+1 35	+3 40	+0 16	+17 40	+2 46
6	+4 10	+1 22	+0 10	+1 3	+2 50	−1 0	+19 30	+2 12
7	+3 20	+0 33	+0 40	+1 9	+3 30	−0 35	+19 5	+0 3
8	+2 30	(−0 11)	0 0	(+0 7)	+4 10	(−0 1)	+18 40	(−1 21)
9	+1 40	−0 47	+0 40	+0 29	+4 30	+0 23	+17 15	−2 57
10	+0 40	−1 30	+0 20	−0 5	+4 40	+0 48	+15 50	−3 45
11	+0 30	−1 17	+0 10	−0 25	+3 30	+0 1	+14 10	−4 1
12	−0 20	−1 40	+0 10	−0 30	+3 20	+0 24	+12 30	−3 35
13	−0 45	−1 34	+0 10	−0 29	+2 0	−0 16	+11 30	−1 53
14	−0 45	−1 2	+0 10	−0 24	+1 30	+0 1	+9 20	−0 51
15	−0 50	−0 35	−1 0	−1 23	+0 20	−0 20	+5 55	−0 44
16	−0 50	(−0 3)	−1 40	(−1 49)	0 0	(+0 11)	+2 30	(−0 22)
17	−0 40	+0 37	−1 0	−0 50	−1 0	+0 3	0 0	+1 1
18	−0 20	+1 24	−0 40	−0 8	−1 50	+0 3	−2 30	+2 21
19	−0 40	+1 26	−0 30	+0 26	−3 10	−0 30	−5 50	+2 41
20	−0 30	+1 53	0 0	+1 23	−3 50	−0 29	−9 10	+2 44
21	−1 0	+1 36	−1 30	+0 20	−4 10	−0 16	−11 52	+2 58
22	−1 20	+1 22	−2 30	−0 13	−4 10	+0 10	−14 35	+2 39
23	−2 0	+0 41	−1 30	+1 11	−4 50	−0 15	−16 47	+2 11
24	−2 30	(+0 5)	−2 10	(+0 53)	−4 30	(+0 11)	−19 0	(+0 57)
25	−3 0	−0 39	−3 0	+0 21	−4 25	+0 12	−20 22	−0 14
26	−3 20	−1 16	−1 30	+2 5	−4 10	+0 12	−21 45	−2 14
27	−3 30	−1 49	−3 20	+0 25	−4 0	−0 1	−22 10	−4 3
28	−3 0	−1 46	−5 30	−1 40	−3 0	+0 26	−20 0	−3 59
29	−2 0	−1 17	−5 0	−1 11	−2 40	+0 6	−16 30	−3 11
30	−1 30	−1 19	−5 50	−2 6	−1 40	+0 19	−13 0	−2 53
31	0 0	−0 21	−5 0	−1 27	−1 0	+0 10	−7 32	−0 57
Quadrantal coefficients	+1° 38′ −1 41 +1 48 −1 41		+1° 21′ −0 33 +0 26 −0 43		−0° 13′ +0 12 −0 15 +0 17		+3° 57′ −3 33 +3 19 −3 30	
Mean quadrantal coefficient	+1 42		+0 46		−0 14		+3 35	
True modulus	·049		·040		·077		·352	
True starboard angle	1^p·49		27^p·60		31^p·93		31^p·33	

No.	25.		26.		27.	
Ship's name	Trident.		Vulcan.			
Place of observation	Rio Janeiro.		Portsmouth.		Simon's Bay.	
Time of observation	1852, November.		1852, July.		1853, February.	
Index correction	+0° 8′		+0° 13′		−1° 8′	
Approximate modulus	·216		·155		·271	
Approx. starboard angle	31ᵖ·22		15ᵖ·58		15ᵖ·73	
	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.
Apparent azimuth of ship's head. p 0	− 1° 0′	(+0° 59′)	− 0° 45′	(−1° 16′)	+ 3° 20′	(+1° 23′)
1	+ 2 0	+1 35	− 0 55	+0 19	+ 3 0	+4 5
2	+ 5 0	+2 11	− 1 5	+1 51	+ 2 0	+6 3
3	+ 8 30	+3 24	− 1 22	+3 9	+ 0 35	+7 25
4	+11 0	+3 47	− 1 40	+4 19	− 3 5	+6 13
5	+13 0	+3 59	− 3 22	+3 49	− 8 0	+3 24
6	+13 40	+3 10	− 5 4	+3 5	−13 10	−0 9
7	+13 20	+1 47	− 6 46	+2 0	−18 35	−4 30
8	+12 0	(−0 12)	− 8 30	(+0 36)	−17 49	(−3 16)
9	+11 0	−1 19	− 9 47	−0 43	−16 40	−2 15
10	+ 9 0	−2 58	−11 5	−2 23	−15 15	−1 35
11	+ 8 0	−3 9	−10 32	−2 32	−15 10	−2 51
12	+ 6 30	−3 23	−10 0	−3 0	−14 20	−3 51
13	+ 4 0	−4 15	− 8 50	−3 4	−12 55	−4 42
14	+ 3 10	−3 8	− 7 40	−3 23	− 7 30	−1 56
15	+ 3 0	−1 6	− 5 0	−2 20	− 5 20	−2 37
16	+ 1 20	(−0 23)	− 2 20	(−1 23)	+ 0 55	(+0 36)
17	0 0	+0 41	+ 1 25	+0 37	+ 4 35	+1 14
18	− 0 24	+2 41	+ 5 10	+2 40	+ 8 10	+1 51
19	− 2 20	+3 2	+ 7 20	+3 15	+12 30	+3 24
20	− 4 0	+3 29	+ 9 30	+4 0	+14 35	+3 1
21	− 6 0	+3 17	+10 5	+3 20	+16 55	+3 15
22	− 7 40	+3 6	+10 40	+2 57	+20 30	+5 13
23	− 9 40	+2 9	+ 9 55	+1 35	+17 50	+1 29
24	−11 20	(+1 8)	+ 9 10	(+0 30)	+17 30	(+0 41)
25	−13 20	−0 45	+ 7 40	−0 58	+16 30	−0 11
26	−15 0	−2 46	+ 6 10	−2 6	+12 0	−3 56
27	−15 0	−3 35	+ 4 40	−2 54	+10 25	−4 10
28	−16 0	−5 51	+ 3 10	−3 24	+ 8 30	−4 15
29	−12 0	−3 29	+ 2 2	−3 18	+ 6 50	−3 39
30	− 9 0	−2 26	+ 0 55	−2 56	+ 4 0	−3 50
31	− 6 0	−1 38	+ 0 5	−2 9	+ 3 40	−1 19
Quadrantal coefficients	+3° 59′ −3 52 +3 41 −4 6		+3° 42′ −3 29 +3 41 −3 33		+4° 30′ −3 57 +3 53 −4 16	
Mean quadrantal coefficient	+3 54		+3 36		+4 9	
True modulus	·221		·158		·277	
True starboard angle	31ᵖ·29		15ᵖ·62		15ᵖ·76	

No.	28.		29.	
Ship's name	Simoom.			
Place of observation	Portsmouth.		Simon's Town.	
Time of observation	1852, September.		1853, October.	
Index correction	+0° 7′		+1° 47′	
Approximate modulus	·368		·235	
Approx. starboard angle	30p·20		31p·03	
	Observed deviation uncorrected.	Residual error.	Observed deviation uncorrected.	Residual error.
Apparent azimuth of ship's head. p 0	− 7° 40′	(−0° 14′)	− 4° 0′	(+0° 20′)
1	− 1 35	+1 50	+ 0 10	+1 53
2	+ 4 30	+3 47	+ 5 30	+4 36
3	+ 9 20	+4 31	+ 8 30	+5 4
4	+13 15	+4 31	+10 40	+4 53
5	+17 10	+4 47	+11 0	+3 10
6	+18 10	+2 38	+11 20	+1 49
7	+19 15	+1 6	+11 45	+0 58
8	+20 20	(+0 16)	+10 30	(−1 3)
9	+19 45	−1 26	+ 8 55	−2 53
10	+19 10	−1 40	+ 7 25	−4 6
11	+16 50	−4 0	+ 6 30	−4 13
12	+14 50	−4 34	+ 5 15	−4 11
13	+12 50	−4 22	+ 4 45	−2 47
14	+10 45	−3 36	+ 4 30	−1 10
15	+ 8 40	−2 17	+ 1 45	−1 33
16	+ 7 20	(+0 8)	+ 0 50	(+0 4)
17	+ 4 20	+1 9	− 1 0	+0 51
18	+ 1 20	+2 17	− 2 5	+2 23
19	− 1 40	+3 23	− 4 10	+2 50
20	− 4 40	+4 18	− 5 40	+3 42
21	− 8 10	+4 27	− 7 40	+3 44
22	−11 40	+4 6	− 9 20	+3 45
23	−16 10	+2 13	−11 20	+3 1
24	−20 20	(−0 2)	−15 20	(−0 13)
25	−22 17	−0 52	−16 30	−1 8
26	−24 15	−3 11	−17 0	−1 55
27	−25 10	−4 6	−19 0	−4 43
28	−24 50	−5 12	−18 20	−5 20
29	−22 20	−4 54	−16 20	−5 14
30	−18 30	−3 55	−11 15	−2 1
31	−12 30	−1 19	− 7 40	−0 48
Quadrantal coefficients	+4° 38′ −4 23 +4 23 −4 42		+4° 29′ −4 11 +4 3 −4 14	
Mean quadrantal coefficient	+4 31		+4 14	
True modulus	·375		·240	
True starboard angle	30p·37		31p·12	

I shall first proceed to consider the quadrantal deviations in these ships. It will be remembered that, according to theory, the coefficient of quadrantal deviation in each ship ought to be sensibly the same in all localities; but that the coefficient in one ship may differ in any degree from that in another ship.

Coefficients of Quadrantal Deviation.

1. Wood-built sailing-ships.

In the Erebus, at .	Gillingham (No. 1)	$+0^\circ\ 23'$
	Porto Praya (No. 2)	+0 28
	St. Helena (No. 3)	+0 34
	Cape of Good Hope (No. 4) . .	+0 35
	Kerguelen's Land (No. 5) . . .	+0 38
In the Pandora, at .	Plymouth (No. 14)	+0 38
	Auckland (No. 15)	+0 30
In the Mæander, at .	Sheerness (No. 16)	−0 12
	Simon's Bay (No. 17)	−0 11
In the Spy, at . . .	Sheerness (No. 22)	+0 46
	St. Paul's Loando (No. 23) . .	−0 14

The accordance of the numbers in the three first instances is very remarkable; and the more so because the compound polar magnetism has changed considerably. In the Spy there is a discordance, but not important in nautical experiments. It will be seen, on referring to the columns of Residual Error for the Spy, that there is general irregularity.

2. Wood-built steamers.

In the Virago, at . .	Plymouth (No. 18)	$+1^\circ\ 9'$
	Valparaiso (No. 19)	+1 44
In the Plumper, at .	Portsmouth (No. 20)	+1 30
	St. Catherine's (No. 21) . . .	+1 42

The agreement is sufficiently close.

3. Iron-built steamers.

In the Bloodhound, at	Plymouth (No. 6)	$+3^\circ\ 48'$
	Constantinople (No. 7) . . .	+3 27
	Piræus (No. 8)	+3 59
In the Jackal, at . .	Plymouth (No. 9)	+4 14
	Lisbon (No. 10)	+4 11
	Piræus (No. 11)	+3 32

In the Trident, at	Greenhithe (No. 12)	+4° 11′
	Malta (No. 13)	+3 44
	Greenhithe (No. 24)	+3 35
	Rio de Janeiro (No. 25)	+3 54
In the Vulcan, at	Portsmouth (No. 26)	+3 36
	Simon's Bay (No. 27)	+4 9
In the Simoom, at	Portsmouth (No. 28)	+4 31
	Simon's Town (No. 29)	+4 14

The general accordance here is extremely good. The petty discordances appear to be purely accidental. It is evident, at least, that they are not dependent on the geographical locality. Thus at (8) the Piræus produces the largest and at (11) the smallest in the group; at (12) Greenhithe produces the largest and at (24) the smallest; at (27) Simon's Bay produces the larger and at (29) the smaller. Nor have I been able to connect these differences with any other law. Regarding their accidental character as established, they give a measure of the range of accident in these observations, and they show that that in the Spy, though large, is not excessive.

I think it therefore certain that this part of the theory is entirely supported by the observations; and therefore that this quadrantal deviation may be perfectly neutralized in all localities by a mass of soft iron placed in the manner described at the beginning of this paper, leaving only a deviation which follows accurately in every place, separately considered, the laws of polar-magnet-deviation, and which therefore in every place, separately considered, may be neutralized by the application of permanent steel magnets.

I shall now proceed to consider the deductions from the two elements of polar-magnet-deviation, namely the starboard angle of the polar-magnet-force and the modulus. For this purpose, I premise the following elements of terrestrial magnetism. The forces are expressed in GAUSS's method, adopting as units the English foot and the English grain. For some of the elements I am indebted to the kindness of Colonel SABINE: others were obtained from other sources. None were furnished to me precisely in the form in which they are here exhibited, and some calculation therefore has been required to adapt them to my wants. It is possible that they may be affected by trifling inaccuracies.

	Horizontal Force.	Vertical Force.
1. Gillingham	3·78	+ 9·94
2. Porto Praya	6·26	+ 6·38
3. St. Helena	5·97	− 1·97
4. Cape of Good Hope	4·56	− 6·08
5. Kerguelen's Land	3·88	−10·68
6. Plymouth	3·82	+ 9·67

	Horizontal Force.	Vertical Force.
7. Constantinople	5·63	+ 7·34
8. Piræus	5·85	+ 6·85
9. Plymouth	3·82	+ 9·67
10. Lisbon	4·62	+ 9·19
11. Piræus	5·85	+ 6·85
12. Greenhithe	3·79	+ 9·66
13. Malta	5·74	+ 7·03
14. Plymouth	3·82	+ 9·67
15. Auckland	6·32	− 11·32
16. Sheerness	3·78	+ 9·67
17. Simon's Bay	4·46	− 6·19
18. Plymouth	3·82	+ 9·67
19. Valparaiso	7·17	− 5·23
20. Portsmouth	3·83	+ 9·63
21. St. Catherine's	6·48	− 2·54
22. Sheerness	3·78	+ 9·67
23. St. Paul's Loando	5·57	− 3·09
24. Greenhithe	3·79	+ 9·66
25. Rio de Janeiro	6·49	− 1·49
26. Portsmouth	3·83	+ 9·63
27. Simon's Bay	4·46	− 6·19
28. Portsmouth	3·83	+ 9·63
29. Simon's Town	4·46	− 6·19

From these, with the starboard angle and the modulus, the resolved polar-magnet-force in the two directions of "headward" and "starboard" are formed by the rules given above. It will be remembered that H is the ship's subpermanent magnetism headward, S the subpermanent magnetism to the starboard side, and N a constant of capacity for induced magnetism peculiar to each ship. As the total directive force at Greenhithe is 3·79, a change of disturbing force represented by $\frac{3·79}{57}$, or ·066 nearly, would produce at Greenhithe a change of disturbance whose maximum is 1° nearly; and so, *mutatis mutandis*, at other places.

1. *Wood-built Sailing Ships.*

The Erebus.

1.	$H+N\times 9·94=+0·261$	$S=+0·020$
2.	$H+N\times 6·38=+0·202$	$S=+0·037$
3.	$H-N\times 1·97=+0·043$	$S=+0·017$
4.	$H-N\times 6·08=-0·090$	$S=+0·018$
5.	$H-N\times 10·68=-0·262$	$S=+0·010$

H and S have small positive values, but so small that their combination would not produce, in the Thames, an error of half a degree. N has a well-marked positive value. In this ship the magnetism would be sensibly corrected, by placing (by trial) a mass of soft iron abaft the compass and at a lower level, in such a position as to correct the deviation with head east and head west, and then placing a mass of soft iron at the level of the compass, starboard or larboard, so as to correct the combination of the original quadrantal deviation and the new quadrantal deviation produced by the first corrector; and this would be sensibly effective, without change, in all localities. Or, permanent magnets might be applied to neutralize the errors at the cardinal points, and soft iron at the level of the compass, starboard or larboard, to correct the quadrantal deviation; the soft iron would then be effective in all latitudes, but one of the magnets would require alteration in different latitudes, and would require reversion in opposite hemispheres.

The Pandora.

14. $H+N\times 9{\cdot}67=+0{\cdot}166 \qquad S=+0{\cdot}009$
15. $H-N\times 11{\cdot}32=-0{\cdot}272 \qquad S=+0{\cdot}082$

S has changed sufficiently to produce at Auckland (No. 15) a disturbance whose maximum is 40′. If this be neglected, the compass may be sensibly corrected in the same manner as for the Erebus. H is small.

The Mæander.

16. $H+N\times 9{\cdot}67=+0{\cdot}070 \qquad S=+0{\cdot}034$
17. $H-N\times 6{\cdot}19=-0{\cdot}195 \qquad S=+0{\cdot}008$

H would appear to have a value of $-0{\cdot}091$, which at Sheerness would produce a deviation of 1° 20′. S is practically insensible. This ship would require for perfect correction a weak magnet with its marked end towards the stern, in addition to the soft iron as in the Erebus and Pandora. It will probably be better to use, for the polar-magnet-correction, a magnet alone, adjusting or reversing it as may be necessary.

The Spy.

22. $H+N\times 9{\cdot}67=+0{\cdot}097 \qquad S=-0{\cdot}113$
23. $H-N\times 3{\cdot}09=+0{\cdot}431 \qquad S=-0{\cdot}006$

If the headward subpermanent magnetism has not changed, H is positive and N is negative. These forces would require correctors in positions opposite to those of the Mæander. S seems to have changed, to an amount which would produce at Sheerness a deviation of 1° 30′. It will probably be best to correct the polar-magnet-deviation by adjustible magnets.

2. *Wood-built Steamers.*

The Virago.

18. $H+N\times 9{\cdot}67=+0{\cdot}493$ $S=+0{\cdot}055$
19. $H-N\times 5{\cdot}23=+0{\cdot}029$ $S=+0{\cdot}079$

S presents no sufficient evidence of change. H and N are both positive: and the correction of polar-magnet-deviation would be made, either by a magnet with marked end towards the head, and soft iron as is described for the Erebus, or by an adjustible magnet.

The Plumper.

20. $H+N\times 9{\cdot}63=+0{\cdot}402$ $S=+0{\cdot}069$
21. $H-N\times 2{\cdot}54=+0{\cdot}301$ $S=+0{\cdot}091$

The remarks on the Virago apply also to the Plumper.

3. *Iron-built Steamers.*

The Bloodhound.

6. $H+N\times 9{\cdot}67=+0{\cdot}997$ $S=-0{\cdot}153$
7. $H+N\times 7{\cdot}34=+1{\cdot}043$ $S=-0{\cdot}209$
8. $H+N\times 6{\cdot}85=+0{\cdot}931$ $S=-0{\cdot}220$

The evidence for a distinct value of N is not very clear. There seem to have been changes in the values of H and S (subpermanent magnetism) which on the whole would produce, at the stations Nos. 7 and 8, a maximum deviation of nearly 1°. The correction of polar-magnet-deviation should be made by adjustible magnets.

The Jackal.

9. $H+N\times 9{\cdot}67=+1{\cdot}157$ $S=-0{\cdot}123$
10. $H+N\times 9{\cdot}19=+1{\cdot}005$ $S=-0{\cdot}162$
11. $H+N\times 6{\cdot}85=+1{\cdot}084$ $S=-0{\cdot}082$

The remarks on the Bloodhound will nearly apply to the Jackal. The changes in H and S would produce at the stations Nos. 10 and 11 a maximum deviation of a little more than 1°.

The Trident.

12. $H+N\times 9{\cdot}66=+1{\cdot}419$ $S=+0{\cdot}080$
13. $H+N\times 7{\cdot}03=+1{\cdot}402$ $S=-0{\cdot}091$
24. $H+N\times 9{\cdot}66=+1{\cdot}323$ $S=-0{\cdot}175$
25. $H-N\times 1{\cdot}49=+1{\cdot}418$ $S=-0{\cdot}198$

There seems to be good reason for thinking that N is insensible, that H has not sensibly changed, but that S has changed gradually, in the course of several years,

by a quantity which at station No. 25 would produce a maximum effect of 2°. The change in the voyage between No. 24 and No. 25 is nearly insensible.

The Vulcan.

26. $H+N\times 9{\cdot}63=-0{\cdot}603$ $\quad S=+0{\cdot}046$

27. $H-N\times 6{\cdot}19=-1{\cdot}236$ $\quad S=+0{\cdot}059$

In this instance, as in some others, we feel greatly the want of observations after the ship's return, to inform us whether the ship's subpermanent magnetism has really undergone a change. Assuming that it has not (and it is certain that S has not sensibly changed), then H has a sensible negative value and N a sensible positive value: and the correction for all stations would be effected by magnets and masses of iron as has been described for the Mæander. But it would probably be better to rely solely on adjustible magnets for the correction of the polar-magnet-disturbance.

The Simoom.

28. $H+N\times 9{\cdot}63=+1{\cdot}364$ $\quad S=-0{\cdot}451$

29. $H-N\times 6{\cdot}19=+1{\cdot}055$ $\quad S=-0{\cdot}184$

The change in the value of S would produce at the station No. 29 a maximum error of about 3° 40′. If the whole change in the compound headward force depended upon H, that change would also produce an error of nearly the same magnitude; and the combination of the two would produce, as the total result of the change of subpermanent magnetism, an error of about 5° 30′. But it is probable that N has some positive value, and that the change of H is not so great.

I shall now state what appear to be the just practical inferences from the preceding investigations.

1. At any place, the deviation of the compass in any ship, whether wood-built or iron-built, may be accurately represented as the effect of the combination of two forces, of which one alone would produce a disturbance following the law of polar-magnet-deviation, and the other alone would produce a disturbance following the law of quadrantal deviation.

2. Consequently, at any place the deviation of the compass may be accurately corrected by well-known mechanical methods; namely by a magnet in the athwart-ship direction, fixed at a distance determined by trial, for correcting the deviation when the ship's head is N. or S.; by a magnet in the head-and-stern direction, also at a distance determined by trial, for correcting the deviation when the ship's head is E. or W.; and by a mass of unmagnetized iron, at the same level as the compass, in the athwart-ship line or in the head-and-stern line according to circumstances (usually in the former), also at a distance determined by trial, for correcting the deviation when the ship's head is N.E., S.E., S.W. or N.W.

3. For the same ship, the mass of unmagnetized iron, if adjusted at one port, will produce its due effect at all parts of the world, without ever requiring change or adjustment. The quadrantal deviation may thus be accurately corrected without difficulty, leaving only the polar-magnet-deviation uncorrected.

4. The elements of polar-magnet-deviation are liable to changes, but of very different amounts in different ships. In some (as the Trident), even in the voyage of an iron steamer from the Thames to Rio Janeiro, the ship's subpermanent magnetism is so little altered, that, if the compass were rigorously corrected in the Thames, it would (as to sense) be rigorously correct at Rio Janeiro; in others there is such a change, in going to the Cape of Good Hope, that the compass might be in error 5° or 6°. This is nearly the greatest error that appears in the observations discussed above.

5. It is therefore imperatively the duty of every captain of a ship, particularly of an iron-built ship, to examine the state of the compasses at every opportunity. For the correctness of the compasses may be vitiated, not only by the changes in the polar-magnetism of the ship, but also by changes in the intensity of the magnets used for the correction. But as the correction of the quadrantal deviation is not liable to any doubt whatever, it is sufficient, for ascertaining the existence and recording the amount of error of the polar-magnet-deviation, to observe the error when the ship's head is N. or S., and when it is E. or W.

6. From whatever cause the changes in the elements of polar-magnet-deviation arise (whether from a real change in the subpermanent magnetism of the ship, or from the variation of that part of induced magnetism which is similar to polar-magnetism but which changes in different magnetic latitudes), they may be precisely corrected by readjusting the position of the magnets, leaving the unmagnetized iron undisturbed. And the change (if there is any) in the intensity of the correcting magnets will also be corrected, as to its effect on the compass, by the same readjustment of position.

7. It is therefore highly desirable that the magnets should be mounted in such a manner that their distance from the compass can be delicately changed. And, as the easiest way of preserving a register of the ship's magnetism, it is desirable that there should be means of registering the positions of the magnets.

8. In a ship's first voyage, there are no means of correcting the errors of the compass at different parts of the earth, except by such adjustment of the distances of the magnets. But if, on the ship's return to England, her subpermanent magnetism is found to be unaltered (which affords presumption that it has been unaltered during the voyage), and if the elements of magnetism have been registered either by record of the positions of the correcting magnets or by such discussions as those which occupy this Memoir, then it will be possible to correct by magnets that part of the polar-magnet-deviation which is due to subpermanent magnetism only (and which, alone, would be sensible at the magnetic equator), and to correct the remain-

ing part by unmagnetized iron, as is described above for the Erebus; and then the correction would be complete in all parts of the earth.

9. But, practically, it will perhaps always be easier and safer to readjust the positions of the magnets (as in art. 6.) whenever the directions of one of the magnetic points N. and S., and one of the magnetic points E. and W., can be truly ascertained. This can always be done in harbour, in a very short time. Probably this can also be done at sea, in fine weather, by reference to a compass carried high up the ships' masts. It can also be done with the aid of astronomical observations and of a knowledge of the local "variation" or "declination." In all cases, the mere adjustment of the magnets is an extremely rapid process.

10. On reviewing the results of the preceding examinations, I think that I am justified in denouncing any system of navigating a ship by forming a table of compass-deviations at the starting port, and using that table until means of correction can be obtained from observations, as dangerous; and I think that it ought to be at once discontinued. It does not in the smallest degree provide against the effects of possible change in the ship's subpermanent magnetism during the interval in which no observations are obtained (which, with sometimes a minute change in the powers of the magnets, is the only risk to which the method of mechanical correction is liable); and, as it does not recognize the effect of the variation in the magnitude of terrestrial horizontal magnetism at different places (which alters the compass-deviation by changing the proportion of the ship's subpermanent magnetism to the terrestrial horizontal magnetism, upon which proportion the compass-deviation depends), it gratuitously introduces a class of errors which are entirely avoided by correcting the compass by magnets and soft iron. Thus, in the instance of the Trident (24) and (25), sailing from Greenhithe to Rio Janeiro: suppose that there had been no good opportunity of making observations of azimuth on the voyage; on the ship's arrival at Rio Janeiro, the table of deviations formed at Greenhithe would have been found erroneous by 6° or 7° in one direction with head eastward, and erroneous by 8° or 9° in the opposite direction with head westward. But if the compass had been corrected by magnets and soft iron at Greenhithe, it would have been correct at Rio Janeiro without an error approaching to a single degree. The change of compass-deviation, in fact, has been produced, not by the change of the ship's subpermanent magnetism (which has been sensibly constant), but by the change in the magnitude of the earth's directive magnetism, which change has altered the proportion of the ship's invariable magnetism to the earth's variable magnetism; and if this proportion had been reduced to zero by neutralization of the ship's magnetism by means of magnets, the variation of the proportion as depending on the variation of the earth's magnetism would also have been destroyed. What has been said in regard to the errors arising during the whole voyage, applies, in a proportionate degree, to the errors arising during a part of the voyage: if there had been valid observations

after making half the voyage, the errors perhaps would have been only half as great; but these errors would have been equally gratuitous.

In other instances, such as that of the Simoom, in which the change of subpermanent magnetism is real and unusually great, the tabular method (supposing, for illustration, that there had been no opportunity of sufficiently investigating the errors during the whole voyage) would have united the gratuitous errors with the errors produced by the real change, and would have produced at the Cape of Good Hope an error of 11°; whereas, if the correction by magnets had been used, the error would have been under 6°. At intermediate places, as the neighbourhood of St. Helena, where the earth's directive force differs still more from that in England, the gratuitous error would have been much greater, and the error really depending on change in the ship would probably have been less, as occurring in an earlier part of the voyage.

The mere comparison of magnitudes of errors, however, in this way, does not sufficiently exhibit the disadvantage of the method of "Tables of Deviations." It is an important defect that no good new table can be formed, without observations for the error on numerous points of azimuth; whereas the operations for readjustment of magnets require observations on only two points of azimuth. And, I apprehend, that the necessity of using a table at all (that is, of steering by one nominal course when another course is intended) is, especially in difficult channels, a very serious evil, from which the method of steering by a corrected compass is entirely free.

11. I have alluded above to the possible changes in the energy of the correcting magnets; but I am bound to state that these changes (when ordinary care is taken for the conservation of the magnets) are, to the best of my knowledge, extremely minute. It is known, as a matter of experience, that the diminution of the subpermanent magnetism of a new iron ship, though small, is usually greater than that of the magnets; inasmuch as it usually becomes necessary to increase the distance of the magnets from the compass.

I subjoin the Table of Polar Magnet Deviations, which has been used in the preceding investigations, and which may perhaps be useful, in future, for similar investigations.

TABLE

OF

POLAR-MAGNET-DEVIATIONS.

Table of Polar-Magnet-Deviations,

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·00	·01	·02	·03	·04	·05	·06	·07	·08	·09	·10
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0·0	16·0	16·0	32·0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
0·1	15·9	16·1	31·9	0 0	0 1	0 1	0 2	0 3	0 4	0 4	0 5	0 6	0 7	0 7
0·2	15·8	16·2	31·8	0 0	0 2	0 3	0 4	0 6	0 7	0 8	0 10	0 11	0 13	0 14
0·3	15·7	16·3	31·7	0 0	0 2	0 4	0 6	0 9	0 11	0 13	0 15	0 17	0 19	0 21
0·4	15·6	16·4	31·6	0 0	0 3	0 5	0 8	0 11	0 14	0 16	0 19	0 22	0 25	0 27
0·5	15·5	16·5	31·5	0 0	0 4	0 7	0 10	0 14	0 17	0 20	0 24	0 27	0 31	0 34
0·6	15·4	16·6	31·4	0 0	0 4	0 8	0 12	0 16	0 20	0 24	0 28	0 32	0 36	0 40
0·7	15·3	16·7	31·3	0 0	0 5	0 9	0 14	0 19	0 24	0 28	0 33	0 38	0 43	0 47
0·8	15·2	16·8	31·2	0 0	0 5	0 11	0 16	0 22	0 27	0 32	0 38	0 43	0 48	0 53
0·9	15·1	16·9	31·1	0 0	0 6	0 12	0 18	0 24	0 30	0 36	0 42	0 48	0 54	1 0
1·0	15·0	17·0	31·0	0 0	0 7	0 13	0 20	0 27	0 34	0 40	0 47	0 54	1 1	1 7
1·1	14·9	17·1	30·9	0 0	0 8	0 15	0 22	0 30	0 37	0 44	0 52	0 59	1 7	1 14
1 2	14·8	17·2	30·8	0 0	0 8	0 16	0 24	0 32	0 40	0 48	0 56	1 4	1 12	1 20
1·3	14·7	17·3	30·7	0 0	0 9	0 17	0 26	0 35	0 44	0 52	1 1	1 9	1 18	1 27
1·4	14·6	17·4	30·6	0 0	0 9	0 19	0 28	0 38	0 47	0 56	1 6	1 15	1 24	1 33
1·5	14·5	17·5	30·5	0 0	0 10	0 20	0 30	0 40	0 50	1 0	1 10	1 20	1 30	1 40
1·6	14·4	17·6	30·4	0 0	0 11	0 21	0 32	0 43	0 54	1 4	1 15	1 25	1 36	1 46
1·7	14·3	17·7	30·3	0 0	0 11	0 22	0 34	0 45	0 56	1 7	1 19	1 30	1 41	1 53
1·8	14·2	17·8	30·2	0 0	0 12	0 24	0 36	0 48	1 0	1 11	1 23	1 35	1 47	1 59
1·9	14·1	17·9	30·1	0 0	0 13	0 25	0 38	0 50	1 3	1 15	1 28	1 40	1 53	2 6
2·0	14·0	18·0	30·0	0 0	0 13	0 26	0 40	0 53	1 6	1 19	1 32	1 45	1 59	2 12
2·1	13·9	18·1	29·9	0 0	0 14	0 27	0 41	0 55	1 9	1 22	1 36	1 50	2 4	2 18
2·2	13·8	18·2	29·8	0 0	0 15	0 29	0 44	0 58	1 12	1 26	1 41	1 55	2 10	2 24
2·3	13·7	18·3	29·7	0 0	0 15	0 30	0 45	1 0	1 15	1 30	1 45	2 0	2 15	2 30
2·4	13·6	18·4	29·6	0 0	0 16	0 31	0 47	1 3	1 19	1 34	1 50	2 5	2 21	2 36
2·5	13·5	18·5	29·5	0 0	0 16	0 32	0 49	1 5	1 21	1 37	1 54	2 10	2 26	2 42
2·6	13·4	18·6	29·4	0 0	0 17	0 34	0 51	1 8	1 25	1 41	1 58	2 15	2 32	2 48
2·7	13·3	18·7	29·3	0 0	0 18	0 35	0 53	1 10	1 27	1 44	2 2	2 19	2 37	2 54
2·8	13·2	18·8	29·2	0 0	0 18	0 36	0 54	1 12	1 30	1 48	2 6	2 24	2 42	3 0
2·9	13·1	18·9	29·1	0 0	0 19	0 37	0 56	1 15	1 34	1 52	2 11	2 29	2 48	3 6
3·0	13·0	19·0	29·0	0 0	0 19	0 38	0 58	1 17	1 36	1 55	2 14	2 33	2 52	3 11
3·1	12·9	19·1	28·9	0 0	0 20	0 39	0 59	1 19	1 39	1 58	2 18	2 37	2 57	3 17
3·2	12·8	19·2	28·8	0 0	0 20	0 40	1 1	1 21	1 41	2 1	2 22	2 42	3 2	3 22
3·3	12·7	19·3	28·7	0 0	0 21	0 41	1 2	1 23	1 44	2 4	2 25	2 46	3 7	3 28
3·4	12·6	19·4	28·6	0 0	0 21	0 42	1 4	1 25	1 46	2 7	2 29	2 50	3 12	3 33
3·5	12·5	19·5	28·5	0 0	0 22	0 44	1 6	1 28	1 50	2 11	2 33	2 55	3 17	3 39
3·6	12·4	19·6	28·4	0 0	0 23	0 45	1 8	1 30	1 52	2 14	2 37	2 59	3 22	3 44
3·7	12·3	19·7	28·3	0 0	0 23	0 46	1 9	1 32	1 55	2 17	2 40	3 3	3 26	3 49
3·8	12·2	19·8	28·2	0 0	0 24	0 47	1 11	1 34	1 57	2 20	2 44	3 7	3 31	3 54
3·9	12·1	19·9	28·1	0 0	0 24	0 48	1 12	1 36	2 0	2 23	2 47	3 11	3 35	3 59
4·0	12·0	20·0	28·0	0 0	0 24	0 49	1 13	1 38	2 2	2 26	2 51	3 15	3 39	4 4
+	+	−	−											

for Azimuths referred to the Disturbed Compass.

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·00	·01	·02	·03	·04	·05	·06	·07	·08	·09	·10
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+	+	−	−											
p	p	p	p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
4·0	12·0	20·0	28·0	0 0	0 24	0 49	1 13	1 38	2 2	2 26	2 51	3 15	3 39	4 4
4·1	11·9	20·1	27·9	0 0	0 25	0 50	1 15	1 40	2 4	2 29	2 54	3 19	3 44	4 9
4·2	11·8	20·2	27·8	0 0	0 25	0 51	1 16	1 41	2 6	2 32	2 57	3 22	3 48	4 13
4·3	11·7	20·3	27·7	0 0	0 26	0 52	1 17	1 43	2 9	2 35	3 0	3 26	3 52	4 18
4·4	11·6	20·4	27·6	0 0	0 26	0 52	1 18	1 44	2 11	2 37	3 3	3 29	3 55	4 22
4·5	11·5	20·5	27·5	0 0	0 26	0 53	1 19	1 46	2 13	2 40	3 6	3 33	3 59	4 26
4·6	11·4	20·6	27·4	0 0	0 27	0 54	1 21	1 48	2 15	2 42	3 9	3 36	4 3	4 30
4·7	11·3	20·7	27·3	0 0	0 27	0 55	1 22	1 50	2 17	2 45	3 12	3 40	4 7	4 35
4·8	11·2	20·8	27·2	0 0	0 28	0 56	1 23	1 51	2 19	2 47	3 15	3 43	4 11	4 39
4·9	11·1	20·9	27·1	0 0	0 28	0 57	1 24	1 53	2 21	2 50	3 18	3 46	4 14	4 43
5·0	11·0	21·0	27·0	0 0	0 28	0 57	1 25	1 54	2 23	2 52	3 21	3 49	4 18	4 47
5·1	10·9	21·1	26·9	0 0	0 29	0 58	1 27	1 56	2 25	2 55	3 24	3 53	4 22	4 51
5·2	10·8	21·2	26·8	0 0	0 29	0 59	1 28	1 58	2 26	2 57	3 26	3 56	4 25	4 54
5·3	10·7	21·3	26·7	0 0	0 29	0 59	1 28	1 58	2 28	2 58	3 28	3 58	4 27	4 57
5·4	10·6	21·4	26·6	0 0	0 30	1 0	1 30	2 0	2 30	3 0	3 30	4 0	4 30	5 0
5·5	10·5	21·5	26·5	0 0	0 30	1 1	1 31	2 1	2 31	3 2	3 32	4 3	4 33	5 4
5·6	10·4	21·6	26·4	0 0	0 30	1 1	1 31	2 2	2 33	3 4	3 34	4 5	4 36	5 7
5·7	10·3	21·7	26·3	0 0	0 31	1 2	1 32	2 4	2 35	3 6	3 37	4 8	4 39	5 10
5·8	10·2	21·8	26·2	0 0	0 31	1 3	1 34	2 5	2 37	3 8	3 39	4 10	4 41	5 13
5·9	10·1	21·9	26·1	0 0	0 31	1 3	1 34	2 6	2 38	3 10	3 41	4 13	4 44	5 16
6·0	10·0	22·0	26·0	0 0	0 32	1 4	1 35	2 7	2 39	3 11	3 43	4 15	4 46	5 18
6·1	9·9	22·1	25·9	0 0	0 32	1 4	1 36	2 8	2 40	3 13	3 45	4 17	4 49	5 21
6·2	9·8	22·2	25·8	0 0	0 32	1 5	1 37	2 9	2 41	3 14	3 46	4 18	4 50	5 23
6·3	9·7	22·3	25·7	0 0	0 32	1 5	1 37	2 10	2 42	3 15	3 47	4 20	4 52	5 25
6·4	9·6	22·4	25·6	0 0	0 32	1 5	1 38	2 11	2 44	3 16	3 48	4 21	4 54	5 27
6·5	9·5	22·5	25·5	0 0	0 33	1 6	1 39	2 12	2 45	3 18	3 50	4 23	4 56	5 29
6·6	9·4	22·6	25·4	0 0	0 33	1 6	1 39	2 12	2 45	3 19	3 52	4 25	4 58	5 31
6·7	9·3	22·7	25·3	0 0	0 33	1 7	1 40	2 13	2 46	3 20	3 53	4 26	4 59	5 33
6·8	9·2	22·8	25·2	0 0	0 33	1 7	1 40	2 14	2 47	3 21	3 54	4 28	5 1	5 35
6·9	9·1	22·9	25·1	0 0	0 33	1 7	1 40	2 14	2 48	3 22	3 55	4 29	5 3	5 37
7·0	9·0	23·0	25·0	0 0	0 34	1 8	1 41	2 15	2 49	3 23	3 56	4 30	5 4	5 38
7·1	8·9	23·1	24·9	0 0	0 34	1 8	1 41	2 15	2 49	3 23	3 57	4 31	5 5	5 39
7·2	8·8	23·2	24·8	0 0	0 34	1 8	1 42	2 16	2 50	3 24	3 58	4 32	5 6	5 40
7·3	8·7	23·3	24·7	0 0	0 34	1 8	1 42	2 16	2 50	3 25	3 59	4 33	5 7	5 41
7·4	8·6	23·4	24·6	0 0	0 34	1 8	1 42	2 16	2 50	3 25	3 59	4 33	5 7	5 42
7·5	8·5	23·5	24·5	0 0	0 34	1 8	1 42	2 17	2 51	3 25	3 59	4 33	5 7	5 42
7·6	8·4	23·6	24·4	0 0	0 34	1 9	1 43	2 17	2 51	3 26	4 0	4 34	5 8	5 43
7·7	8·3	23·7	24·3	0 0	0 34	1 9	1 43	2 17	2 51	3 26	4 0	4 34	5 8	5 43
7·8	8·2	23·8	24·2	0 0	0 34	1 9	1 43	2 18	2 52	3 26	4 0	4 35	5 9	5 44
7·9	8·1	23·9	24·1	0 0	0 34	1 9	1 43	2 18	2 52	3 26	4 1	4 35	5 9	5 44
8·0	8·0	24·0	24·0	0 0	0 34	1 9	1 43	2 18	2 52	3 26	4 1	4 35	5 10	5 44
+	+	−	−											
Mean				0 0	0 22	0 44	1 6	1 28	1 49	2 11	2 33	2 55	3 17	3 39

Table of Polar-Magnet-Deviations,

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·10	·11	·12	·13	·14	·15	·16	·17	·18	·19	·20
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0·0	16·0	16·0	32·0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
0·1	15·9	16·1	31·9	0 7	0 8	0 9	0 9	0 10	0 10	0 11	0 12	0 12	0 13	0 14
0·2	15·8	16·2	31·8	0 14	0 16	0 17	0 18	0 19	0 20	0 22	0 23	0 24	0 26	0 27
0·3	15·7	16·3	31·7	0 21	0 23	0 25	0 27	0 29	0 31	0 33	0 35	0 36	0 39	0 41
0·4	15·6	16·4	31·6	0 27	0 30	0 33	0 36	0 39	0 41	0 44	0 47	0 49	0 52	0 54
0·5	15·5	16·5	31·5	0 34	0 38	0 41	0 45	0 48	0 51	0 54	0 58	1 1	1 5	1 8
0·6	15·4	16·6	31·4	0 40	0 45	0 49	0 53	0 57	1 1	1 5	1 9	1 13	1 17	1 21
0·7	15·3	16·7	31·3	0 47	0 52	0 56	1 1	1 6	1 11	1 16	1 21	1 25	1 30	1 35
0·8	15·2	16·8	31·2	0 53	0 59	1 4	1 10	1 16	1 21	1 27	1 32	1 27	1 43	1 48
0·9	15·1	16·9	31·1	1 0	1 6	1 12	1 19	1 25	1 31	1 37	1 43	1 49	1 55	2 1
1·0	15·0	17·0	31·0	1 7	1 14	1 20	1 27	1 34	1 41	1 48	1 55	2 1	2 8	2 14
1·1	14·9	17·1	30·9	1 14	1 21	1 28	1 36	1 43	1 51	1 59	2 6	2 13	2 21	2 28
1·2	14·8	17·2	30·8	1 20	1 28	1 36	1 45	1 53	2 1	2 10	2 18	2 26	2 34	2 41
1·3	14·7	17·3	30·7	1 27	1 36	1 44	1 53	2 2	2 11	2 20	2 29	2 38	2 46	2 54
1·4	14·6	17·4	30·6	1 33	1 43	1 52	2 2	2 11	2 20	2 30	2 40	2 49	2 58	3 7
1·5	14·5	17·5	30·5	1 40	1 50	2 0	2 10	2 20	2 30	2 41	2 51	3 1	3 11	3 20
1·6	14·4	17·6	30·4	1 46	1 57	2 8	2 19	2 29	2 40	2 51	3 2	3 12	3 23	3 33
1·7	14·3	17·7	30·3	1 53	2 5	2 16	2 28	2 39	2 50	3 1	3 12	3 23	3 35	3 46
1·8	14·2	17·8	30·2	1 59	2 11	2 23	2 36	2 48	2 59	3 11	3 23	3 34	3 46	3 58
1·9	14·1	17·9	30·1	2 6	2 19	2 31	2 44	2 56	3 9	3 21	3 33	3 45	3 58	4 10
2·0	14·0	18·0	30·0	2 12	2 25	2 38	2 51	3 4	3 18	3 31	3 44	3 57	4 10	4 23
2·1	13·9	18·1	29·9	2 18	2 32	2 46	3 0	3 13	3 27	3 41	3 55	4 8	4 22	4 35
2·2	13·8	18·2	29·8	2 24	2 39	2 53	3 8	3 22	3 36	3 50	4 5	4 19	4 34	4 48
2·3	13·7	18·3	29·7	2 30	2 45	3 0	3 15	3 30	3 45	4 0	4 15	4 30	4 45	5 0
2·4	13·6	18·4	29·6	2 36	2 52	3 7	3 23	3 38	3 54	4 10	4 26	4 41	4 57	5 12
2·5	13·5	18·5	29·5	2 42	2 59	3 15	3 31	3 47	4 4	4 20	4 36	4 52	5 8	5 24
2·6	13·4	18·6	29·4	2 48	3 5	3 22	3 39	3 55	4 12	4 29	4 46	5 3	5 20	5 36
2·7	13·3	18·7	29·3	2 54	3 12	3 29	3 47	4 4	4 22	4 39	4 56	5 13	5 31	5 48
2·8	13·2	18·8	29·2	3 0	3 18	3 36	3 54	4 12	4 30	4 48	5 6	5 24	5 42	6 0
2·9	13·1	18·9	29·1	3 6	3 24	3 42	4 1	4 19	4 38	4 57	5 16	5 34	5 53	6 12
3·0	13·0	19·0	29·0	3 11	3 30	3 49	4 9	4 27	4 47	5 6	5 25	5 44	6 4	6 23
3·1	12·9	19·1	28·9	3 17	3 37	3 56	4 16	4 35	4 55	5 15	5 35	5 54	6 14	6 34
3·2	12·8	19·2	28·8	3 22	3 44	4 3	4 23	4 43	5 4	5 24	5 44	6 4	6 25	6 45
3·3	12·7	19·3	28·7	3 28	3 50	4 9	4 30	4 51	5 12	5 33	5 54	6 14	6 35	6 56
3·4	12·6	19·4	28·6	3 33	3 55	4 16	4 38	4 59	5 21	5 42	6 3	6 24	6 46	7 7
3·5	12·5	19·5	28·5	3 39	4 1	4 23	4 45	5 7	5 29	5 51	6 13	6 34	6 56	7 18
3·6	12·4	19·6	28·4	3 44	4 7	4 29	4 52	5 14	5 37	5 59	6 21	6 43	7 6	7 28
3·7	12·3	19·7	28·3	3 49	4 12	4 35	4 58	5 21	5 44	6 7	6 30	6 52	7 15	7 38
3·8	12·2	19·8	28·2	3 54	4 18	4 41	5 5	5 28	5 52	6 15	6 38	7 1	7 25	7 48
3·9	12·1	19·9	28·1	3 59	4 23	4 47	5 11	5 35	5 59	6 23	6 47	7 10	7 34	7 58
4·0	12·0	20·0	28·0	4 4	4 28	4 53	5 17	5 42	6 6	6 30	6 55	7 19	7 44	8 8
+	+	−	−											

for Azimuths referred to the Disturbed Compass.

Apparent Azimuth from Neutral Position in points and decimals.				Modulus. ·10	·11	·12	·13	·14	·15	·16	·17	·18	·19	·20
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
4·0	12·0	20·0	28·0	4 4	4 28	4 53	5 17	5 42	6 6	6 30	6 55	7 19	7 44	8 8
4·1	11·9	20·1	27·9	4 9	4 33	4 58	5 23	5 48	6 13	6 38	7 3	7 28	7 53	8 18
4·2	11·8	20·2	27·8	4 13	4 38	5 4	5 29	5 55	6 20	6 46	7 11	7 36	8 1	8 27
4·3	11·7	20·3	27·7	4 18	4 43	5 9	5 35	6 1	6 27	6 53	7 18	7 44	8 10	8 36
4·4	11·6	20·4	27·6	4 22	4 48	5 15	5 41	6 7	6 33	7 0	7 26	7 52	8 18	8 45
4·5	11·5	20·5	27·5	4 26	4 53	5 20	5 46	6 13	6 40	7 7	7 33	8 0	8 26	8 54
4·6	11·4	20·6	27·4	4 30	4 57	5 24	5 51	6 18	6 46	7 13	7 40	8 7	8 34	9 2
4·7	11·3	20·7	27·3	4 35	5 2	5 29	5 57	6 24	6 52	7 20	7 47	8 15	8 42	9 10
4·8	11·2	20·8	27·2	4 39	5 7	5 35	6 3	6 31	6 59	7 27	7 54	8 22	8 50	9 18
4·9	11·1	20·9	27·1	4 43	5 11	5 40	6 8	6 36	7 4	7 33	8 1	8 29	8 57	9 26
5·0	11·0	21·0	27·0	4 47	5 15	5 44	6 13	6 41	7 10	7 39	8 8	8 36	9 5	9 34
5·1	10·9	21·1	26·9	4 51	5 20	5 49	6 18	6 47	7 16	7 46	8 15	8 44	9 13	9 42
5·2	10·8	21·2	26·8	4 54	5 23	5 53	6 22	6 52	7 21	7 51	8 20	8 50	9 19	9 49
5·3	10·7	21·3	26·7	4 57	5 27	5 57	6 27	6 57	7 27	7 57	8 26	8 56	9 26	9 56
5·4	10·6	21·4	26·6	5 0	5 30	6 1	6 31	7 1	7 31	8 2	8 32	9 2	9 32	10 3
5·5	10·5	21·5	26·5	5 4	5 34	6 5	6 35	7 6	7 37	8 8	8 38	9 9	9 39	10 10
5·6	10·4	21·6	26·4	5 7	5 38	6 9	6 40	7 11	7 42	8 13	8 43	9 14	9 45	10 16
5·7	10·3	21·7	26·3	5 10	5 41	6 13	6 44	7 15	7 46	8 18	8 49	9 20	9 51	10 22
5·8	10·2	21·8	26·2	5 13	5 44	6 16	6 47	7 19	7 50	8 22	8 53	9 25	9 56	10 28
5·9	10·1	21·9	26·1	5 16	5 47	6 19	6 51	7 23	7 55	8 27	8 58	9 30	10 2	10 34
6·0	10·0	22·0	26·0	5 18	5 50	6 22	6 54	7 26	7 58	8 31	9 3	9 35	10 7	10 39
6·1	9·9	22·1	25·9	5 21	5 53	6 25	6 57	7 30	8 2	8 35	9 7	9 39	10 11	10 44
6·2	9·8	22·2	25·8	5 23	5 55	6 28	7 0	7 33	8 6	8 39	9 11	9 44	10 16	10 49
6·3	9·7	22·3	25·7	5 25	5 58	6 31	7 4	7 37	8 10	8 43	9 15	9 48	10 21	10 54
6·4	9·6	22·4	25·6	5 27	6 0	6 33	7 6	7 39	8 12	8 46	9 19	9 52	10 25	10 58
6·5	9·5	22·5	25·5	5 29	6 2	6 36	7 9	7 42	8 15	8 49	9 22	9 56	10 29	11 2
6·6	9·4	22·6	25·4	5 31	6 4	6 38	7 11	7 45	8 18	8 52	9 25	9 59	10 32	11 6
6·7	9·3	22·7	25·3	5 33	6 7	6 41	7 14	7 48	8 21	8 55	9 29	10 3	10 36	11 10
6·8	9·2	22·8	25·2	5 35	6 9	6 43	7 17	7 51	8 24	8 58	9 32	10 6	10 39	11 13
6·9	9·1	22·9	25·1	5 37	6 11	6 45	7 19	7 53	8 27	9 1	9 35	10 8	10 42	11 16
7·0	9·0	23·0	25·0	5 38	6 12	6 46	7 20	7 54	8 28	9 3	9 37	10 11	10 45	11 19
7·1	8·9	23·1	24·9	5 39	6 13	6 48	7 22	7 56	8 30	9 5	9 39	10 14	10 48	11 22
7·2	8·8	23·2	24·8	5 40	6 14	6 49	7 23	7 58	8 32	9 7	9 41	10 16	10 50	11 24
7·3	8·7	23·3	24·7	5 41	6 15	6 50	7 24	7 59	8 33	9 8	9 42	10 17	10 51	11 26
7·4	8·6	23·4	24·6	5 42	6 16	6 51	7 25	8 0	8 34	9 9	9 43	10 18	10 52	11 27
7·5	8·5	23·5	24·5	5 42	6 17	6 52	7 26	8 1	8 35	9 10	9 45	10 20	10 54	11 29
7·6	8·4	23·6	24·4	5 43	6 17	6 52	7 26	8 1	8 36	9 11	9 46	10 21	10 55	11 30
7·7	8·3	23·7	24·3	5 43	6 17	6 52	7 26	8 1	8 36	9 11	9 46	10 21	10 56	11 31
7·8	8·2	23·8	24·2	5 44	6 18	6 53	7 27	8 2	8 37	9 12	9 47	10 22	10 57	11 32
7·9	8·1	23·9	24·1	5 44	6 18	6 53	7 27	8 2	8 37	9 12	9 47	10 22	10 57	11 32
8·0	8·0	24·0	24·0	5 44	6 19	6 53	7 28	8 2	8 37	9 12	9 47	10 22	10 57	11 32
+	+	−	−											
Mean				3 39	4 1	4 23	4 45	5 7	5 29	5 52	6 14	6 36	6 58	7 20

Table of Polar-Magnet-Deviations,

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·20	·21	·22	·23	·24	·25	·26	·27	·28	·29	·30
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0·0	16·0	16·0	32·0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
0·1	15·9	16·1	31·9	0 14	0 15	0 15	0 16	0 16	0 17	0 18	0 19	0 19	0 20	0 20
0·2	15·8	16·2	31·8	0 27	0 29	0 30	0 31	0 32	0 34	0 35	0 36	0 37	0 39	0 40
0·3	15·7	16·3	31·7	0 41	0 43	0 45	0 47	0 49	0 51	0 53	0 55	0 57	0 59	1 1
0·4	15·6	16·4	31·6	0 54	0 57	1 0	1 3	1 5	1 8	1 11	1 14	1 16	1 19	1 21
0·5	15·5	16·5	31·5	1 8	1 11	1 14	1 18	1 21	1 25	1 28	1 31	1 34	1 38	1 41
0·6	15·4	16·6	31·4	1 21	1 25	1 29	1 33	1 37	1 41	1 45	1 49	1 53	1 57	2 1
0·7	15·3	16·7	31·3	1 35	1 40	1 44	1 49	1 53	1 58	2 3	2 8	2 12	2 17	2 21
0·8	15·2	16·8	31·2	1 48	1 54	1 59	2 4	2 9	2 15	2 20	2 25	2 30	2 36	2 41
0·9	15·1	16·9	31·1	2 1	2 7	2 13	2 19	2 25	2 31	2 37	2 43	2 49	2 55	3 1
1·0	15·0	17·0	31·0	2 14	2 21	2 27	2 34	2 41	2 48	2 54	3 1	3 7	3 14	3 21
1·1	14·9	17·1	30·9	2 28	2 35	2 42	2 50	2 57	3 5	3 12	3 19	3 26	3 34	3 41
1·2	14·8	17·2	30·8	2 41	2 49	2 57	3 5	3 13	3 21	3 29	3 37	3 45	3 53	4 1
1·3	14·7	17·3	30·7	2 54	3 3	3 11	3 20	3 28	3 37	3 46	3 55	4 3	4 12	4 21
1·4	14·6	17·4	30·6	3 7	3 17	3 26	3 35	3 44	3 54	4 3	4 12	4 21	4 31	4 40
1·5	14·5	17·5	30·5	3 20	3 30	3 40	3 50	4 0	4 10	4 20	4 30	4 40	4 50	5 0
1·6	14·4	17·6	30·4	3 33	3 44	3 54	4 5	4 15	4 26	4 37	4 48	4 58	5 9	5 19
1·7	14·3	17·7	30·3	3 46	3 57	4 8	4 20	4 31	4 43	4 54	5 5	5 16	5 28	5 39
1·8	14·2	17·8	30·2	3 58	4 10	4 22	4 34	4 46	4 58	5 10	5 22	5 34	5 46	5 58
1·9	14·1	17·9	30·1	4 10	4 23	4 36	4 49	5 1	5 14	5 27	5 40	5 52	6 5	6 17
2·0	14·0	18·0	30·0	4 23	4 37	4 50	5 3	5 16	5 30	5 43	5 56	6 9	6 23	6 36
2·1	13·9	18·1	29·9	4 35	4 49	5 3	5 17	5 31	5 45	5 59	6 13	6 27	6 41	6 55
2·2	13·8	18·2	29·8	4 48	5 3	5 17	5 32	5 46	6 1	6 15	6 30	6 44	6 59	7 13
2·3	13·7	18·3	29·7	5 0	5 15	5 30	5 45	6 0	6 16	6 31	6 46	7 1	7 16	7 31
2·4	13·6	18·4	29·6	5 12	5 28	5 44	6 0	6 15	6 31	6 47	7 3	7 18	7 34	7 50
2·5	13·5	18·5	29·5	5 24	5 41	5 57	6 14	6 30	6 47	7 3	7 18	7 35	7 52	8 8
2·6	13·4	18·6	29·4	5 36	5 53	6 10	6 27	6 44	7 1	7 18	7 35	7 52	8 9	8 26
2·7	13·3	18·7	29·3	5 48	6 6	6 23	6 41	6 58	7 16	7 34	7 52	8 9	8 27	8 44
2·8	13·2	18·8	29·2	6 0	6 18	6 36	6 54	7 12	7 31	7 49	8 7	8 25	8 43	9 1
2·9	13·1	18·9	29·1	6 12	6 31	6 49	7 8	7 26	7 45	8 4	8 23	8 41	9 0	9 19
3·0	13·0	19·0	29·0	6 23	6 43	7 2	7 21	7 40	8 0	8 19	8 38	8 57	9 17	9 36
3·1	12·9	19·1	28·9	6 34	6 54	7 14	7 34	7 54	8 14	8 34	8 54	9 13	9 33	9 53
3·2	12·8	19·2	28·8	6 45	7 6	7 26	7 47	8 7	8 28	8 48	9 9	9 29	9 50	10 10
3·3	12·7	19·3	28·7	6 56	7 17	7 38	7 59	8 20	8 41	9 2	9 23	9 44	10 5	10 26
3·4	12·6	19·4	28·6	7 7	7 28	7 50	8 12	8 33	8 55	9 16	9 38	9 59	10 21	10 42
3·5	12·5	19·5	28·5	7 18	7 40	8 2	8 24	8 46	9 8	9 30	9 52	10 14	10 36	10 58
3·6	12·4	19·6	28·4	7 28	7 51	8 13	8 36	8 58	9 21	9 44	10 7	10 29	10 52	11 14
3·7	12·3	19·7	28·3	7 38	8 1	8 24	8 47	9 10	9 34	9 57	10 20	10 43	11 7	11 30
3·8	12·2	19·8	28·2	7 48	8 12	8 35	8 59	9 22	9 46	10 10	10 34	10 57	11 21	11 45
3·9	12·1	19·9	28·1	7 58	8 22	8 46	9 10	9 34	9 59	10 23	10 47	11 11	11 36	12 0
4·0	12·0	20·0	28·0	8 8	8 33	8 57	9 22	9 46	10 11	10 36	11 1	11 25	11 50	12 15
+	+	−	−											

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·20	·21	·22	·23	·24	·25	·26	·27	·28	·29	·30
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
4·0	12·0	20·0	28·0	8 8	8 33	8 57	9 22	9 46	10 11	10 36	11 1	11 25	11 50	12 15
4·1	11·9	20·1	27·9	8 18	8 43	9 8	9 33	9 58	10 23	10 48	11 13	11 38	12 4	12 30
4·2	11·8	20·2	27·8	8 27	8 52	9 18	9 43	10 9	10 34	11 0	11 26	11 52	12 18	12 44
4·3	11·7	20·3	27·7	8 36	9 2	9 28	9 54	10 20	10 46	11 12	11 38	12 5	12 31	12 58
4·4	11·6	20·4	27·6	8 45	9 11	9 38	10 4	10 31	10 57	11 24	11 50	12 17	12 44	13 11
4·5	11·5	20·5	27·5	8 54	9 20	9 47	10 14	10 41	11 8	11 36	12 3	12 30	12 57	13 25
4·6	11·4	20·6	27·4	9 2	9 29	9 57	10 24	10 52	11 19	11 47	12 14	12 42	13 10	13 38
4·7	11·3	20·7	27·3	9 10	9 38	10 6	10 34	11 2	11 30	11 58	12 26	12 54	13 22	13 51
4·8	11·2	20·8	27·2	9 18	9 46	10 15	10 43	11 12	11 40	12 9	12 37	13 6	13 34	14 3
4·9	11·1	20·9	27·1	9 26	9 55	10 24	10 53	11 22	11 50	12 19	12 48	13 17	13 46	14 15
5·0	11·0	21·0	27·0	9 34	10 3	10 32	11 1	11 31	12 0	12 29	12 58	13 28	13 57	14 27
5·1	10·9	21·1	26·9	9 42	10 11	10 41	11 10	11 40	12 9	12 39	13 8	13 38	14 8	14 38
5·2	10·8	21·2	26·8	9 49	10 18	10 48	11 18	11 48	12 18	12 48	13 18	13 48	14 18	14 49
5·3	10·7	21·3	26·7	9 56	10 26	10 56	11 26	11 56	12 27	12 57	13 27	13 58	14 29	15 0
5·4	10·6	21·4	26·6	10 3	10 33	11 3	11 33	12 4	12 35	13 6	13 37	14 8	14 39	15 10
5·5	10·5	21·5	26·5	10 10	10 40	11 11	11 42	12 13	12 44	13 15	13 46	14 18	14 49	15 20
5·6	10·4	21·6	26·4	10 16	10 47	11 18	11 49	12 21	12 52	13 24	13 56	14 27	14 58	15 30
5·7	10·3	21·7	26·3	10 22	10 53	11 25	11 57	12 28	13 0	13 32	14 4	14 36	15 8	15 40
5·8	10·2	21·8	26·2	10 28	11 0	11 32	12 4	12 36	13 8	13 40	14 12	14 44	15 16	15 49
5·9	10·1	21·9	26·1	10 34	11 6	11 38	12 10	12 43	13 15	13 47	14 19	14 52	15 24	15 57
6·0	10·0	22·0	26·0	10 39	11 11	11 44	12 16	12 49	13 21	13 54	14 26	14 59	15 32	16 5
6·1	9·9	22·1	25·9	10 44	11 16	11 49	12 22	12 55	13 27	14 1	14 34	15 7	15 40	16 13
6·2	9·8	22·2	25·8	10 49	11 21	11 54	12 27	13 0	13 33	14 7	14 40	15 14	15 47	16 21
6·3	9·7	22·3	25·7	10 54	11 27	12 0	12 33	13 6	13 39	14 13	14 47	15 21	15 54	16 28
6·4	9·6	22·4	25·6	10 58	11 31	12 5	12 38	13 12	13 45	14 19	14 53	15 27	16 1	16 35
6·5	9·5	22·5	25·5	11 2	11 35	12 9	12 43	13 17	13 50	14 24	14 58	15 33	16 7	16 41
6·6	9·4	22·6	25·4	11 6	11 39	12 13	12 47	13 21	13 55	14 29	15 3	15 38	16 12	16 47
6·7	9·3	22·7	25·3	11 10	11 44	12 18	12 52	13 26	14 0	14 34	15 8	15 43	16 18	16 53
6·8	9·2	22·8	25·2	11 13	11 47	12 21	12 55	13 30	14 4	14 39	15 13	15 48	16 23	16 58
6·9	9·1	22·9	25·1	11 16	11 50	12 25	12 59	13 34	14 8	14 43	15 18	15 53	16 28	17 3
7·0	9·0	23·0	25·0	11 19	11 53	12 28	13 2	13 37	14 12	14 47	15 22	15 57	16 32	17 7
7·1	8·9	23·1	24·9	11 22	11 56	12 31	13 5	13 40	14 15	14 50	15 25	16 0	16 35	17 11
7·2	8·8	23·2	24·8	11 24	11 58	12 33	13 8	13 43	14 18	14 53	15 28	16 3	16 38	17 14
7·3	8·7	23·3	24·7	11 26	12 1	12 36	13 11	13 46	14 21	14 56	15 31	16 6	16 41	17 17
7·4	8·6	23·4	24·6	11 27	12 2	12 38	13 13	13 48	14 23	14 58	15 33	16 9	16 44	17 20
7·5	8·5	23·5	24·5	11 29	12 4	12 39	13 14	13 50	14 25	15 0	15 35	16 11	16 46	17 22
7·6	8·4	23·6	24·4	11 30	12 5	12 40	13 15	13 51	14 26	15 2	15 37	16 13	16 48	17 24
7·7	8·3	23·7	24·3	11 31	12 6	12 41	13 16	13 52	14 27	15 3	15 38	16 14	16 50	17 26
7·8	8·2	23·8	24·2	11 32	12 6	12 42	13 17	13 53	14 28	15 4	15 39	16 15	16 51	17 27
7·9	8·1	23·9	24·1	11 32	12 7	12 43	13 18	13 54	14 29	15 5	15 40	16 16	16 51	17 27
8·0 +	8·0 +	24·0 −	24·0 −	11 32	12 7	12 43	13 18	13 54	14 29	15 5	15 40	16 16	16 51	17 27
Mean				7 20	7 42	8 4	8 26	8 49	9 11	9 34	9 56	10 19	10 41	11 4

Table of Polar-Magnet-Deviations,

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·30	·31	·32	·33	·34	·35	·36	·37	·38	·39	·40
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+	+	−	−											
p	p	p	p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0·0	16·0	16·0	32·0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
0·1	15·9	16·1	31·9	0 20	0 21	0 22	0 22	0 23	0 24	0 25	0 25	0 26	0 26	0 27
0·2	15·8	16·2	31·8	0 40	0 42	0 43	0 44	0 46	0 47	0 49	0 50	0 51	0 52	0 54
0·3	15·7	16·3	31·7	1 1	1 3	1 5	1 7	1 9	1 11	1 13	1 15	1 17	1 19	1 21
0·4	15·6	16·4	31·6	1 21	1 23	1 26	1 28	1 31	1 34	1 37	1 39	1 42	1 45	1 48
0·5	15·5	16·5	31·5	1 41	1 44	1 47	1 50	1 54	1 57	2 1	2 4	2 8	2 11	2 15
0·6	15·4	16·6	31·4	2 1	2 5	2 9	2 13	2 17	2 21	2 26	2 30	2 34	2 38	2 42
0·7	15·3	16·7	31·3	2 21	2 25	2 30	2 35	2 40	2 45	2 50	2 55	3 0	3 4	3 9
0·8	15·2	16·8	31·2	2 41	2 46	2 52	2 57	3 3	3 8	3 14	3 19	3 25	3 30	3 36
0·9	15·1	16·9	31·1	3 1	3 7	3 13	3 19	3 25	3 31	3 38	3 44	3 50	3 56	4 3
1·0	15·0	17·0	31·0	3 21	3 28	3 35	3 41	3 48	3 55	4 2	4 8	4 15	4 22	4 29
1·1	14·9	17·1	30·9	3 41	3 48	3 56	4 3	4 11	4 18	4 26	4 33	4 41	4 48	4 56
1·2	14·8	17·2	30·8	4 1	4 9	4 17	4 25	4 33	4 41	4 50	4 58	5 6	5 14	5 22
1·3	14·7	17·3	30·7	4 21	4 29	4 38	4 46	4 55	5 4	5 13	5 21	5 30	5 39	5 48
1·4	14·6	17·4	30·6	4 40	4 49	4 59	5 8	5 18	5 27	5 37	5 46	5 55	6 4	6 14
1·5	14·5	17·5	30·5	5 0	5 10	5 20	5 30	5 40	5 50	6 0	6 10	6 20	6 30	6 40
1·6	14·4	17·6	30·4	5 19	5 30	5 40	5 50	6 1	6 12	6 23	6 33	6 44	6 55	7 6
1·7	14·3	17·7	30·3	5 39	5 50	6 1	6 12	6 24	6 35	6 47	6 58	7 9	7 20	7 32
1·8	14·2	17·8	30·2	5 58	6 10	6 22	6 34	6 46	6 58	7 10	7 21	7 33	7 45	7 57
1·9	14·1	17·9	30·1	6 17	6 29	6 42	6 54	7 7	7 20	7 33	7 45	7 58	8 10	8 23
2·0	14·0	18·0	30·0	6 36	6 49	7 2	7 15	7 28	7 41	7 55	8 8	8 21	8 34	8 48
2·1	13·9	18·1	29·9	6 55	7 8	7 22	7 36	7 50	8 4	8 18	8 31	8 45	8 59	9 13
2·2	13·8	18·2	29·8	7 13	7 27	7 42	7 56	8 11	8 25	8 40	8 54	9 9	9 23	9 38
2·3	13·7	18·3	29·7	7 31	7 46	8 2	8 17	8 32	8 47	9 2	9 17	9 32	9 47	10 2
2·4	13·6	18·4	29·6	7 50	8 5	8 21	8 36	8 52	9 8	9 24	9 39	9 55	10 11	10 27
2·5	13·5	18·5	29·5	8 8	8 24	8 41	8 57	9 14	9 30	9 47	10 3	10 19	10 35	10 52
2·6	13·4	18·6	29·4	8 26	8 43	9 0	9 17	9 34	9 51	10 8	10 25	10 42	10 59	11 16
2·7	13·3	18·7	29·3	8 44	9 1	9 19	9 36	9 54	10 12	10 30	10 47	11 5	11 22	11 40
2·8	13·2	18·8	29·2	9 1	9 19	9 37	9 55	10 13	10 32	10 50	11 8	11 26	11 44	12 3
2·9	13·1	18·9	29·1	9 19	9 37	9 56	10 15	10 34	10 53	11 12	11 30	11 49	12 8	12 27
3·0	13·0	19·0	29·0	9 36	9 55	10 15	10 34	10 54	11 13	11 33	11 52	12 11	12 30	12 50
3·1	12·9	19·1	28·9	9 53	10 13	10 33	10 53	11 13	11 33	11 53	12 13	12 33	12 53	13 13
3·2	12·8	19·2	28·8	10 10	10 30	10 51	11 11	11 32	11 53	12 14	12 34	12 55	13 15	13 36
3·3	12·7	19·3	28·7	10 26	10 47	11 9	11 30	11 51	12 12	12 34	12 55	13 16	13 37	13 58
3·4	12·6	19·4	28·6	10 42	11 4	11 26	11 47	12 9	12 31	12 53	13 15	13 36	13 58	14 20
3·5	12·5	19·5	28·5	10 58	11 20	11 43	12 5	12 28	12 50	13 13	13 35	13 57	14 19	14 42
3·6	12·4	19·6	28·4	11 14	11 37	12 0	12 23	12 46	13 9	13 32	13 54	14 17	14 40	15 3
3·7	12·3	19·7	28·3	11 30	11 53	12 17	12 40	13 4	13 27	13 51	14 14	14 37	15 0	15 24
3·8	12·2	19·8	28·2	11 45	12 9	12 33	12 57	13 21	13 45	14 9	14 33	14 57	15 21	15 45
3·9	12·1	19·9	28·1	12 0	12 24	12 49	13 13	13 38	14 3	14 28	14 52	15 17	15 41	16 6
4·0	12·0	20·0	28·0	12 15	12 40	13 5	13 30	13 55	14 20	14 45	15 10	15 36	16 1	16 26
+	+	−	−											

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·30	·31	·32	·33	·34	·35	·36	·37	·38	·39	·40
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
4·0	12·0	20·0	28·0	12 15	12 40	13 5	13 30	13 55	14 20	14 45	15 10	15 36	16 1	16 26
4·1	11·9	20·1	27·9	12 30	12 55	13 21	13 46	14 11	14 37	15 2	15 28	15 54	16 20	16 46
4·2	11·8	20·2	27·8	12 44	13 10	13 36	14 1	14 27	14 53	15 19	15 45	16 12	16 38	17 5
4·3	11·7	20·3	27·7	12 58	13 24	13 51	14 17	14 43	15 10	15 36	16 3	16 30	16 57	17 24
4·4	11·6	20·4	27·6	13 11	13 38	14 5	14 32	14 59	15 26	15 53	16 20	16 47	17 14	17 42
4·5	11·5	20·5	27·5	13 25	13 52	14 20	14 47	15 14	15 42	16 9	16 36	17 4	17 32	18 0
4·6	11·4	20·6	27·4	13 38	14 6	14 34	15 1	15 29	15 57	16 25	16 53	17 21	17 49	18 18
4·7	11·3	20·7	27·3	13 51	14 19	14 48	15 15	15 44	16 12	16 41	17 9	17 38	18 7	18 36
4·8	11·2	20·8	27·2	14 3	14 32	15 1	15 29	15 58	16 27	16 56	17 25	17 54	18 23	18 53
4·9	11·1	20·9	27·1	14 15	14 44	15 14	15 42	16 12	16 41	17 11	17 40	18 10	18 40	19 10
5·0	11·0	21·0	27·0	14 27	14 56	15 26	15 55	16 25	16 55	17 25	17 55	18 25	18 55	19 26
5·1	10 9	21·1	26·9	14 38	15 8	15 38	16 8	16 38	17 9	17 39	18 9	18 40	19 10	19 41
5·2	10·8	21·2	26·8	14 49	15 19	15 50	16 20	16 51	17 22	17 53	18 23	18 54	19 25	19 56
5·3	10·7	21·3	26·7	15 0	15 31	16 2	16 32	17 3	17 35	18 6	18 37	19 8	19 39	20 11
5·4	10·6	21·4	26·6	15 10	15 41	16 13	16 44	17 15	17 47	18 19	18 50	19 22	19 54	20 26
5·5	10·5	21·5	26·5	15 20	15 52	16 24	16 55	17 27	17 59	18 31	19 3	19 35	20 7	20 40
5·6	10·4	21·6	26·4	15 30	16 2	16 34	17 6	17 38	18 10	18 43	19 15	19 48	20 20	20 53
5·7	10·3	21·7	26·3	15 40	16 12	16 44	17 16	17 49	18 21	18 54	19 27	20 0	20 33	21 6
5·8	10·2	21·8	26·2	15 49	16 21	16 54	17 26	17 59	18 32	19 5	19 38	20 11	20 44	21 18
5·9	10·1	21·9	26·1	15 57	16 30	17 3	17 36	18 9	18 42	19 16	19 49	20 23	20 56	21 30
6·0	10·0	22·0	26·0	16 5	16 38	17 12	17 45	18 18	18 52	19 26	19 59	20 33	21 7	21 41
6·1	9·9	22·1	25·9	16 13	16 47	17 21	17 54	18 27	19 1	19 35	20 9	20 43	21 17	21 52
6·2	9·8	22·2	25·8	16 21	16 55	17 29	18 2	18 36	19 10	19 44	20 18	20 53	21 27	22 2
6·3	9·7	22·3	25·7	16 28	17 2	17 37	18 10	18 45	19 19	19 53	20 27	21 2	21 37	22 12
6·4	9·6	22·4	25·6	16 35	17 9	17 44	18 18	18 53	19 27	20 2	20 37	21 12	21 47	22 22
6·5	9·5	22·5	25·5	16 41	17 16	17 51	18 25	19 0	19 35	20 10	20 45	21 20	21 55	22 31
6·6	9·4	22·6	25·4	16 47	17 22	17 57	18 32	19 7	19 42	20 17	20 52	21 28	22 3	22 39
6·7	9·3	22·7	25·3	16 53	17 28	18 3	18 38	19 13	19 48	20 24	20 59	21 35	22 10	22 46
6·8	9·2	22·8	25·2	16 58	17 33	18 8	18 43	19 18	19 54	20 30	21 5	21 41	22 17	22 53
6·9	9·1	22·9	25·1	17 3	17 38	18 13	18 48	19 23	19 59	20 35	21 11	21 47	22 23	23 0
7·0	9·0	23·0	25·0	17 7	17 42	18 18	18 53	19 28	20 4	20 40	21 16	21 53	22 29	23 6
7·1	8·9	23·1	24·9	17 11	17 46	18 22	18 57	19 33	20 9	20 45	21 21	21 58	22 34	23 11
7·2	8·8	23·2	24·8	17 14	17 50	18 26	19 1	19 37	20 13	20 50	21 26	22 3	22 39	23 16
7·3	8·7	23·3	24·7	17 17	17 53	18 29	19 5	19 41	20 17	20 54	21 30	22 7	22 43	23 20
7·4	8·6	23·4	24·6	17 20	17 56	18 32	19 8	19 44	20 20	20 57	21 33	22 10	22 47	23 24
7·5	8·5	23·5	24·5	17 22	17 58	18 34	19 10	19 47	20 23	21 0	21 36	22 13	22 50	23 27
7·6	8·4	23·6	24·4	17 24	18 0	18 36	19 12	19 49	20 25	21 2	21 39	22 16	22 53	23 30
7·7	8·3	23·7	24·3	17 26	18 2	18 38	19 14	19 51	20 27	21 4	21 41	22 18	22 55	23 32
7·8	8·2	23·8	24·2	17 27	18 3	18 39	19 15	19 52	20 28	21 5	21 42	22 19	22 56	23 34
7·9	8·1	23·9	24·1	17 27	18 3	18 40	19 16	19 53	20 29	21 6	21 43	22 20	22 57	23 35
8·0	8·0	24·0	24·0	17 27	18 3	18 40	19 16	19 53	20 29	21 6	21 43	22 21	22 58	23 35
+	+	−	−											
Mean				11 4	11 27	11 50	12 13	12 35	12 58	13 21	13 44	14 7	14 30	14 53

Table of Polar-Magnet-Deviations,

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·40	·41	·42	·43	·44	·45	·46	·47	·48	·49	·50
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0·0	16·0	16·0	32·0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
0·1	15·9	16·1	31·9	0 27	0 27	0 28	0 29	0 30	0 31	0 31	0 31	0 32	0 33	0 34
0·2	15·8	16·2	31·8	0 54	0 55	0 57	0 58	1 0	1 1	1 2	1 3	1 4	1 5	1 7
0·3	15·7	16·3	31·7	1 21	1 23	1 25	1 27	1 29	1 31	1 33	1 35	1 37	1 39	1 41
0·4	15·6	16·4	31·6	1 48	1 50	1 53	1 56	1 59	2 1	2 4	2 6	2 9	2 12	2 15
0·5	15·5	16·5	31·5	2 15	2 18	2 21	2 24	2 28	2 31	2 35	2 38	2 42	2 45	2 49
0·6	15·4	16·6	31·4	2 42	2 46	2 50	2 54	2 58	3 2	3 6	3 10	3 14	3 18	3 22
0·7	15·3	16·7	31·3	3 9	3 14	3 19	3 24	3 29	3 33	3 38	3 42	3 47	3 51	3 56
0·8	15·2	16·8	31·2	3 36	3 41	3 47	3 52	3 58	4 3	4 8	4 13	4 18	4 23	4 29
0·9	15·1	16·9	31·1	4 3	4 9	4 15	4 21	4 27	4 33	4 39	4 45	4 51	4 57	5 3
1·0	15·0	17·0	31·0	4 29	4 35	4 42	4 49	4 56	5 2	5 9	5 15	5 22	5 29	5 36
1·1	14·9	17·1	30·9	4 56	5 3	5 10	5 17	5 25	5 32	5 40	5 47	5 54	6 1	6 9
1·2	14·8	17·2	30·8	5 22	5 30	5 38	5 46	5 54	6 2	6 10	6 18	6 26	6 34	6 42
1·3	14·7	17·3	30·7	5 48	5 57	6 6	6 15	6 24	6 32	6 41	6 49	6 58	7 6	7 15
1·4	14·6	17·4	30·6	6 14	6 23	6 33	6 42	6 52	7 1	7 11	7 20	7 29	7 38	7 48
1·5	14·5	17·5	30·5	6 40	6 50	7 0	7 10	7 21	7 31	7 41	7 51	8 1	8 11	8 21
1·6	14·4	17·6	30·4	7 6	7 16	7 27	7 37	7 48	7 59	8 10	8 20	8 31	8 42	8 53
1·7	14·3	17·7	30·3	7 32	7 43	7 54	8 5	8 17	8 28	8 40	8 51	9 2	9 13	9 25
1·8	14·2	17·8	30·2	7 57	8 9	8 21	8 33	8 45	8 57	9 10	9 22	9 34	9 46	9 58
1·9	14·1	17·9	30·1	8 23	8 35	8 48	9 0	9 13	9 26	9 39	9 51	10 4	10 17	10 30
2·0	14·0	18·0	30·0	8 48	9 1	9 15	9 28	9 42	9 55	10 9	10 22	10 35	10 48	11 2
2·1	13·9	18·1	29·9	9 13	9 27	9 41	9 55	10 9	10 23	10 38	10 52	11 6	11 20	11 34
2·2	13·8	18·2	29·8	9 38	9 52	10 7	10 21	10 36	10 51	11 6	11 20	11 35	11 50	12 5
2·3	13·7	18·3	29·7	10 2	10 17	10 33	10 48	11 4	11 19	11 35	11 50	12 5	12 21	12 36
2·4	13·6	18·4	29·6	10 27	10 43	10 59	11 15	11 31	11 47	12 3	12 19	12 35	12 51	13 7
2·5	13·5	18·5	29·5	10 52	11 8	11 25	11 41	11 58	12 15	12 32	12 48	13 5	13 21	13 38
2·6	13·4	18·6	29·4	11 16	11 33	11 50	12 7	12 24	12 41	12 59	13 16	13 33	13 50	14 8
2·7	13·3	18·7	29·3	11 40	11 57	12 15	12 33	12 51	13 9	13 27	13 44	14 2	14 20	14 38
2·8	13·2	18·8	29·2	12 3	12 21	12 40	12 58	13 17	13 35	13 54	14 12	14 31	14 49	15 8
2·9	13·1	18·9	29·1	12 27	12 46	13 5	13 24	13 43	14 2	14 22	14 41	15 0	15 19	15 38
3·0	13·0	19·0	29·0	12 50	13 10	13 30	13 49	14 9	14 29	14 49	15 8	15 28	15 48	16 8
3·1	12·9	19·1	28·9	13 13	13 33	13 54	14 14	14 35	14 55	15 16	15 36	15 56	16 16	16 37
3·2	12·8	19·2	28·8	13 36	13 57	14 18	14 39	15 0	15 21	15 42	16 3	16 24	16 45	17 6
3·3	12·7	19·3	28·7	13 58	14 19	14 41	15 2	15 24	15 46	16 8	16 29	16 51	17 12	17 34
3·4	12·6	19·4	28·6	14 20	14 42	15 4	15 26	15 48	16 10	16 33	16 55	17 17	17 39	18 2
3·5	12·5	19·5	28·5	14 42	15 4	15 27	15 50	16 13	16 36	16 59	17 21	17 44	18 7	18 30
3·6	12·4	19·6	28·4	15 3	15 26	15 50	16 13	16 37	17 0	17 24	17 47	18 10	18 33	18 57
3·7	12·3	19·7	28·3	15 24	15 48	16 12	16 36	17 0	17 24	17 48	18 12	18 36	19 0	19 24
3·8	12·2	19·8	28·2	15 45	16 9	16 34	16 58	17 23	17 47	18 12	18 36	19 1	19 25	19 50
3·9	12·1	19·9	28·1	16 6	16 31	16 56	17 21	17 46	18 11	18 36	19 1	19 26	19 51	20 16
4·0	12·0	20·0	28·0	16 26	16 51	17 17	17 42	18 8	18 33	18 59	19 25	19 50	20 16	20 42
+	+	−	−											

for Azimuths referred to the Disturbed Compass.

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·40	·41	·42	·43	·44	·45	·46	·47	·48	·49	·50
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
4·0	12·0	20·0	28·0	16 26	16 51	17 17	17 42	18 8	18 33	18 59	19 25	19 50	20 16	20 42
4·1	11·9	20·1	27·9	16 46	17 12	17 38	18 4	18 30	18 56	19 22	19 48	20 14	20 40	21 7
4·2	11·8	20·2	27·8	17 5	17 32	17 58	18 25	18 51	19 18	19 45	20 11	20 38	21 5	21 32
4·3	11·7	20·3	27·7	17 24	17 51	18 18	18 45	19 12	19 40	20 7	20 34	21 2	21 29	21 57
4·4	11·6	20·4	27·6	17 42	18 10	18 37	19 5	19 33	20 1	20 29	20 57	21 25	21 53	22 21
4·5	11·5	20·5	27·5	18 0	18 29	18 57	19 25	19 53	20 22	20 50	21 18	21 47	22 15	22 44
4·6	11·4	20·6	27·4	18 18	18 47	19 15	19 44	20 13	20 42	21 11	21 40	22 9	22 38	23 7
4·7	11·3	20·7	27·3	18 36	19 5	19 34	20 3	20 32	21 2	21 31	22 0	22 30	22 59	23 29
4·8	11·2	20·8	27·2	18 53	19 23	19 52	20 22	20 51	21 21	21 51	22 21	22 51	23 21	23 51
4·9	11·1	20·9	27·1	19 10	19 40	20 10	20 40	21 10	21 40	22 10	22 40	23 11	23 42	24 13
5·0	11·0	21·0	27·0	19 26	19 56	20 26	20 57	21 27	21 58	22 29	23 0	23 31	24 2	24 34
5·1	10·9	21·1	26·9	19 41	20 12	20 43	21 14	21 45	22 16	22 48	23 19	23 51	24 22	24 54
5·2	10·8	21·2	26·8	19 56	20 28	20 59	21 31	22 2	22 34	23 6	23 38	24 10	24 42	25 14
5·3	10·7	21·3	26·7	20 11	20 43	21 15	21 47	22 19	22 51	23 23	23 55	24 28	25 0	25 33
5·4	10·6	21·4	26·6	20 26	20 58	21 30	22 2	22 34	23 7	23 40	24 13	24 46	25 19	25 52
5·5	10·5	21·5	26·5	20 40	21 12	21 44	22 17	22 50	23 23	23 56	24 29	25 3	25 36	26 10
5·6	10·4	21·6	26·4	20 53	21 26	21 59	22 32	23 5	23 38	24 12	24 45	25 19	25 53	26 27
5·7	10·3	21·7	26·3	21 6	21 39	22 12	22 46	23 19	23 53	24 27	25 1	25 35	26 9	26 44
5·8	10·2	21·8	26·2	21 18	21 52	22 25	22 59	23 33	24 7	24 42	25 16	25 51	26 25	27 0
5·9	10·1	21·9	26·1	21 30	22 4	22 38	23 12	23 46	24 21	24 56	25 31	26 6	26 41	27 16
6·0	10·0	22·0	26·0	21 41	22 16	22 50	23 25	23 59	24 34	25 9	25 44	26 20	26 55	27 31
6·1	9·9	22·1	25·9	21 52	22 27	23 1	23 36	24 11	24 46	25 22	25 57	26 33	27 9	27 45
6·2	9·8	22·2	25·8	22 2	22 37	23 12	23 47	24 22	24 58	25 34	26 10	26 46	27 22	27 59
6·3	9·7	22·3	25·7	22 12	22 48	23 23	23 58	24 33	25 9	25 46	26 22	26 59	27 35	28 12
6·4	9·6	22·4	25·6	22 22	22 58	23 33	24 9	24 44	25 20	25 57	26 33	27 10	27 47	28 24
6·5	9·5	22·5	25·5	22 31	23 7	23 42	24 18	24 54	25 30	26 7	26 44	27 21	27 58	28 35
6·6	9·4	22·6	25·4	22 39	23 15	23 51	24 27	25 3	25 40	26 17	26 54	27 31	28 8	28 46
6·7	9·3	22·7	25·3	22 46	23 23	23 59	24 36	25 12	25 49	26 26	27 3	27 41	28 18	28 56
6·8	9·2	22·8	25·2	22 53	23 30	24 6	24 43	25 20	25 57	26 35	27 12	27 50	28 27	29 5
6·9	9·1	22·9	25·1	23 0	23 37	24 13	24 50	25 27	26 4	26 42	27 20	27 58	28 36	29 14
7·0	9·0	23·0	25·0	23 6	23 43	24 20	24 57	25 34	26 11	26 49	27 27	28 5	28 43	29 22
7·1	8·9	23·1	24·9	23 11	23 48	24 25	25 2	25 39	26 17	26 56	27 34	28 12	28 50	29 29
7·2	8·8	23·2	24·8	23 16	23 53	24 30	25 8	25 45	26 23	27 2	27 40	28 19	28 57	29 36
7·3	8·7	23·3	24·7	23 20	23 58	24 35	25 13	25 50	26 28	27 7	27 45	28 24	29 2	29 41
7·4	8·6	23·4	24·6	23 24	24 2	24 39	25 17	25 55	26 32	27 11	27 49	28 28	29 7	29 46
7·5	8·5	23·5	24·5	23 27	24 5	24 42	25 20	25 58	26 36	27 15	27 53	28 32	29 11	29 50
7·6	8·4	23·6	24·4	23 30	24 8	24 45	25 23	26 1	26 39	27 18	27 57	28 36	29 15	29 54
7·7	8·3	23·7	24·3	23 32	24 10	24 48	25 26	26 3	26 41	27 20	27 59	28 38	29 17	29 56
7·8	8·2	23·8	24·2	23 34	24 11	24 49	25 27	26 5	26 43	27 22	28 1	28 40	29 19	29 58
7·9	8·1	23·9	24·1	23 35	24 12	24 50	25 28	26 6	26 44	27 23	28 2	28 41	29 20	29 59
8·0 +	8·0 +	24·0 −	24·0 −	23 35	24 13	24 51	25 29	26 7	26 45	27 24	28 3	28 42	29 21	30 0
Mean				14 53	15 16	15 39	16 2	16 26	16 49	17 13	17 37	18 1	18 25	18 49

Table of Polar-Magnet-Deviations,

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·50	·51	·52	·53	·54	·55	·56	·57	·58	·59	·60
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0·0	16·0	16·0	32·0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
0·1	15·9	16·1	31·9	0 34	0 34	0 35	0 36	0 36	0 37	0 38	0 38	0 39	0 40	0 41
0·2	15·8	16·2	31·8	1 7	1 8	1 10	1 11	1 13	1 14	1 16	1 17	1 18	1 19	1 21
0·3	15·7	16·3	31·7	1 41	1 43	1 45	1 47	1 49	1 52	1 54	1 56	1 58	2 0	2 2
0·4	15·6	16·4	31·6	2 15	2 17	2 20	2 22	2 25	2 28	2 31	2 33	2 36	2 39	2 42
0·5	15·5	16·5	31·5	2 49	2 52	2 55	2 58	3 2	3 5	3 9	3 12	3 16	3 19	3 23
0·6	15·4	16·6	31·4	3 22	2 26	3 30	3 34	3 38	3 42	3 47	3 51	3 55	3 59	4 3
0·7	15·3	16·7	31·3	3 56	4 0	4 5	4 9	4 14	4 19	4 24	4 28	4 33	4 38	4 43
0·8	15·2	16·8	31·2	4 29	4 34	4 40	4 45	4 51	4 56	5 2	5 7	5 12	5 17	5 23
0·9	15·1	16·9	31·1	5 3	5 9	5 15	5 21	5 27	5 33	5 39	5 45	5 51	5 57	6 3
1·0	15·0	17·0	31·0	5 36	5 42	5 49	5 55	6 2	6 9	6 16	6 22	6 29	6 36	6 43
1·1	14·9	17·1	30·9	6 9	6 16	6 24	6 31	6 39	6 46	6 54	7 1	7 8	7 15	7 23
1·2	14·8	17·2	30·8	6 42	6 50	6 58	7 6	7 14	7 22	7 31	7 39	7 47	7 55	8 3
1·3	14·7	17·3	30·7	7 15	7 23	7 32	7 41	7 50	7 59	8 8	8 16	8 25	8 34	8 43
1·4	14·6	17·4	30·6	7 48	7 57	8 7	8 16	8 26	8 35	8 45	8 54	9 3	9 12	9 22
1·5	14·5	17·5	30·5	8 21	8 31	8 41	8 51	9 1	9 11	9 22	9 32	9 42	9 52	10 2
1·6	14·4	17·6	30·4	8 53	9 4	9 15	9 25	9 36	9 47	9 58	10 8	10 19	10 30	10 41
1·7	14·3	17·7	30·3	9 25	9 36	9 48	9 59	10 11	10 22	10 34	10 45	10 57	11 8	11 20
1·8	14·2	17·8	30·2	9 58	10 10	10 22	10 34	10 46	10 58	11 11	11 23	11 35	11 47	11 59
1·9	14·1	17·9	30·1	10 30	10 43	10 56	11 8	11 21	11 34	11 47	11 59	12 12	12 25	12 38
2·0	14·0	18·0	30·0	11 2	11 15	11 29	11 42	11 56	12 9	12 23	12 36	12 49	13 2	13 16
2·1	13·9	18·1	29·9	11 34	11 48	12 2	12 16	12 30	12 44	12 58	13 12	13 26	13 40	13 54
2·2	13·8	18·2	29·8	12 5	12 20	12 35	12 49	13 4	13 19	13 34	13 48	14 3	14 18	14 33
2·3	13·7	18·3	29·7	12 36	12 51	13 7	13 22	13 38	13 53	14 9	14 25	14 40	14 55	15 11
2·4	13·6	18·4	29·6	13 7	13 23	13 39	13 55	14 11	14 27	14 44	15 0	15 16	15 32	15 48
2·5	13·5	18·5	29·5	13 38	13 54	14 11	14 27	14 44	15 1	15 18	15 35	15 51	16 8	16 25
2·6	13·4	18·6	29·4	14 8	14 25	14 43	15 0	15 18	15 35	15 53	16 10	16 28	16 45	17 3
2·7	13·3	18·7	29·3	14 38	14 56	15 14	15 32	15 50	16 8	16 27	16 45	17 3	17 21	17 40
2·8	13·2	18·8	29·2	15 8	15 26	15 45	16 4	16 23	16 42	17 1	17 19	17 38	17 57	18 16
2·9	13·1	18·9	29·1	15 38	15 57	16 17	16 36	16 56	17 15	17 35	17 54	18 13	18 32	18 52
3·0	13·0	19·0	29·0	16 8	16 28	16 48	17 8	17 28	17 48	18 8	18 28	18 48	19 8	19 28
3·1	12·9	19·1	28·9	16 37	16 58	17 18	17 38	17 59	18 20	18 41	19 1	19 22	19 43	20 4
3·2	12·8	19·2	28·8	17 6	17 27	17 48	18 9	18 31	18 52	19 14	19 35	19 56	20 17	20 39
3·3	12·7	19·3	28·7	17 34	17 56	18 18	18 40	19 2	19 24	19 46	20 8	20 30	20 52	21 14
3·4	12·6	19·4	28·6	18 2	18 24	18 47	19 9	19 32	19 55	20 18	20 40	21 3	21 25	21 48
3·5	12·5	19·5	28·5	18 30	18 53	19 16	19 39	20 2	20 26	20 49	21 12	21 35	21 58	22 22
3·6	12·4	19·6	28·4	18 57	19 21	19 45	20 9	20 33	20 57	21 21	21 44	22 8	22 32	22 56
3·7	12·3	19·7	28·3	19 24	19 48	20 13	20 37	21 2	21 26	21 51	22 15	22 40	23 4	23 29
3·8	12·2	19·8	28·2	19 50	20 15	20 41	21 6	21 31	21 56	22 22	22 47	23 12	23 37	24 2
3·9	12·1	19·9	28·1	20 16	20 42	21 8	21 34	21 59	22 25	22 51	23 17	23 43	24 8	24 34
4·0	12·0	20·0	28·0	20 42	21 8	21 34	22 1	22 27	22 53	23 20	23 46	24 13	24 39	25 6
+	+	−	−											

for Azimuths referred to the Disturbed Compass.

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·50	·51	·52	·53	·54	·55	·56	·57	·58	·59	·60
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+	+	−	−											
p	p	p	p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
4·0	12·0	20·0	28·0	20 42	21 8	21 34	22 1	22 27	22 53	23 20	23 46	24 13	24 39	25 6
4·1	11·9	20·1	27·9	21 7	21 34	22 1	22 27	22 54	23 21	23 49	24 16	24 43	25 10	25 37
4·2	11·8	20·2	27·8	21 32	21 59	22 27	22 54	23 22	23 49	24 17	24 44	25 12	25 40	26 8
4·3	11·7	20·3	27·7	21 57	22 25	22 53	23 20	23 48	24 16	24 45	25 13	25 42	26 10	26 39
4·4	11·6	20·4	27·6	22 21	22 49	23 18	23 46	24 14	24 43	25 12	25 41	26 10	26 39	27 9
4·5	11·5	20·5	27·5	22 44	23 13	23 42	24 11	24 40	25 9	25 39	26 8	26 38	27 8	27 38
4·6	11·4	20·6	27·4	23 7	23 36	24 6	24 35	25 5	25 35	26 5	26 35	27 6	27 36	28 7
4·7	11·3	20·7	27·3	23 29	23 59	24 30	25 0	25 30	26 0	26 31	27 2	27 33	28 4	28 35
4·8	11·2	20·8	27·2	23 51	24 22	24 53	25 24	25 54	26 25	26 56	27 27	27 59	28 30	29 2
4·9	11·1	20·9	27·1	24 13	24 44	25 16	25 47	26 18	26 49	27 21	27 53	28 25	28 57	29 29
5·0	11·0	21·0	27·0	24 34	25 6	25 38	26 9	26 41	27 13	27 46	28 18	28 51	29 23	29 56
5·1	10·9	21·1	26·9	24 54	25 26	25 59	26 31	27 3	27 36	28 9	28 42	29 15	29 48	30 22
5·2	10·8	21·2	26·8	25 14	25 47	26 20	26 52	27 25	27 58	28 32	29 5	29 39	30 12	30 46
5·3	10·7	21·3	26·7	25 33	26 6	26 40	27 13	27 46	28 20	28 54	29 28	30 2	30 36	31 10
5·4	10·6	21·4	26·6	25 52	26 26	27 0	27 33	28 7	28 41	29 16	29 50	30 25	30 59	31 34
5·5	10·5	21·5	26·5	26 10	26 44	27 19	27 53	28 27	29 1	29 37	30 12	30 47	31 22	31 57
5·6	10·4	21·6	26·4	26 27	27 2	27 37	28 11	28 46	29 21	29 57	30 32	31 8	31 43	32 19
5·7	10·3	21·7	26·3	26 44	27 19	27 54	28 29	29 4	29 40	30 16	30 52	31 28	32 4	32 40
5·8	10·2	21·8	26·2	27 0	27 35	28 11	28 46	29 22	29 58	30 35	31 11	31 48	32 24	33 1
5·9	10·1	21·9	26·1	27 16	27 51	28 27	29 3	29 39	30 15	30 53	31 30	32 7	32 44	33 21
6·0	10·0	22·0	26·0	27 31	28 7	28 43	29 19	29 55	30 32	31 10	31 47	32 25	33 2	33 40
6·1	9·9	22·1	25·9	27 45	28 21	28 58	29 34	30 11	30 48	31 26	32 4	32 42	33 20	33 58
6·2	9·8	22·2	25·8	27 59	28 36	29 13	29 49	30 26	31 3	31 41	32 19	32 58	33 36	34 15
6·3	9·7	22·3	25·7	28 12	28 49	29 26	30 3	30 40	31 18	31 56	32 35	33 14	33 53	34 32
6·4	9·6	22·4	25·6	28 24	29 2	29 39	30 16	30 54	31 32	32 11	32 50	33 29	34 8	34 48
6·5	9·5	22·5	25·5	28 35	29 13	29 51	30 29	31 7	31 45	32 25	33 4	33 44	34 23	35 3
6·6	9·4	22·6	25·4	28 46	29 24	30 3	30 41	31 19	31 58	32 38	33 17	33 57	34 36	35 16
6·7	9·3	22·7	25·3	28 56	29 34	30 13	30 51	31 30	32 10	32 50	33 29	34 9	34 49	35 29
6·8	9·2	22·8	25·2	29 5	29 43	30 22	31 1	31 41	32 20	33 0	33 40	34 20	35 0	35 41
6·9	9·1	22·9	25·1	29 14	20 52	30 31	31 11	31 51	32 30	33 10	33 50	34 31	35 11	35 52
7·0	9·0	23·0	25·0	29 22	30 0	30 39	31 19	31 59	32 39	33 19	34 0	34 41	35 22	36 3
7·1	8·9	23·1	24·9	29 29	30 8	30 47	31 27	32 7	32 47	33 27	54 8	34 50	35 31	36 13
7·2	8·8	23·2	24·8	29 36	30 15	30 54	31 34	32 14	32 54	33 35	34 16	34 58	35 39	36 21
7·3	8·7	23·3	24·7	29 41	30 20	31 0	31 40	32 20	33 0	33 42	34 23	35 5	35 46	36 28
7·4	8·6	23·4	24·6	29 46	30 25	31 5	31 45	32 26	33 6	33 47	34 28	35 10	35 52	36 34
7·5	8·5	23·5	24·5	29 50	30 29	31 9	31 50	32 31	33 11	33 52	34 33	35 15	35 57	36 39
7·6	8·4	23·6	24·4	29 54	30 33	31 13	31 53	32 34	33 15	33 56	34 38	35 20	36 2	36 44
7·7	8·3	23·7	24·3	29 56	30 36	31 16	31 56	32 37	33 18	33 59	34 41	35 23	36 5	36 48
7·8	8·2	23·8	24·2	29 58	30 38	31 18	31 58	32 39	33 20	34 1	34 43	35 25	36 8	36 50
7·9	8·1	23·9	24·1	29 59	30 39	31 20	32 0	32 41	33 21	34 2	34 44	35 26	36 9	36 52
8·0	8·0	24·0	24·0	30 0	30 40	31 21	32 1	32 42	33 22	34 3	34 45	35 27	36 9	36 52
+	+	−	−											
Mean				18 49	19 13	19 37	20 1	20 26	20 51	21 15	21 40	22 5	22 30	22 55

Table of Polar-Magnet-Deviations,

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·60	·61	·62	·63	·64	·65	·66	·67	·68	·69	·70
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+	+	−	−											
p	p	p	p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0·0	16·0	16·0	32·0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
0·1	15·9	16·1	31·9	0 41	0 41	0 42	0 42	0 43	0 44	0 45	0 45	0 46	0 46	0 47
0·2	15·8	16·2	31·8	1 21	1 22	1 24	1 25	1 26	1 27	1 29	1 30	1 31	1 32	1 34
0·3	15·7	16·3	31·7	2 2	2 4	2 6	2 8	2 10	2 12	2 14	2 15	2 17	2 19	2 21
0·4	15·6	16·4	31·6	2 42	2 44	2 47	2 49	2 52	2 55	2 58	3 0	3 3	3 5	3 8
0·5	15·5	16·5	31·5	3 23	3 26	3 29	3 32	3 36	3 39	3 43	3 46	3 49	3 52	3 56
0·6	15·4	16·6	31·4	4 3	4 7	4 11	4 15	4 19	4 23	4 27	4 31	4 35	4 39	4 43
0·7	15·3	16·7	31·3	4 43	4 48	4 53	4 57	5 2	5 7	5 12	5 16	5 21	5 25	5 30
0·8	15·2	16·8	31·2	5 23	5 28	5 34	5 39	5 45	5 50	5 56	6 1	6 6	6 11	6 17
0·9	15·1	16·9	31·1	6 3	6 9	6 15	6 21	6 27	6 33	6 40	6 46	6 52	6 58	7 4
1·0	15·0	17·0	31·0	6 43	6 50	6 57	7 3	7 10	7 17	7 24	7 30	7 37	7 44	7 51
1·1	14·9	17·1	30·9	7 23	7 30	7 38	7 45	7 53	8 0	8 8	8 15	8 23	8 30	8 38
1·2	14·8	17·2	30·8	8 3	8 11	8 19	8 27	8 35	8 43	8 52	9 0	9 8	9 16	9 24
1·3	14·7	17·3	30·7	8 43	8 51	9 0	9 8	9 17	9 26	9 35	9 43	9 52	10 1	10 10
1·4	14·6	17·4	30·6	9 22	9 31	9 41	9 50	10 0	10 9	10 19	10 28	10 38	10 47	10 57
1·5	14·5	17·5	30·5	10 2	10 12	10 22	10 32	10 42	10 53	11 3	11 13	11 23	11 33	11 43
1·6	14·4	17·6	30·4	10 41	10 52	11 3	11 13	11 24	11 35	11 46	11 56	12 7	12 18	12 29
1·7	14·3	17·7	30·3	11 20	11 31	11 43	11 54	12 6	12 17	12 29	12 40	12 52	13 3	13 15
1·8	14·2	17·8	30·2	11 59	12 11	12 23	12 35	12 47	12 59	13 12	13 24	13 36	13 48	14 1
1·9	14·1	17·9	30·1	12 38	12 50	13 3	13 16	13 29	13 42	13 55	14 8	14 21	14 34	14 47
2·0	14·0	18·0	30·0	13 16	13 29	13 43	13 56	14 10	14 24	14 38	14 51	15 5	15 18	15 32
2·1	13·9	18·1	29·9	13 54	14 8	14 23	14 37	14 51	15 5	15 20	15 34	15 49	16 3	16 17
2·2	13·8	18·2	29·8	14 33	14 48	15 3	15 18	15 33	15 48	16 3	16 17	16 32	16 47	17 2
2·3	13·7	18·3	29·7	15 11	15 26	15 42	15 57	16 13	16 29	16 45	17 0	17 16	17 31	17 47
2·4	13·6	18·4	29·6	15 48	16 4	16 21	16 37	16 53	17 9	17 26	17 42	17 58	18 14	18 31
2·5	13·5	18·5	29·5	16 25	16 42	17 0	17 17	17 34	17 51	18 8	18 25	18 42	18 59	19 16
2·6	13·4	18·6	29·4	17 3	17 20	17 38	17 55	18 13	18 31	18 49	19 6	19 24	19 42	20 0
2·7	13·3	18·7	29·3	17 40	17 58	18 16	18 34	18 53	19 11	19 30	19 48	20 7	20 25	20 44
2·8	13·2	18·8	29·2	18 16	18 35	18 54	19 13	19 32	19 51	20 11	20 30	20 49	21 8	21 27
2·9	13·1	18·9	29·1	18 52	19 12	19 32	19 51	20 11	20 31	20 51	21 10	21 30	21 50	22 10
3·0	13·0	19·0	29·0	19 28	19 48	20 9	20 29	20 50	21 10	21 31	21 51	22 12	22 32	22 53
3·1	12·9	19·1	28·9	20 4	21 25	20 46	21 7	21 28	21 49	22 11	22 32	22 53	23 14	23 36
3·2	12·8	19·2	28·8	20 39	21 0	21 22	21 44	22 6	22 28	22 50	23 12	23 34	23 56	24 18
3·3	12·7	19·3	28·7	21 14	21 36	21 58	22 20	22 43	23 6	23 29	23 51	24 14	24 37	25 0
3·4	12·6	19·4	28·6	21 48	22 11	22 34	22 57	23 21	23 44	24 8	24 31	24 54	25 17	25 41
3·5	12·5	19·5	28·5	22 22	22 45	23 9	23 33	23 57	24 21	24 46	25 10	25 34	25 58	26 22
3·6	12·4	19·6	28·4	22 56	23 20	23 44	24 9	24 34	24 59	25 24	25 48	26 13	26 38	27 3
3·7	12·3	19·7	28·3	23 29	23 54	24 19	24 44	25 10	25 35	26 1	26 26	26 52	27 17	27 43
3·8	12·2	19·8	28·2	24 2	24 27	24 53	25 19	25 45	26 11	26 38	27 4	27 30	27 56	28 22
3·9	12·1	19·9	28·1	24 34	25 0	25 27	25 53	26 20	26 47	27 14	27 40	28 7	28 34	29 1
4·0	12·0	20·0	28·0	25 6	25 33	26 0	26 28	26 55	27 22	27 50	28 17	28 45	29 12	29 40
+	+	−	−											

for Azimuths referred to the Disturbed Compass.

Apparent Azimuth from Neutral Position in points and decimals.				Modulus. ·60	·61	·62	·63	·64	·65	·66	·67	·68	·69	·70
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+	+	−	−											
p	p	p	p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
4·0	12·0	20·0	28·0	25 6	25 33	26 0	26 28	26 55	27 22	27 50	28 17	28 45	29 12	29 40
4·1	11·9	20·1	27·9	25 37	26 5	26 33	27 1	27 29	27 56	28 24	28 53	29 22	29 50	30 18
4·2	11·8	20·2	27·8	26 8	26 36	27 5	27 33	28 2	28 30	28 59	29 28	29 58	30 27	30 56
4·3	11·7	20·3	27·7	26 39	27 8	27 37	28 6	28 35	29 4	29 34	30 4	30 34	31 3	31 33
4·4	11·6	20·4	27·6	27 9	27 38	28 8	28 38	29 8	29 37	30 8	30 38	31 9	31 39	32 10
4·5	11·5	20·5	27·5	27 38	28 8	28 38	29 9	29 40	30 10	30 41	31 12	31 43	32 14	32 46
4·6	11·4	20·6	27·4	28 7	28 38	29 9	29 40	30 11	30 42	31 13	31 45	32 17	32 49	33 21
4·7	11·3	20·7	27·3	28 35	29 6	29 38	30 9	30 41	31 13	31 45	32 17	32 50	33 23	33 56
4·8	11·2	20·8	27·2	29 2	29 34	30 6	30 38	31 11	31 44	32 17	32 50	33 23	33 56	34 30
4·9	11·1	20·9	27·1	29 29	30 1	30 34	31 7	31 40	32 14	32 48	33 21	33 55	34 29	35 3
5·0	11·0	21·0	27·0	29 56	30 29	31 2	31 35	32 9	32 43	33 18	33 52	34 26	35 1	35 36
5·1	10·9	21·1	26·9	30 22	30 55	31 29	32 3	32 37	33 11	33 46	34 21	34 56	35 32	36 8
5·2	10·8	21·2	26·8	30 46	31 20	31 55	32 29	33 4	33 39	34 14	34 50	35 26	36 2	36 39
5·3	10·7	21·3	26·7	31 10	31 45	32 20	32 55	33 30	34 6	34 42	35 18	35 55	36 32	37 9
5·4	10·6	21·4	26·6	31 34	32 9	32 45	33 20	33 56	34 32	35 9	35 46	36 24	37 2	37 39
5·5	10·5	21·5	26·5	31 57	32 33	33 9	33 45	34 21	34 58	35 36	36 14	36 52	37 30	38 8
5·6	10·4	21·6	26·4	32 19	32 55	33 32	34 9	34 46	35 23	36 1	36 39	37 18	37 56	38 35
5·7	10·3	21·7	26·3	32 40	33 17	33 55	34 32	35 10	35 47	36 25	37 4	37 43	38 22	39 2
5·8	10·2	21·8	26·2	33 1	33 38	34 16	34 54	35 32	36 10	36 48	37 28	38 8	38 48	39 28
5·9	10·1	21·9	26·1	33 21	33 59	34 37	35 15	35 54	36 32	37 11	37 51	38 32	39 12	39 53
6·0	10·0	22·0	26·0	33 40	34 18	34 57	35 36	36 15	36 54	37 33	38 14	38 55	39 36	40 18
6·1	9·9	22·1	25·9	33 58	34 37	35 16	35 55	36 35	37 14	37 54	38 35	39 17	39 59	40 41
6·2	9·8	22·2	25·8	34 15	34 54	35 34	36 14	36 54	37 34	38 14	38 56	39 39	40 21	41 3
6·3	9·7	22·3	25·7	34 32	35 12	35 52	36 32	37 12	37 53	38 34	39 16	39 59	40 41	41 24
6·4	9·6	22·4	25·6	34 48	35 28	36 9	36 49	37 30	38 11	38 53	39 35	40 17	41 0	41 44
6·5	9·5	22·5	25·5	35 3	35 43	36 24	37 5	37 46	38 28	39 10	39 52	40 35	41 19	42 3
6·6	9·4	22·6	25·4	35 16	35 57	36 38	37 19	38 1	38 44	39 26	40 9	40 52	41 36	42 21
6·7	9·3	22·7	25·3	35 29	36 10	36 52	37 33	38 15	38 59	39 42	40 25	41 9	41 53	42 38
6·8	9·2	22·8	25·2	35 41	36 23	37 5	37 47	38 29	39 12	39 56	40 40	41 24	42 9	42 54
6·9	9·1	22·9	25·1	35 52	36 34	37 16	37 58	38 41	39 24	40 8	40 53	41 37	42 22	43 8
7·0	9·0	23·0	25·0	36 3	36 45	37 27	38 9	38 52	39 36	40 20	41 4	41 49	42 35	43 21
7·1	8·9	23·1	24·9	36 13	36 55	37 37	38 19	39 2	39 46	40 30	41 15	42 0	42 46	43 33
7·2	8·8	23·2	24·8	36 21	37 3	37 46	38 29	39 12	39 56	40 40	41 25	42 11	42 57	43 44
7·3	8·7	23·3	24·7	36 28	37 11	37 54	38 37	39 21	40 5	40 49	41 35	42 21	43 7	43 54
7·4	8·6	23·4	24·6	36 34	37 17	38 1	38 44	39 28	40 12	40 57	41 42	42 28	43 15	44 2
7·5	8·5	23·5	24·5	36 39	37 22	38 6	38 50	39 34	40 18	41 3	41 49	42 35	43 22	44 9
7·6	8·4	23·6	24·4	36 44	37 27	38 11	38 55	39 39	40 23	41 8	41 54	42 41	43 28	44 15
7·7	8·3	23·7	24·3	36 48	37 31	38 15	38 59	39 43	40 27	41 12	41 58	42 45	43 32	44 20
7·8	8·2	23·8	24·2	36 50	37 33	38 17	39 1	39 45	40 29	41 15	42 1	42 48	43 35	44 23
7·9	8·1	23·9	24·1	36 52	37 35	38 19	39 2	39 46	40 31	41 17	42 3	42 50	43 37	44 25
8·0	8·0	24·0	24·0	36 52	37 35	38 19	39 3	39 47	40 32	41 18	42 4	42 51	43 38	44 26
+	+	−	−											
Mean				22 55	23 20	23 46	24 12	24 38	25 4	25 30	25 56	26 23	26 50	27 17

Table of Polar-Magnet-Deviations,

Apparent Azimuth from Neutral Position n points and decimals.				Modulus.										
				·70	·71	·72	·73	·74	·75	·76	·77	·78	·79	·80
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+	+	−	−											
p	p	p	p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0·0	16·0	16·0	32·0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
0·1	15·9	16·1	31·9	0 47	0 47	0 48	0 48	0 49	0 50	0 51	0 51	0 52	0 53	0 54
0·2	15·8	16·2	31·8	1 34	1 35	1 37	1 38	1 40	1 41	1 43	1 44	1 45	1 46	1 48
0·3	15·7	16·3	31·7	2 21	2 23	2 25	2 27	2 29	2 31	2 34	2 36	2 38	2 40	2 42
0·4	15·6	16·4	31·6	3 8	3 11	3 14	3 16	3 19	3 22	3 25	3 27	3 30	3 33	3 36
0·5	15·5	16·5	31·5	3 56	3 59	4 2	4 5	4 9	4 12	4 16	4 19	4 23	4 26	4 30
0·6	15·4	16·6	31·4	4 43	4 47	4 51	4 55	4 59	5 3	5 8	5 12	5 16	5 20	5 24
0·7	15·3	16·7	31·3	5 30	5 34	5 39	5 44	5 49	5 54	5 59	6 3	6 8	6 13	6 18
0·8	15·2	16·8	31·2	6 17	6 22	6 28	6 33	6 39	6 44	6 50	6 55	7 1	7 6	7 12
0·9	15·1	16·9	31·1	7 4	7 1	7 16	7 22	7 28	7 34	7 41	7 47	7 53	7 59	8 6
1·0	15·0	17·0	31·0	7 51	7 57	8 4	8 11	8 18	8 25	8 32	8 38	8 45	8 52	8 59
1·1	14·9	17·1	30·9	8 38	8 45	8 52	8 59	9 7	9 15	9 23	9 30	9 38	9 45	9 53
1·2	14·8	17·2	30·8	9 24	9 32	9 40	9 48	9 56	10 4	10 13	10 21	10 29	10 37	10 46
1·3	14·7	17·3	30·7	10 10	10 19	10 28	10 37	10 46	10 55	11 4	11 13	11 22	11 31	11 40
1·4	14·6	17·4	30·6	10 57	11 6	11 16	11 25	11 35	11 45	11 55	12 4	12 14	12 23	12 33
1·5	14·5	17·5	30·5	11 43	11 53	12 4	12 14	12 24	12 34	12 45	12 55	13 5	13 15	13 26
1·6	14·4	17·6	30·4	12 29	12 41	12 51	13 2	13 13	13 24	13 35	13 46	13 57	14 8	14 19
1·7	14·3	17·7	30·3	13 15	13 27	13 39	13 50	14 2	14 14	14 26	14 37	14 49	15 0	15 12
1·8	14·2	17·8	30·2	14 1	14 13	14 26	14 38	14 50	15 2	15 15	15 27	15 39	15 52	16 4
1·9	14·1	17·9	30·1	14 47	15 0	15 13	15 26	15 39	15 52	16 5	16 18	16 31	16 44	16 57
2·0	14·0	18·0	30·0	15 32	15 45	15 59	16 12	16 26	16 40	16 54	17 8	17 22	17 35	17 49
2·1	13·9	18·1	29·9	16 17	16 31	16 46	17 0	17 15	17 29	17 44	17 58	18 13	18 27	18 42
2·2	13·8	18·2	29·8	17 2	17 17	17 32	17 47	18 2	18 17	18 33	18 48	19 3	19 18	19 34
2·3	13·7	18·3	29·7	17 47	18 2	18 18	18 34	18 50	19 6	19 22	19 38	19 54	20 10	20 26
2·4	13·6	18·4	29·6	18 31	18 47	19 4	19 20	19 37	19 54	20 11	20 27	20 44	21 1	21 18
2·5	13·5	18·5	29·5	19 16	19 33	19 50	20 7	20 25	20 42	21 0	21 17	21 35	21 52	22 10
2·6	13·4	18·6	29·4	20 0	20 18	20 36	20 54	21 12	21 30	21 49	22 7	22 25	22 43	23 1
2·7	13·3	18·7	29·3	20 44	21 2	21 21	21 40	21 59	22 18	22 37	22 55	23 14	23 33	23 52
2·8	13·2	18·8	29·2	21 27	21 46	22 6	22 25	22 45	23 5	23 25	23 44	24 4	24 23	24 43
2·9	13·1	18·9	29·1	22 10	22 30	22 51	23 11	23 31	23 51	24 12	24 32	24 52	25 12	25 33
3·0	13·0	19·0	29·0	22 53	23 14	23 35	23 56	24 17	24 38	24 59	25 20	25 41	26 2	26 23
3·1	12·9	19·1	28·9	23 36	23 57	24 19	24 40	25 2	25 24	25 46	26 7	26 29	26 51	27 13
3·2	12·8	19·2	28·8	24 18	24 40	25 2	25 24	25 47	26 9	26 32	26 54	27 17	27 39	28 2
3·3	12·7	19·3	28·7	25 0	25 22	25 45	26 8	26 32	26 55	27 19	27 42	28 5	28 28	28 52
3·4	12·6	19·4	28·6	25 41	26 4	26 28	26 52	27 16	27 40	28 5	28 29	28 53	29 17	29 41
3·5	12·5	19·5	28·5	26 22	26 46	27 10	27 35	28 0	28 25	28 50	29 15	29 40	30 5	30 30
3·6	12·4	19·6	28·4	27 3	27 27	27 52	28 17	28 43	29 9	29 35	30 0	30 26	30 52	31 18
3·7	12·3	19·7	28·3	27 43	28 8	28 34	29 0	29 27	29 53	30 20	30 46	31 13	31 39	32 6
3·8	12·2	19·8	28·2	28 22	28 48	29 15	29 42	30 9	30 36	31 4	31 31	31 58	32 25	32 53
3·9	12·1	19·9	28·1	29 1	29 28	29 56	30 24	30 52	31 20	31 48	32 16	32 44	33 12	33 40
4·0	12·0	20·0	28·0	29 40	30 8	30 37	31 5	31 34	32 2	32 31	33 0	33 29	33 58	34 27
+	+	−	−											

for Azimuths referred to the Disturbed Compass.

Apparent Azimuth from Neutral Position in points and decimals.				Modulus.										
				·70	·71	·72	·73	·74	·75	·76	·77	·78	·79	·80
				Corresponding Polar-Magnet-Deviation, in degrees and minutes.										
+ p	+ p	− p	− p	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
4·0	12·0	20·0	28·0	29 40	30 8	30 37	31 5	31 34	32 2	32 31	33 0	33 29	33 58	34 27
4·1	11·9	20·1	27·9	30 18	30 47	31 17	31 46	32 15	32 44	33 14	33 43	34 13	34 43	35 13
4·2	11·8	20·2	27·8	30 56	31 26	31 56	32 26	32 56	33 25	33 56	34 26	34 57	35 28	35 59
4·3	11·7	20·3	27·7	31 33	32 3	32 34	33 5	33 36	34 6	34 37	35 8	35 40	36 12	36 44
4·4	11·6	20·4	27·6	32 10	32 41	33 12	33 43	34 15	34 46	35 18	35 50	36 23	36 55	37 28
4·5	11·5	20·5	27·5	32 46	33 17	33 49	34 21	34 54	35 26	35 59	36 32	37 5	37 38	38 12
4·6	11·4	20·6	27·4	33 21	33 53	34 26	34 59	35 32	36 5	36 39	37 13	37 47	38 21	38 55
4·7	11·3	20·7	27·3	33 56	34 29	35 2	35 36	36 10	36 43	37 18	37 53	38 28	39 3	39 38
4·8	11·2	20·8	27·2	34 30	35 4	35 38	36 12	36 47	37 21	37 57	38 32	39 8	39 44	40 20
4·9	11·1	20·9	27·1	35 3	35 38	36 13	36 48	37 23	37 58	38 35	39 11	39 47	40 24	41 1
5·0	11·0	21·0	27·0	35 36	36 11	36 47	37 23	37 59	38 35	39 12	39 49	40 26	41 4	41 42
5·1	10·9	21·1	26·9	36 8	36 44	37 20	37 57	38 34	39 10	39 48	40 26	41 4	41 43	42 22
5·2	10·8	21·2	26·8	36 39	37 16	37 53	38 30	39 8	39 45	40 24	41 3	41 42	42 21	43 1
5·3	10·7	21·3	26·7	37 9	37 47	38 25	39 3	39 41	40 19	40 59	41 39	42 19	42 59	43 39
5·4	10·6	21·4	26·6	37 39	38 17	38 56	39 35	40 14	40 53	41 34	42 14	42 54	43 35	44 16
5·5	10·5	21·5	26·5	38 8	38 47	39 26	40 5	40 45	41 25	42 7	42 47	43 28	44 10	44 52
5·6	10·4	21·6	26·4	38 35	39 15	39 55	40 35	41 16	41 56	42 38	43 20	44 2	44 45	45 28
5·7	10·3	21·7	26·3	39 2	39 42	40 23	41 4	41 45	42 26	43 9	43 52	44 35	45 19	46 3
5·8	10·2	21·8	26·2	39 28	40 9	40 50	41 32	42 14	42 56	43 39	44 23	45 6	45 51	46 36
5·9	10·1	21·9	26·1	39 53	40 34	41 16	41 59	42 42	43 25	44 8	44 52	45 36	46 22	47 8
6·0	10·0	22·0	26·0	40 18	41 0	41 42	42 25	43 8	43 52	44 36	45 21	46 6	46 52	47 39
6·1	9·9	22·1	25·9	40 41	41 24	42 7	42 50	43 34	44 18	45 3	45 49	46 35	47 22	48 9
6·2	9·8	22·2	25·8	41 3	41 46	42 29	43 13	43 58	44 43	45 28	46 15	47 2	47 50	48 38
6·3	9·7	22·3	25·7	41 24	42 7	42 51	43 36	44 21	45 7	45 53	46 40	47 28	48 17	49 6
6·4	9·6	22·4	25·6	41 44	42 28	43 12	43 57	44 43	45 30	46 17	47 5	47 53	48 42	49 32
6·5	9·5	22·5	25·5	42 3	42 47	43 32	44 18	45 4	45 51	46 39	47 28	48 17	49 7	49 57
6·6	9·4	22·6	25·4	42 21	43 6	43 51	44 37	45 24	46 12	47 0	47 49	48 39	49 30	50 21
6·7	9·3	22·7	25·3	42 38	43 23	44 9	44 56	45 43	46 31	47 20	48 10	49 0	49 51	50 43
6·8	9·2	22·8	25·2	42 54	43 40	44 26	45 13	46 1	46 49	47 39	48 29	49 20	50 12	51 4
6·9	9·1	22·9	25·1	43 8	43 54	44 41	45 29	46 17	47 6	47 56	48 47	49 38	50 30	51 23
7·0	9·0	23·0	25·0	43 21	44 8	44 55	45 43	46 32	47 21	48 12	49 3	49 55	50 48	51 41
7·1	8·9	23·1	24·9	43 33	44 20	45 8	45 56	46 45	47 35	48 26	49 18	50 10	51 3	51 57
7·2	8·8	23·2	24·8	43 44	44 31	45 19	46 8	46 57	47 48	48 39	49 31	50 24	51 18	52 12
7·3	8·7	23·3	24·7	43 54	44 41	45 29	46 18	47 8	47 59	48 51	49 43	50 36	51 30	52 25
7·4	8·6	23·4	24·6	44 2	44 50	45 38	46 27	47 17	48 8	49 0	49 53	50 46	51 41	52 36
7·5	8·5	23·5	24·5	44 9	44 57	45 46	46 35	47 25	48 16	49 8	50 1	50 55	51 50	52 46
7·6	8·4	23·6	24·4	44 15	45 3	45 52	46 42	47 32	48 23	49 15	50 8	51 2	51 58	52 54
7·7	8·3	23·7	24·3	44 20	45 8	45 57	46 47	47 37	48 28	49 21	50 14	51 8	52 4	53 0
7·8	8·2	23·8	24·2	44 23	45 11	46 0	46 50	47 40	48 32	49 25	50 19	51 13	52 8	53 4
7·9	8·1	23·9	24·1	44 25	45 13	46 2	46 52	47 42	48 34	49 27	50 21	51 15	52 10	53 7
8·0	8·0	24·0	24·0	44 26	45 14	46 3	46 53	47 44	48 35	49 28	50 21	51 16	52 11	53 8
+	+	−	−											
Mean				27 17	27 44	28 11	28 39	29 7	29 35	30 4	30 33	31 2	31 32	32 2

VI. *A Second Memoir upon Quantics.* By ARTHUR CAYLEY, *Esq.*

Received April 14,—Read May 24, 1855.

THE present memoir is intended as a continuation of my Introductory Memoir upon Quantics, t. 144. (1854) p. 245, and must be read in connexion with it; the paragraphs of the two Memoirs are numbered continuously. The special subject of the present memoir is the theorem referred to in the Postscript to the Introductory Memoir, and the various developments arising thereout in relation to the number and form of the covariants of a binary quantic.

25. I have already spoken of asyzygetic covariants and invariants, and I shall have occasion to speak of irreducible covariants and invariants. Considering in general a function u determined like a covariant or invariant by means of a system of partial differential equations, it will be convenient to explain what is meant by an asyzygetic integral and by an irreducible integral. Attending for greater simplicity only to a single set $(a, b, c\ldots)$, which in the case of the covariants or invariants of a single function will be as before the coefficients or elements of the function, it is assumed that the system admits of integrals of the form $u=\mathrm{P}$, $u=\mathrm{Q}$, &c., or as we may express it, of integrals P, Q, &c., where P, Q, &c. are rational and integral homogeneous functions of the set $(a, b, c..)$, and moreover that the system is such that P, Q, &c. being integrals, $\phi(\mathrm{P}, \mathrm{Q}..)$ is also an integral. Then considering only the integrals which are rational and integral homogeneous functions of the set $(a, b, c..)$, integrals P, Q, R,.. not connected by any linear equation or syzygy (such as $\lambda\mathrm{P}+\mu\mathrm{Q}+\nu\mathrm{R}..=0$*), are said to be asyzygetic; but in speaking of the asyzygetic integrals of a particular degree, it is implied that the integrals are a system such that every other integral of the same degree can be expressed as a linear function (such as $\lambda\mathrm{P}+\mu\mathrm{Q}+\nu\mathrm{R}..$) of these integrals; and any integral P not expressible as a rational and integral homogeneous function of integrals of inferior degrees is said to be an irreducible integral.

26. Suppose now that $\mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3$, &c. denote the number of asyzygetic integrals of the degrees 1, 2, 3, &c. respectively, and let $\alpha_1, \alpha_2, \alpha_3$, &c. be determined by the equations

$$\mathrm{A}_1=\alpha_1$$

$$\mathrm{A}_2=\frac{1}{2}\alpha_1(\alpha_1+1)+\alpha_2$$

$$\mathrm{A}_3=\frac{1}{6}\alpha_1(\alpha_1+1)(\alpha_1+2)+\alpha_1\alpha_2+\alpha_3$$

$$\mathrm{A}_4=\frac{1}{24}\alpha_1(\alpha_1+1)(\alpha_1+2)(\alpha_1+3)+\frac{1}{2}\alpha_1(\alpha_1+1)\alpha_2+\alpha_1\alpha_3+\frac{1}{2}\alpha_2(\alpha_2+1)+\alpha_4, \text{ \&c.},$$

* It is hardly necessary to remark, that the multipliers $\lambda, \mu, \nu..$, and generally any coefficients or quantities not expressly stated to contain the set $(a, b, c..)$, are considered as independent of the set, or to use a convenient word, are considered as 'trivials.'

or what is the same thing, suppose that

$$1+A_1x+A_2x^2+\&c.=(1-x)^{-\alpha_1}(1-x^2)^{-\alpha_2}(1-x^3)^{-\alpha_3}\ldots$$

A little consideration will show that α_r represents the number of irreducible integrals of the degree r less the number of linear relations or syzygies between the composite or non-irreducible integrals of the same degree. In fact the asyzygetic integrals of the degree 1 are necessarily irreducible, *i. e.* $A_1=\alpha_1$. Represent for a moment the irreducible integrals of the degree 1 by X, X', &c., then the composite integrals X^2, XX', &c., the number of which is $\frac{1}{2}\alpha_1(\alpha_1+1)$, must be included among the asyzygetic integrals of the degree 2; and if the composite integrals in question were asyzygetic, there would remain $A_2-\frac{1}{2}\alpha_1(\alpha_1+1)$ for the number of irreducible integrals of the degree 2; but if there exist syzygies between the composite integrals in question, the number to be subtracted from A_2 will be $\frac{1}{2}\alpha_1(\alpha_1+1)$ less the number of these syzygies, and we shall have $A_2-\frac{1}{2}\alpha_1(\alpha_1+1)$, *i. e.* α_2 equal to the number of the irreducible integrals of the degree 2 less the number of syzygies between the composite integrals of the same degree. Again, suppose that α_2 is negative $=-\beta_2$, we may for simplicity suppose that there are no irreducible integrals of the degree 2, but that the composite integrals of this degree, X^2, XX', &c., are connected by β_2 syzygies, such as $\lambda X^2+\mu XX'+\&c.=0$, $\lambda_1X^2+\mu_1XX'+\&c.=0$. The asyzygetic integrals of the degree 4 include X^4, X^3X', &c., the number of which is $\frac{1}{24}\alpha_1(\alpha_1+1)(\alpha_1+2)(\alpha_1+3)$; but these composite integrals are not asyzygetic, they are connected by syzygies which are augmentatives of the β_2 syzygies of the second degree, viz. by syzygies such as

$$(\lambda X^2+\mu XX'..)X^2=0,\ (\lambda X^2+\mu XX'..)XX'=0\ \&c.\ (\lambda_1X^2+\mu_1XX'..)X^2=0,$$
$$(\lambda_1X^2+\mu_1XX'..)XX'=0,\ \&c.,$$

the number of which is $\frac{1}{2}\alpha_1(\alpha_1+1)\beta_2$. And these syzygies are themselves not asyzygetic, they are connected by secondary syzygies such as

$$\lambda_1(\lambda X^2+\mu XX'..)X^2+\mu_1(\lambda X^2+\mu XX'..)XX'+\&c.$$
$$-\lambda(\lambda_1X^2+\mu_1XX'..)X^2-\mu(\lambda_1X^2+\mu_1XX'..)XX'-\&c.=0,\ \&c.\ \&c.,$$

the number of which is $\frac{1}{2}\beta_2(\beta_2-1)$. The real number of syzygies between the composite integrals X^4, X^3X', &c. (*i. e.* of the syzygies arising out of the β_2 syzygies between X^2, XX', &c.), is therefore $\frac{1}{2}\alpha_1(\alpha_1+1)\beta_2-\frac{1}{2}\beta_2(\beta_2-1)$, and the number of integrals of the degree 4, arising out of the integrals and syzygies of the degrees 1 and 2 respectively, is therefore

$$\frac{1}{24}\alpha_1(\alpha_1+1)(\alpha_1+2)(\alpha_1+3)-\frac{1}{2}\alpha_1(\alpha_1+1)\beta_2+\frac{1}{2}\beta_2(\beta_2-1);$$

or writing $-\alpha_2$ instead of β_2, the number in question is

$$\frac{1}{24}\alpha_1(\alpha_1+1)(\alpha_1+2)(\alpha_1+3)+\frac{1}{2}\alpha_1(\alpha_1+1)\alpha_2+\frac{1}{2}\alpha_2(\alpha_2+1).$$

The integrals of the degrees 1 and 3 give rise to $\alpha_1\alpha_3$ integrals of the degree 4; and if all the composite integrals obtained as above were asyzygetic, we should have

$$A_4-\frac{1}{24}\alpha_1(\alpha_1+1)(\alpha_1+2)(\alpha_1+3)-\frac{1}{2}\alpha_1(\alpha_1+1)\alpha_2-\frac{1}{2}\alpha_2(\alpha_2+1)-\alpha_1\alpha_3,$$

i. e. α_4 as the number of irreducible integrals of the degree 4; but if there exist any further syzygies between the composite integrals, then α_4 will be the number of the irreducible integrals of the degree 4 less the number of such further syzygies, and the like reasoning is in all cases applicable.

27. It may be remarked, that for any given partial differential equation, or system of such equations, there will be always a finite number ν such that given ν independent integrals every other integral is a function (in general an irrational function only expressible as the root of an equation) of the ν independent integrals; and if to these integrals we join a single other integral not a rational function of the ν integrals, it is easy to see that every other integral will be a rational function of the $\overline{\nu+1}$ integrals; but every such other integral will not in general be a rational and integral function of the $\overline{\nu+1}$ integrals; and there is not in general any finite number whatever of integrals, such that every other integral is a rational and integral function of these integrals, *i. e.* the number of irreducible integrals is in general infinite; and it would seem that this is in fact the case in the theory of covariants.

28. In the case of the covariants, or the invariants of a binary quantic, A_2 is given (this will appear in the sequel) as the coefficient of x^n in the development, in ascending powers of x, of a rational fraction $\frac{\phi x}{fx}$, where fx is of the form

$$(1-x)^{\beta_1}(1-x^2)^{\beta_2}..(1-x^k)^{\beta_k},$$

and the degree of ϕx is less than that of fx. We have therefore

$$1+A_1x+A_2x^2+..=\frac{\phi x}{fx},$$

and consequently $\quad \phi x=(1-x)^{\beta_1-\alpha_1}(1-x^2)^{\beta_2-\alpha_2}..(1-x^k)^{\beta_k-\alpha_k}(1-x^{k+1})^{-\alpha_{k+1}}...$

Now every rational factor of a binomial $1-x^m$ is the irreducible factor of $1-x^{m'}$, where m' is equal to or a submultiple of m. Hence in order that the series α_1, α_2, α_3.. may terminate, ϕx must be made up of factors each of which is the irreducible factor of a binomial $1-x^m$, or if ϕx be itself irreducible, then ϕx must be the irreducible factor of a binomial $1-x^m$. Conversely, if ϕx be not of the form in question, the series α_1, α_2, α_3, &c. will go on *ad infinitum*, and it is easy to see that there is no point in the series such that the terms beyond that point are all of them negative, *i. e.* there will be irreducible covariants or invariants of indefinitely high degrees; and the

number of covariants or invariants will be infinite. The number of invariants is first infinite in the case of a quantic of the seventh order, or septic; the number of covariants is first infinite in the case of a quantic of the fifth order, or quintic.

29. Resuming the theory of binary quantics, I consider the quantic

$$(a, b, .. b', a' \between x, y)^m.$$

Here writing

$$\{y\partial_x\} = a\partial_b + 2b\partial_c .. + mb'\partial_{a'} = \mathrm{X}$$

$$\{x\partial_y\} = mb\partial_a + \overline{m-1}c\partial_b .. + a'\partial_{b'} = \mathrm{Y},$$

any function which is reduced to zero by each of the operations $\mathrm{X}-y\partial_x$, $\mathrm{Y}-x\partial_y$ is a covariant of the quantic. But a covariant will always be considered as a rational and integral function separately homogeneous in regard to the facients (x, y) and to the coefficients $(a, b, .. b', a')$. And the words order and degree will be taken to refer to the facients and to the coefficients respectively.

I commence by proving the theorem enunciated, No. 23. It follows at once from the definition, that the covariant is reduced to zero by the operation

$$\overline{\mathrm{X}-y\partial_x}.\overline{\mathrm{Y}-x\partial_y}-\overline{\mathrm{Y}-x\partial_y}.\overline{\mathrm{X}-y\partial_x},$$

which is equivalent to

$$\mathrm{X}.\mathrm{Y}-\mathrm{Y}.\mathrm{X}+y\partial_y-x\partial_x.$$

Now

$$\mathrm{X}.\mathrm{Y}=\mathrm{XY}+\mathrm{X}(\mathrm{Y})$$

$$\mathrm{Y}.\mathrm{X}=\mathrm{YX}+\mathrm{Y}(\mathrm{X}),$$

where XY and YX are equivalent operations, and

$$\mathrm{X}(\mathrm{Y}) = 1ma\partial_a + 2\overline{m-1}b\partial_b ... + m1b'\partial_{b'}$$

$$\mathrm{Y}(\mathrm{X}) = \qquad m1b\partial_b .. + 2\overline{m-1}b'\partial_{b'} + 1ma'\partial_{a'},$$

whence

$$\mathrm{X}(\mathrm{Y})-\mathrm{Y}(\mathrm{X}) = ma\partial_a + \overline{m-2}b\partial_b .. - \overline{m-2}b'\partial_{b'} - ma'\partial_{a'}$$

$$= k \text{ suppose},$$

and the covariant is therefore reduced to zero by the operation

$$k + y\partial_y - x\partial_x.$$

Now as regards a term $a^\alpha b^\beta .. b'^{\beta'} a'^{\alpha'} . x^i y^j$, we have

$$k = m\alpha + \overline{m-2}\beta .., -\overline{m-2}\beta' - m\alpha'$$

$$y\partial_y - x\partial_x = j - i;$$

and we see at once that for each term of the covariant we must have

$$m\alpha + \overline{m-2}\beta .. - \overline{m-2}\beta' - m\alpha' + j - i = 0,$$

i. e. if (x, y) are considered as being of the weights $\tfrac{1}{2}$, $-\tfrac{1}{2}$ respectively, and $(a, b, .. b', a')$ as being of the weights $-\tfrac{1}{2}m$, $-\tfrac{1}{2}m+1$, $..\tfrac{1}{2}m-1$, $\tfrac{1}{2}m$ respectively, then the weight of each term of the covariant is zero.

But if (x, y) are considered as being of the weights 1, 0 respectively, and $(a, b, \ldots b', a')$ as being of the weights $0, 1, \ldots \overline{m-1}, m$ respectively, then writing the equation under the form

$$m(\alpha+\beta\ldots+\beta'+\alpha')+j+i-2(\beta+\ldots+\overline{m-1}\beta'+m\alpha'+i)=0,$$

and supposing that the covariant is of the order μ and of the degree θ, each term of the covariant will be of the weight $\frac{1}{2}(m\theta+\mu)$.

I shall in the sequel consider the weight as reckoned in the last-mentioned manner. It is convenient to remark, that as regards any function of the coefficients of the degree θ and of the weight q, we have

$$X.Y-Y.X=m\theta-2q.$$

30. Consider now a covariant

$$(A, B, \ldots B', A' \between x, y)^{\mu}$$

of the order μ and of the degree θ; the covariant is reduced to zero by each of the operations $X-y\partial_x$, $Y-x\partial_y$, and we are thus led to the systems of equations

$$\begin{aligned}
XA &= 0\\
XB &= \mu A\\
XC &= \overline{\mu-1}B\\
&\cdot\\
&\cdot\\
XB' &= 2C'\\
XA' &= B'
\end{aligned}$$

and

$$\begin{aligned}
YA &= B\\
YB &= 2C\\
&\cdot\\
&\cdot\\
YC' &= \overline{\mu-1}B'\\
YB' &= \mu A'\\
YA' &= 0.
\end{aligned}$$

Conversely if these equations are satisfied the function will be a covariant.

I assume that A is a function of the degree θ and of the weight $\frac{1}{2}(m\theta-\mu)$, satisfying the condition

$$XA=0.$$

And I represent by YA, Y^2A, Y^3A, &c. the results obtained by successive operations with Y upon the function A. The function Y^sA will be of the degree θ and of the weight $\frac{1}{2}(m\theta-\mu)+s$. And it is clear that in the series of terms YA, Y^2A, Y^3A, &c., we must at last come to a term which is equal to zero. In fact, since m is the greatest weight of any coefficient, the weight of Y^s is at most equal to $m\theta$, and therefore if $\frac{1}{2}(m\theta-\mu)+s>m\theta$, or $s>\frac{1}{2}(m\theta+\mu)$, we must have $Y^sA=0$.

Now writing for greater simplicity XY instead of X.Y, and so in similar cases, we have, as regards Y^sA,

$$XY-YX=\mu-2s.$$

Hence

$$(XY-YX)A=\mu A,$$

and consequently

$$XYA=YXA+\mu A=\mu A.$$

Similarly

$$(XY-YX)YA=\overline{\mu-2}YA,$$

and therefore

$$\begin{aligned}XY^2A&=YXYA+\overline{\mu-2}YA\\&=\mu YA+\overline{\mu-2}YA=2(\mu-1)YA.\end{aligned}$$

And again,

$$(XY-YX)Y^2A=\overline{\mu-4}Y^2A,$$

and therefore

$$\begin{aligned}XY^3A&=YXY^2A+\overline{\mu-4}Y^2A\\&=2\overline{\mu-1}Y^2A+\overline{\mu-4}Y^2A=3(\mu-2)Y^2A.\end{aligned}$$

or generally

$$XY^sA=s(\mu-s+1)Y^sA.$$

Hence putting $s=\mu+1$, $\mu+2$, &c., we have

$$\begin{aligned}&XY^{\mu+1}A=0\\&XY^{\mu+2}A=-(\mu+2)1.Y^{\mu+1}A\\&XY^{\mu+3}A=-(\mu+3)2.Y^{\mu+2}A\\&\text{\&c.,}\end{aligned}$$

equations which show that

$$Y^{\mu+1}A=0\,;$$

for unless this be so, *i. e.* if $Y^{\mu+1}A\neq0$, then from the second equation $XY^{\mu+2}A\neq0$, and therefore $Y^{\mu+2}A\neq0$, from the third equation $XY^{\mu+3}\neq0$, and therefore $Y^{\mu+3}A\neq0$, and so on *ad infinitum*, *i.e.* we must have $Y^{\mu+1}A=0$.

31. The suppositions which have been made as to the function A, give therefore the equations

$$\begin{aligned}&XA=0\\&XYA=\mu A\\&XY^2A=2(\mu-1)YA\\&\quad\vdots\\&XY^\mu A=\mu Y^{\mu-1}A\\&Y^{\mu+1}A=0.\end{aligned}$$

And if we now assume

$$B=YA,\ C=\tfrac{1}{2}YB,\ ..A'=\frac{1}{\mu}YB',$$

the system becomes

$$\begin{aligned} XA &= 0 \\ XB &= \mu A \\ XC &= \overline{\mu-1}B \\ &\cdot \\ &\cdot \\ XA' &= B' \\ YA' &= 0\,; \end{aligned}$$

so that the entire system of equations which express that $(A, B..B', A' \between x, y)^\mu$ is a covariant is satisfied; hence

Theorem. Given a quantic $(a, b, ..b', a' \between x, y)^m$; if A be a function of the coefficients of the degree θ and of the weight $\frac{1}{2}(m\theta-\mu)$ satisfying the condition $XA=0$, and if B, C, ..B', A' are determined by the equations

$$B=YA, \quad C=\tfrac{1}{2}YB, \quad ..A'=\frac{1}{\mu}YB',$$

then will

$$(A, B, ..B', A' \between x, y)^\mu$$

be a covariant.

In particular, a function A of the degree θ and of the weight $\frac{1}{2}m\theta$, satisfying the condition $XA=0$, will (also satisfy the equation $YA=0$ and will) be an invariant.

32. I take now for A the most general function of the coefficients, of the degree θ and of the weight $\frac{1}{2}(m\theta-\mu)$; then XA is a function of the degree θ and of the weight $\frac{1}{2}(m\theta-\mu)-1$, and the arbitrary coefficients in the function A are to be determined so that $XA=0$. The number of arbitrary coefficients is equal to the number of terms in A, and the number of the equations to be satisfied is equal to the number of terms in XA; hence the number of the arbitrary coefficients which remains indeterminate is equal to the number of terms in A less the number of terms in XA; and since the covariant is completely determined when the leading coefficient is known, the difference in question is equal to the number of the asyzygetic covariants, *i. e.* the number of the asyzygetic covariants of the order μ and the degree θ is equal to the number of terms of the degree θ and weight $\frac{1}{2}(m\theta-\mu)$, less the number of terms of the degree θ and weight $\frac{1}{2}(m\theta-\mu)-1$.

33. I shall now give some instances of the calculation of covariants by the method just explained. It is very convenient for this purpose to commence by forming the literal parts by ARBOGAST's Method of Derivations: we thus form tables such as the following:—

a	b	c

a^2	ab	ac b^2	bc	b^2

a	b	c	d

a^2	ab	ac b^2	ad bc	bd c^2	cd	d^2

a^3	a^2b	a^2c ab^2	a^2d abc b^3	abd ac^2 b^2c	acd b^2d bc^2	ad^2 bcd c^3	bd^2 c^2d	cd^2	d^3

a^4	a^3b	a^3c a^2b^2	a^3d a^2bc ab^3	a^2bd a^2c^2 ab^2c b^4	a^2cd ab^2d abc^2 b^3c	a^2d^2 $abcd$ ac^3 b^3d b^2c^2	abd^2 ac^2d b^2cd bc^3	acd^2 b^2d^2 bc^2d c^4	ad^3 bcd^2 c^3d	bd^3 c^2d^2	cd^3	d^4

a	b	c	d	e

a^2	ab	ac b^2	ad bc	ae bd c^2	be cd	bd c^2	cd	d^2

a^3	a^2b	a^2c ab^2	a^2d abc b^3	a^2e abd ac^2 b^2c	abe acd b^2d bc^2	ace ad^2 b^2e bcd c^3	ade bce bd^2 c^2d	ae^2 bde c^2e cd^2	be^2 cde d^3	ce^2 d^2e	de^2	e^3

34. Thus in the case of a cubic $(a, b, c, d\between x, y)^3$, the tables show that there will be a single invariant of the degree 4. Represent this by

$$\begin{aligned}&\mathrm{A}a^2d^2\\ &+\mathrm{B}abcd\\ &+\mathrm{C}ac^3\\ &+\mathrm{D}b^3d\\ &+\mathrm{E}b^2c^2,\end{aligned}$$

which is to be operated upon with $a\partial_b+2b\partial_c+3c\partial_d$. This gives

$+\mathrm{B}$		$+6\mathrm{A}$	a^2cd
$+3\mathrm{D}$	$+2\mathrm{B}$		ab^2d
$+2\mathrm{E}$	$+6\mathrm{C}$	$+3\mathrm{B}$	abc^2
	$+4\mathrm{E}$	$+3\mathrm{D}$	b^3c

i. e. $B+6A=0$, $3D+2B=0$, &c.; or putting $A=1$, we find $B=-6$, $C=4$, $D=4$, $E=-3$, and the invariant is

$$\begin{array}{l} a^2d^2 \\ -6abcd \\ +4ac^3 \\ +4b^3d \\ -3b^2c^2. \end{array}$$

Again, there is a covariant of the order 3 and the degree 3. The coefficient of x^3 or leading coefficient is

$$\begin{array}{l} \mathrm{A}a^2d \\ +\mathrm{B}abc \\ +\mathrm{C}b^3, \end{array}$$

which operated upon with $a\partial_b+2b\partial_c+3c\partial_d$, gives

$+\mathrm{B}$		$+3\mathrm{A}$	a^2c
$+3\mathrm{C}$	$+2\mathrm{B}$		ab^2

i. e. $B+3A=0$, $3C+2B=0$; or putting $A=1$, we have $B=-3$, $C=2$, and the leading coefficient is

$$\begin{array}{l} a^2d \\ -3abc \\ +2b^3. \end{array}$$

The coefficient of x^2y is found by operating upon this with $(3b\partial_a+2c\partial_b+d\partial_c)$, this gives

$+6$		-3	abd
	-6		ac^2
-9	$+12$		b^2c

i. e. the required coefficient of x^2y is

$$\begin{array}{l} 3abd \\ -6ac^2 \\ +3b^2c; \end{array}$$

and by operating upon this with $\frac{1}{2}(3b\partial_a+2c\partial_b+d\partial_c)$, we have for the coefficient of xy^2.

	$+3$	-6	acd
$+\frac{9}{2}$		$+\frac{3}{2}$	b^2d
-9	$+6$		bc^2

i. e. the coefficient of xy^2 is

$$\begin{array}{l} -3acd \\ +6b^2d \\ -3bc^2. \end{array}$$

Finally, operating upon this with $\frac{1}{3}(3b\partial_a+2c\partial_b+d\partial_c)$, we have for the coefficient of y^3,

		-1	ad^2
-3	$+8$	-2	bcd
	-2		c^3

i. e. the coefficient of y^3 is

$$\begin{array}{l} -ad^2 \\ +3bcd \\ -2c^3, \end{array}$$

and the covariant is

$$\left(\begin{array}{c|c|c|c} a^2d & 3abd & -3acd & -ad^2 \\ -3abc & -6ac^2 & +6b^2d & +3bcd \\ +2b^3 & +3b^2c & -3bc^2 & -2c^3 \end{array}\right)(x, y)^3$$

I have worked out the example in detail as a specimen of the most convenient method for the actual calculation of more complicated covariants *.

35. The number of terms of the degree θ and of the weight q is obviously equal to the number of ways in which q can be made up as a sum of θ terms with the elements $(0, 1, 2, \ldots m)$, a number which is equal to the coefficient of $x^q z^\theta$ in the development of

$$\frac{1}{(1-z)(1-xz)(1-x^2z)\ldots(1-x^mz)};$$

and the number of the asyzygetic covariants of any particular degree for the quantic

* Note added Feb. 7, 1856.—The following method for the calculation of an invariant or of the leading coefficient of a covariant, is easily seen to be identical in principle with that given in the text. Write down all the terms of the weight next inferior to that of the invariant or leading coefficient, and operate on each of these separately with the symbol

$$\text{ind. } b\cdot\frac{b}{a}+2\text{ ind. } c\cdot\frac{c}{b}\cdots+\overline{m-1}\text{ ind. } b'\cdot\frac{b'}{a'},$$

where we are first to multiply by the fraction, rejecting negative powers, and then by the index of the proper letter in the term so obtained. Equating the results to zero, we obtain equations between the terms of the invariant or leading coefficient, and replacing in these equations each term by its numerical coefficient in the invariant or leading coefficient, we have the equations of connexion of these numerical coefficients. Thus, for the discriminant of a cubic, the terms of the next inferior weight are a^2cd, ab^2d, abc^2, b^3c, and operating on each of these separately with the symbol

$$\text{ind. } b\cdot\frac{b}{a}+2\text{ ind. } c\cdot\frac{c}{b}+3\text{ ind. } d\cdot\frac{d}{c},$$

we find

$abcd$		$+6\ a^2d^2$
$3\ b^3d$	$+2\ abcd$	
$2\ b^2c^2$	$+6\ ac^3$	$+3\ abcd$
	$+4\ b^2c^2$	$+3\ b^3d$

and equating the horizontal lines to zero, and assuming $a^2d^2=1$, we have $a^2d^2=1$, $abcd=-6$, $ac^3=4$, $b^3d=4$, $b^2c^2=-3$, or the value of the discriminant is that given in the text.

$(*\between x, y)^m$ can therefore be determined by means of this development. In the case of a cubic, for example, the function to be developed is

$$\frac{1}{(1-z)(1-xz)(1-x^2z)(1-x^3z)},$$

which is equal to

$$1+z(1+x+x^2+x^3)+z^2(1+x+2x^2+2x^3+2x^4+x^5+x^6)+\&c.,$$

where the coefficients are given by the following table; on account of the symmetry, the series of coefficients for each power of z is continued only to the middle term or middle of the series.

									1	(0)
								1	1	(1)
						1	1	2	2	(2)
					1	1	2	3	3	(3)
			1	1	2	3	4	4	5	(4)
		1	1	2	3	4	5	6	6	(5)
1	1	2	3	4	5	7	7	8	8	(6)

and from this, by subtracting from each coefficient the coefficient which immediately precedes it, we form the table

									1	(0)
								1	0	(1)
						1	0	1	0	(2)
					1	0	1	1	0	(3)
			1	0	1	1	1	0	1	(4)
		1	0	1	1	1	1	1	0	(5)
1	0	1	1	1	1	2	0	1	0	(6)

The successive lines fix the number and character of the covariants of the degrees 0, 1, 2, 3, &c. The line (0), if this were to be interpreted, would show that there is a single covariant of the degree 0; this covariant is of course merely the absolute constant unity, and may be excluded. The line (1) shows that there is a single covariant

of the degree 1, viz. a covariant of the order 3; this is the cubic itself, which I represent by U. The line (2) shows that there are two asyzygetic covariants of the degree 2, viz. one of the order 6, this is merely U^2, and one of the order 2, this I represent by H. The line (3) shows that there are three asyzygetic covariants of the degree 3, viz. one of the order 9, this is U^3; one of the order 5, this is UH, and one of the order 3, this I represent by Φ. The line (4) shows that there are five asyzygetic covariants of the degree 4, viz. one of the order 12, this is U^4; one of the order 8, this is U^2H; one of the order 6, this is H^2; and one of the order 0, *i. e.* an invariant, this I represent by ∇. The line (5) shows that there are six asyzygetic covariants of the degree 5, viz. one of the order 15, this is U^5; one of the order 11, this is U^3H; one of the order 9, this is $U^2\Phi$; one of the order 7, this is UH^2; one of the order 5, this is $H\Phi$; and one of the order 3, this is ∇U. The line (6) shows that there are 8 asyzygetic covariants of the degree 6, viz. one of the order 18, this is U^6; one of the order 14, this is U^4H; one of the order 12, this is $U^3\Phi$; one of the order 10, this is U^2H^2; one of the order 8, this is $UH\Phi$; two of the order 6 (*i. e.* the three covariants H^3, Φ^2 and ∇U^2 are not asyzygetic, but are connected by a single linear equation or syzygy), and one of the order 2, this is ∇H. We are thus led to the irreducible covariants U, H, Φ, ∇ connected by a linear equation or syzygy between H^3, Φ^2 and ∇U^2, and this is in fact the complete system of irreducible covariants; ∇ is therefore the only invariant.

36. The asyzygetic covariants are of the form $U^pH^q\nabla^r$, or else of the form $U^pH^q\nabla^r\Phi$; and since U, H, ∇ are of the degrees 1, 2, 4 respectively, and Φ is of the degree 3, the number of asyzygetic covariants of the degree m of the first form is equal to the coefficient of x^m in $1\div(1-x)(1-x^2)(1-x^4)$, and the number of the asyzygetic covariants of the degree m of the second form is equal to the coefficient of x^m in $x^3\div(1-x)(1-x^2)(1-x^4)$. Hence the total number of asyzygetic covariants is equal to the coefficient of x^m in $(1+x^3)\div(1-x)(1-x^2)(1-x^4)$, or what is the same thing, in

$$\frac{1-x^6}{(1-x)(1-x^2)(1-x^3)(1-x^4)};$$

and conversely, if this expression for the number of the asyzygetic covariants of the degree m were established independently, it would follow that the irreducible invariants were four in number, and of the degrees 1, 2, 3, 4 respectively, but connected by an equation of the degree 6. As regards the invariants, every invariant is of the form ∇^p, *i. e.* the number of asyzygetic invariants of the degree m is equal to the coefficient of x^m in $\frac{1}{1-x^4}$, and conversely, from this expression it would follow that there was a single irreducible invariant of the degree 4.

37. In the case of a quartic, the function to be developed is

$$\frac{1}{(1-z)(1-xz)(1-x^2z)(1-x^3z)(1-x^4z)},$$

and the coefficients are given by the table.

												1	(0)
										1	1	1	(1)
								1	1	2	2	3	(2)
						1	1	2	3	4	4	5	(3)
				1	1	2	3	5	5	7	7	8	(4)
		1	1	2	3	5	6	8	9	11	11	12	(5)
1	1	2	3	5	6	9	10	13	14	16	16	18	(6)

And subtracting from each coefficient the coefficient immediately preceding it, we have the table—

												1	(0)
										1	0	0	(1)
								1	0	1	0	1	(2)
						1	0	1	1	1	0	1	(3)
				1	0	1	1	2	0	2	0	1	(4)
		1	0	1	1	2	1	2	1	2	0	1	(5)
1	0	1	1	2	1	3	1	3	1	2	0	2	(6)

the examination of which will show that we have for the quartic the following irreducible covariants, viz. the quartic itself U; an invariant of the degree 2, which I represent by I; a covariant of the order 4 and of the degree 2, which I represent by H; an invariant of the degree 3, which I represent by J; and a covariant of the order 6 and the degree 3, which I represent by Φ; but that the irreducible covariants are connected by an equation of the degree 6, viz. there is a linear equation or syzygy between Φ^2, I^3H^3, I^2JH^2U, IJ^2HU^2 and J^3U^3; this is in fact the complete system of the irreducible covariants of the quartic: the only irreducible invariants are the invariants I, J.

38. The asyzygetic covariants are of the form $U^pI^qH^rJ^s$, or else of the form $U^pI^qH^rJ^s\Phi$, and the number of the asyzygetic covariants of the degree m is equal to the coefficient of x^m in $(1+x^3)\div(1-x)(1-x^2)^2(1-x^3)$, or what is the same thing, in

$$\frac{1-x^6}{(1-x)(1-x^2)^2(1-x^3)^2}.$$

and the asyzygetic invariants are of the form I^pJ^q, and the number of the asyzygetic invariants of the degree m is equal to the coefficient of x^m in $1\div(1-x^2)(1-x^3)$. Conversely, if these formulæ were established, the preceding results as to the form of the system of the irreducible covariants or of the irreducible invariants, would follow.

39. In the case of a quintic, the function to be developed is

$$\frac{1}{(1-z)(1-xz)(1-x^2z)(1-x^3z)(1-x^4z)(1-x^5z)};$$

and the coefficients are given by the table:

												1	(0)
										1	1	1	(1)
							1	1	2	2	3	3	(2)
					1	1	2	3	4	5	6	6	(3)
		1	1	2	3	5	6	8	9	11	11	12	(4)
1	1	2	3	5	7	9	11	14	16	18	19	20	(5)
													⋮

and subtracting from each coefficient the one which immediately precedes it, we have the table:

												1	(0)
										1	0	0	(1)
							1	0	1	0	1	0	(2)
					1	0	1	1	1	1	1	0	(3)
		1	0	1	1	2	1	2	1	2	0	1	(4)
1	0	1	1	2	2	2	2	3	2	2	1	1	(5)
													⋮

We thus obtain the following irreducible covariants, viz.—

Of the degree 1; a single covariant of the order 5, this is the quintic itself.

Of the degree 2; two covariants, viz. one of the order 6, and one of the order 2.

Of the degree 3; three covariants, viz. one of the order 9, one of the order 5, and one of the order 3.

Of the degree 4; three covariants, viz. one of the order 6, one of the order 4, and one of the order 0 (an invariant).

Of the degree 5; three covariants, viz. one of the order 7, one of the order 3, and one of the order 1 (a linear covariant).

And these covariants are connected by a single syzygy of the degree 5 and of the order 11; in fact, the table shows that there are only two asyzygetic covariants of this degree and order; but we may, with the above-mentioned irreducible covariants of the degrees 1, 2, 3 and 4, form three covariants of the degree 5 and the order 11; there is therefore a syzygy of this degree and order.

40. I represent the number of ways in which q can be made up as a sum of m terms with the elements $0, 1, 2, \dots m$, each element being repeatable an indefinite number of times by the notation

$$P(0, 1, 2, .. m)^\theta q,$$

and I write for shortness

$$P'(0, 1, 2, .. m)^\theta q = P(0, 1, 2 .. m)^\theta q - P(0, 1, 2 \dots m)^\theta q - 1.$$

Then for a quantic of the order m, the number of asyzygetic covariants of the degree θ and of the order μ is

$$P'(0, 1, 2 \dots m)^\theta \tfrac{1}{2}(m\theta - \mu).$$

In particular, the number of asyzygetic invariants of the degree θ is

$$P'(0, 1, 2 .. m)^\theta \tfrac{1}{2} m\theta.$$

To find the total number of the asyzygetic covariants of the degree θ, suppose first that $m\theta$ is even; then, giving to μ the successive values $0, 2, 4, .. m\theta$, the required number is

$$\begin{aligned} &P(\tfrac{1}{2}m\theta) \quad\ \ - P(\tfrac{1}{2}m\theta - 1) \\ &+ P(\tfrac{1}{2}m\theta - 1) - P(\tfrac{1}{2}m\theta - 2) \\ &\quad \vdots \\ &+ P(2) \qquad\quad - P(1) \\ &+ P(1) \\ &= P(\tfrac{1}{2}m\theta), \end{aligned}$$

i. e. when $m\theta$ is even, the number of the asyzygetic covariants of the degree θ is

$$P(0, 1, 2 .. m)^\theta \tfrac{1}{2} m\theta;$$

and similarly, when $m\theta$ is odd, the number of the asyzygetic covariants of the degree θ is

$$P(0, 1, 2, .. m)^\theta \tfrac{1}{2}(m\theta - 1).$$

But the two formulæ may be united into a single formula; for when $m\theta$ is odd $\tfrac{1}{2}m\theta$ is a fraction, and therefore $P(\tfrac{1}{2}m\theta)$ vanishes, and so when $m\theta$ is even $\tfrac{1}{2}(m\theta - 1)$ is a fraction, and $P\tfrac{1}{2}(m\theta - 1)$ vanishes; we have thus the theorem, that for a quantic of the order m,—

The number of the asyzygetic covariants of the degree θ is

$$P(0, 1, 2 \dots m)^\theta \tfrac{1}{2} m\theta + P(0, 1, 2, .. m)^\theta \tfrac{1}{2}(m\theta - 1).$$

41. The functions $P\tfrac{1}{2}m\theta$, &c. may, by the method explained in my "Researches

on the Partition of Numbers," *post.* p. 34, be determined as the coefficients of x^θ in certain functions of x; I have calculated the following particular cases:—
Putting, for shortness,

$$P'(0, 1, 2, ..m)^\theta \tfrac{1}{2}m\theta = \text{coefficient } x^\theta \text{ in } \phi m,$$

then
$$\phi 2=\frac{1}{1-x^2}$$
$$\phi 3=\frac{1}{1-x^4}$$
$$\phi 4=\frac{1}{(1-x^2)(1-x^3)}$$
$$\phi 5=\frac{1-x^6+x^{12}}{(1-x^4)(1-x^6)(1-x^8)}$$
$$\phi 6=\frac{(1-x)(1+x-x^3-x^4-x^5+x^7+x^8)}{(1-x^2)^2(1-x^3)(1-x^4)(1-x^5)}$$
$$\phi 7=\frac{1-x^6+2x^8-x^{10}+5x^{12}+2x^{14}+6x^{16}+2x^{18}+5x^{20}-x^{22}+2x^{24}-x^{26}+x^{32}}{(1-x^4)(1-x^6)(1-x^8)(1-x^{10})(1-x^{12})}$$
$$\phi 8=\frac{(1-x)(1+x-x^3-x^4+x^6+x^7+x^8+x^9+x^{10}-x^{13}+x^{15}+x^{16})}{(1-x^2)^2(1-x^3)^2(1-x^4)(1-x^5)(1-x^7)};$$

$$P(0, 1, 2, ..m)^\theta \tfrac{1}{2}m\theta = \text{coefficient of } x^\theta \text{ in } \psi m,$$

then
$$\psi 2=\frac{1}{(1-x)(1-x^2)}$$
$$\psi 3=\frac{1+x^4}{(1-x^2)^2(1-x^4)}$$
$$\psi 4=\frac{1-x+x^2}{(1-x)^2(1-x^2)(1-x^3)}$$
$$\psi 5=\frac{1+x^2+6x^4+9x^6+12x^8+9x^{10}+6x^{12}+x^{14}+x^{16}}{(1-x^2)^2(1-x^4)(1-x^6)(1-x^8)};$$

$$P(0, 1, 2, ..m)^\theta \tfrac{1}{2}(m\theta-1) = \text{coefficient of } x \quad \text{n } \psi_{,} m,$$

then
$$\psi_{,}3=\frac{x+x^3}{(1-x^2)^2(1-x^4)}$$
$$\psi_{,}5=\frac{x+4x^3+8x^5+10x^7-10x^9+8x^{11}+4x^{13}+x^{15}}{(1-x^2)^2(1-x^4)(1-x^6)(1-x^8)}.$$

And from what has preceded, it appears that for a quantic of the order m, the number of asyzygetic covariants of the degree θ is for m even, coefficient x^θ in ψm, and for m odd, coefficient x^θ in $(\psi m+\psi_{,}m)$; and that the number of asyzygetic invariants of the degree θ is coefficient x^θ in ϕm. Attending first to the invariants,—

42. For a quadric, the number of asyzygetic invariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{1}{1-x^2},$$

which leads to the conclusion that there is a single irreducible invariant of the degree 2.

43. For a cubic, the number of asyzygetic invariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{1}{1-x^4},$$

i. e. there is a single irreducible invariant of the degree 4.

44. For a quartic, the number of asyzygetic invariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{1}{(1-x^2)(1-x^3)},$$

i. e. there are two irreducible invariants of the degrees 2 and 3 respectively.

45. For a quintic, the number of asyzygetic invariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{1-x^6+x^{12}}{(1-x^4)(1-x^6)(1-x^8)}.$$

The numerator is the irreducible factor of $1-x^{36}$, *i. e.* it is equal to $(1-x^{36})(1-x^6) \div (1-x^{18})(1-x^{12})$; and substituting this value, the number becomes

$$\text{coefficient } x^\theta \text{ in } \frac{1-x^{36}}{(1-x^4)(1-x^8)(1-x^{12})(1-x^{18})},$$

i. e. there are in all four irreducible invariants, which are of the degrees 4, 8, 12 and 18 respectively; but these are connected by an equation of the degree 36, *i. e.* the square of the invariant of the degree 18 is a rational and integral function of the other three invariants; a result, the discovery of which is due to M. HERMITE.

46. For a sextic, the number of asyzygetic invariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{(1-x)(1+x-x^3-x^4-x^5+x^7+x^8)}{(1-x^2)^2(1-x^3)(1-x^4)(1-x^5)}:$$

the second factor of the numerator is the irreducible factor of $1-x^{30}$, *i. e.* it is equal to $(1-x^{30})(1-x^5)(1-x^3)(1-x^2) \div (1-x^{15})(1-x^{10})(1-x^6)(1-x)$; and substituting this value, the number becomes

$$\text{coefficient } x^{30} \text{ in } \frac{1-x^{30}}{(1-x^2)(1-x^4)(1-x^6)(1-x^{10})(1-x^{15})},$$

i. e. there are in all five irreducible invariants, which are of the degrees 2, 4, 6, 10 and 15 respectively; but these are connected by an equation of the degree 30, *i. e.* the square of the invariant of the degree 15 is a rational and integral function of the other four invariants.

47. For a septic, the number of asyzygetic invariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{1-x^6+2x^8-x^{10}+5x^{12}+2x^{14}+6x^{16}+2x^{18}+5x^{20}-x^{22}+2x^{24}-x^{26}+x^{32}}{(1-x^4)(1-x^6)(1-x^8)(1-x^{10})(1-x^{12})},$$

the numerator is equal to

$$(1-x^6)(1-x^8)^{-2}(1-x^{10})(1-x^{12})^{-5}(1-x^{14})^{-4}\ldots,$$

where the series of factors does not terminate; hence the number of irreducible inva-

riants is infinite; substituting the preceding value, the number of asyzygetic invariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } (1-x^4)^{-1}(1-x^8)^{-3}(1-x^{12})^{-6}(1-x^{14})^{-4}\ldots$$

The first four indices give the number of irreducible invariants of the corresponding degrees, *i. e.* there are 1, 3, 6 and 4 irreducible invariants of the degrees 4, 8, 12 and 14 respectively, but there is no reason to believe that the same thing holds with respect to the indices of the subsequent terms. To verify this it is to be remarked, that there are 1, 4, 10 and 4 asyzygetic invariants of the degrees in question respectively; there is therefore one irreducible invariant of the degree 4; calling this X_4, there is only one composite invariant of the degree 8, viz. X_4^2; there are therefore three irreducible invariants of this degree, say X_8, X_8', X_8''. The composite invariants of the degree 12 are four in number, viz. X_4^3, X_4X_8, X_4X_8', X_4X_8'', *and these cannot be connected by any syzygy*, for if they were so, X_4^2, X_8, X_8', X_8'' would be connected by a syzygy, or there would be less than 3 irreducible invariants of the degree 8. Hence there are precisely 6 irreducible invariants of the degree 12. And since the irreducible invariants of the degrees 4, 8 and 12 do not give rise to any composite invariant of the degree 14, there are precisely 4 irreducible invariants of the degree 14.

48. For an octavic, the number of the asyzygetic invariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{(1-x)(1+x-x^3-x^4+x^6+x^7+x^8+x^9+x^{10}-x^{12}-x^{13}+x^{15}+x^{16})}{(1-x^2)^2(1-x^3)^2(1-x^4)(1-x^5)(1-x^7)};$$

and the second factor of the numerator is

$$(1-x)^{-1}(1-x^2)(1-x^3)^{-1}(1-x^6)^{-1}(1-x^8)^{-1}(1-x^9)^{-1}(1-x^{10})^{-1}(1-x^{16})(1-x^{17})(1-x^{18})\ldots,$$

where the series of factors does not terminate, hence the number of irreducible invariants is infinite. Substituting the preceding value, the number of the asyzygetic invariants of the degree θ is

$$\text{coeff. } x^\theta \text{ in } (1-x^2)^{-1}(1-x^3)^{-1}(1-x^4)^{-1}(1-x^5)^{-1}(1-x^6)^{-1}(1-x^7)^{-1}(1-x^8)^{-1}(1-x^9)^{-1}(1-x^{10})^{-1}(1-x^{16})(1-x^{17})(1-x^{18}),.$$

There is certainly one, and only one irreducible invariant for each of the degrees 2, 3, 4, 5 and 6 respectively; but the formula does not show the number of the irreducible invariants of the degrees 7, &c.; in fact, representing the irreducible invariants of the degrees 2, 3, 4, 5 and 6 by X_2, X_3, X_4, X_5, X_6, these give rise to 3 composite invariants of the degree 7, viz. $X_2X_2X_3$, X_2X_5, X_3X_4, which may or may not be connected by a syzygy; if they are not connected by a syzygy, there will be a single irreducible invariant of the degree 7; but if they are connected by a syzygy, there will be two irreducible invariants of the degree 7; it is useless at present to pursue the discussion further.

Considering next the covariants,—

49. For a quadric, the number of asyzygetic covariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{1}{(1-x)(1-x^2)},$$

i. e. there are two irreducible covariants of the degrees 1 and 2 respectively; these are of course the quadric itself and the invariant.

50. For a cubic, the number of the asyzygetic covariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{(1+x)(1+x^2)}{(1-x^2)^2(1-x^4)}.$$

The first factor of the numerator is the irreducible factor of

$$1-x^2, =(1-x^2)\div(1-x),$$

and the second factor of the numerator is the irreducible factor of

$$1-x^4, =(1-x^4)\div(1-x^2);$$

substituting these values, the number is

$$\text{coefficient } x^\theta \text{ in } \frac{1-x^6}{(1-x)(1-x^2)(1-x^3)(1-x^4)},$$

i. e. there are 4 irreducible covariants of the degrees 1, 2, 3, 4 respectively; but these are connected by an equation of the degree 6; the covariant of the degree 1 is the cubic itself U, the other covariants are the covariants already spoken of and represented by the letters H, Φ and ∇ respectively (H is of the degree 2 and the order 3, Φ of the degree 3 and the order 3, and ∇ is of the degree 4 and the order 0, *i. e.* it is an invariant).

51. For a quartic, the number of the asyzygetic covariants of the degree θ is

$$\text{coefficient } x^\theta \text{ in } \frac{1-x+x^2}{(1-x)^2(1-x^2)(1-x^3)},$$

the numerator of which is the irreducible factor of $1-x^6$, *i. e.* it is equal to $(1-x^6)(1-x)\div(1-x^2)(1-x^3)$. Making this substitution, the number is

$$\text{coefficient } x^\theta \text{ in } \frac{1-x^6}{(1-x)(1-x^2)^2(1-x^3)^2},$$

i. e. there are five irreducible covariants, one of the degree 1, two of the degree 2, and two of the degree 3, but these are connected by an equation of the degree 6. The irreducible covariant of the degree 1 is of course the quartic itself U, the other irreducible covariants are those already spoken of and represented by I, H, J, Φ respectively (I is of the degree 2 and the order 0, and J is of the degree 3 and the order 0, *i. e.* I and J are invariants, H is of the degree 2 and the order 4, Φ is of the degree 3 and the order 6).

52. For a quintic, the number of irreducible covariants of the degree θ is

$$\text{coeff. } x^\theta \text{ in } \frac{1+x+x^2+4x^3+6x^4+8x^5+9x^6+10x^7+12x^8+10x^9+9x^{10}+8x^{11}+6x^{12}+4x^{13}+x^{14}+x^{15}+x^{16}}{(1-x^2)^2(1-x^4)(1-x^6)(1-x^8)},$$

the numerator of which is

$$(1+x)^2(1-x+2x^2+x^3+2x^4+3x^5+x^6+5x^7+x^8+3x^9+2x^{10}+x^{11}+2x^{12}-x^{13}+x^{14});$$

the first factor is $(1-x)^{-2}(1-x^2)^2$, the second factor is

$$(1-x)(1-x^2)^{-2}(1-x^3)^{-3}(1-x^4)^{-2}(1-x^5)^{-2}(1-x^6)^5(1-x^7)^5(1-x^8)^7(1-x^9)^1(1-x^{10})^{-9}(1-x^{11})^{-19}\ldots,$$

which does not terminate; the number of irreducible covariants is therefore infinite. Substituting the preceding values, the expression for the number of the asyzygetic covariants of the degree θ is

$$\text{coeff.}\, x^{\theta} \text{ in } (1-x)^{-1}(1-x^2)^{-2}(1-x^3)^{-3}(1-x^4)^{-3}(1-x^5)^{-2}(1-x^6)^{4}(1-x^7)^{5}(1-x^8)^{6}(1-x^9)^{1}(1-x^{10})^{-9}(1-x^{11})^{-19}\ldots$$

which agrees with a previous result: the numbers of irreducible covariants for the degrees 1, 2, 3, 4 are 1, 2, 3 and 3 respectively, and for the degree 5, the number of irreducible covariants is three, but there is one syzygy between the composite covariants of the degree in question; the difference $3-1=2$ is the index taken with its sign reversed of the factor $(1-x^5)^{-2}$.

53. I consider a system of the asyzygetic covariants of any particular degree and order of a given quantic, the system may of course be replaced by a system the terms of which are any linear functions of those of the original system, and it is necessary to inquire what covariants ought to be selected as most proper to represent the system of asyzygetic covariants; the following considerations seem to me to furnish a convenient rule of selection. Let the literal parts of the terms which enter into the coefficients of the highest power of x or leading coefficients be represented by M_α, M_β, M_γ,.. these quantities being arranged in the natural or alphabetical order; the first in order of these quantities M, which enters into the leading coefficient of a particular covariant, may for shortness be called the leading term of such covariant, and a covariant the leading term of which is posterior in order to the leading term of another covariant, may be said to have a lower leading term.

It is clear, that by properly determining the multipliers of the linear functions we may form a covariant the leading term of which is lower than the leading term of any other covariant (the definition implies that there is but one such covariant); call this Θ. We may in like manner form a covariant such that its leading term is lower than the leading term of every other covariant except Θ_1; or rather we may form a system of such covariants, since if Φ_2 be a covariant having the property in question, $\Phi_2+k\Theta_1$ will have the same property, but k may be determined so that the covariant shall not contain the leading term of Θ_1, *i. e.* we may form a covariant Θ_2 such that its leading term is lower than the leading term of every other covariant excepting Θ_1, and that the leading term of Θ_1 does not enter into Θ_2; and there is but one such covariant, Θ_2. Again, we may form a covariant Θ_3 such that its leading term is lower than the leading term of every other covariant excepting Θ_1 and Θ_2, and that the leading terms of Θ_1 and Θ_2 do not either of them enter into Θ_3; and there is but one such covariant, Θ_3. And so on, until we arrive at a covariant the leading term of which is higher than the leading terms of the other covariants, and which does not contain the leading terms of the other covariants. We have thus a series of covariants Θ_1, Θ_2, Θ_3, &c. containing the proper number of terms, and which covariants may be taken to represent the asyzygetic covariants of the degree and order in question.

In order to render the covariants Θ definite as well numerically as in regard to sign, we may suppose that the covariant is in its least terms (*i. e.* we may reject numerical factors common to all the terms), and we may make the leading term positive. The leading term with the proper numerical coefficient, if different from unity and with the proper power of x, or the order of the function annexed, will, when the covariants of a quantic are tabulated, be sufficient to indicate, without any ambiguity whatever, the particular covariant referred to. I subjoin a table of the covariants of a quadric, a cubic and a quartic, and of the covariants of the degrees 1, 2, 3, 4 and 5 respectively of a quintic, and also two other invariants of a quintic.

Covariant Tables (Nos. 1 to 26).

No. 1.

$$\left(\begin{array}{|c|c|c|} +1\ a & +2\ b & +1\ c \end{array} \between x, y\right)^2$$

No. 2.

$$\begin{array}{|l|} +1\ ac \\ -1\ b^2 \end{array}$$

The tables Nos. 1 and 2 are the covariants of a binary quadric. No. 1 is the quadric itself; No. 2 is the quadrinvariant, which is also the discriminant.

No. 3.

$$\left(\begin{array}{|c|c|c|c|} +1\ a & +3\ b & +3\ c & +1\ d \end{array} \between x, y\right)^3.$$

No. 4.

$$\left(\begin{array}{|l|l|l|} +1\ ac & +1\ ad & +1\ bd \\ -1\ b^2 & -1\ bc & -1\ c^2 \end{array} \between x, y\right)^2$$

No. 5.

$$\left(\begin{array}{|l|l|l|l|} +1\ a^2d & +3\ abd & -3\ acd & -1\ ad^2 \\ -3\ abc & -6\ ac^2 & +6\ b^2d & +3\ bcd \\ +2\ b^3 & +3\ b^2c & -3\ bc^2 & -2\ c^3 \end{array} \between x, y\right)^3.$$

No. 6.

$$\begin{array}{|l|} +1\ a^2d^2 \\ -6\ abcd \\ +4\ ac^3 \\ +4\ b^3d \\ -3\ b^2c^2 \end{array}$$

The tables Nos. 3, 4, 5 and 6 are the covariants of a binary cubic. No. 3 is the cubic itself; No. 4 is the quadricovariant, or Hessian; No. 5 is the cubicovariant: No. 6 is the invariant, or discriminant. And if we write

$$\text{No. } 3 = \text{U},$$
$$\text{No. } 4 = \text{H},$$
$$\text{No. } 5 = \Phi,$$
$$\text{No. } 6 = \nabla,$$

then identically,

$$\Phi^2 - \nabla \text{U}^2 + 4\text{H}^3 = 0.$$

No. 7.

(	$+1\ a$	$+4\ b$	$+6\ c$	$+4\ d$	$+1\ e$	$)(x, y)^4$

No. 8.

$+1\ ae$ $-4\ bd$ $+3\ c^2$

No. 9.

(	$+1\ ac$ $-1\ b^2$	$+2\ ad$ $-2\ bc$	$+1\ ae$ $+2\ bd$ $-3\ c^2$	$+2\ be$ $-2\ cd$	$+1\ bd$ $-1\ c^2$	$)(x, y)^4$

No. 10.

$+1\ ace$ $-1\ ad^2$ $-1\ b^2e$ $+2\ bcd$ $-1\ c^3$

No. 11.

(	$+1\ a^2d$ $-3\ abc$ $+2\ b^3$	$+1\ a^2e$ $+2\ abd$ $-9\ ac^2$ $+6\ b^2c$	$+\ 5\ abe$ $-15\ acd$ $+10\ b^2d$ $\backsim\ bc^2$	$\backsim\ ace$ $-10\ ad^2$ $+10\ b^2e$ $\backsim\ bcd$ $\backsim\ c^3$	$-\ 5\ ade$ $+15\ bce$ $-10\ bd^2$ $\backsim\ c^2d$	$-1\ ae^2$ $-2\ bde$ $+9\ c^2e$ $-6\ cd^2$	$-1\ be^2$ $+3\ cde$ $-2\ d^3$	$)(x, y)^6$

No. 12.

$+\ 1\ a^3e^3$ $-\ 12\ a^2bde^2$ $-\ 18\ a^2c^2e^2$ $+\ 54\ a^2cd^2e$ $-\ 27\ a^2d^4$ $+\ 54\ ab^2ce^2$ $-\ 6\ ab^2d^2e$ $-180\ abc^2de$ $+108\ abcd^3$	$+\ 81\ ac^4e$ $-\ 54\ ac^3d^2$ $-\ 27\ b^4e^2$ $+108\ b^3cde$ $-\ 64\ b^3d^3$ $-\ 54\ b^2c^3e$ $+\ 36\ b^2c^2d^2$ $\backsim\ bc^4d$ $\backsim\ c^6$

The tables Nos. 7, 8, 9, 10 and 11 are the irreducible covariants of a quartic. No. 7 is the quartic itself; No. 8 is the quadrinvariant; No. 9 is the quadricovariant, or Hessian; No. 10 is the cubinvariant; and No. 11 is the cubicovariant. The table No. 12 is the discriminant. And if we write

$$\begin{aligned} \text{No. } 7 &= \mathrm{U}, \\ \text{No. } 8 &= \mathrm{I}, \\ \text{No. } 9 &= \mathrm{H}, \\ \text{No. } 10 &= \mathrm{J}, \\ \text{No. } 11 &= \Phi, \\ \text{No. } 12 &= \nabla, \end{aligned}$$

then the irreducible covariants are connected by the identical equation

$$\mathrm{JU}^3 - \mathrm{IU}^2\mathrm{H} + 4\mathrm{H}^3 + \Phi^2 = 0,$$

and we have $$\nabla = \mathrm{I}^3 - 27\mathrm{J}^2.$$

No. 13.

$$\left(\begin{array}{c|c|c|c|c|c} +1\ a & +5\ b & +10\ c & +10\ d & +5\ e & +1\ f \end{array}\between x, y\right)^5$$

No. 14.

$$\left(\begin{array}{c|c|c} +1\ ae & +1\ af & +1\ bf \\ -4\ bd & -3\ be & -4\ ce \\ +3\ c^2 & +2\ cd & +3\ d^2 \end{array}\between x, y\right)^2$$

No. 15.

$$\left(\begin{array}{c|c|c|c|c|c|c} +1\ ac & +3\ ad & +3\ ae & +1\ af & +3\ bf & +3\ cf & +1\ df \\ -1\ b^2 & -3\ bc & +3\ bd & +7\ be & +3\ ce & -3\ de & -1\ e^2 \\ & & -6\ c^2 & -8\ cd & -6\ d^2 & & \end{array}\between x, y\right)^6$$

No. 16.

$$\left(\begin{array}{c|c|c|c} +1\ ace & +1\ acf & +1\ adf & +1\ bdf \\ -1\ ad^2 & -1\ ade & -1\ ae^2 & -1\ be^2 \\ -1\ b^2e & -1\ b^2f & -1\ bcf & -1\ c^2f \\ +2\ bcd & +1\ bce & +1\ bde & +2\ cde \\ -1\ c^3 & +1\ bd^2 & +1\ c^2e & -1\ d^3 \\ & -1\ c^2d & -1\ cd^2 & \end{array}\between x, y\right)^3$$

No. 17.

$$\left(\begin{array}{c|c|c|c|c|c} +1\ a^2f & +5\ abf & +2\ acf & -2\ adf & -5\ aef & -1\ af^2 \\ -5\ abe & -16\ ace & -12\ ade & -8\ ae^2 & +16\ bdf & +5\ bef \\ +2\ acd & +6\ ad^2 & +8\ b^2f & +12\ bcf & +9\ be^2 & -2\ cdf \\ +8\ b^2d & -9\ b^2e & -38\ bce & +38\ bde & -6\ c^2f & -8\ ce^2 \\ -6\ bc^2 & +38\ bcd & +72\ bd^2 & -72\ c^2e & -38\ cde & +6\ d^2e \\ & -24\ c^3 & -32\ c^2d & +32\ cd^2 & +24\ d^3 & \end{array}\between x, y\right)^5$$

No. 18.

$$\left(\begin{array}{c|c|c|c|c|c|c|c|c|c} +1\ a^2d & +2\ a^2e & +1\ a^2f & +7\ abf & +5\ acf & -5\ adf & -7\ aef & -1\ af^2 & -2\ bf^2 & -1\ cf^2 \\ -3\ abc & +1\ abd & +11\ abe & -8\ ace & -40\ ade & -16\ ade & +8\ bdf & -11\ bef & -1\ cef & +3\ def \\ +2\ b^3 & -12\ ac^2 & -34\ acd & -34\ cd^2 & +16\ b^2f & +40\ b^2f & -29\ be^2 & +34\ cdf & +12\ d^2f & -2\ e^3 \\ & +9\ b^2c & +16\ b^2d & +29\ b^2e & +47\ bce & -47\ bce & +34\ c^2f & -16\ ce^2 & -9\ de^2 & \\ & & +6\ bc^2 & -2\ bcd & -44\ bd^2 & +44\ bd^2 & +2\ cde & -6\ d^2e & & \\ & & & +8\ c^3 & +16\ c^2d & -16\ c^2d & -8\ d^3 & & & \end{array}\between x, y\right)^9$$

No. 19.

$+1\ a^2f^2$
$-10\ abef$
$+4\ acdf$
$+16\ ace^2$
$-12\ ad^2e$
$+16\ b^2df$
$+9\ b^2e^2$
$-12\ bc^2f$
$-76\ bcde$
$+48\ bd^3$
$+48\ c^3e$
$-32\ c^2d^2$

No. 20.

(

$+1\ a^2df$	$+2\ a^2ef$	$+1\ a^2f^2$	$+2\ abf^2$	$+1\ acf^2$
$\backsim\ a^2e^2$	$-4\ abdf$	$-4\ abef$	$-4\ acef$	$-3\ adef$
$-3\ abcf$	$-10\ abe^2$	$-2\ acdf$	$-2\ ad^2f$	$+2\ ae^3$
$-5\ abde$	$-2\ ac^2f$	$+4\ ace^2$	$+4\ ade^2$	$\backsim\ b^2f^2$
$+10\ ac^2e$	$+24\ acde$	$\backsim\ ad^2e$	$-10\ b^2ef$	$-5\ bcef$
$-4\ acd^2$	$-12\ ad^3$	$+4\ b^2df$	$+24\ bcdf$	$+10\ bd^2f$
$+2\ b^3f$	$+4\ b^2cf$	$-9\ b^2e^2$	$+16\ bce^2$	$-5\ bde^2$
$-5\ b^2ce$	$+16\ b^2de$	$\backsim\ bc^2f$	$-22\ bd^2e$	$-4\ c^2df$
$+14\ b^2d^2$	$-22\ bc^2e$	$+50\ bcde$	$-12\ c^3f$	$+14\ c^2e^2$
$-16\ bc^2d$	$-4\ bcd^2$	$-36\ bd^3$	$-4\ c^2de$	$-16\ cd^2e$
$+6\ c^4$	$+8\ c^3d$	$-36\ c^3e$	$+8\ cd^3$	$+6\ d^4$
		$+28\ c^2d^2$		

$\c(x, y)^4$

No. 21.

(

$+1\ a^2cf$	$+2\ a^2df$	$\backsim\ a^2ef$	$\backsim\ a^2f^2$	$\backsim\ abf^2$	$-2\ acf^2$	$-1\ adf^2$
$-1\ a^2de$	$-2\ a^2e^2$	$+2\ abdf$	$\backsim\ abef$	$-2\ acef$	$\backsim\ adef$	$+1\ ae^2f$
$-1\ ab^2f$	$-10\ abcf$	$-2\ abe^2$	$\backsim\ acdf$	$+1\ ad^2f$	$+2\ ae^3$	$+1\ bcf^2$
$-2\ abce$	$+10\ abde$	$-1\ ac^2f$	$-20\ ace^2$	$+1\ ade^2$	$+2\ b^2f^2$	$+2\ bdef$
$+4\ abd^2$	$\backsim\ ac^2e$	$-2\ acde$	$+20\ ad^2e$	$+2\ b^2ef$	$\backsim\ bcef$	$-3\ be^3$
$-1\ ac^2d$	$\backsim\ acd^2$	$+3\ ad^3$	$+20\ b^2df$	$+2\ bcdf$	$+10\ bd^2f$	$-4\ c^2ef$
$+3\ b^3e$	$-2\ b^3f$	$-1\ b^2cf$	$\backsim\ b^2e^2$	$-5\ bce^2$	$-14\ bde^2$	$+1\ cd^2f$
$-6\ b^2cd$	$+14\ b^2ce$	$+5\ b^2de$	$-20\ bc^2f$	$-1\ bd^2e$	$-10\ c^2df$	$+6\ cde^2$
$+3\ bc^3$	$+2\ b^2d^2$	$+1\ bc^2e$	$\backsim\ bcde$	$-3\ c^3f$	$-2\ c^2e^2$	$-3\ d^3e$
	$-26\ bc^2d$	$-9\ bcd^2$	$-20\ bd^3$	$+9\ c^2de$	$+26\ cd^2e$	
	$+12\ c^4$	$+4\ c^3d$	$+20\ c^3e$	$-4\ cd^3$	$-12\ d^4$	
			$\backsim\ c^2d^2$			

$\c(x, y)^6$.

No. 22.

(

$+1\ a^2cf^2$	$+1\ a^2df^2$
$-2\ a^2def$	$-1\ a^2e^2f$
$+1\ a^2e^3$	$-2\ abcf^2$
$-1\ ab^2f^2$	$-4\ abdef$
$-4\ abcef$	$+6\ abe^3$
$+8\ abd^2f$	$+8\ ac^2ef$
$-2\ abde^2$	$-2\ acd^2f$
$-2\ ac^2df$	$-12\ acde^2$
$+14\ ac^2e^2$	$+6\ ad^3e$
$-22\ acd^2e$	$+1\ b^3f^2$
$+9\ ad^4$	$-2\ b^2cef$
$+6\ b^3ef$	$+14\ b^2d^2f$
$-12\ b^2cdf$	$-15\ b^2de^2$
$-15\ b^2ce^2$	$-22\ bc^2df$
$+10\ b^2d^2e$	$+10\ bc^2e^2$
$+6\ bc^3f$	$+30\ bcd^2e$
$+30\ bc^2de$	$-15\ bd^4$
$-20\ bcd^3$	$+9\ c^4f$
$-15\ c^4e$	$-20\ c^3de$
$+10\ c^3d^2$	$+10\ c^2d^3$

$\c(x, y)^1$

No. 23.

∽ a^2bf^2	+ 1 a^2cf^2	− 1 a^2df^2	∽ a^2ef^2
+ 1 a^2cef	− 5 a^2def	+ 1 a^2e^2f	− 1 $abdf^2$
− 3 a^2d^2f	+ 4 a^2e^3	+ 5 $abcf^2$	+ 1 abe^2f
+ 2 a^2de^2	− 1 ab^2f^2	− 8 $abdef$	+ 3 ac^2f^2
− 1 ab^2ef	+ 8 $abcef$	+ 3 abe^3	−14 $acdef$
+14 $abcdf$	+11 abd^2f	−11 ac^2ef	+ 8 ace^3
−11 $abce^2$	−17 $abde^2$	+11 acd^2f	+ 9 ad^3f
− 1 abd^2e	−11 ac^2df	+ 6 $acde^2$	− 6 ad^2e^2
− 9 ac^3f	−16 ac^2e^2	− 6 ad^3e	− 2 b^2cf^2
+14 ac^2de	+44 acd^2e	− 4 b^3f^2	+11 b^2def
− 6 acd^3	−18 ad^4	+17 b^2cef	− 9 b^2e^3
− 8 b^3df	− 3 b^3ef	+16 b^2d^2f	+ 1 bc^2ef
+ 9 b^3e^2	− 6 b^2cdf	−21 b^2de^2	−14 bcd^2f
+ 6 b^2c^2f	+21 b^2ce^2	−44 bc^2df	+16 $bcde^2$
−16 b^2cde	− 5 b^2d^2e	+ 5 bc^2e^2	− 3 bd^3e
+ 8 b^2d^3	+ 6 bc^3f	+39 bcd^2e	+ 6 c^3df
+ 3 bc^3e	−39 bc^2de	−12 bd^4	− 8 c^3e^2
− 2 bc^2d^2	+22 bcd^3	+18 c^4f	+ 2 c^2d^2e
∽ c^4d	+12 c^4e	−22 c^3de	∽ cd^4
	− 8 c^3d^2	+ 8 c^2d^3	

$)(x, y)^8$

No. 24.

∽ a^3ef	∽ a^3f^2	∽ a^2bf^2	− 1 a^2cf^2	+ 1 a^2df^2	∽ a^2ef^2	∽ a^2f^3	∽ abf^3
∽ a^2bdf	∽ a^2bef	− 3 a^2cef	+ 7 a^2def	− 1 a^2e^2f	+ 3 $abdf^2$	∽ $abef^2$	∽ $acef^2$
∽ a^2be^2	+ 7 a^2cdf	+12 a^2d^2f	− 6 a^2e^3	− 7 $abcf^2$	− 3 abe^2f	− 7 $acdf^2$	−2 ad^2f^2
+2 a^2c^2f	−10 a^2ce^2	− 9 a^2de^2	+ 1 ab^2f^2	+26 $abdef$	−12 ac^2f^2	+ 7 ace^2f	+4 ade^2f
−5 a^2cde	+ 3 a^2d^2e	+ 3 ab^2ef	−26 $abcef$	−19 abe^3	+18 $acdef$	+ 7 ad^2ef	−2 ae^4
+3 a^2d^3	− 7 ab^2df	−18 $abcdf$	+32 abd^2f	−32 ac^2ef	+ 6 ace^4	− 7 ade^3	∽ b^2ef^2
−4 ab^2cf	+10 ab^2e^2	−18 $abce^2$	− 8 $abde^2$	+18 acd^2f	+ 3 ad^3f	+10 b^2df^2	+5 $bcdf^2$
+5 ab^2de	− 7 abc^2f	+30 abd^2e	−18 ac^2df	+53 $acde^2$	−15 ad^2e^2	−10 b^2e^2f	−5 bce^2f
+5 abc^2e	− 8 $abcde$	− 3 ac^3f	+ 6 ac^2e^2	−39 ad^3e	+ 9 b^2cf^2	− 3 bc^2f^2	−5 bd^2ef
−7 $abcd^2$	+ 9 abd^3	+45 ac^2de	+52 acd^2e	+ 6 b^3f^2	+18 b^2def	+ 8 $bcdef$	+5 bde^3
+1 ac^3d	+22 ac^3e	−39 acd^3	−39 ad^4	+ 8 b^2cef	−27 b^2e^3	− 2 bce^3	−3 c^2f^2
+2 b^4f	−19 ac^2d^2	− 6 b^3df	+19 b^3ef	− 6 b^2d^2f	−30 bc^2ef	−22 bd^3f	+7 c^2def
−5 b^3ce	+ 7 b^3cf	+27 b^3e^2	−53 b^2cdf	−20 b^2de^2	−45 bcd^2f	+19 bd^2e^2	+2 c^2e^3
−2 b^3d^2	+ 2 b^3de	+15 b^2c^2f	+20 b^2ce^2	+45 bc^2df	+87 $bcde^2$	− 9 c^3ef	−1 cd^3f
+8 b^2c^2d	−19 b^2c^2e	−87 b^2cde	−25 b^2d^2e	+25 bc^2e^2	−12 bd^3e	+19 c^2d^2f	−8 cd^2e^2
−3 bc^4	−11 b^2cd^2	+ 6 b^2d^3	+39 bc^3f	−52 bcd^2e	+39 c^3df	+11 c^2de^2	+3 d^4e
	+33 bc^3d	+12 bc^3e	−45 bc^2de	∽ bd^4	− 6 c^3e^2	−33 cd^3e	
	−12 c^5	+57 bc^2d^2	+65 bcd^3	+39 c^4f	−57 c^2d^2e	+12 d^5	
		−24 c^4d	∽ c^4e	−65 c^3de	+24 cd^4		
			−20 c^3d^2	+20 c^2d^3			

$)(x, y)^7$

No. 25.

∽ a^4f^4	+ 12 $a^2c^3ef^2$	− 18 ab^2d^3ef	∽ acd^6	+ 8 $b^2c^2d^3f$
∽ a^3bcf^3	− 21 $a^2c^2d^2f^2$	+ 3 $ab^2d^2e^3$	− 2 b^5f^3	+25 $b^2c^2d^2e^2$
+ 1 a^3cdf^3	− 34 $a^2c^2de^2f$	+ 78 abc^3df^2	+ 15 b^4cef^2	−57 b^2cd^4e
− 1 $a^3ce^2f^2$	+ 22 $a^2c^2e^4$	− 18 abc^3e^2f	+ 18 $b^4d^2f^2$	+18 b^2d^6
− 3 $a^3d^2ef^2$	+ 78 a^2cd^3ef	−210 abc^2d^2ef	− 54 b^4de^2f	− 9 bc^5ef
+ 5 a^3de^3f	− 48 $a^2cd^2e^3$	+106 abc^2de^3	+ 27 b^4e^4	+ 6 bc^4d^2f
− 2 a^3e^5	− 27 a^2d^5f	+ 93 $abcd^4f$	− 48 $b^3c^2df^2$	−57 bc^4de^2
− 1 $a^2b^2df^3$	+ 18 $a^2d^4e^2$	− 30 $abcd^3e^2$	+ 3 b^3ce^2f	+38 bc^3d^3e
+ 1 $a^2b^2e^2f^2$	+ 5 ab^3cf^3	− 9 abd^5e	+106 b^3cd^2ef	−24 bc^2d^5
− 3 $a^2bc^2f^3$	− 5 ab^3def^2	− 17 ac^5f^2	− 81 b^3cde^3	∽ c^6df
+11 a^2bcdef^2	∽ ab^3e^3f	+ 93 ac^4def	− 38 b^3d^4f	+18 c^6e^2
− 5 a^2bce^3f	− 30 $ab^2c^2ef^2$	− 38 ac^4e^3	+ 38 $b^3d^3e^2$	−24 c^5d^2e
+12 $a^2bd^3f^2$	− 34 $ab^2cd^2f^2$	− 42 ac^3d^3f	+ 18 $b^2c^4f^2$	+ 8 c^4d^4
−30 $a^2bd^2e^2f$	+133 ab^2cde^2f	+ 8 $ac^3d^2e^2$	− 30 b^2c^3def	
+15 a^2bde^4	− 54 ab^2ce^4	+ 6 ac^2d^4e	+ 38 $b^2c^3e^3$	

No. 26.

$+\ 1$	a^4f^4	-10080	a^2cd^3ef	$+\ 7200$	$abcd^3e^3$	$+6400$	b^3d^4f
$-\ 20$	a^3bef^3	$+\ 5760$	$a^2cd^2e^3$	$\backsim$	abd^5e	-4000	$b^3d^3e^2$
$-\ 120$	a^3cdf^3	$+\ 3456$	a^2d^5f	$+\ 3456$	ac^5f^2	-2160	$b^2c^4f^2$
$+\ 160$	$a^3ce^2f^2$	$-\ 2160$	$a^2d^4e^2$	-11520	ac^4def	$+7200$	b^2c^3def
$+\ 360$	$a^3d^2ef^2$	$-\ 640$	ab^3cf^3	$+\ 6400$	ac^4e^3	-4000	$b^2c^3e^3$
$-\ 640$	a^3de^3f	$+\ 320$	ab^3def^2	$+\ 5120$	ac^3d^3f	-3200	$b^2c^2d^3f$
$+\ 256$	a^3e^5	$-\ 180$	ab^3e^3f	$-\ 3200$	$ac^3d^2e^2$	$+2000$	$b^2c^2d^2e^2$
$+\ 160$	$a^2b^2df^3$	$+\ 4080$	$ab^2c^2ef^2$	$\backsim$	ac^2d^4e	$\backsim$	b^2cd^4e
$-\ 10$	$a^2b^2e^2f^2$	$+\ 4480$	$ab^2cd^2f^2$	$\backsim$	acd^6	$\backsim$	b^2d^6
$+\ 360$	$a^2bc^2f^3$	-14920	ab^2cde^2f	$+\ 256$	b^5f^3	$\backsim$	bc^5ef
$-\ 1640$	a^2bcdef^2	$+\ 7200$	ab^2ce^4	$-\ 1920$	b^4cef^2	$\backsim$	bc^4d^2f
$+\ 320$	a^2bce^3f	$+\ 960$	ab^2d^3ef	$-\ 2560$	$b^4d^2f^2$	$\backsim$	bc^4de^2
$-\ 1440$	$a^2bd^3f^2$	$-\ 600$	$ab^2d^2e^3$	$+\ 7200$	b^4de^2f	$\backsim$	bc^3d^3e
$+\ 4080$	$a^2bd^2e^2f$	-10080	abc^3df^2	$-\ 3375$	b^4e^4	$\backsim$	bc^2d^5
$-\ 1920$	a^2bde^4	$+\ 960$	abc^3e^2f	$+\ 5760$	$b^3c^2df^2$	$\backsim$	c^6df
$-\ 1440$	$a^2c^3ef^2$	$+28480$	abc^2d^2ef	$-\ 600$	b^3ce^2f	$\backsim$	c^6e^2
$+\ 2640$	$a^2c^2d^2f^2$	-16000	abc^2de^3	-16000	b^3cd^2ef	$\backsim$	c^5d^2e
$+\ 4480$	$a^2c^2de^2f$	-11520	$abcd^4f$	$+\ 9000$	b^3cde^3	$\backsim$	c^4d^4
$-\ 2560$	$a^2c^2e^4$						

The tables Nos. 13 to 24 are the irreducible covariants of the degrees 1, 2, 3, 4 and 5 of a quintic. No. 13 is the quintic itself; No. 15 is the Hessian; No. 19 is the quartinvariant; No. 22 is the linear covariant; the other covariants can be referred to by their degree and order, or simply by the number of the table. The foregoing covariants are connected by the equation of the degree 5 and order 11,

$$(\text{No. } 13)(\text{No. } 21)+(\text{No. } 14)(\text{No. } 18)-(\text{No. } 15)(\text{No. } 17)=0.$$

The table No. 25 is the simplest octinvariant, and the table No. 26 is the discriminant; we have

$$(\text{No. } 26)=(\text{No. } 19)^2-1152(\text{No. } 25).$$

VII. *Researches on the Partition of Numbers.* By ARTHUR CAYLEY, *Esq.*

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I PROPOSE to discuss the following problem: "To find in how many ways a number q can be made up of the elements $a, b, c, ..$ each element being repeatable an indefinite number of times." The required number of partitions is represented by the notation

$$P(a, b, c, ..)q, .$$

and we have, as is well known,

$$P(a, b, c, ..)q = \text{coefficient } x^q \text{ in } \frac{1}{(1-x^a)(1-x^b)(1-x^c)..},$$

where the expansion is to be effected in ascending powers of x.

It may be as well to remark that each element is to be considered as a separate and distinct element, notwithstanding any equalities which may exist between the numbers $a, b, c, ..$; thus, although $a=b$, yet $a+a+a+$ &c. and $a+a+b+$ &c. are to be considered as two different partitions of the number q, and so in all similar cases.

The solution of the problem is thus seen to depend upon the theory, to which I now proceed, of the expansion of algebraical fractions.

Consider an algebraical fraction $\frac{\phi x}{fx}$,

where the denominator is the product of any number of factors (the same or different) of the form $1-x^m$. Suppose in general that $[1-x^m]$ denotes the irreducible factor of $1-x^m$, *i. e.* the factor which, equated to zero, gives the prime roots of the equation $1-x^m=0$. We have

$$1-x^m = \Pi[1-x^{m'}],$$

where m' denotes any divisor whatever of m (unity and the number m itself not excluded). Hence, if a represent a divisor of one or more of the indices m, and k be the number of the indices of which a is a divisor, we have

$$fx = \Pi[1-x^a]^k.$$

Now considering apart from the others one of the multiple factors $[1-x^a]^k$, we may write $fx = [1-x^a]^k f_1x$.

Suppose that the fraction $\frac{\phi x}{fx}$ is decomposed into simpler fractions, in the form

$$\begin{aligned}\frac{\phi x}{fx} = & \, \mathrm{I}(x) \\ & + (x\partial_x)^{k-1}\frac{\theta x}{[1-x^a]} + (x\partial_x)^{k-2}\frac{\theta_1 x}{[1-x^a]} \cdot\cdot + \frac{\theta_{k-1} x}{[1-x^a]} \\ & + \&c.,\end{aligned}$$

where $\mathrm{I}(x)$ denotes the integral part, and the &c. refers to the fractional terms depending upon the other multiple factors, such as $[1-x^a]^k$. The functions θx are to be considered as functions with indeterminate coefficients, the degree of each such function being inferior by unity to that of the corresponding denominator; and it is proper to remark that the number of the indeterminate coefficients in all the functions θx together is equal to the degree of the denominator fx.

The term $(x\partial_x)^{k-1}\dfrac{\theta x}{[1-x^a]}$ may be reduced to the form

$$\frac{gx}{[1-x^a]^k}+\frac{g_1x}{[1-x^a]^{k-1}}+\text{\&c.},$$

the functions gx being of the same degree as θx, and the coefficients of these functions being linearly connected with those of the function θx. The first of the foregoing terms is the only term on the right-hand side which contains the denominator $[1-x^a]^k$; hence, multiplying by this denominator and then writing $[1-x^a]=0$, we find

$$\frac{\phi x}{f_{\prime}x}=gx,$$

which is true when x is any root whatever of the equation $[1-x^a]=0$. Now by means of the equation $[1-x^a]=0, \dfrac{\phi x}{f_{\prime}x}$ may be expressed in the form of a rational and integral function $\mathrm{G}x$, the degree of which is less by unity than that of $[1-x^a]$. We have therefore $\mathrm{G}x=gx$, an equation which is satisfied by each root of $[1-x^a]=0$, and which is therefore an identical equation; gx is thus determined, and the coefficients of θx being linear functions of those of gx, the function θx may be considered as determined. And this being so, the function

$$\frac{\phi x}{fx}-(x\partial_x)^{k-1}\frac{\theta x}{[1-x^a]}$$

will be a fraction the denominator of which does not contain any power of $[1-x^a]$ higher than $[1-x^a]^{k-1}$; and therefore $\theta_1 x$ can be found in the same way as θx, and similarly $\theta_2 x$, and so on. And the fractional parts being determined, the integral part may be found by subtracting from $\dfrac{\phi x}{fx}$ the sum of the fractional parts, so that the fraction $\dfrac{\phi x}{fx}$ can by a direct process be decomposed in the above-mentioned form.

Particular terms in the decomposition of certain fractions may be obtained with great facility. Thus m being a prime number, assume

$$\frac{1}{(1-x^2)(1-x^3)\ldots(1-x^m)}=\text{\&c.}+\frac{\theta x}{[1-x^m]};$$

then observing that $(1-x^m)=(1-x)[1-x^m]$, we have for $[1-x^m]=0$,

$$\theta x=\frac{1}{(1-x)(1-x^2)\ldots(1-x^{m-1})}.$$

Now u being any quantity whatever and x being a root of $[1-x^m]=0$, we have identically

$$[1-u^m]=(u-x)(u-x^2)..(u-x^{m-1});$$

and therefore putting $u=1$, we have $\quad m=(1-x)(1-x^2)..(1-x^{m-1}),$
and therefore

$$\theta x=\frac{1}{m},$$

whence

$$\frac{1}{(1-x^2)(1-x^3)..(1-x^m)}=\&c.+\frac{1}{m}\,\frac{1}{[1-x^m]}.$$

Again, m being as before a prime number, assume

$$\frac{1}{(1-x)(1-x^2)..(1-x^m)}=\&c.+\frac{\theta x}{[1-x^m]},$$

we have in this case for $[1-x^m]=0$,

$$\theta x=\frac{1}{(1-x)^2(1-x^2)..(1-x^{m-1})},$$

which is immediately reduced to $\theta x=\frac{1}{m}\,\frac{1}{1-x}$. Now

$$\frac{[1-u^m]}{u-x}=\frac{[1-u^m]-[1-x^m]}{u-x}=(1+u+..+u^{m-2})+(1+u..+u^{m-3})x..+(1+u)x^{m-3}+x^{m-2};$$

or putting $u=1$,

$$\frac{m}{1-x}=\overline{m-1}+\overline{m-2}x..+x^{m-2};$$

and substituting this in the value of θx, we find

$$\frac{1}{(1-x)(1-x^2)..(1-x^m)}=\&c.+\frac{1}{m^2}\,\frac{(m-1)+(m-2)x..+x^{m-2}}{[1-x^m]}.$$

The preceding decomposition of the fraction $\frac{\phi x}{fx}$ gives very readily the expansion of the fraction in ascending powers of x. For, consider a fraction such as

$$\frac{\theta x}{[1-x^a]},$$

where the degree of the numerator is in general less by unity than that of the denominator; we have

$$1-x^a=[1-x^a]\Pi[1-x^{a'}],$$

where a' denotes any divisor of a (including unity, but not including the number a itself). The fraction may therefore be written under the form

$$\frac{\theta x\Pi[1-x^{a'}]}{1-x^a};$$

where the degree of the numerator is in general less by unity than that of the denominator, *i. e.* is equal to $\overline{a-1}$. Suppose that b is any divisor of a (including unity, but not including the number a itself), then $1-x^b$ is a divisor of $\Pi[1-x^{a'}]$, and

therefore of the numerator of the fraction. Hence representing this numerator by

$$A_0+A_1x\ldots+A_{a-1}x^{a-1},$$

and putting $a=bc$, we have (corresponding to the case $b=1$)

$$A_0+A_1+A_2\ldots+A_{a-1}=0,$$

and generally for the divisor b,

$$\begin{aligned}&A_0+A_b\ldots+A_{(c-1)b}=0\\&A_1+A_{b+1}\ldots+A_{(c-1)b+1}=0\\&\quad\vdots\\&A_{b-1}+A_{2b-1}\ldots+A_{cb-1}=0.\end{aligned}$$

Suppose now that a_q denotes a circulating element to the period a, *i. e.* write

$$\begin{aligned}&a_q=1 \quad q=0 \text{ (mod. } a)\\&a_q=0 \text{ in every other case.}\end{aligned}$$

A function such as

$$A_0a_q+A_1a_{q-1}\ldots+A_{a-1}a_{q-a+1}$$

will be a circulating function, or circulator to the period a, and may be represented by the notation

$$(A_0,\ A_1,\ \ldots A_{a-1}) \text{ circlor } a_q.$$

In the case however where the coefficients A satisfy, for each divisor b of the number a, the above-mentioned equations, the circulating function is what I call a prime circulator, and I represent it by the notation

$$(A_0,\ A_1,\ \ldots A_{a-1}) \text{ pcr } a_q.$$

By means of this notation we have at once

$$\text{coefficient } x_q \text{ in } \frac{\theta x}{[1-x^a]}=(A_0,\ A_1\ldots A_{a-1}) \text{ pcr } a_q,$$

and thence also

$$\text{coefficient } x_q \text{ in } (x\partial_x)^r\frac{\theta x}{[1-x^a]}=q^r(A_0,\ A_1\ldots A_{a-1}) \text{ pcr } a_q.$$

Hence assuming that in the fraction $\frac{\phi x}{fx}$ the degree of the numerator is less than that of the denominator (so that there is not any integral part), we have

$$\text{coefficient } x_q \text{ in } \frac{\phi x}{fx}=\Sigma\ q^r(A_0,\ A_1,\ \ldots A_{a-1}) \text{ pcr } a_q\,;$$

or, if we wish to put in evidence the non-circulating part arising from the divisor $a=1$,

$$\begin{aligned}\text{coefficient } x_q \text{ in } \frac{\phi x}{fx}=&Aq^{k-1}+Bq^{k-2}\ldots+Lq+M\\&+\Sigma\ q^r(A_0,\ A_1\ldots A_{a-1}) \text{ pcr } a_q\,;\end{aligned}$$

where k denotes the number of the factors $1-x^m$ in the denominator fx, a is any divisor (unity excluded) of one or more of the indices m; and for each value of a r extends from $r=0$ to $r=k-1$, where k denotes the number of indices m of which

a is a divisor. The particular results previously obtained show, that m being a prime number,

$$\text{coefficient } x^q \text{ in } \frac{1}{(1-x^2)(1-x^3)\ldots(1-x^m)}=\text{\&c.}+\frac{1}{m}(1,-1,0,0,\ldots) \text{ per } m_q,$$

and

$$\text{coefficient } x^q \text{ in } \frac{1}{(1-x)(1-x^2)\ldots(1-x^m)}=\text{\&c.}+\frac{1}{m^2}(m-1,-1,-1,\ldots) \text{ per } m_q.$$

Suppose, as before, that the degree of ϕx is less than that of fx, and let the analytical expression above obtained for the coefficient of x^q in the expansion in ascending powers of x of the fraction $\frac{\phi x}{fx}$ be represented by Fq, it is very remarkable that if we expand $\frac{\phi x}{fx}$ in descending powers of x, then the coefficient of x^q in this new expansion (q is here of course negative, since the expansion contains only negative powers of x) is precisely equal to $-Fq$; this is in fact at once seen to be the case with respect to each of the partial fractions into which $\frac{\phi x}{fx}$ has been decomposed, and it is consequently the case with respect to the fraction itself*. This gives rise to a result of some importance. Suppose that ϕx and fx are respectively of the degrees N and D: it is clear from the form of fx that we have $f\left(\frac{1}{x}\right)=(-)^{D}x^{-D}fx$; and I suppose that ϕx is also such that $\phi\left(\frac{1}{x}\right)=(\pm)^{N}x^{-N}\phi x$; then writing $D-N=h$, and supposing that $\frac{\phi x}{fx}$ is expanded in descending powers of x, so that the coefficient of x^q in the expansion is $-Fq$, it is in the first place clear that the expansion will commence with the term x^{-h}, and we must therefore have

$$Fq=0$$

for all values of q from $q=-1$ to $q=-(h-1)$.

Consider next the coefficient of a term x^{-h-q}, where q is 0 or positive; the coefficient in question, the value of which is $-F(-h-q)$, is obviously equal to the coefficient of x^{h+q} in the expansion in ascending powers of x of $\frac{\phi\frac{1}{x}}{f\left(\frac{1}{x}\right)}$, *i. e.* to

$$(\pm)^{N}(-)^{D} \text{ coefficient } x^{h+q} \text{ in } \frac{x^h\phi x}{fx},$$

or what is the same thing, to

$$(\pm)^{N}(-)^{D} \text{ coefficient } x^{q} \text{ in } \frac{\phi x}{fx};$$

and we have therefore, q being zero or positive,

$$F(-h-q)=-(\pm)^{N}(-)^{D}Fq.$$

In particular, when $\phi x=1$, $\qquad Fq=0$

* The property is a fundamental one in the general theory of developments.

for all values of q from $q=-1$ to $q=-(D-1)$; and q being 0 or positive,

$$F(-D-q)=(-)^{D-1}Fq.$$

The preceding investigations show the general form of the function $P(a, b, c, \ldots)q$, viz. that

$$P(a, b, c, ..)q=Aq^{k-1}+Bq^{k-2}..+Lq+M+\Sigma\ q^r(A_0, A_1, .. A_{l-1}) \text{ pcr } l_q,$$

a formula in which k denotes the number of the elements $a, b, c, ..$ &c., and l is any divisor (unity excluded) of one or more of these elements; the summation in the case of each such divisor extends from $r=0$ to $r=k-1$, where k is the number of the elements $a, b, c, \ldots$ &c. of which l is a divisor; and the investigations indicate how the values of the coefficients A of the prime circulators are to be obtained. It has been moreover in effect shown, that if $D=a+b+c+..$, then, writing for shortness $P(q)$ instead of $P(a, b, c, ..)q$, we have

$$P(q)=0$$

for all values of q from $q=-1$ to $q=-(D-1)$, and that q being 0 or positive,

$$P(-D-q)=(-)^{D-1}P(q);$$

these last theorems are however uninterpretable in the theory of partitions, and they apply only to the analytical expression for $P(q)$.

I have calculated the following particular results:—

$$P(1, 2)q = \frac{1}{4}\left\{\begin{array}{l} 2q+3 \\ +(1, -1) \text{ pcr } 2_q \end{array}\right\}$$

$$P(1, 2, 3)q = \frac{1}{72}\left\{\begin{array}{l} 6q^2+36q+47 \\ +9(1, -1) \text{ pcr } 2_q \\ +8(2, -1, -1) \text{ pcr } 3_q \end{array}\right\}$$

$$P(1, 2, 3, 4)q = \frac{1}{288}\left\{\begin{array}{l} 2q^3+30q^2+135q+175 \\ +(9q+45)(1, -1) \text{ pcr } 2_q \\ +32\ (1, 0, -1) \text{ pcr } 3_q \\ +36\ (1, 0, -1, 0) \text{ pcr } 4_q \end{array}\right\}$$

$$P(1, 2, 3, 4, 5)q = \frac{1}{86400}\left\{\begin{array}{l} 30q^4+900q^3+9300q^2+38250q+50651 \\ +(1350q+10125)\ (1, -1) \text{ pcr } 2_q \\ +3200\ (2, -1, -1) \text{ pcr } 3_q \\ +5400\ (1, 1, -1, -1) \text{ pcr } 4_q \\ +3456(4, -1, -1, -1, -1) \text{ pcr } 5_q \end{array}\right\}$$

$$P(2)q = \frac{1}{2}\left\{\begin{array}{l} 1 \\ +(1, -1) \text{ pcr } 2_q \end{array}\right\}$$

$$\mathrm{P}(2,3)q = \frac{1}{12}\left\{\begin{aligned}&2q+5\\&+3\quad(1,-1)\text{ per }2_q\\&+4(1,-1,0)\text{ per }3_q\end{aligned}\right\}$$

$$\mathrm{P}(2,3,4)q = \frac{1}{288}\left\{\begin{aligned}&6q^2+54q+107\\&+(18q+81)(1,-1)\text{ per }2_q\\&+32\quad(2,-1,-1)\text{ per }3_q\\&+36\ (1,-1,-1,1)\text{ per }4_q\end{aligned}\right\}$$

$$\mathrm{P}(2,3,4,5)q = \frac{1}{1440}\left\{\begin{aligned}&2q^3+42q^2+267q+497\\&+(45q+315)(1,-1)\text{ per }2_q\\&+160\qquad(1,-1,0)\text{ per }3_q\\&+180\quad(1,0,-1,0)\text{ per }4_q\\&+288\ (1,-1,0,0,0)\text{ per }5_q\end{aligned}\right\}$$

$$\mathrm{P}(2,3,4,5,6)q = \frac{1}{172800}\left\{\begin{aligned}&10q^4+400q^3+5550q^2+31000q+56877\\&+(450q^2+9000q+39075)(1,-1)\text{ per }2_q\\&+3200q\qquad(1,-1,0)\text{ per }3_q\\&+1600\qquad(21,-19,-2)\text{ per }3_q\\&+10800\qquad(1,0,-1,0)\text{ per }4_q\\&+6912\quad(4,-1,-1,-1,-1)\text{ per }5_q\\&+4800\quad(1,-1,-2,-1,1,2)\text{ per }6_q\end{aligned}\right\}$$

$$\mathrm{P}(1,2,3,5)q = \frac{1}{720}\left\{\begin{aligned}&4q^3+66q^2+324q+451\\&+45\qquad(1,-1)\text{ per }2_q\\&+80\quad(1,-1,0)\text{ per }3_q\\&+144(1,0,0,0,-1)\text{ per }5_q\end{aligned}\right\}$$

$$\mathrm{P}(1,2,2,3,4)q = \frac{1}{6912}\left\{\begin{aligned}&6q^4+144q^3+1194q^2+3960q+4267\\&+(54q^2+648q+1701)(1,-1)\text{ per }2_q\\&+256\qquad(2,-1,-1)\text{ per }3_q\\&+432\qquad(1,0,-1,0)\text{ per }4_q\end{aligned}\right\}$$

$$\mathrm{P}(8)q = \frac{1}{8}\left\{\begin{aligned}&1\\&+1\qquad(1,-1)\text{ per }2_q\\&+2\quad(1,0,-1,0)\text{ per }4_q\\&+8(1,0,0,0,-1,0,0,0)\text{ per }8_q\end{aligned}\right\}$$

$$P(7,8)q=\frac{1}{112}\left\{\begin{array}{ll}2q+43 & \\ +7 & (1,-1)\text{ pcr }2_q \\ +14 & (1,-1,-1,1)\text{ pcr }4_q \\ +16 & (3,2,1,0,-1,-2,-3)\text{ pcr }7_q \\ +56 & (0,-1,-1,0,0,1,1,0)\text{ pcr }8_q\end{array}\right\},$$

which are, I think, worth preserving.

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I proceed to discuss the following problem: "To find in how many ways a number q can be made up as a sum of m terms with the elements $0, 1, 2, \ldots k$, each element being repeatable an indefinite number of times." The required number of partitions is represented by

$$P(0, 1, 2, ..k)^m q,$$

and the number of partitions of q less the number of partitions of $q-1$ is represented by

$$P'(0, 1, 2, ..k)^m q.$$

We have, as is well known,

$$P(0, 1, 2, ..k)^m q = \text{coefficient } x^q z^m \text{ in } \frac{1}{(1-z)(1-xz)..(1-x^k z)},$$

where the expansion is to be effected in ascending powers of z. Now

$$\frac{1}{(1-z)(1-xz)..(1-x^k z)}=1+\frac{1-x^{k+1}}{1-x}z+\frac{(1-x^{k+1})(1-x^{k+2})}{(1-x)(1-x^2)}z^2+\&\text{c.},$$

the general term being

$$\frac{(1-x^{k+1})(1-x^{k+2})..(1-x^{k+m})}{(1-x)(1-x^2)..(1-x^m)}z^m,$$

or, what is the same thing,

$$\frac{(1-x^{m+1})(1-x^{m+2})..(1-x^{m+k})}{(1-x)(1-x^2)..(1-x^k)}z^m,$$

and consequently

$$P(0, 1, 2, ..k)^m q = \text{coefficient } x^q \text{ in } \frac{(1-x^{m+1})(1-x^{m+2})..(1-x^{m+k})}{(1-x)(1-x^2)..(1-x^k)};$$

to transform this expression I make use of the equation

$$(1+xz)(1+x^2z)..(1+x^kz)=1+\frac{x(1-x^k)}{1-x}z+\frac{x^3(1-x^k)(1-x^{k-1})}{(1-x)(1-x^2)}z^2+\&\text{c.},$$

where the general term is

$$x^{\frac{1}{2}s(s+1)}\frac{(1-x^k)(1-x^{k-1})..(1-x^{k-s+1})}{(1-x)(1-x^2)..(1-x^s)}z^s,$$

and the series is a finite one, the last term being that corresponding to $s=k$, viz. $x^{\frac{1}{2}k(k+1)}z^k$. Writing $-x^m$ for z, and substituting the resulting value of

$$(1-x^{m+1})(1-x^{m+2})..(1-x^{m+k})$$

in the formula for $P(0, 1, 2, ..k)^m q$, we have

$$P(0, 1, 2, ..k)^m q=\Sigma_s\left\{(-)^s \text{ coefficient } x^q \text{ in } \frac{x^{sm+\frac{1}{2}s.s+1}}{(1-x)(1-x^2)..(1-x^s)(1-x)(1-x^2)..(1-x^{k+s})}\right\},$$

where the summation extends from $s=0$ to $s=k$; but if for any value of s between these limits $sm+\frac{1}{2}s(s+1)$ becomes greater than q, then it is clear that the summation need only be extended from $s=0$ to the last preceding value of s, or what is the same thing, from $s=0$ to the greatest value of s, for which $q-sm-\frac{1}{2}s(s+1)$ is positive or zero.

It is obvious, that if $q>km$, then

$$P(0, 1, 2..k)^m q=0;$$

and moreover, that if $\theta \not> \frac{1}{2}km$, then

$$P(0, 1, 2, ..k)^m\theta=P(0, 1, 2..k)^m.km-\theta,$$

so that we may always suppose $q \not> \frac{1}{2}km$. I write therefore $q=\frac{1}{2}(km-\alpha)$ where α is zero or a positive integer not greater than km, and is even or odd according as km is even or odd. Substituting this value of q and making a slight change in the form of the result, we have

$$P(0, 1, 2..k)^m\tfrac{1}{2}(km-\alpha)=\Sigma_s\left\{(-)^s \text{coeff.}\, x^{(\frac{1}{2}k-s)m} \text{ in } \frac{x^{\frac{1}{2}\alpha+\frac{1}{2}s.s+1}}{(1-x)(1-x^2)..(1-x^s)(1-x)(1-x^2)..(1-x^{k-s})}\right\},$$

where the summation extends from $s=0$ to the greatest value of s, for which $(\frac{1}{2}k-s)m-\frac{1}{2}\alpha-\frac{1}{2}s(s+1)$ is positive or zero. But we may, if we please, consider the summation as extending, when k is even, from $s=0$ to $s=\frac{1}{2}k-1$, and when k is odd, from $s=0$ to $s=\frac{1}{2}(k-1)$, the terms corresponding to values of s greater than the greatest value for which $(\frac{1}{2}k-s)m-\frac{1}{2}\alpha-\frac{1}{2}s(s+1)$ is positive or zero, being of course equal to zero. It may be noticed, that the fraction will be a proper one if $\alpha<(k-s)(k-s+1)$; or substituting for s its greatest value, the fraction will be a proper one for all values of s, if, when k is even, $\alpha<\frac{1}{4}k(k+2)$, and when k is odd, $\alpha<\frac{1}{4}(k+1)(k+3)$.

We have in a similar manner,

$$P'(0, 1, 2 \dots k)^m q=\text{coefficient } x^q z^m \text{ in } \frac{1-x}{(1-z)(1-xz)..(1-x^k z)},$$

which leads to

$$P'(0, 1, 2..k)^m\tfrac{1}{2}(km-\alpha)=\Sigma_s\left\{(-)^s \text{coeff.}\, x^{(\frac{1}{2}k-s)m} \text{ in } \frac{x^{\frac{1}{2}\alpha+s(s+1)}}{(1-x^2)..(1-x^s)(1-x)(1-x^2)..(1-x^{k-s})}\right\},$$

where the summation extends, as in the former case, from $s=0$ to the greatest value of s, for which $(\frac{1}{2}k-s)m-\frac{1}{2}\alpha-\frac{1}{2}s(s+1)$ is positive or zero, or, if we please, when k is even, from $s=0$ to $s=\frac{1}{2}k-1$, and when s is odd, from $s=0$ to $s=\frac{1}{2}(k-1)$. The condition, in order that the fraction may be a proper one for all values of s, is, when k is even, $\alpha+1<\frac{1}{4}k(k+2)$, and when k is odd, $\alpha+1<\frac{1}{4}(k+1)(k+3)$.

To transform the preceding expressions, I write when k is odd x^2 instead of x, and I put for shortness θ instead of $\frac{1}{2}k-s$ or $2(\frac{1}{2}k-s)$, and γ instead of $\frac{1}{2}\alpha+\frac{1}{2}s(s+1)$ or $\alpha+s(s+1)$; we have to consider an expression of the form

$$\text{coefficient } x^{\theta m} \text{ in } \frac{x^{\gamma}}{\mathrm{F}x},$$

where $\mathrm{F}x$ is the product of factors of the form $1-x^a$. Suppose that a' is the least common multiple of a and θ, then $(1-x^{a'})\div(1-x^a)$ is an integral function of x, equal χx suppose, and $1\div(1-x^a)=\chi x\div(1-x^{a'})$. Making this change in all the factors of $\mathrm{F}x$ which require it (*i. e.* in all the factors except those in which a is a multiple of θ), the general term becomes

$$\text{coefficient } x^{\theta m} \text{ in } \frac{x^{\gamma}\mathrm{H}x}{\mathrm{G}x},$$

where $\mathrm{G}x$ is a product of factors of the form $1-x^{a'}$, in which a' is a multiple of θ, *i. e.* $\mathrm{G}x$ is a rational and integral function of x^{θ}. But in the numerator $x^{\gamma}\mathrm{H}x$ we may reject, as not contributing to the formation of the coefficient of $x^{\theta m}$, all the terms in which the indices are not multiples of θ; the numerator is thus reduced to a rational and integral function of x^{θ}, and the general term is therefore of the form

$$\text{coefficient } x^{\theta m} \text{ in } \frac{\lambda(x^{\theta})}{\varkappa(x^{\theta})},$$

or what is the same thing, of the form

$$\text{coefficient } x^{m} \text{ in } \frac{\lambda x}{\varkappa x}.$$

Where $\varkappa x$ is the product of factors of the form $1-x^a$, and λx is a rational and integral function of x, the particular value of the fraction depends on the value of s; and uniting the different terms, we have an expression

$$\text{coefficient } x^{m} \text{ in } \mathrm{S}_s\,(-)^s\frac{\lambda x}{\varkappa x},$$

which is equivalent to

$$\text{coefficient } x^{m} \text{ in } \frac{\phi x}{fx},$$

where fx is a product of factors of the form $1-x^a$, and ϕx is a rational and integral function of x. And it is clear that the fraction will be a proper one when each of the fractions in the original expression is a proper fraction, *i. e.* in the case of $\mathrm{P}(0,1,2\,..\,k)^m\frac{1}{2}(km-\alpha)$, when for k even $\alpha<\frac{1}{4}k(k+2)$, and for k odd $\alpha<\frac{1}{4}(k+1)(k+3)$; and in the case of $\mathrm{P}'(0,1,2\,..\,k)^m\frac{1}{2}(km-\alpha)$, when for k even $\alpha+1<\frac{1}{4}k(k+2)$, and for k odd $\alpha+1<\frac{1}{4}(k+1)(k+3)$.

We see, therefore, that

$$\mathrm{P}(0,1,2\,..\,k)^m\tfrac{1}{2}(km-\alpha),$$

and

$$\mathrm{P}'(0,1,2\,..\,k)^m\tfrac{1}{2}(km-\alpha),$$

are each of them of the form

$$\text{coefficient } x^{m} \text{ in } \frac{\phi x}{fx},$$

where fx is the product of factors of the form $1-x^{a}$, and up to certain limiting values of α the fraction is a proper fraction. When the fraction $\frac{\phi x}{fx}$ is known, we may therefore obtain by the method employed in the former part of this Memoir, analytical expressions (involving prime circulators) for the functions P and P′.

As an example, take $$\mathrm{P}(0,1,2,3)^{m}\tfrac{3}{2}m,$$
which is equal to

$$\text{coefficient } x^{3m} \text{ in } \frac{1}{(1-x^{2})(1-x^{4})(1-x^{6})}$$

$$-\text{coefficient } x^{m} \text{ in } \frac{1}{(1-x^{2})(1-x^{2})(1-x^{4})}.$$

The multiplier for the first fraction is

$$\frac{(1-x^{6})(1-x^{12})}{(1-x^{2})(1-x^{4})},$$

which is equal to $$1+x^{2}+2x^{4}+x^{6}+2x^{8}+x^{10}+x^{12}.$$

Hence, rejecting in the numerator the terms the indices of which are not divisible by 3, the first term becomes

$$\text{coefficient } x^{3m} \text{ in } \frac{1+x^{6}+x^{12}}{(1-x^{6})(1-x^{12})(1-x^{6})},$$

or what is the same thing, the first term is

$$\text{coefficient } x^{m} \text{ in } \frac{1+x^{2}+x^{4}}{(1-x^{2})^{2}(1-x^{4})};$$

and the second term being

$$-\text{coefficient } x^{m} \text{ in } \frac{x^{2}}{(1-x^{2})^{2}(1-x^{4})},$$

we have $$\mathrm{P}(0,1,2,3)^{m}\tfrac{3}{2}m=\text{coefficient } x^{m} \text{ in } \frac{1+x^{4}}{(1-x^{2})^{2}(1-x^{4})}.$$

And similarly it may be shown, that

$$\mathrm{P}(0,1,2,3)^{m}\tfrac{1}{2}(3m-1)=\text{coefficient } x^{m} \text{ in } \frac{x+x^{3}}{(1-x^{2})^{2}(1-x^{4})}.$$

As another example, take $$\mathrm{P}'(0,1,2,3,4,5)\tfrac{5}{2}m,$$
which is equal to

$$\text{coefficient } x^{5m} \text{ in } \frac{1}{(1-x^{4})(1-x^{6})(1-x^{8})(1-x^{10})}$$

$$-\text{coefficient } x^{3m} \text{ in } \frac{x^{2}}{(1-x^{2})(1-x^{4})(1-x^{6})(1-x^{8})}$$

$$+\text{coefficient } x^{m} \text{ in } \frac{x^{6}}{(1-x^{2})(1-x^{4})(1-x^{4})(1-x^{6})}.$$

The multiplier for the first fraction is

$$\frac{(1-x^{20})(1-x^{30})(1-x^{40})}{(1-x^{4})(1-x^{6})(1-x^{8})},$$

which is a function of x^2 of the order 36, the coefficients of which are

1, 0, 1, 1, 2, 1, 3, 2, 4, 3, 4, 4, 6, 4, 6, 5, 7, 5, 7, 5, 7, 5, 6, 4, 6, 4, 4, 3, 4, 2, 3, 1, 2, 1, 1, 0, 1,

and the first part becomes therefore

$$\text{coefficient } x^m \text{ in } \frac{1+x^2+4x^4+5x^6+7x^8+4x^{10}+3x^{12}}{(1-x^2)(1-x^4)(1-x^6)(1-x^8)}.$$

The multiplier for the second fraction is

$$\frac{(1-x^6)(1-x^{12})(1-x^{24})}{(1-x^2)(1-x^4)(1-x^8)},$$

which is a function of x^2 of the order 14, the coefficients of which are

1, 1, 2, 1, 3, 2, 3, 1, 3, 2, 3, 1, 2, 1, 1;

and the second term becomes

$$-\text{coefficient } x^m \text{ in } \frac{2x^2+2x^4+3x^6+x^8+x^{10}}{(1-x^2)^2(1-x^4)(1-x^8)};$$

and the third term is $\quad$ coefficient x^m in $\dfrac{x^6}{(1-x^2)(1-x^4)^2(1-x^6)}$.

Now the fractions may be reduced to a common denominator

$$(1-x^2)(1-x^4)(1-x^6)(1-x^8)$$

by multiplying the terms of the second fraction by $\frac{1-x^6}{1-x^2}(=1+x^2+x^4)$, and the terms of the third fraction by $\frac{1-x^8}{1-x^4}(=1+x^4)$; performing the operations and adding, the numerator and denominator of the resulting fraction will each of them contain the factor $1-x^2$; and casting this out, we find

$$P(0, 1, 2, 3, 4, 5)^m \tfrac{5}{2}m = \text{coefficient } x^m \text{ in } \frac{1-x^6+x^{12}}{(1-x^4)(1-x^6)(1-x^8)}.$$

I have calculated by this method several other particular cases, which are given in my "Second Memoir upon Quantics;" the present researches were in fact made for the sake of their application to that theory.

Received April 20,—Read May 3 and 10, 1855.

Since the preceding portions of the present Memoir were written, Mr. SYLVESTER has communicated to me a remarkable theorem which has led me to the following additional investigations*.

Let $\frac{\phi x}{fx}$ be a rational fraction, and let $(x-x_1)^k$ be a factor of the denominator fx, then if

$$\left\{\frac{\phi x}{fx}\right\}_{x_1}$$

* Mr. SYLVESTER's researches are published in the Quarterly Mathematical Journal, July 1855, and he has there given the general formula as well for the circulating as the non-circulating part of the expression for the number of partitions.—Added 23rd February, 1856.—A. C.

denote the portion which is made up of the simple fractions having powers of $x-x_1$ for their denominators, we have by a known theorem

$$\left\{\frac{\phi x}{fx}\right\}_{x_1} = \text{coefficient } \frac{1}{z} \text{ in } \frac{1}{x-x-z}\frac{\phi(x_1+z)}{f(x_1+z)}.$$

Now by a theorem of JACOBI'S and CAUCHY'S,

$$\text{coefficient } \frac{1}{z} \text{ in } \mathrm{F}z = \text{coefficient } \frac{1}{t} \text{ in } \mathrm{F}(\psi t)\psi' t\,;$$

whence, writing $x_1+z=x_1e^{-t}$, we have

$$\left\{\frac{\phi x}{fx}\right\}_{x_1} = \text{coefficient } \frac{1}{t} \text{ in } \frac{x_1}{x_1-xe^t}\frac{\phi(x_1e^{-t})}{f(x_1e^{-t})}.$$

Now putting for a moment $x=x_1e^\theta$, we have

$$\frac{1}{x_1-xe^t}=\frac{1}{x_1(1-e^{\theta+t})}=\frac{1}{x_1(1-e^\theta)}+\partial_\theta\,\frac{1}{x_1(1-e^\theta)}+\dots$$

and $\partial_\theta=x\partial_x$, whence

$$\frac{1}{x_1-xe^t}=\frac{1}{x_1-x}+\frac{t}{1}x\partial_x\frac{1}{x_1-x}+\frac{t^2}{1.2}(x\partial_x)^2\,\frac{1}{x_1-x}+\dots,$$

the general term of which is

$$\frac{t^{s-1}}{\Pi(s-1)}(x\partial_x)^{s-1}\frac{1}{x_1-x}.$$

Hence representing the general term of

$$\frac{x_1\phi(x_1e^{-t})}{f(x_1e^{-t})}$$

by $\chi x_1 t^{-s}$, so that

$$\chi x_1 = \text{coefficient } \frac{1}{t} \text{ in } t^{s-1}\frac{x_1\phi(x_1e^{-t})}{f(x_1e^{-t})},$$

we find, writing down only the general term

$$\left\{\frac{\phi x}{fx}\right\}_{x_1} = \dots+\frac{1}{\Pi(s-1)}\,(x\partial_x)^{s-1}\frac{\chi x_1}{x_1-x}+\dots$$

where the value of χx_1 depends upon that of s, and where s extends from $s=1$ to $s=k$.

Suppose now that the denominator is made up of factors (the same or different) of the form $1-x^m$. And let a be any divisor of one or more of the indices m, and let k be the number of the indices of which a is a divisor. The denominator contains the divisor $[1-x^a]^k$, and consequently if ϱ be any root of the equation $[1-x^a]=0$, the denominator contains the factor $(\varrho-x)^k$. Hence writing ϱ for x_1 and taking the sum with respect to all the roots of the equation $[1-x^a]=0$, we find

$$\left\{\frac{\phi x}{fx}\right\}_{[1-x^a]} = \dots+\frac{1}{\Pi(s-1)}(x\partial_x)^{s-1}\mathrm{S}\frac{\chi\varrho}{\varrho-x}+\dots$$

$$=\dots+\frac{1}{\Pi(s-1)}\,(x\partial_x)^{s-1}\frac{\theta x}{[1-x^a]}+\dots,$$

where

$$\chi\varrho = \text{coefficient } \frac{1}{t} \text{ in } t^{s-1}\,\frac{\varrho\phi(\varrho e^{-t})}{f(\varrho e^{-t})},$$

and as before s extends from $s=1$ to $s=k$. We have thus the actual value of the function θx made use of in the memoir.

A preceding formula gives

$$\left\{\frac{\phi x}{fx}\right\}_1 = \text{coefficient } \frac{1}{t} \text{ in } \frac{1}{1-xe^t}\,\frac{\phi(e^{-t})}{f(e^{-t})},$$

which is a very simple expression for the non-circulating part of the fraction $\frac{\phi x}{fx}$. This is, in fact, Mr. SYLVESTER's theorem above referred to.

VIII. *Examination of select Vegetable Products from India.*

By JOHN STENHOUSE, *LL.D., F.R.S.*

Received November 14,—Read December 6, 1855.

PART I.

THROUGH the kindness of my esteemed friend Dr. ROYLE, I have been permitted to select such vegetable products from the extensive collection at the India House as seemed most likely to repay the trouble of investigation. My attention during the last twelve months has been chiefly directed to three of these vegetable substances; and the results of their examination I now take the liberty of submitting to the Royal Society, to be followed by those of the others as they may be completed.

Datisca cannabina.

The first of these substances which I examined consisted of a quantity of the roots of the *Datisca cannabina*, from Lahore, where this plant is employed to dye silk of a fast yellow colour. The roots, which had been cut into pieces about six or eight inches long, were from one-half to three-quarters of an inch in thickness. They had a deep yellow colour. The leaves and smaller branches of the *Datisca cannabina* from the Levant have long been employed for a similar purpose in the South of France. A decoction of the leaves of the *Datisca cannabina* was examined by BRACONNOT in 1816, who discovered in it a crystallizable principle to which he gave the name of *datiscine*. BRACONNOT, of course, did not subject this substance to analysis, but he described its appearance and properties in an exceedingly accurate manner*. The observations of BRACONNOT had fallen into such entire oblivion, however, that for many years past, we find in most of the larger systems of chemistry the term *datiscine* used as synonymous with *inuline*. Thus in BRANDE's 'Chemistry,' vol. ii. page 1168, we find it stated that a variety of names had been given to inuline, such as "dahline, datiscine," &c. In LÖWIG's 'Chemistry of Organic Compounds,' vol. i. page 359, the same error is repeated, where, under the article "inuline," the synonyms given are "dahline and datiscine."

The bruised roots were extracted in a MOHR's apparatus by long-continued digestion with wood-spirit. The liquor obtained, which had a dark brown colour, was concentrated by distilling off a portion of the wood-spirit. The brown syrupy liquid remaining in the retort, on being poured into open vessels and standing for some

* Annales de Chimie et de Physique (1830), iii. 277.

time, deposited a resinous matter containing merely traces of a crystalline substance. When this syrupy liquid, however, was treated with about half its bulk of hot water, the greater portion of the brown resin was rapidly deposited, and the mother-liquor, having been poured off and left to slow spontaneous evaporation, deposited a considerable quantity of an imperfectly crystallizable substance resembling grape-sugar. These crystals are impure *datiscine*, still retaining a considerable amount of resinous matter, to which the dark brown colour is owing.

They may be purified in various ways, advantage being taken of the greater solubility of the resinous matter in alcohol or ether, than that of the datiscine. By repeated crystallizations, therefore, from either of these liquids, the datiscine may be rendered almost perfectly colourless, the impurities remaining in the mother-liquor. The following is the method which I have found, on the whole, most convenient. The crude datiscine, while still moist, is strongly pressed between folds of blotting-paper; it is then dissolved in alcohol, and again treated with water, which throws down the resin. The diluted alcoholic solution, after standing some time, yields the datiscine in a much finer state. On repeating this operation several times, the datiscine may be obtained perfectly pure. In order to separate any traces of tannic acid which might have been present, I repeatedly added a concentrated solution of gelatine to the alcoholic solution of the datiscine, and after careful filtration, precipitated the datiscine in the way already described. I could not, however, observe that the crystals of the datiscine, when gelatine had been employed in its preparation, were in the least degree dissimilar from the datiscine obtained in the usual way.

Properties of Datiscine.—Datiscine, when quite pure, is perfectly colourless, but unless great attention is paid to its purification, it usually has more or less of a yellow colour, varying from pale to deep yellow, according to circumstances. It is very soluble in alcohol, even in the cold, boiling alcohol dissolving almost any amount of it.

By slow spontaneous evaporation, its alcoholic solutions yield small silky needles arranged in groups. Cold water does not dissolve much of it, but it is tolerably soluble in boiling water, the hot solutions on cooling depositing it in shining scales. Ether does not dissolve much datiscine, but an ethereal solution, when allowed to evaporate as slowly as possible, yielded it in larger crystals than I could have obtained in any other way. When water is added to an alcoholic solution of datiscine, no precipitate is immediately formed, unless the solution is greatly concentrated; but on standing for some time, the datiscine separates in a very pure state, consisting of fine crystals having a pale yellow colour. The datiscine prepared by this process was found to contain but a minute trace of ash. When datiscine is heated to about 180° C., it melts, and if the heat is still further increased, it burns, evolving an odour of caramel, and leaves a voluminous charcoal. If datiscine be heated in a close vessel while a stream of dry air is slowly passed over it, a small quantity of a crystalline substance sublimes. Datiscine and its solutions have a very bitter taste; and

though it does not produce any change upon test paper, I think there is reason to regard it as a feebly acid body. It dissolves in solutions of the fixed alkalies and ammonia, as well as in lime and baryta water. Their solutions have a deep yellow colour, which they lose on the addition of an acid, when the datiscine is precipitated, even acetic acid precipitating datiscine. When its alkaline solutions are not too concentrated, and are neutralized while hot, the datiscine on cooling is deposited in small crystals. The aqueous solution of datiscine is precipitated by neutral and basic acetates of lead, or chloride of tin. These precipitates have a bright yellow colour. Salts of copper produce greenish precipitates; those of peroxide of iron deep brownish-green precipitates. The datiscine employed in these experiments had been purified with gelatine.

The lead salts form such gelatinous precipitates, that they could not be washed, and therefore I was unable to employ them in determining the equivalents of datiscine.

Action of dilute Sulphuric Acid on Datiscine.

When an aqueous solution of datiscine is treated with a small quantity of sulphuric acid, the clear liquid, after being boiled for a few minutes, becomes turbid, and deposits a crystalline substance. This is collected on a filter, and the clear liquid which passes through, after it has been made strongly alkaline by the addition of potash, and after being heated to 212°, throws down suboxide of copper. When the excess of sulphuric acid is removed from another portion of the solution, by neutralizing with carbonate of lead or baryta, it acquires a sweet taste; and when evaporated to the consistence of a syrup, on standing for some time, it formed a semi-crystalline mass, closely resembling honey. This experiment showed therefore that datiscine, like salicine and similar bodies, belongs to the class of glucosides, and is a copulated compound of sugar and another substance which I shall call *datiscetine.*

Datiscetine.—Datiscetine in its general appearance and properties closely resembles datiscine. On a closer examination, however, these two substances are found to differ essentially, both in composition and properties. Datiscine, when prepared by boiling a solution of pure datiscine with dilute sulphuric acid, precipitates in the state of fine needles, which are nearly colourless. It is easily soluble in alcohol; a hot alcoholic solution, on cooling, depositing the greater portion of it in crystalline groups. It is almost insoluble in water, and consequently datiscetine is abundantly precipitated from its alcoholic solutions by the addition of water. It dissolves in ether to almost any extent, and is deposited, on the evaporation of that liquid, in needles.

These properties of datiscetine enable us to obtain it in a tolerably pure state when even very impure datiscine is employed in its preparation. The mother-liquors out of which datiscine has been crystallized, and which retain a large amount of impurities, can be used in the following way for the preparation of datiscetine. These

liquors are first treated with basic acetate of lead, the precipitate is collected on a filter and washed, and then, having been distributed through water, is decomposed by sulphuretted hydrogen. The solution which has filtered from the precipitated sulphide of lead, is then concentrated, and boiled with dilute sulphuric acid. The datiscetine produced in this way can be easily separated from the resinous matter, as the latter is precipitated before the formation of the datiscetine by the acid, and adheres to the bottom of the flask. By dissolving the datiscetine thus obtained in alcohol, and precipitating it by the addition of water, it is rendered tolerably pure.

Properties of Datiscetine.—Datiscetine has no taste. When heated it melts like datiscine, but the heat required is much higher than is necessary for that body. It crystallizes again on cooling. By operating very cautiously, a portion of the datiscetine may be sublimed in crystals. This sublimate, however, appears to be altered datiscetine, for, when recrystallized out of ether, it has a sweet taste. Datiscetine on burning does not emit the smell of caramel.

Datiscetine, like datiscine, dissolves in alkaline solutions, and is reprecipitated by the addition of an acid. When an alcoholic solution of acetate of lead is added to one of datiscetine, also dissolved in alcohol, a finely coloured deep yellow precipitate is obtained, which can be easily washed both by alcohol and water.

This precipitate therefore was subjected to analysis.

Analysis of Datiscine and Datiscetine.—The lead salt of datiscetine, when subjected to analysis, gave the following results:—

I. 0·4555 grm. gave 0·2060 grm. PbO.
   0·2655 grm. gave 0·3515 grm. CO_2 and 0·0405 grm. HO.
II. 0·4310 grm. gave 0·1945 grm. PbO.
III. (from another preparation).
   0·4170 grm. gave 0·1907 grm. PbO.
   0·3170 grm. gave 0·4195 grm. CO_2 and 0·0460 grm. HO.

These analyses give the formula $C_{30}H_8O_{10}+2PbO$.

Required.			Found.		
			I.	II.	III.
C_{30}	= 180·0	36·63	36·11	——	36·09
H_8	= 8·0	1·63	1·69	——	1·61
O_{10}	= 80·0	16·28	——	——	——
2PbO	= 223·4	45·46	45·22	45·13	45·73

Analysis of Datiscetine.—The different preparations were all dried at 100° C. in a current of dry air, and burned in the gas furnace.

Datiscetine crystallized out of Alcohol.

I. 0·3970 grm. gave 0·9155 grm. CO_2 and 0·1280 grm. HO.
II. 0·4455 grm. gave 1·0275 grm. CO_2 and 0·1485 grm. HO.

Datiscine prepared from its alcoholic solution by the addition of Water.

III. 0·2045 grm. gave 0·4715 grm. CO_2 and 0·0665 grm. HO.
IV. 0·3680 grm. gave 0·8505 grm. CO_2 and 0·1245 grm. HO.

The per-centage results calculated from these analyses agree very closely with the formula deduced from the analyses of the lead salt, as shown in the following Table:—

Required.		Found. I.	II.	III.	IV.
C_{30}=180	62·94	62·89	62·90	62·88	63·03
H_{10}= 10	3·49	3·58	3·70	3·61	3·76
O_{12}= 96	33·57	—	—	—	—

Analysis of Datiscine dried in a LIEBIG'S *drying tube.*

A. Datiscine crystallized out of alcohol.

I. 0·3890 grm. gave 0·7815 grm. CO_2 and 0·1780 grm. HO.
II. 0·3395 grm. gave 0·6770 grm. CO_2 and 0·1585 grm. HO.

B. Datiscine crystallized out of alcohol from another preparation.

I. 0·2785 grm. gave 0·5595 grm. CO_2 and 0·1315 grm. HO.
II. 0·2905 grm. gave 0·5830 grm. CO_2 and 0·1385 grm. HO.

C. Datiscine prepared with gelatine and out of alcohol.

0·4145 grm. gave 0·8355 grm. CO_2 and 0·1925 grm. HO.

D. Datiscine separated from its alcoholic solution by the addition of water.

I. 0·1580 grm. gave 0·3135 grm. CO_2 and 0·0770 grm. HO.
II. 0·1980 grm. gave 0·3955 grm. CO_2 and 0·0925 grm. HO.

E. Datiscine precipitated from its potash solution by the addition of acetic acid.

0·2985 grm. gave 0·5940 grm. CO_2 and 0·1375 grm. HO.

The substance analysed in A. contained 0·3 to 0·4 per cent. of ash; that in B. 0·2 to 0·25 per cent.; and that in C. 0·12 per cent. No allowance, however, was made for these amounts of ash in calculating the following numbers.

The datiscine used for the analyses D. and E. did not contain any appreciable quantity of ash.

The following are the *per-centage* numbers calculated from the above analyses:—

	A. I.	A. II.	B. I.	B. II.	C.	D. I.	D. II.	E.
C......	54·79	54·38	54·79	54·73	54·97	54·11	54·48	54·27
H......	5·08	5·19	5·25	5·29	5·16	5·41	5·19	5·12
O......	—	—	—	—	—	—	—	—

When the decomposition of datiscine into datiscetine and sugar is taken into consideration, it seems probable that the formula for datiscine is—

$$\text{Datiscetine}+\text{Sugar} = \text{Datiscine}$$
$$C_{30}H_{10}O_{12} + C_{12}H_{12}O_{12} = C_{42}H_{22}O_{24}.$$

That formula requires

$C_{42}=252$	54·08
$H_{22}=22$	4·72
$O_{24}=192$	41·20

If the formula $C_{42}H_{22}O_{24}$ be correct, the decomposition of datiscine would be analogous to salicine, which, when treated with dilute acid, gives

$$\text{Salicine} = \text{Sugar} + \text{Salicetine}$$
$$C_{26}H_{18}O_{14} = C_{12}H_{12}O_{12} + C_{14}H_{6}O_{2};$$

whilst when treated with emulsine,

$$\text{Salicine} = \text{Sugar} + \text{Saligenine} - 2HO.$$
$$C_{26}H_{18}O_{14} = C_{12}H_{12}O_{12} + C_{14}H_{8}O_{4} - 2HO.$$

The above formula for datiscine is confirmed by the following determinations of sugar formed by treating a weighed quantity of datiscine with dilute sulphuric acid. The quantity of sugar produced was estimated by FEHLING's method with a standard solution of sulphate of copper. Four determinations made in this way gave the following quantities of sugar for 100 parts of dry datiscine:—

I.	41·6
II.	39 5
III.	39·3
IV.	37·8.

The calculation for the formula $C_{42}H_{22}O_{24}$ requires 38·6 per cent. of sugar.

An attempt was made to confirm these determinations by the direct estimation of the datiscetine formed during the decomposition of a given weight of datiscine by dilute sulphuric acid. The numbers obtained, however, during different experiments did not correspond with each other, and were all much lower than theory required. This however is not to be wondered at, as the datiscetine, though very differently soluble, is by no means absolutely insoluble in water. Dilute hydrochloric acid was found, like sulphuric acid, to decompose the datiscine into sugar and datiscetine, and even on boiling an aqueous solution of pure datiscine for some hours traces of sugar could be detected; thus showing that a small portion of the datiscine had been decomposed. It has been shown by previous experiments, and analysis No. 8, that datiscine may be dissolved in cold solutions of potash without decomposition. When boiled, however, with a concentrated potash lye for some time, decomposition takes place, and the precipitate, thrown down by the addition of an acid, has all the properties of datiscetine. In this respect, therefore, datiscine agrees with tannin and

similar glucosides, which yield the same products when acted upon by acids and alkalies; tannin from nut-galls being equally resolved by acid and alkalies into gallic acid and grape-sugar. A solution of datiscetine, when left for some time in a warm place in contact with yeast, did not ferment, and though emulsine was also tried no separation of datiscetine was observed.

Action of Nitric Acid on Datiscine and Datiscetine.

When datiscetine is treated in the cold with nitric acid of ordinary strength, a violent reaction takes place, brown vapours are disengaged, and a resinous substance produced. The heat evolved by the reaction is sufficient to continue it till all the resinous matter is dissolved and a dark red liquid produced, which after boiling becomes yellow, and when cautiously evaporated, deposits on cooling crystals of nitropicric acid. If the reaction is not carried too far no oxalic acid is formed, and the nitropicric acid obtained is very pure, forming large crystals of a pale yellow colour. The following experiments show that these crystals are really nitropicric acid.

When treated in the cold with a concentrated solution of hypochlorite of lime, the very characteristic smell of chloropicrine is instantaneously observed. The addition of a solution of potash to a second portion of the acid threw down crystals of very pure nitropicrate of potash, which, when subjected to analysis, gave 18·04 per cent. of potash, the calculated quantity required for nitropicrate of potash being 17·66 per cent. Datiscine treated in the same way yields nitropicric acid as well as oxalic acid. The formation of nitropicric acid by the action of strong nitric acid on datiscine and datiscetine, rendered it highly probable, that, by employing dilute nitric acid, less highly oxidized products of decomposition might be obtained. Datiscetine was therefore boiled with dilute nitric acid, *i. e.* nitric acid with ten parts of water; the crystals of datiscetine soon dissolved, and a yellow liquid was obtained, which when treated with a solution of perchloride of iron gave a blood-red colour. When the original solution had cooled, pale yellow crystals were deposited. These crystals were very soluble in hot water, and recrystallized on cooling.

They were likewise very soluble in alcohol and ether, and when deposited by slow evaporation from their alcoholic solutions, they formed nearly colourless crystals of considerable size, having a fine silky lustre. When treated with hypochlorite of lime they did not evolve chloropicrine in the cold, but readily produced that compound when gently heated. When they were cautiously heated between two watch-glasses, a portion of the substance sublimed in colourless needles. When this acid was heated with an insufficient quantity of water to dissolve it, it melted and recrystallized on cooling. These properties agree with those of nitrosalicylic acid. In order to confirm this hypothesis the following salts were prepared.

On saturating a solution of the acid with carbonate of baryta, the baryta salt was obtained in yellow crystals. From their solution ammonia threw down another

yellow-coloured baryta salt. A solution of the acid likewise produced a crystallized lead salt. The ammoniacal salt was obtained in orange-coloured needles by saturating the acid with ammonia and crystallizing *in vacuo*. By double decomposition with the ammoniacal salt I prepared the silver combination, which, like all the others, agreed in its properties with MARCHAND's description of nitrosalicylic acid salts. When subjected to analysis, 0·1695 grm. of the silver salt gave

0·0630 grm. of silver, equal to 37·17 per cent.,

the quantity of silver in nitrosalicylate of silver being 37·24 per cent. When datiscine was allowed to stand in contact with dilute nitric acid in the cold it gradually dissolved, and the liquid assumed a yellow colour. The solution, when left to evaporate *in vacuo*, was found to contain a mixture of oxalic and nitropicric acids.

Action of Potash on Datiscine and Datiscetine.

It was stated in a previous part of this paper that datiscine and datiscetine dissolve in solutions of caustic alkalies without decomposition, and that datiscine, when boiled for some time, is decomposed with the formation of datiscetine. It only remained therefore to try the action of fused hydrate of potash. Datiscetine, when added in small successive portions to fused hydrate of potash, assumed a deep orange colour, and then dissolved with the evolution of hydrogen gas. When the disengagement of hydrogen ceased, the mass was dissolved in water and supersaturated with hydrochloric acid. A partly crystalline resinous substance separated, which, by sublimation, yielded perfectly colourless long crystals, closely resembling benzoic acid. Their solution in water assumed, on the addition of perchloride of iron, that deep violet tint which disappears on the addition of hydrochloric acid, and is characteristic of salicylic acid.

Action of Chromic Acid.

Datiscetine was likewise distilled with bichromate of potash and sulphuric acid; the liquid which came over did not contain oily drops, but had the smell of salicylous acid, and formed with persalts of iron a purple-coloured solution.

A trace of salicylous acid appeared therefore to have been produced.

It follows therefore, I think, from the experiment already detailed, that datiscine, like salicine, phloridzine, &c., is a glucoside, and that it approaches nearer to salicine than any other glucoside, with the exception of populine, yet known. In fact I am not aware of any glucoside, with the exception of salicine and populine, which, when treated with nitric acid, yields nitrosalicylic or even nitropicric acid. Phloridzine and phloretine, for instance, when treated with nitric acid, are stated by different experimenters to yield only oxalic acid. I repeated the experiment with phloretine, and obtained much oxalic acid, while the residual liquor yielded not a trace of chloropicrine when treated with hypochlorite of lime, and consequently contained no nitropicric acid. Quercitrine, when likewise treated with nitric acid, was also found

to yield, as Rigaud states, only oxalic acid. I had no opportunity of trying esculetine, but it is stated by those who have investigated it to yield only oxalic acid.

I will conclude this account of datiscine by proposing the following practical application. As is well known, the colouring matter of madder when boiled with dilute sulphuric acid is changed into sugar and garancine, a new dye-stuff, which for many purposes is found superior to that originally present in the madder.

Within the last twelve months, Mr. Lieshing, by treating the colouring matters in weld and quercitron bark with dilute sulphuric acid, has resolved them into new colouring matters, which are but slightly soluble in water, and are found nearly three times more powerful as dye-stuffs than the original colouring matters from which they had been produced. As datiscine, when boiled with dilute sulphuric acid, undergoes a perfectly similar transformation, being resolved into sugar and datiscetine, which has a much higher colouring power than the datiscine which has produced it, I have not the least doubt that silk dyers, who may hereafter employ solutions of *Datisca cannabina*, will find it highly advantageous to convert their datiscine into datiscetine by boiling it with dilute sulphuric acid; as the process is an extremely simple one, and as the datiscetine, from its sparing solubility in water, can be very readily obtained in a state of comparative purity.

Ptychotis Ajowan.

The *Ptychotis Ajowan* is an umbelliferous plant, well known in India for its aromatic and carminative properties. Its seeds, which very much resemble in appearance those of the caraway, only being much smaller, have a very agreeable odour, resembling oil of thyme.

On distilling these seeds repeatedly with water, the essential oil is very easily obtained, amounting to between five and six per cent. of the weight of the seeds. This oil has a light brown colour, and possesses an agreeable aromatic odour. Its specific gravity is 0·896 at 12° C., and upon leaving the oil for some time in an open dish to spontaneous evaporation, the temperature at the time being comparatively low, large beautiful crystals were deposited, which on examination were found to be identical with the stearopten brought from India by the late Dr. Stocks, and described by me in a short notice published in the number of the 'Pharmaceutical Journal' for December 1854. When the crude oil is submitted to distillation, it begins to boil at 160° C., the thermometer rising rapidly to 174° C. The thermometer then rises to 220° C., and the oil which comes over at this temperature crystallizes on cooling. The residue, which does not immediately crystallize, on remaining at rest for some time, solidifies into a crystalline mass, the crystals having precisely the same form as those obtained by spontaneous evaporation, and amounting, in weight, to from one-third to one-fourth of the crude oil. As it seemed probable from these experiments, therefore, that Ptychotis oil, like many other essential oils, is a

mixture of a liquid hydrocarbon and of a less volatile oxygenated stearopten, I proceeded to separate these compounds.

The Hydrocarbon.—The more volatile portion of the oil was redistilled, that part of it which boiled at 176° C. being separately collected. After having been dried with chloride of calcium, it was distilled over caustic potash. It was then repeatedly treated with sodium, and again cautiously rectified. The hydrocarbon thus obtained was perfectly colourless, refracted light strongly, and had a pungent, aromatic odour, quite dissimilar, however, from that of oil of thyme. Its boiling-point was found to be 172° C., a thermometer being placed in the vapour, and its specific gravity 0·854 at 12° C. The following analyses show that it is isomeric with oil of turpentine.

I. 0·1280 grm. gave 0·4145 grm. carbonic acid and 0·1335 grm. water.

II. 0·1765 grm. gave 0·5705 grm. carbonic acid and 0·1825 grm. water.

	Required.		Found. I.	Found. II.
C_{10} =	60	88.23	88·31	88·15
H_8 =	8	11·77	11·59	11·49

When the oil was treated with hydrochloric acid no crystalline compound was obtained, but a brown mobile liquid, having an agreeable smell similar to the oil itself.

The Stearopten.—I have already mentioned that the portion remaining in the retort from the distillation of the crude oil, solidified on cooling into a crystalline mass. When cautiously rectified it began to boil at 218° C., the thermometer, towards the latter part of the distillation, rising to 225° C., and even higher. The greater portion, however, came over at about 222° C. The first portion which came over was quite colourless, and had a mild aromatic smell, but the subsequent portions had a more pungent odour, and a yellowish colour. The more volatile and by far the larger portion of the distillate crystallized on cooling, especially when agitated, the crystals assuming a rhombohedral form. When it was kept quite quiet, however, it remained liquid for several days, but on being plunged into a freezing mixture it immediately solidified. The less volatile and more coloured portion of the distillate could not be made to crystallize, even when kept in a mixture of snow and salt. The form of the crystals, obtained by the solidification of the distilled stearopten, appeared at first sight to differ from that of the stearopten from India given me by Dr. Stocks; but, upon these crystals being dissolved in the hydrocarbon of the oil, they were obtained in forms precisely similar to those of the Indian stearopten. This was also the case when either the crude or the distilled crystals were deposited from their solutions in alcohol or ether.

Through the kindness of Professor Miller of Cambridge, I am enabled to submit the following very accurate description and measurements of these crystals.

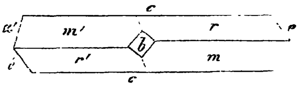
"The white crystals, which were the purer portions of the distilled stearopten, have the following figure and measurements:—

"*Oblique*:—Symbols of the simple forms:—

a 100,	*b* 010,	*c* 001,
m 110,	*e* 210,	*r'* 111.

Angles between normals to the faces:—

	°	'
bc	90	0
ca	103	23
ab	90	0
ea	42	17
ec	61	6
ma	49	21
mb	40	39
mc	98	40
cm'	81	20
rc	49	18
rm	49	22
em	61	11
re	42	13
rm'	76	36
mm'	81	18
rr'	108	50



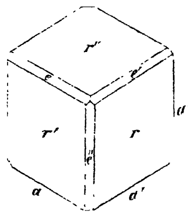
"*Cleavage*:—*m*, *c* very perfect.

"The brown crystals.

"*Rhombohedral*:—Symbols of the simple forms, *a* $01\bar{1}$, *r* 100, *e* 011.

Angles between normals to the faces:—

	°	'
aa'	60	0
rr'	98	38
re'	49	19
ra'	40	41



"*Cleavage*:—*r* very perfect. The faces *e*, *a*, are very narrow.

"The white crystals are very thin in a direction perpendicular to the faces *c*. It is remarkable, that though they have the character of crystals of the oblique system, the cleavages make with each other very nearly the same angles as the cleavages of the brown crystals. The differences fall within the probable errors of observation: for the crystals, being very tender, and frequently having lost a portion of the original surface by evaporation, do not admit of very accurate measurement.

"In the brown crystals the cleavages are parallel to the faces *r*, *r'*, *r''*, and the

angle between normals to rr' is 98° 38′. In the white crystals the cleavages are parallel to the faces c, m, m'. The angle between normals to m, c is 98° 40′, and the angle between normals to m and the face parallel to m' is 98° 42′. I should almost be inclined to suppose the brown crystals to have actually the same form as the white ones, the difference of the angles rr', $r'r''$, rr'' escaping observation, and as a consequence, that they have the same chemical constitution, a slight amount of impurity excepted."

The following are the results of the analyses of the stearopten obtained from different sources.

A. *Stearopten from India*.*

I. 0·2385 grm. gave 0·6955 grm. CO_2 and 0·2025 grm. HO.
II. 0·3405 grm. gave 0·9910 grm. CO_2 and 0·2815 grm. HO.
III. 0·1870 grm. gave 0·5440 grm. CO_2 and 0·1550 grm. HO.

B. *Stearopten obtained by the spontaneous evaporation of the oil.*

0·3355 grm. gave 0·9780 grm. CO_2 and 0·2735 grm. HO.

C. *Distilled stearopten recrystallized from the oil.*

I. 0·2720 grm. gave 0·8015 grm. CO_2 and 0·2225 grm. HO.
II. 0·3115 grm. gave 0·9175 grm. CO_2 and 0·2630 grm. HO.

The most simple formula agreeing with the numbers obtained by the above analyses is $C_{10}H_7O$, as shown by the following Table:—

Required.		Found.					
		A.			B.	C.	
		I.	II.	III.		I.	II.
C_{10} = 60	80·00	79·53	79·38	79·34	79·50	80·36	80·33
H_7 = 7	9·33	9·43	9·19	9·19	9·06	9·09	9·38
O = 8	10·67	——	——	——	——		

D. *Analysis of the distilled stearopten not recrystallized out of the oil.*

I. 0·4125 grm. gave 1·1885 CO_2 and 0·3375 grm. HO.
II. 0·3030 grm. gave 0·8655 CO_2 and 0·2475 grm. HO.

E. *Analysis of the same crystals recrystallized out of ether and pressed between folds of blotting-paper.*

I. 0·5915 grm. gave 1·7100 grm. CO_2 and 0·4860 grm. HO.
II. 0·2385 grm. gave 0·6900 grm. CO_2 and 0·1975 grm. HO.

* In the December Number of the 'Pharmaceutical Journal' for 1854 I published an analysis of the stearopten made from a small portion of the substance given me by the late Dr. STOCKS. This I have subsequently found to be inaccurate, and the analyses given above are substituted in its stead.

F. *Analysis of the last (now crystallizable) part of the distillation.*

I. 0·3980 grm. gave 1·1195 grm. CO_2 and 0·3395 grm. HO.

II. 0·3875 grm. gave 1·0930 grm. CO_2 and 0·3225 grm. HO.

From the foregoing analyses the following per-centage results were obtained:—

D.		E.		F.	
I.	II.	I.	II.	I.	II.
C = 78·58 H = 9·09	78·26 9·06	78·84 9·09	78·90 9·20	76·71 9·48	76·92 9·25

It will be seen, by the above analyses, that, during distillation, the stearopten undergoes partial decomposition, the amount of carbon being thus decreased.

The analyses D. and E. were evidently made with an impure substance, but the analysis F. agrees in a surprising manner with the numbers calculated from the formula $C_{15} H_{11} O_2$, which requires

$$\begin{array}{ll} C_{15}=90 & 76{\cdot}92 \\ H_{11}=11 & 9{\cdot}40 \\ O_2=16 & \end{array}$$

I do not however wish it to be supposed that the formula given above, or the double of it, viz. $C_{30} H_{22} O_4$, is the true formula for that substance; nevertheless I may observe that it bears a very simple relation to the formula of the stearopten itself. The one may be regarded as

$$3C_{10} H_6+4HO,$$

and the other

$$3C_{10} H_6+3HO,$$

that is, as two different hydrates of the same hypothetical hydrocarbon $C_{10} H_6$, or $C_{20} H_{12}$, which perhaps might have been obtained by distilling the stearopten with anhydrous phosphoric acid.

All the experiments which I made, with a view of determining the basic or acid properties of the stearopten, gave negative results. When an alcoholic solution of the crystals was mixed with one of acetate of lead, no precipitate was formed, and the same was the case with nitrate and ammonio-nitrate of silver. The crystals of the stearopten dissolve when gently warmed in oil of vitriol, heat being evolved, and a purple colour produced. On standing the whole solidified, and the mass thus obtained was very soluble in water, yielding a colourless solution, with drops of a red oily liquid floating in it, produced probably by the action of the sulphuric acid on some of the hydrocarbon adhering to the stearopten. When dissolved in a small quantity of hot water the copulated acid was deposited, on the cooling of the solution, in fine colourless scaly crystals. I regret that the small quantity of the substance at my disposal prevented me from determining the equivalent of the stearopten by means of the acid or one of its salts. I obtained both the baryta and the lead salt in a crystallized

state by neutralizing the solutions of the acid with the carbonates of those bases. Their analysis, however, gave irregular results, as, owing to the smallness of their quantity, I could not prepare them in a state of perfect purity. In the former notice of the stearopten, already mentioned, it was stated that by long-continued digestion with concentrated nitric acid, the stearopten was dissolved, and a colourless crystalline acid produced. This acid is neither oxalic acid, nor, apparently, any of the nitrogenated acids. At least when warmed with hypochlorite of lime it gives off no chloropicrine. I suspect, therefore, that it will be found to be a new acid, the examination of which I hope ere long to lay before the Society. From the physical properties and the elementary composition of the stearopten of the *Ptychotis Ajowan*, it struck me that it was very similar to, if not identical with, the solid portion of oil of thyme, described by LALLEMAND in his recent papers on that substance*. This idea induced me to distil the stearopten dissolved in oil of vitriol with an excess of peroxide of manganese. As anticipated, besides formic acid, there came over yellow drops which solidified on cooling, forming large crystals, having a peculiar odour somewhat resembling iodine or kinone, and agreeing in every respect with the substance described by LALLEMAND as thymol.

Before LALLEMAND has published his experiments in detail, which are now only known from the two notes in the 'Comptes Rendus,' I do not think it possible to decide with perfect certainty whether thymol and the stearopten of the *Ptychotis* are really identical substances, and even then, perhaps, it will be found necessary to make more complete experiments with the stearopten of the *Ptychotis*. The hypothesis of their identity, however, appears to me highly probable from the great similarity existing in the most important properties of those substances. I may add, that I agree with GERHARDT† in considering the crytallizable substance obtained by ARPPE‡ from the essential oil of the Horse-mint (*Monarda punctata, huile de monarde*), to be identical with LALLEMAND's thymol and DOVERI's§ less volatile portion of the oil of thyme.

All these substances gave nearly the same numerical results when subjected to analysis.

DOVERI observed two boiling-points, one between 175° and 180° C., and the other between 230° and 235° C.

LALLEMAND found the boiling-point of his thymène to be 165° C., and obtained a liquid hydrochloric acid compound.

ARPPE's crystallizable substance from the horse-mint oil, was found to melt at 48° C., to solidify at 38° C., and to boil at 224° C.

The crystals were rhombohedral, having one angle of 97° 30′ and another of 82° 30′.

* Compt. Rend. de l'Acad. xxxvii. 498, and xxxviii. 1022.

† Traité de Chimie Organique, iii. 610.

‡ Annalen der Chemie und Pharmacie, lviii. 41; Chem. Gaz. December 1846.

§ Annales de Chimie et de Physique, [3] xx. 174.

The Ptychotis oil, according to my experiments, contains a hydrocarbon boiling at 172°, and forming a liquid hydrochloric acid compound, and a crystallizable substance melting at 44° C., and boiling at about 222° C. The principal angles are stated by Professor MILLER to be—

"Crystallized on cooling . . .	$rr' = 98$	38
Indian crystals	$mc = 98$	40
Indian crystals	$mm' = 81$	18
Indian crystals	$m'c = 81$	20."

Two properties, however, I observed different from those given by LALLEMAND, viz. that the stearopten of the *Ptychotis* is precipitated (in a liquid state) from its alcoholic solutions by water, and that potash does not dissolve it, but merely causes it to assume the liquid state.

From the results of these experiments, therefore, I think we may confidently infer that the stearopten of the Ptychotis oil, and the crystallizable oxygenated portion of oil of thyme, examined by LALLEMAND, if not identical, as I apprehend they are, are certainly extremely similar bodies.

Gum of the Gardenia lucida, ROXB. (*the Decamalee Gum of Scinde*).

The specimen of this gum on which I operated was evidently very old. It formed a hard, dry mass, of a dark brown colour, with numerous patches of greenish-yellow matter disseminated through it. It had but a faint odour, unless freshly fractured or gently heated, when it smelt like the urine of the cat. A comparatively recent specimen of this gum, which I saw in the hands of the late Dr. STOCKS, had nearly the consistence of candied honey, and an exceedingly offensive odour. Dr. STOCKS informed me that the recent gum was employed as a dressing for wounds, as it kept off the flies.

The resin was digested in strong spirits of wine till a saturated solution was obtained. This, on cooling, immediately deposited some yellow, amorphous flocks. These were separated by filtration, and the clear liquid slowly evaporated *in vacuo*. On standing a few days, it deposited slender golden-yellow crystals, about half an inch in length. The crystals had considerable lustre, and were very brittle. To this crystalline substance I propose giving the provisional name of *Gardenine*.

Gardenine is nearly insoluble both in cold and hot water. It dissolves pretty readily in alcohol, but much less easily in ether, yielding bright yellow solutions, out of which it crystallizes on cooling. Alkalies, such as ammonia, do not appear to increase its solubility. It is slightly soluble in hot hydrochloric acid. Strong oil of vitriol dissolves gardenine in the cold with the production of a beautiful dark red colour. On adding water to this solution the gardenine is precipitated apparently unchanged. Its alcoholic solutions give no precipitates with ammonio-nitrate of silver, or with basic acetate of lead. When gardenine is digested with concentrated

nitric acid, it is rapidly decomposed, nitropicric acid, but apparently no oxalic acid, being produced.

Unfortunately, from the very small quantity of resin at my disposal, I was unable to prepare a sufficient amount of the gardenine, either to subject it to analysis, or to examine it more particularly. Dr. ROYLE has, however, already commissioned a large quantity of the resin from India, which I trust will ere long enable me to complete its examination.

Gardenine appears to belong to the tolerably numerous class of indifferent crystallizable resins, of which it is certainly one of the most beautiful.

St. Bartholomew's Hospital,
November 14*th*, 1855.

Notes on *Datisca cannabina*, *Ajowan* and Decamalee Gum, received from Dr. ROYLE, March 12, 1856.

Ikl-beer, the stems and roots of *Datisca cannabina*.

"*Datisca* is found both in the Old and New World, existing in the latter in Pennsylvania. *D. cannabina*, so named from its great resemblance to the Hemp-plant, extends from the south of Europe and Asia Minor through Iberia to the valleys of the Himalaya. I have obtained it from Cashmere and Kunawm, and found it at the foot of the Choor and Kedarkanta mountains. It spreads also to Nepal; *D. nepalensis*, Don, being the same species."—ROYLE, *Himal. Bot.* p. 340.

The *Ikl-beer* is much esteemed in the Punjab for dyeing silk of a yellow colour. It is probable that some of the silk scarves of a lemon-yellow colour, which were much admired by artists for the delicacy of their colour, had been dyed by this substance, as it is there esteemed for dyeing silk of this colour.

Ajwain.

"The remaining Umbelliferæ, which are known in India, are those found only in a cultivated state; but this from so remote a period as to have become perfectly naturalized, and known to the natives, as well as to have names given to them in the languages of different parts of the country; some also have not as yet been found in other parts of the world;"—"as *Ptychotis Ajowan*, known everywhere by the name *Ajwain*, slightly varied in different districts."—"In addition to these, *Ptychotis sylvestris*, nob., called *arub ajwain*, is used as a carminative by the natives."—ROYLE, *Himalaya Bot.* p. 229.

"Dr. ROXBURGH, in describing *Ligusticum Ajowan*, states, he cannot conceive that this famous Indian plant, aromatic in smell, pungent in taste, used both by natives and Europeans for culinary and medicinal purposes, can be unknown to

European botanists. To *Ajwain* Persian authors assign *nankhwah* as the Arabic name. This is the نانخواه (Nankhwah) of AVICENNA, written *nanachua* and *nanachue* in the marginal translation of Ammi, in the Latin edition of his works; which names are quoted under Ammi by MATHIOLUS, in his Commentaries on DIOSCORIDES. But in Persian works on Materia Medica, Aammi is also given as the Greek synonym of *nankhwah*, that is, of *Ajwain*, which Dr. ROXBURGH justly supposed could not be unknown to Europeans. This plant has been referred by M. DECANDOLLE to the genus *Ptychotis*, which brings it near *P. copticum*, called at one time *Ammi copticum*: the Ammi itself is called *Cuminum Æthiopicum* and *regium*; the latter name is translated by KUMOON MULLOOKEE, and is given as a synonym of *nankhwah* in Persian authors."—*L. c.* p. 230. It is probable therefore that this Indian species of *Ptychotis* has long been employed as the Aammi of the Greeks.

Decamalee.

"This is the gum of the *Gardenia lucida*. It exudes in amber-coloured transparent drops, at the ends of the young shoots, and from thence is collected. It is most useful in preventing vermin from breeding in wounds. It is brought to Bombay from the interior."—*Bombay List of Articles for Exhibition of* 1851, p. 30.

IX. *Experimental Researches in Electricity.—Thirtieth Series.* By MICHAEL FARADAY, *Esq., D.C.L., F.R.S., Fullerian Prof. Chem. Royal Institution, Foreign Associate of the Acad. Sciences, Paris, Ord. Boruss. Pour le Mérite, Eq., Memb. Royal and Imp. Acadd. of Sciences, Petersburgh, Florence, Copenhagen, Berlin, Göttingen, Modena, Stockholm, Munich, Bruxelles, Vienna, Bologna, Commander of the Legion of Honour, &c. &c.*

Received October 24,—Read November 15 and 22, 1855.

§ 38. *Constancy of differential magnecrystallic force in different media.*
§ 39. *Action of heat on magnecrystals.*
§ 40. *Effect of heat upon the absolute magnetic force of bodies.*

3363. WHILST using lines of force as a true, searching, and as yet, never-failing representative of the one form of power possessed by paramagnets, diamagnets, and electric currents,—and whilst endeavouring simultaneously to make the principle of representation a key to new phenomena, and subjecting the principle itself to rigid cross examination,—I have had occasion to examine the action of certain magnetic bodies in different media and at different temperatures; and as the results are true, and must, therefore, be valuable in any view of the cause of magnetic action, I have thought them worthy of presentation to the Royal Society.

3364. When an unmagnetized but magnetic body, placed in a magnetic field, is affected by the forces thrown upon it, and under their action *sets*, or takes up a definite position, the effect may depend upon its peculiar molecular condition, or upon its relation to the surrounding medium, or upon both conjointly, or upon one or both combined with temperature. Some of each of these conditions have been the objects of my investigations.

§ 38. *Constancy of differential magnecrystallic force in different media.*

3365. When a sphere or cylinder formed from a crystal of calcareous spar or of bismuth is suspended in a particular direction in the magnetic field, it points with considerable power, whereas if formed out of amorphous or granular carbonate of lime or bismuth, it has no such tendency. In the latter case, it (by reason of its relation to the surrounding medium) is urged to move from stronger to weaker places of force; in the former case, also, it has the same tendency, but the power of pointing, which it possesses in addition, has no relation to the medium about it, but only to the difference in strength of the magnetic force, as developed in different directions within the sphere itself. Such effects constitute the branch of science known under

the name of *magnecrystallic action*. Early experiments (2499–2501.) showed that, with respect to bismuth, the relation between the magnetic force in the axial and equatorial directions, was unchanged by varying the surrounding medium from water to a solution of sulphate of iron. This equality has, perhaps, been confirmed by other philosophers, and, it may be, with other substances; but not being aware of any strict investigation, I found it needful in relation to my own views, which required just now proofs more certain than those quoted, to enlarge and extend the experimental results.

3366. The method I have employed to compare the possible variations of force produced by different circumstances, has been to suspend the object, a magnecrystal for instance, by a torsion fibre or wire;—to place it in the magnetic field;—to adjust the torsion index so that it should be at zero when the crystal had taken its position of stable equilibrium;—then to put on right-handed torsion until the crystal had attained the point of unstable equilibrium, or the upsetting point, on that side; and after having noted the torsion required, to reverse the motion and put on left-handed force until the upsetting point on the opposite side was attained. Either of these forces, minus the deflection, is the measure of the upsetting force; and therefore the sum of these two observations, minus the number of degrees through which the crystal has moved in passing from one upsetting point to the other, may be considered as expressing the force which solicits the crystal to retain its stable position of rest. By thus making the observations on both sides of zero, the effect of set in the suspending torsion thread could be included in a regular and compensatory manner;—two definite starting-points (the upsetting positions) were ensured;—and also a large, *i. e.* a sensitive expression of the force to be measured was obtained.

3367. It is evident, that, when a magnecrystal is suspended in the magnetic field, and torsion force is gradually applied to deflect it from its position of rest, that force will grow up and carry the crystal round, until at last the latter will attain a position at which the setting forces of the crystal are equal to the torsion force, but beyond which the former will, by further motion of the crystal, fall more rapidly than the latter, so that the least additional torsion force will carry the crystal past that position and cause it to revolve through many degrees. This position (the upsetting point) being one of unstable equilibrium, is easily observed experimentally; and, by careful manipulation with the torsion index, it is also easily attained. When the two upsetting points are observed, the whole number of degrees of torsion required to proceed from one to the other, is an excellent measure of the setting force of the object. When the crystal is in the form of a cube or sphere and suspended between flat-faced poles, the two upsetting points are nearly at an angle of 45° with the axial line, and the angle between them is near upon 90°. I have found this angle to vary for the different objects employed; but, whilst the position of the poles, &c. remained the same, not for the same object, however the force might vary. It has therefore to be ascertained, experimentally, for each object, in any one series of observations where

the force only varies; for being included in the motion of the torsion index it has to be subtracted, as a constant quantity, from the observed result, and then leaves the true expression of the torsion force exerted between the upsetting points.

3368. The magnet employed was that great one constructed by LOGEMAN, and sent to the Exhibition of 1851. It could sustain a weight of 430 lbs., and is, I believe, very constant in power. It, with the torsion balance now used, is described in the Proceedings of the Royal Institution*. The sliding pole-pieces were of square iron, and presented either pointed terminations towards each other, or two flat faces, 1·7 inch square, which could be brought up to the opposite sides of the troughs or vessels containing the different fluids and media required for the experiments. These vessels were of various sizes and kinds; but the outer ones were usually of copper, with flat sides, that the pole-pieces might bear against them, and be thus preserved in their position during the progress of a single experiment or a series of comparative results.

3369. The torsion suspender was about 24·5 inches in length; and was either a fine platinum wire, of which 28·5 inches weighed 1 grain, or a finer wire of silver; or a bundle of cocoon silk fibres. The last was useful for certain delicate experiments, but could not be employed except in limited cases; for its torsion force is liable to much variation under the influence of the vapour of water, camphine, &c. All these suspenders are liable to more or less of set, and that varies with the vibrations to which the apparatus is subject; but, by equalizing the time, by paying attention, and especially by alternating and combining right- and left-handed observations (3366.), the effect of this set may be obviated to a very great degree. The torsion wire terminated below by a hook, made out of a flat piece of copper foil, intended to receive on its edge a corresponding hook, attached to the object submitted to experiment.

3370. The crystal, or other object, was held by one turn of a fine copper wire, which was continued upwards for 5·7 inches, and terminated by a flat hook like that just described. In this way different objects could be attached to the torsion wire, yet without any possibility of loose or uncertain motion about the point of attachment. Each loop had a horizontal bristle fixed to it, and this, by its position, not only showed the place of the crystal or other object beneath, but being retained between moveable stops associated with a horizontal scale, it indicated when the crystal was approaching the upsetting points, and being held within, and governed by, the stops, allowed them, through it, to govern the crystal.

3371. The balance was enclosed by glass, to shut out currents of air as much as possible and prevent their production.

3372. Experiments on the differential magnecrystallic force of bodies surrounded by different media required the bodies to be immersed in those media; and, as the latter varied from one another in specific gravity, so they exerted different degrees of buoyancy upon the same crystal, and thereby caused different degrees of tension

* January 21, 1853, vol. i. p. 230.

upon the torsion wire:—thus, a crystal of tourmaline, which in air hung with the weight of 40·4 grains upon the wire, would in water hang with the weight of 27·3 grains, and in phosphorus with the weight of only 15·5 grains. As this variation would slightly change the value of the torsion degrees, compensating weights of pure copper were added at the lower end of the torsion wire (3369.), *i.e.* 5·7 inches above the place of the crystal and magnetic poles.

3373. *Bismuth crystal.*—A piece of uniformly crystallized bismuth was reduced to the form of an octagonal prism, the height of which was 0·45, and its average diameter about 0·28 of an inch; its weight was 77 grains. When suspended perpendicularly its magnecrystallic axis was horizontal, and therefore set in the magnetic field, which existed between the flat faces of the pole-pieces (3368.) fixed at the distance of 1 inch apart. The torsion suspender was, in this case, a bundle of ten cocoon silk fibres, only 5 inches in length. The temperature was 68° FAHR. The torsion between the upsetting points was as follows, for four different media, differing much in their magnetic relation:—

Air	2250°
Absolute alcohol	2269
Water	2230
Saturated solution of protosulphate of iron	2234

In another set of experiments, carbonic acid gas was compared with alcohol and water, and the result was the same in it as in them.

3374. Desiring to include a highly diamagnetic medium in the list, I employed phosphorus; but as the heat required to melt it is competent to change the magnetic force of the bismuth crystal (3399.), it was requisite to compare it with water at a like temperature:—this was done, the temperature of 160° FAHR. being chosen. The results came forth as follows:—torsion force in water 1945°; in melted phosphorus 1950°; and are therefore to be considered alike.

3375. The liquids employed as surrounding media were contained in wide thin glass tubes, placed within the copper trough against which the magnetic poles rested (3368). Much care was requisite in the use of phosphorus. This substance was covered with water, and when the bismuth was passed through the water into the phosphorus, although it did not wet with phosphorus, still the latter acted slightly upon it, producing a few minute bubbles of adhering gas. These were not found on after occasions, when the same crystal was employed. It was also necessary that the phosphorus should be perfectly clean and good. Films soon form in it, and more especially at the contact of the phosphorus and the covering water; and these, clinging to the suspending wire, embarrass the vibrations of the immersed crystal and render them uncertain; the least portion of burnt phosphorus makes these films abundant. Whenever they appeared, fresh clean phosphorus was employed.

3376. From these results and from many others not described, it follows that the differential magnecrystallic force, *i.e.* the relations of the magnetic force in different

directions in a crystal of bismuth, is not altered by great changes in the magnetic character of the medium surrounding it; since they remain the same in phosphorus, alcohol, water, carbonic acid gas, air, and solution of protosulphate of iron;—a list which includes both diamagnetic and paramagnetic substances.

3377. *Tourmaline.*—As a paramagnecrystal, and therefore in contrast with bismuth, a black tourmaline was selected, regular in form, and nearly 0·37 of an inch in diameter. A piece, 0·36 in length, was cut off with flat ends; its weight was 40·4 grains, and its specific gravity 3·076. When suspended between the flat-faced poles with the axis of the prism horizontal, that axis set strongly in the equatorial direction, by virtue of the differential magnecrystallic force. On using a silk suspension (3369.) the necessary upsetting torsion force was as follows:—

	Temp.	Torsion force.
In air	57°	2534
In alcohol	56	2546
In water	56	2541
In solution of sulphate of iron saturated	57	2632

3378. These results sufficiently indicate that the torsion force, and therefore the differential magnecrystallic force, was alike for the same temperature, whatever the character of the surrounding medium. But to give more certainty, the fine silver torsion wire (3369.) was employed, and with the following results:—

	Temp.	Torsion force.
Water	65°	1082
Olive oil	65	1085
Alcohol	65	1081
Air	65	1079
Saturated solution of protosulphate of iron	65	1081

which sufficiently prove that the magnecrystallic force remained the same in degree, notwithstanding great variations in the character of the surrounding media. The angle between the upsetting points was 90°; but it has not been abstracted from the experimental results, inasmuch as that correction would make no difference in their character.

3379. The native *protocarbonate of iron* is very magnecrystallic, being also as a whole highly paramagnetic. A rhomboid was selected, and, being placed with its greatest length vertical and its shortest axis horizontal, was reduced, by grinding at the sides, to a rough octagonal prism, having an upright length of 0·6 of an inch, and an average horizontal breadth of 0·37;—the weight was 47·5 grains and the magnecrystallic axis horizontal. The magnetic force of this crystal was so great, that though the fine silver torsion wire (3369.) was employed with it, the pole-pieces had to be opened to the full extent of the magnet, *i. e.* to 4·7 inches, before the torsion force was sufficiently reduced to render the *set* of the crystal manageable. To lessen the power of the magnet by a cross bar of iron at the sides, I considered objection-

able; inasmuch as that bar might take more or less of charge during the course of the experiments, and so render the magnetic power of the field in some degree variable. When the crystal was placed in succession in different media at the temperature of 66° FAHR., the results were as follows:—

In water . . . . 542 of torsion,
In air . . . . 543 of torsion,
In saturated solution of protosulphate of iron . 542 of torsion,

results which perfectly accord with those obtained in the former cases.

3380. The *red ferroprussiate of potassa* is a prismatic salt, which sets most strongly in the magnetic field, when the axis of the prism is horizontal and the plane passing through the obtuse linear angles is vertical. A crystal, reduced in length and width until these were nearly alike, and having therefore little or no mere paramagnetic or diamagnetic set (for the salt is very slightly paramagnetic in air), was placed in the magnetic field, surrounded first by air and then by camphine; the results were as follows:—in air the tension force was 314, and in camphine 316; the accordance being most close with the results before obtained.

3381. Thus the old conclusions (2499–2501.) are confirmed; there appears to be no experimental difference in the proportion of the force developed in different directions in a magnecrystal by the action of magnetic induction, whatever the nature of the medium surrounding it, and whatever the difference in paramagnetic and diamagnetic character of the crystals, or the media employed; crystals differing as much as bismuth and carbonate of iron, and media differing as greatly as phosphorus and saturated solution of sulphate of iron, having been employed.

3382. Theoretically, however, there ought to be small differences produced, and according to my view of the lines of force, as true representations of the magnetic power, they ought to be of the following nature. If a magnecrystal be subjected to the action of a constant magnet, whilst the magnetic field, and the whole space around the magnet, are occupied by a common medium, as air, and then a small part of the field around the crystal be occupied by another medium as in the experiments described, then, if the medium be a better conductor of the force, *i. e.* be more paramagnetic than the former medium, it ought to determine more force across that place; and that increase of force would be the same as if a stronger magnet had been employed, and so the magnecrystal should show a variation:—acting as if more highly affected than before, its differential power in two directions should appear greater. Or, if the part of the medium around the crystal were replaced by a medium more diamagnetic, *i. e.* a worse conductor, then less force would pass in that direction and the magnecrystal should appear weaker than before, and so point with less force. Even the very shape of these partial substitutions should have an influence, according as it might extend in the axial or the equatorial direction.

3383. But if *all* the medium reached by the powers of the magnet were changed at once, and not that part only about the magnecrystal in the magnetic field, then

the use of a more paramagnetic or better conducting medium should have a contrary effect and make the magnecrystal appear less affected; for the transmission of power would be increased (proportionally) everywhere else, more than through it. On the contrary, the use of a more diamagnetic medium would have the reverse effect, and the transmission of force decreasing everywhere else more than through the crystal, would make the latter appear to increase in its peculiar condition of force. I am assuming that the magnet is unchangeable in power, and therefore *must* exert the like external force in every case; and after all, I conclude that these effects would be so small, as not to be observable except by the use of media differing far more from each other than those we at present possess. For my own part, I feel, even now, that the hypothesis of magnetic fluids cannot exist in the presence of conjoint paramagnetic and diamagnetic phenomena; but considerations such as those above, may be able to do good service in arranging hypotheses in their right places and giving them their true value.

3384. The aptitude of a magnecrystal, when in the magnetic field, to assume a maximum conductive state in a given direction, makes it similar in action to a permanently magnetized sphere; and therefore, however diamagnetic it may be, and however slight its magnecrystallic condition, still it will set in a definite direction, *i. e.* with its chief magnecrystallic axis* parallel to the magnetic axis of the field, even if it could be surrounded by a fluid medium having a paramagnetic condition equal to that of iron. And here I wish to correct an expression which has been allowed in a former series of these Researches (3158.), where it is said, that "an ordinary magnetic needle cannot show polarity in a field of equal force." It cannot of course exist in association with a field of equal force, for it would itself destroy the equality of the force, unless the medium around it were iron as high in paramagnetic power as itself; but even in such a case it would show polarity when deflected, for its magnetic axis would correspond in quality with a chief magnecrystallic axis, and it would always set or point in the accordant direction, *i. e.* axially in the magnetic field.

3385. Magnecrystals may be employed in experiments to measure magnetic force just as needles are, but in some points of view they are philosophically more accurate. A magnecrystal is equal in quality in all its parts; it appears to take up precisely the same state under the same inductive force and to have no coercitive or retentive faculty; whereas the force of a needle changes easily under inductive action, and when that action ceases the return towards its former condition is uncertain. It is also independent of the surrounding medium. Hence it may, in some cases, be found to supply a more true and certain standard of force, in the amount of tension required for its deflection.

3386. That magnecrystals are attracted or repelled with different degrees of force in different directions, has been long ago established by myself (2841.) and others.

* THOMSON on Magnecrystallic Axis, Philosophical Magazine, 1851, vol. i. p. 177.

As the difference of force remains constant when the surrounding medium is varied (3381.), it follows that the possibility exists of finding a magnecrystal and a medium so related, that the attraction and repulsion of the crystal, as a whole, should be convertible terms depending upon the position of the crystal in regard to the lines of force. I was desirous of verifying this result experimentally, and especially in relation to the case of mere space or a vacuum about the crystal, and therefore selected certain magnecrystals which promised favourable results, and yielded the following illustrations.

3387. The *tourmaline* crystal already employed (3377.) was found paramagnetic, not merely in water and air, but also in a saturated solution of protosulphate of iron; and though the difference in degree of attraction according to its position was very striking in all the media, the substance was for the present dismissed.

3388. *Red ferroprussiate of potassa*, being attracted in water or camphine, was repelled in the solution of iron, and therefore promised the desired result if it could be protected from the action of aqueous solutions*. Some good crystals were selected, and shortened by grinding until the length was little more than the breadth; then the angles were removed until each crystal became a rounded mass, after which they were made fast to suspending copper wires (3370.), 6 inches in length, so that the axis of the crystal prism should be in the horizontal plane, and, when in place in the magnetic field, either axial or equatorial at pleasure. Some wax was melted and kept at a temperature above the fusing-point, and the crystals being introduced and retained in the wax until they were above its fusing temperature, were then removed and carefully hung up. Afterwards a wax bath was prepared, of which part was fluid and part solid, and the cold crystals being suddenly dipped in and removed, brought away a congealed coat of wax which in the course of a few minutes became a compact envelope. Being left for a few hours, I found that they might afterwards be immersed and left in water, or in a solution of iron, for two or three days without any action between the crystal and the medium around it. No varnishing could thus protect them.

3389. A small torsion balance with a single cocoon thread was constructed. The end of the arm intended to sustain the crystal, was bent at an angle of 90°, so that the crystal could be suspended from it in either of two positions at right angles with each other. A counterpoise to the crystal was placed on the other arm, and the balance was covered with a jar to screen it from air currents. The crystal, when in place, hung down below the edge of the jar, descending into a vessel arranged at one pole of a great electro-magnet (2247.), so that it could be surrounded by any medium in which its actions were to be observed. The pole-piece terminated either in a cone, or an upright edge, or a face 1·5 inch square; the cone was the best, and the crystal

* Varnished crystals are not protected; when put into water the salt dissolves through every part of the coat; for, being soluble in alcohol, the coating matter is a mixture of resin and the salt. In solution of iron this substance dissolves in a very interesting manner whether unprotected or imperfectly coated.

was then able to approach the pole until separated only by the thickness of the glass of the vessel.

3390. A saturated solution of protosulphate of iron was prepared at a temperature of 65° Fahr., and a little sulphuric acid added to prevent the occurrence of turbidness upon the addition of water. This solution, more or less diluted, was put into the glass vessel, and when its motion had ceased, the crystal was placed on the balance and adjusted near the pole; then the magnet was excited by a voltaic battery and the effect observed. When the axis of the prism was in the magnetic axis, the crystal was repelled in all solutions stronger than one consisting of about eleven volumes of the saturated solution and six of water; in weaker solutions it was attracted, the force of attraction and repulsion varying, of course, as the medium varied. When the crystal axis was equatorial, *i. e.* when the chief magnecrystallic axis coincided with the magnetic axis of the field, then the crystal was repelled in all solutions stronger than one consisting of about eighteen volumes of the saturated solution and six of water. Hence there is a range of medium, varying in strength from that produced by adding either two or three volumes of the saturated solution to one volume of water, within which the crystal in one position is attracted and in the other repelled; and, as might be expected, a mixture of fourteen or fifteen volumes of solution with six of water, forms a medium in which the attraction and repulsion were nearly equal to each other. It was very easy in any of these media, to find a position for the crystal (by turning it on the vertical axis) in which it was neither attracted nor repelled. A second and a third crystal were, in succession, put upon the balance and gave exactly the same results.

3391. The *red ferroprussiate of potassa* is a crystal so paramagnetic as to be attracted in all positions in space. I therefore turned to *calcareous spar*, which, though it be diamagnetic in air, is not necessarily so in space, since air is, because of its oxygen, a magnetic body (2791.). Possessing a sphere of calcareous spar, given to me by Professor W. Thomson, I tried it first in water, and found, that whether the optic axis was placed equatorial or axial in relation to the magnetic field, the sphere was attracted, though more in the former case than in the latter. Hence the body is less diamagnetic than water and so approaches to a vacuum. In order to compare it with a vacuum I employed carbonic acid as the medium*, but then found that in both positions it was repelled; hence its differential range as a magnecrystal cannot include the magnetic force exerted in space merely.

3392. This sphere is repelled in all positions in alcohol; therefore it would be easy, by the addition either of alcohol or a little solution of iron to water, to obtain a liquid intermediate in force between the forces of the sphere in its two positions.

3393. But though pure calcareous spar will not include space, yet it is probable that some crystals may be found by trial which will do so. I have various specimens of calcareous spar, which contain in combination minute portions of iron, and being

* Royal Institution Proceedings, Jan. 21, 1853, p. 233; or Experimental Researches, 8vo, vol. iii. p. 502.

magnecrystallic, set, in the magnetic field, with the optic axis axially, as TYNDALL and KNOBLAUCH describe*. As wholes they are attracted in every position, whether surrounded by air or carbonic acid (more strongly in the latter than in the former); and when free to revolve they set with the optic axis axially, even in solutions of iron. But there seems no reason why calcareous spars, intermediate between these and the sphere, should not be found by search; nor any reason to doubt that, being crystallized, they would be magnecrystallic. The further suggestions of hypothesis are, however, not very clear, inasmuch as we are not quite sure, without other experiments, whether such bodies may be accepted magnetically as simple bodies, or whether the optic axis would always point axially or equatorially as the body was paramagnetic or diamagnetic in relation to space; or whether the body would disappear from the list of magnecrystals altogether. For suppose such a body coincident in its general magnetic condition (as when pulverized or amorphous) with carbonic acid or space; and that being in its crystallized condition it should be magnecrystallic, and when formed into a sphere should, according to the results just given (3381.), point in the same direction and with the same degree of force, whether surrounded by water, or carbonic acid, or solution of iron;—what direction should the optic axis of such a sphere take? It cannot take that of the pure calcareous spar, and also that of the small rhomboids of ferrocarbonate of lime, for they are at right angles to each other in the magnetic field; neither does any reason appear why it should take one more than the other. I would willingly think that some valuable considerations and evidence regarding the true zero of magnetic force would arise in the investigation of this matter; but the former results with different media make me fear that the subject, when closely examined experimentally, will resolve itself into something of less importance. This matter is carried a little further in the relations of temperature (3416.); for if we consider a true zero as independent of temperature, then one temperature may be assigned for it as well as another; and it will be seen that, in the mixed substance presented by ferrocarbonate of lime, we have a body that can be placed as non-magnecrystallic in carbonic acid at a given temperature; whereas at higher temperatures it is magnecrystallic as carbonate of lime, and at lower temperatures as carbonate of iron.

§ 39. *Action of heat on magnecrystals.*

3394. Heat affects the degree and, perhaps, the disposition of the induced forces in magnecrystals (2569–2573.). There is as yet little or no experimental evidence bearing upon this subject, so that the following contribution for temperatures between 0° and 300° FAHR. may be acceptable. Some new arrangements of apparatus were required, the following brief description of which will probably be sufficient.

3395. Baths for the application of heat and cold were necessary. One, frequently

* Philosophical Magazine, 1850, vol. xxxvi. p. 178.

employed, was a copper vessel 1·15 inch broad in the direction of the magnetic axis, 3·5 long and 7 deep. When in its place between the magnetic poles, they could either bear against its sides or against blocks placed between it and them, so that the pole distances should be unchangeable for the time. The upper part was clothed in flannel and was within the balance box (3368.); the lower part passed through a hole in the magnet table, and could be heated by a spirit-lamp applied below. Oil was most frequently employed in this bath for high temperatures, but sometimes water; and then its surface was covered with oil to prevent evaporation and diminish the production of currents. A thermometer was inserted in this bath at one end to indicate its temperature.

3396. A copper cylinder, 1·1 inch in diameter, 3 inches in depth, closed at the bottom, and expanded at the upper edge, so as to rest on the side edges of the bath, was destined to hold the medium, either camphine, water, or oil, which was immediately around the crystal or other magnetic object. Currents were, of necessity, formed in the fluid, for it could neither be heated nor cooled without them; but the point was to reduce them as much as possible about the object to be observed, and the arrangement described was found very useful for this purpose. It was necessary that the fluid used in this cylinder should be very clean and clear from any filaments or other matters, that might obstruct the motion of the immersed object.

3397. Because of the relative positions of the thermometer and the object to be observed, it is evident, that, with rising temperatures, the former will at the same moment be hotter than the latter, whilst with falling temperatures it will be cooler. The influence of this circumstance was observed in many of the experiments (3408.); but as the cooling was much slower and far more regular than the heating, the chief observations were made as the temperature fell. The greatest source of errors existed in the currents, and could only be overcome, and then only in part, by slow and numerous observations. These currents were often found to have prevalent sets, but these were, to a large extent, remedied by the observations in two positions, *i.e.* at the upsetting points. For low temperatures a smaller trough was employed, well clothed in flannel and filled with an excellent frigorific mixture.

3398. *Bismuth crystal.*—The crystal before described (3373.) was placed on the torsion balance; its upsetting angle was then ascertained to be 105°; and being observed from time to time, it was found to remain the same both for low and high temperatures. The torsion force was measured, first at common temperatures, then as the temperature rose, and also as it fell; it was observed to be greater at the same upper temperature when rising than when falling; an effect referred by particular examination to the fact that the lower the temperature of the bismuth the greater the torsion force; and that, as before said, as the bismuth gained its temperature later than the oil and thermometer in the bath, so it was cooler in the first case than in the second, for the same thermometer indication. As the cooling was purposely rendered slow, that the temperature of the bismuth might be near to that indicated

by the thermometer and the currents in the fluid weaker, so the observations were always considered best when made with a standing or a falling temperature.

3399. The following are the actual observations of one series made with the silver torsion wire as the temperature fell. The crystal was surrounded by oil. The upsetting angle is subtracted as before described (3367.):—

Temp.		Torsion force.	Temp.		Torsion force.	Temp.		Torsion force.	Temp.		Torsion force.
279° F.	...	82	225° F.	...	105	190° F.	...	118	152° F.	...	133
272	...	82	219	...	117	186	...	121	149	...	138
265	...	80	215	...	117	183	...	120	141	...	137
258	...	81	212	...	105	180	...	119	133	...	142
251	...	89	209	...	107	177	...	119	131	...	145
245	...	93	204	...	108	173	...	128	119	...	151
240	...	97	199	...	116	165	...	136	104	...	160
235	...	97	197	...	119	156	...	134	92	...	175
230	...	100	193	...	119						

It will be understood that each of the numbers under torsion force, increased by 105, will give the number of degrees of torsion experimentally observed. The observations at 219° and 215° are, I have no doubt, influenced by currents; but I kept myself purposely ignorant of what might be expected, and I give them as they were obtained. When these numbers are laid down on a scale (Plate III. See bismuth crystal C), having temperature in one direction and torsion force at right angles to it, and when a mean line is drawn through them, it appears to be a straight line; at least there is nothing in the results to justify the assertion, that the change of force at one temperature was different in degree from the change at another, within the range employed. The force, as expressed by such a line, was at 100° equal to 162, and at 280° it was 77; the whole loss within that range being 85, or above half the power it possessed at the lower temperature; in other words, an average alteration of 4·7 for every 10° Fahr.

3400. Another set of observations was made with the same bismuth crystal surrounded by water. Seventy-one observations were taken between 40° and 207°; and, by the substitution of a cold bath, some others were added on for temperatures between 5° and 70°, which were in perfect accordance with the former. These, when laid down (see Plate III., bismuth crystal A and B), also gave a straight mean line, even more close to the various observations than the former one. The force at 5° was 168; at 270° it was 90; at 100° it was 131. The whole change of force between 5° and 207° is 78, or nearly one-half of the force at 5°; it is at the rate of 3·86 for every 10° of temperature, which correspond very closely to the former result; for the rate becomes 4·8 if the force be converted into a scale of number corresponding with that of crystal C.

3401. It is not to be expected (without extreme care) that the numbers in the different series of observations with the same object should coincide. The variation of the medium, which in one case was oil and in the other water, should not, for the

reasons given (3381.), produce any effect; neither, for the same reasons, should their possible variation by change of temperature produce any effect; but any change in the distance of the magnetic poles would produce an alteration, and I have no doubt that, in the first case, the poles were a little nearer to each other, and therefore the force at the same temperature greater. It was very satisfactory also to see that the two mean lines (selected only by the eye) converged towards each other with rising temperatures; thus at 100° the difference of torsion force is 31, whilst at 200° it is 21 nearly; as if they were indeed only the tangents of curves, and, at much higher temperatures, would coincide or become parallel nearly.

3402. These first observations are sufficient to show, that the differential magnecrystallic force of bismuth diminishes with elevation of temperature, and that to a large extent; that this occurs by a regular progression, which presents no appearance of any change of sign within the limits of temperature employed; that the progression appears within these limits to be represented by a straight line, or rather by a portion of a large curve, a supposition favoured by the approximation of the lines at higher temperatures; and that the return of the bismuth to its original degree of power is perfect upon the recurrence of the original temperature.

3403. It is of importance, not merely to examine the effect of temperature upon a crystal of bismuth, as one of different magnecrystals, but to compare the manifestation of magnetic force in bismuth when in this state with the corresponding manifestations when the metal is in other conditions, as in the compressed state; or in the amorphous or granular state, at which time it is affected merely as a diamagnetic body. I therefore proceeded to compress a piece of granular bismuth in one direction, and then cut out of it a short square prism, which, when suspended, was 0·5 of an inch in height and 0·36 in thickness, the line of pressure being horizontal and parallel to two of the sides; when in the magnetic field this line set, of course, equatorially, and the piece therefore, which weighed 128·5 grains, could be subjected to experiment in the same manner as the crystal before it (3398.).

3404. This compressed bismuth acted very well, the difference of torsion force being abundantly large enough for observation. The upsetting angle was 109° and not very definite, so that currents in the surrounding heated medium (oil) interfered more with its exact observance than in the case of the crystal. Such a result was, perhaps, to be expected; for it cannot be supposed that a piece of bismuth, so squeezed in a hydraulic press, should have the line of compressing force of equal intensity and like direction in all its parts, and therefore comparable, in that respect, to a crystal. The results corrected for the upsetting angle were at

70° F. torsion force was . .	157
121° F. torsion force was . .	140
157° F. torsion force was . .	119
194° F. torsion force was . .	116
211° F. torsion force was . .	106

and are laid down upon the plate of lines (see bismuth compressed, D).

3405. It so happens, that the size of the bismuth and the force of the magnet place the observations between those of the crystal before obtained, and the results show how parallel the three are in their direction and nature. These seem, also, to be in a line, straight or nearly so. The force is at 70° equal to 159, and at 210° only to 105, being a loss of 54 for the 140° of difference between the two temperatures. If we take the loss of power for equal differences, at the same temperature, the results are very accordant.

	Torsion force.		Loss of power.
	At 90°	At 207°	
Bismuth crystal, A . .	135	90	. . 45 or $\frac{1}{3}$ of the power at 90°
Bismuth crystal, C . .	167	112	. . 55 or $\frac{1}{3}$ of the power at 90
Bismuth compressed, D.	149	107	. . 42 or $\frac{1}{3\cdot5}$ of the power at 90

3406. The compressed bismuth data are few in number, and do not afford so good an indication as those obtained with the crystal; but the results are such as to give an additional reason to those advanced by TYNDALL, that the magnetic force in compressed bismuth is of precisely the same nature in disposition, &c. as in crystallized bismuth. I have endeavoured to obtain some additional physical evidence, of another kind but in the same direction, by subjecting crystallized and compressed bismuth to the slow dissolving action of dilute nitric acid; but though signs of crystalline structure appear in both cases they are not clear or satisfactory.

3407. *Tourmaline.*—This substance, as a paramagnetic crystal, was then submitted to the action of heat, the crystal employed being that already described (3377.); its upsetting angle was 90°. A series of observations with the crystal in water was made, extending from 39° to 206°, which are entered in the plate of measurements as "Tourmaline crystal I." A second series was made with this crystal in oil, the temperatures reaching from 79° to 289°; they are entered as "Tourmaline crystal K." A third short series with the crystal in brine and extending from 7° to 69°, is entered as "Tourmaline crystal L." These results are recorded, not with the torsion numbers obtained, which, though occurring with the silver torsion wire, ranged from 640 to 1200 degrees of force, but in other numbers which conveniently entered within the range of the table adopted, and which were obtained by reducing the experimental results proportionately. It will be understood, of course, that the different entries in the diagram offer no absolute comparison between one body and another, *that* not being possible for equal bulks or weights of the substances, in a table like the present; but only a result for each particular body during change of temperature, the source of magnetic power and the measure of torsion force remaining invariable for the time.

3408. The precedence which the thermometer takes of the body (3397.), is here very manifest in the first observations of K. The progression of the numbers is generally good, either with rising or falling temperatures. The magnetic force in the crystal diminishes continually with increase of temperature; there is no change of sign. The loss of force between 7° and 289° is nearly half the force possessed by the crystal at the lower temperature; and, therefore, almost as much as that left at

the higher temperature. The force returns perfectly upon the restoration of a lower temperature. There is no permanent disturbance of the specific magnetic capacity, nor anything like magnetic charge. The loss is not in arithmetical progression, but greater for an equal number of degrees at lower than at higher temperatures; and is best represented by a regular curve as the mean line. The two chief series agree very well together, and the third series, at low temperatures, is in near accordance with either. The loss of power at low temperatures, as 0°, is for the same number of degrees of elevation, three times as much as it is for temperatures about 270° or 280°.

3409. The return of this and other crystals to their first condition by return of temperature, combined with the observations made with iron, nickel, &c. (3424.), shows that the magnet, as a source of power, remained unchanged by the variation of temperature from 0° to 300° in the magnetic field.

3410. This tourmaline crystal (3377.) being hung between the poles of the great electro-magnet, was raised by a spirit-lamp to a full red heat, and then set well with its axis equatorially, though with diminished power; so that high temperature does not take away its magnecrystallic character: on cooling it returned to its first higher condition. On a former occasion I found that a like, short, thick, black crystal lost part of its power by the heat of a spirit-lamp flame, but on cooling, the tourmaline became very magnetic, pointed axially, and was strongly attracted. The latter effects were traced to a portion of peroxide of iron on one part of the crystal, which had been reduced to protoxide or even lower, by the vapour and heat of the spirit-lamp: digestion in hydrochloric acid removed this iron and restored the crystal to its first condition. The fact shows, that a temperature which takes away the high paramagnetic condition of iron or its protoxide could not destroy the peculiar condition of tourmaline as a magnecrystal.

3411. *Carbonate of iron.*—The crystal of this substance before described (3379.) was suspended in an oil bath, and carried through temperatures varying from 4° to 293°. Its upsetting angle was 96°. The results are in the diagram marked F. In another set of experiments, results were obtained about 0° and 60°, and are added, being marked G. The whole forms a very consistent series of observations, showing progressive loss of power with elevation of temperature, the diminution being much greater at low than at high degrees; and, on the whole, very great for the range of temperature employed. The loss of power about 0° and 32°, is four times as much as it is at 280° for an equal number of degrees. The whole power at 300° is 135; at 0° it is nearly tripled, being 380.

3412. When carbonate of iron crystallized, is heated, either in air or oil, up to or above a certain high temperature, it is almost sure to fly to pieces like a Rupert's drop. By perseverance, and by selecting the larger fragments when the breaking-up took place in oil, I obtained three pieces, which could be raised by the flame of a spirit-lamp to a red heat. Below a very dull red heat these crystals were *always* magnecrystallic; more at lower temperatures and less at higher:—regaining power

as the heat fell and losing it when temperature increased, and that repeatedly. When the temperature was further raised and continued for a minute or more, the crystal ceased to be magnecrystallic, and lost nearly all magnetic power; but when lowered beneath a certain temperature, it became intensely magnetic, and was found to have lost its carbonic acid and become converted into magnetic oxide of iron.

3413. *Carbonate of lime.*—The sphere of calcareous spar was comparatively so weak in magnetic force, as to give no sufficient indication when a metallic torsion wire was employed with the LOGEMAN magnet; to employ a silk torsion thread would have been unsafe. A very high temperature, amounting to full ignition (being the highest that a spirit-lamp flame could communicate to a small rhomboid), did not take away the magnecrystallic condition of calcareous spar, or interfere with the pointing of the optic axis equatorially; for though the heat was sufficient to convert the exterior of the crystal (to which the aqueous vapour from the flame had access) into quick lime, still the internal crystalline part pointed magnecrystallically, and carried the altered part with it. This permanency, coupled with the low magnecrystallic state possessed by the crystal at common temperatures, shows that the power would decrease at a very low rate and in a very small degree, whilst rising from 0° of temperature to 300°.

3414. When a crystal of *red ferroprussiate of potassa* is heated, either in air or oil, it flies to pieces at a certain high temperature; beneath that degree, however, it retains its magnecrystallic character unaltered, except that the pointing is with less force at the higher temperatures than at the lower.

3415. The *ferrocalcareous spar* before described (3393.) suggested some very curious points of inquiry. It seemed probable that the iron within the crystal would retain its state of chemical combination under the action of heat, if the crystal as a whole should preserve its integrity at high temperatures: and if so, then, because of the slow alteration of calcareous spar by heat, and the much quicker alteration of carbonate of iron, as regarded magnetic force, it seemed further probable, that such a magnecrystal being heated sufficiently, would change its character; and that the axis of magnetic power, which at low temperatures was the maximum, would at high temperatures become the minimum axis, or line of minimum force: which indeed upon investigation proved to be the case.

3416. It was very difficult to raise these crystals above a certain temperature; near a given point, about 300° FAHR., they either broke up suddenly like a RUPERT'S drop, or crumbled to pieces. No previous slow elevation of temperature appeared to prevent this disruption. Nevertheless some pieces were obtained, both in air and in oil, which, though much fissured, still adhered together, so as to represent the crystal. When these were properly suspended in the magnetic field, the short axis of the rhomboid (or optic axis) pointed axially at common temperatures, but when raised by a spirit-lamp to a point clearly below a red heat, the short axis pointed equatorially. When the crystal was allowed to cool, it again pointed axially, and

when reheated its direction was equatorial; and this change could be repeated many times, without the crystal appearing to be at all altered, either by the production of caustic lime or of free oxide of iron:—its state and peculiar qualities were preserved. It was magnecrystallic at high and at low temperatures; but at the upper it was like pure calcareous spar and at the lower like carbonate of iron. At the lower temperature it was, as a whole, paramagnetic in air and therefore in carbonic acid gas. Whether at a given intermediate temperature it would cease altogether to be magnecrystallic, and would in carbonic acid gas be, as a whole, neither paramagnetic nor diamagnetic, and therefore in part, and for a given temperature, answer the inquiry before made (3393.), I cannot say.

3417. In the carbonate of lime and carbonate of iron, the short axis of the rhomboid is, emphatically, a *line* of direction, and points either equatorially or axially according to the nature of the crystal. A plane at right angles to it does not present sensible differences in particular parts, the force appears to be equal in all directions. So the whole change in the ferrocarbonate of lime appears to depend upon the circumstance, that, in the direction of the short axis, the aptness for magnetic induction decreases by heat more rapidly than in the *plane* at right angles to it; so as not merely to overtake the latter but to *pass by it*, and that in cooling it again returns towards, by, and beyond it:—the force in the equatorial plane or direction is probably varied, but nevertheless its whole range appears to be intermediate to that which the axial direction supplies.

3418. It would seem that such crystals as these could not have been formed at a high temperature and common pressures, inasmuch as they cannot sustain such a temperature now. They may even be considered, physically, as different substances at high and low temperatures; for a body which cannot expand and keep its integrity must have a very different arrangement of its molecular forces, when they are just about to burst the mass into particles, to that which exists when they are employed in giving permanency to the state into which they have brought them. The variations in magnetic relations are very striking for the two cases; perhaps some of the optical characters may be found to be affected also. The crystals are, I think, harder than those of calcareous spar, and always more fissured. Such calcareous spar as contains veins of minute crystals of pyrites is almost sure to prove of this peculiar nature.

3419. It would appear that magnecrystals (with the exception of the ferrocarbonate of iron), whether paramagnetic or diamagnetic, are all generally alike in their affections by heat; the differences of force in two given directions diminishing as the temperature is raised, increasing as temperature is lowered, and being constant for a given temperature. Such alterations might take place in various ways; a diminution by heat of the force of the stronger axis would account for it, so also would an increase of the force of the weaker axis; such doubtful points might be settled by combining with results like those I have given, others upon the *whole* paramagnetic or

diamagnetic force of a crystal in a given position at different temperatures. I have little doubt, however, that, according to the general action of heat, the power of the crystal to suffer a certain amount of induction in a given direction through it, is lessened in every direction as the temperature rises, and that the effects I have measured are simply the differences between the whole changes in each of two directions. Experiment only can decide, whether a sphere of tourmaline, or carbonate of lime, would remain affected by the magnet at temperatures which would cause the magnecrystallic character of these bodies to disappear; but it seems almost certain that the diamagnetic force of a granular piece of bismuth must be equal to the sum of the forces of the variously arranged crystalline parts of which it is composed, and would disappear when their magnecrystallic character was taken from them by heat; and it is also certain that the magnecrystallic character of such crystals as can hold together, is retained at very high temperatures.

3420. If the absolute magnetic and the magnecrystallic character of bodies should be found to coincide, then the examination of magnecrystals by heat would acquire increased interest. In many cases we can examine the magnecrystallic disposition of force better than the whole sum of force, and an examination of a part of the rate of diminution might give us a considerable insight as to the nature of the whole. Further, if the magnecrystallic and the magnetic indications agree, so that the one set may be accepted as representing the other, then we have the advantage in magnecrystals, of dismissing the influence (changeable as it is by temperature) of the surrounding medium altogether (3376.). It is remarkable, that as no unmixed body has as yet altered in the character of its magnetism by heat, *i. e.* has not passed by heat from the paramagnetic to the diamagnetic class (assuming space, or its magnetic equivalent carbonic acid gas, to be at zero), or *vice versâ*, so no simple magnecrystal has shown any inversion of this kind; nor have any of the three chief axes of power changed their character or relation to each other. This has to be borne in mind when considering the possible case of a magnecrystal at zero, before referred to (3393.).

It does not appear, from the direction of the lines in the diagram, that much increase in the diamagnetic power of the bismuth is to be expected from the application of any low temperature within our reach. Being the chief diamagnetic substance, and a metal, one would have wished it to have given a curve rather like that of carbonate of iron or even tourmaline.

§ 40. *Effect of heat upon the absolute magnetic force of bodies.*

3421. The time is coming on when we shall require to know the effect of heat on the total magnetic force (under induction) of such bodies as, being paramagnetic or diamagnetic, are near to zero, *i. e.* as near as bismuth or oxygen; so that, amongst other points, we may examine the relation of the *whole* change of power to the change of the differential condition which occurs in magnecrystals. A difficulty, not met

with before, is included in such investigations, by the dependence in a greater or smaller degree of the motions of the body upon the medium around it; for if the latter were to change by differences of temperature, the former would seem to have suffered a change though none might have occurred. The statement made on a former occasion (2359.), that paramagnetic solutions were not affected by heat, can hardly be accepted, without further confirmation, in the present state of the subject. If bodies at magnetic zero suffer no alteration by heat, then a fluid having that condition might be selected as a bath in which to try the changes of solid bodies not at zero; and a solid at zero might in like manner be employed to ascertain the variations by heat in the fluids surrounding it*: further, if paramagnetic solutions suffer no change, they may be employed to exalt the indications of diamagnetic bodies, such as bismuth or phosphorus. In the mean time the following results may be useful and acceptable.

3422. Being very desirous of knowing whether the variation of a piece of *amorphous*, i. e. *granular, bismuth* had the same progression for the same temperatures as a crystal of bismuth, I endeavoured to obtain some measures, but did not satisfy myself. I employed a bar of the metal about 0·55 of an inch long and 0·12 of an inch thick, between pointed poles; but the force of the bismuth under the influence of the LOGEMAN magnet was not sensible with a metallic torsion wire, and when a silk suspender was used (uncertain in itself), the indications were altogether overcome by currents in the surrounding fluid. A *tourmaline* crystal was just as unfavourable under the like circumstances; besides which, it must be understood that as tourmalines differ much from each other, specimens from the same crystal can only properly be compared.

3423. *Carbonate of iron.*—I found this substance sufficiently paramagnetic to supply indications with the LOGEMAN magnet, when the pointed poles were employed and placed 1·95 inch apart. The crystal formerly described (3379.) was therefore reduced by grinding to a plate, which being suspended with the optic axis or short diameter vertical, was then 0·6 of an inch in length, 0·17 in breadth, and 0·37 in height; a small copper cube was hung to it beneath, so as to give it weight in the oil-bath, and prevent its approach as a whole to either one or the other pole. The results obtained are entered in the diagram of curves (see carbonate of iron bar, E). They are not accordant with those given by the same substance as a magnecrystal. In the ascending part of the series the force at 126° is 157, and at 288° it is 133; the diminution 24 being only $\frac{1}{6\cdot54}$ of the force at 126°. In the descending part, the force at 96° is 182, and at 292° it is 125, the difference 57 being $\frac{1}{3\cdot2}$ of the force at 96°: both differences are much less than that with the crystal of carbonate of iron, for then the force at 96° was 255, and at 292° was 137, the difference 118 being almost half of the force at 96°. It is evident therefore that the forces of the bar do not diminish in the same ratio as the forces of the crystal; or else that the medium alters

* Royal Institution Proceedings, Jan. 1853, p. 232; or Experimental Researches, 8vo. vol. iii. p. 500.

importantly though in an unknown manner; or else that the bar as a whole exhibits some peculiar change connected perhaps with the crossing of the diagram lines indicating the ascending and descending results. If the oil of the bath had *lost* diamagnetic power by elevation of temperature (and gain is not to be expected), then the carbonate of iron should, on that account alone, have seemed to suffer a loss which would be added to its own loss; such an effect would have tended to give a result the reverse of that which in reality appears.

3424. Experiments were then made, as additions to former results*, upon the metals iron, nickel, cobalt, and with much facility, in the following manner. A copper cube 0·25 of an inch in the side, had a fine hole made through it, in a direction perpendicular to two of the faces, and a piece of clean soft *iron* wire 0·05 of an inch in length and 0·0166 in diameter, was placed in the middle of the hole. The cube, which weighed 46 grains, was then suspended as before between the magnetic poles, they being 4·86 inches apart. The power of the iron, thus subjected to the magnet, was such as to permit the employment of the platinum torsion wire (3369.), and so remove every objection as to any possible change of the torsion force. The upsetting points were very definite and were 108° apart. The results of the observations, at temperatures varying from 30° to 288°, are entered in the chart (see iron bar, P), and it will be seen that they present no sensible variation; as if the inductive force in the iron underwent no change during this alteration of temperature, but had obtained and kept its maximum degree. We know from other experiments, that at higher temperatures the force would decrease; that at a certain temperature the decrease, though progressive and not instantaneous (2345.), would be very rapid; and that at still higher temperatures it would again become slow and at last almost insensible: and we have reason to suppose that with tourmaline and carbonate of iron we should have a like inversion of the curvature if we could descend to very low temperatures.

3425. *Nickel.*—A small square bar of pure nickel was prepared; it was 0·09 of an inch in length, and 0·036 in thickness. A cube of copper like the former (3424.) had a cavity formed in its upper surface, in which the nickel was placed, and retained immoveable by the suspending wire, and the whole was submitted to changes of temperature. The upsetting angle was 112°. The results are given in the chart marked "Nickel bar M," beginning at the higher temperatures and descending to lower. It will be seen that there is a diminution of force at the upper temperature, which accords with the general effect of heat; and as we know that the temperature of boiling oil is enough to render even large masses of nickel insensible to the action of common magnets, we may believe that a very rapid and interesting series of changes would come on between 300° and 600°.

3426. *Cobalt.*—A small bar of pure cobalt, 0·08 of an inch in length and 0·027 square, was in like manner attached to a cube of copper, and subjected to the action of the magnet and heat. The upsetting angle was 118°. One reason for the differ-

* Experimental Researches, 2344–2347; also 8vo, vol. iii. p. 444.

ence of the upsetting angles of these metals, was the different proportions of length and thickness; another reason will appear further on (3427.). Two sets of results are entered in the chart of lines (marked N and O), both of which present an important indication. It may be observed, in the first place, of the results O, that ascending from 66° and then returning to nearly the same temperature, the cobalt seems to have gained a permanent increase of power of about 30 degrees of torsion force, the whole being about 380. This was ultimately referred to charge or coercitive power; for when, after the observations at 79°, the cube with the cobalt was turned round 180°, so as to reverse the ends as respects the magnetic poles, and then brought back into their first position and observed, the power of the whole seemed to have fallen, as is seen by the six results marked R on the scale; and this condition the cobalt retained though left in the last position for some time. I found, indeed, that small pieces of the iron, nickel, and cobalt, when ignited to remove charge, and then held for a moment in any position in the magnetic field, acquired a charge, which they retained when out of the field. It would seem, that, even when afterwards reversed in the magnetic field, this first charge, or the effect of it, is in part retained, but that at high temperatures the metal loses more or less of it; and hence the difference between the results at the beginning and end of the series of observations marked O. In those marked N the metal was, probably, either in such a condition as to have no permanent loss occasioned by heat, or not to have had the heat (of 290° only) continued long enough for the purpose.

3427. I think it very probable that iron and nickel would show like phenomena as the cobalt if they were sought for; and also that this quality of charge may affect the upsetting angle of pieces of metal differing in their proportions of length and thickness (3426.).

3428. Admitting all the effect of this charge, there is still another result evident in both the cobalt series, and in both parts of the series O; that is, the *increase* of power with elevation of temperature. This is, I believe, the first instance in which such a result has been recognized; and even though we might think for a moment that, whilst ascending from 66° to 300°, the higher temperatures had set the metal more free to give up adverse charge, as above supposed (3426.), still that would not account on descending for a *diminution* of force, without admitting that heat was also able to make the metal more favourable to receive charge; which is in fact to say that the power is greater at higher than at lower temperatures. This effect cannot depend upon any change in the surrounding medium, for, such is the enormous disproportion between it and the cobalt in equal volumes, that if its powers were either annihilated or doubled, the effect would be insensible amongst the results. If such be the truth with cobalt, then it is probable that a like result would occur with iron and nickel at some temperatures, and that in passing to lower temperatures than those employed we should arrive at one presenting the maximum magnetic induction for each, and below which their inductive force would diminish. Within

the range employed, *i. e.* from 0° to 300°, the three metals seem to be in different parts of their course; nickel has passed the period of its maximum force, iron is in it, and cobalt has not yet attained to it; and this accords with the further change by temperature; for by greater elevation nickel first loses its distinctive power at about 635° FAHR.*; then iron at a moderate red heat†, and cobalt at a far higher temperature than either, near the melting-point of copper. Such a view as this increases very much the interest of the relation between heat and magnetism; especially as, if it be well founded, it will probably apply to substances in all states; to gases as oxygen as well as to metals like cobalt; in which case it may be that all bodies, whether paramagnetic or diamagnetic, have a certain temperature at which their induced magnetic condition being most favoured is a maximum, and above or below which their state diminishes.

3429. The effect of heat upon iron and steel, and therefore upon magnets generally, will have hereafter to be distinguished into that which it may produce in the case of iron considered as perfectly soft; and that which it may produce in the case of perfectly hard steel whether charged magnetically or not. It may be that its action upon a magnet, consisting of parts all equally hard and equally charged, may be very different from its action upon another magnet, having superficial or terminal parts harder and more charged than the rest; or as is usually the case, of which the parts are not, as steel, exactly alike, but give a resultant of many different actions

3430. In considering these remarkable effects of heat, the question still recurs, can substances be made to pass each other magnetically by any change of temperature? It does not appear as yet that any of them, being unmixed, can pass the zero presented by a vacuum or carbonic acid gas, *i. e.* none can be converted from the paramagnetic to the diamagnetic state, or *vice versâ*, these states being defined by that zero; and so far that would appear to be a true and natural zero. The further question may be asked, whether, if equal volumes of different bodies in the same shape were subjected to an equal magnetic force, at various temperatures, so that their forces might be expressed in their full and true relation, upon one diagram scale, would the lines expressing these forces ever cross each other? as far as I can see they would not; but the results are as yet far too few in number, and too imperfect in their nature, to justify any serious conclusion.

* Experimental Researches, 8vo, vol. ii. p. 219.

† Ibid. vol. iii. p. 444.

Royal Institution, October 9th, 1855.

X. *Researches on the Foraminifera.*

By William B. Carpenter, *M.D., F.R.S., F.G.S. &c.*

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PART I.

CONTAINING GENERAL INTRODUCTION, AND MONOGRAPH OF THE GENUS ORBITOLITES.

General Introduction.

THE tribe of Animals in regard to certain members of which I propose to offer to the Royal Society the results of my detailed investigations, is almost unknown, save by name, to many well-informed Naturalists; and has only of late received even a small measure of that attention, which its zoological and its geological importance, and its physiological interest, may fairly claim for it. Of the bodies which are now, on account of their general conformity to a certain plan of organization, grouped together under the designation *Foraminifera*, the greater proportion, being spiral multilocular shells, were long associated with *Nautili* and *Ammonites*, and were thus ranked as *Cephalopodous Mollusks* in the classifications of Lamarck and Cuvier; whilst a few others, bearing more resemblance to certain kinds of corals, seemed naturally to take their places among *Zoophytes*.

Down to a comparatively recent period, scarcely anything has been known beyond the external characters presented by these testaceous or coral-like bodies; their internal organization has been very imperfectly examined; of their intimate structure, no attempt at elucidation has been made; and as to the nature of the animals of which they are the skeletons, the grossest misapprehensions and errors have prevailed. And though many important steps have been recently made in this direction, it is obvious that the methods which have been proposed for the classification of the large number of forms (apparently distinct from each other) that have been collected, described,

and systematized, must be provisional merely. No general foundation has yet been laid for that due appreciation of the *value of characters*, which, in the case of every natural group, must be based on the careful study of its own plan of organization, and of the modifications which this may undergo, and which cannot be safely deduced by analogy, from the study of any other group, however closely related. Far less can any such analogy be truly available, that is drawn from the higher forms of *Mollusca*, and applied to one of those simple types of animal structure, which are now commonly included under the general designation *Protozoa*.

Having been myself convinced, by the careful examination and extensive comparison which I have had the opportunity of making, as to the external characters and internal structure of certain Foraminifera, that neither in regard to the limitation of *species*, nor the association of these into *genera*, nor the grouping of genera into *Families* and *Orders*, is such an analogy to be in the least degree relied upon, I am prepared to show that the whole fabric which has been erected on the basis of it, is utterly insecure; and that every attempt to erect a new classification of the group, without a far more intimate knowledge of the anatomical structure and of the physiological history of the animals composing it, than has yet been sought for, must necessarily be premature and therefore unsound.

If, therefore, notwithstanding the large amount of labour which has been given to the study of this group, we are really as yet only at the very commencement of an exact acquaintance with it, I venture to think that any contribution towards a more intimate knowledge will be welcomed, by all such as consider that systematic arangements can only be of value, when based on an extensive comparison, not merely of the external forms, but of the internal organization, of the objects to be classified, and when carried out under the guidance of a competent knowledge of their Physiology as living beings. It may be well for me here to state, that the greater part of the results which I purpose to communicate in successive Memoirs, are based upon the examination, not of a limited number of individuals, but of three very extensive suites of specimens, which have been liberally placed at my disposal; the first of these series having been formed from the dredgings of Mr. J. Beete Jukes on the coast of Australia, into the possession of which I came through the instrumentality of my late friend Professor E. Forbes; the second having been furnished by the collection of Foraminifera made by Mr. Hugh Cuming on the shores of the Philippine Islands, and unreservedly given up to me by that gentleman for the purposes of scientific investigation; and the third being the admirably-arranged series in the possession of my friend Mr. W. K. Parker, who has for several years been patiently and industriously bringing together from various sources a set of illustrations of this group, which in many departments may be safely pronounced to be quite unique. The importance of the first two of these collections consists, not in the number of *species* they include, for this is comparatively limited; but, on the one hand, in the extraordinary development as to *size* which most of these species present; and on the other,

in the immense aggregation of *individuals* they contain, thus presenting the materials for a most full and complete investigation into that hitherto comparatively neglected subject, *the range of variation within the limits of species.* And the great value of Mr. PARKER's collection lies in his having followed out this inquiry (first suggested to him by myself as the result of my examination of the other two collections) in regard to a much larger number of species, and with materials brought together from a much wider geographical area.

The question has been anxiously considered by me, in what mode I might render these materials most beneficial to the progress of science; and I have come to the conclusion, with the full concurrence of some of the most eminent Naturalists both of this country and of the continent, that I should be doing the most acceptable service, by instituting *a searching investigation into the entire history of such typical forms* of this remarkable group, as should be most fitted to afford satisfactory data for reasoning with regard to the rest;—including in this history, a minute examination of the structure, not only of the testaceous skeletons, but also (where this might be possible) of the soft animal body; a careful survey of the differences of form and aspect presented at different ages, so as to enable the history of any individual to be traced from its origin to its decline; a comparison of the variations both in external form and internal structure, presented by different specimens, sufficiently extensive to admit of the determination how far these variations are of value as *specific* characters, or how far they must be accounted merely individual departures from the ordinary type; a like comparison between the series collected in different geographical areas, so as to afford a safe basis for the determination of the range of the species, and the amount of its variation in *space*; and a further extension of this comparison to similar series obtained from the various geological strata in which the like forms may present themselves, so as to ascertain the range of the species and the extent of its variation in *time*. And if I should seem, in following out this plan, to have erred on the side of *over-minuteness*, I trust to receive credit for the real motive which has induced me to bestow so much elaboration on the work I have undertaken,—namely, the desire to furnish a thoroughly secure foundation, on the basis of which the labours of others, in whatever department of the inquiry, may be safely utilized in building up a complete and harmonious superstructure.

It had been my intention to preface the account of my own Researches, with a summary of the History of our knowledge of the group generally. This subject, however, has been so fully and satisfactorily worked out, in two of the most recent treatises on the subject,—that of MM. D'ARCHIAC and HAIME on Nummulites*, and that of Professor SCHULTZE on certain forms of existing Foraminifera†, the latter

* See their "Monographie des Nummulites," forming part of their "Description des Animaux Fossiles du Groupe Nummulitique de l'Inde," Paris, 1853.

† Über den Organismus der Polythalamien (Foraminiferen) nebst Bermerkungen über die Rhizopoden im allgemeinen. Leipzig, 1854.

containing a most complete Bibliography,—that I do not feel it necessary to offer more than a mere sketch, marking out the principal periods into which this History may be divided.

The *first* period includes all the observations made and published in regard to these minute polythalamous bodies, from the time when they first attracted attention, down to the date (1825) when M. D'ORBIGNY grouped them together, as constituting a distinct type of structure. The observers whose labours during this period did most to prepare the way for their successors, were SOLDANI*, FICHTEL and MOLL†, MONTAGUE‡, and DENYS DE MONTFORT§.

The *second* period commences with the presentation to the Académie des Sciences, in 1825, of M. D'ORBIGNY's "Tableau Méthodique de la Classe des Cephalopodes;" in which he first separated these chambered shells, under the title of *Foraminifera*, from the *Siphonifera*; still retaining the former, however, like the latter, as an order of Cephalopods. A large number of new forms were added by M. D'ORBIGNY to the list of those previously known; and he laid the basis of the classification which he has since more fully elaborated. No suspicion appears then to have crossed his mind, that the place of these bodies might be amongst the lowest, instead of among the highest, of the Invertebrata; and if his determination of their Molluscous nature was based on any actual observations of these animals in their living state, it is certain that such observations must have been of the most superficial character.

The *third* period, with which our knowledge of the true nature of the Foraminifera really commenced, is inaugurated by the discovery, first announced by M. DUJARDIN to the Académie des Sciences in June 1835, of the *Rhizopodous* nature of the animal of certain simple forms of Foraminifera, and, by inference, therefore, of that of the group generally; and in the following year he demonstrated the essential identity between the *Amœba* and other simple freshwater Rhizopods (described by Professor EHRENBERG among the Polygastric Animalcules) and the *Cristellaria* and similar composite forms of marine Foraminifera, which had been previously ranked among Cephalopod Mollusks‖.

The general results of M. DUJARDIN's observations was, that the animal body consists, in every instance, of a mass of *sarcode*,—a gelatinous, somewhat granular substance, not enclosed in a distinct membrane, and capable of extending itself into threads of extreme tenuity; that there is neither mouth nor digestive cavity, but that alimentary particles, received into the very substance of the body, are gradually incorporated with it; and that both the introduction of these particles, and the move-

* Saggio orittographico, overo osservazioni sopra la terra Nautiliche e Ammonitiche della Toscana. Sienne, 1780.

† Testacea microscopica aliaque minuta ex generibus Argonauta et Nautilus, ad naturam picta et descripta. Vindob. 1798, 1803.

‡ Testacea Britannica, or Natural History of British Shells. London, 1803–1808.

§ Conchyologie Systematique. Paris, 1808.

‖ See Annales des Sciences Naturelles, 2 sér., Zool. tom. iii. p. 312.

ments of locomotion, are effected by means of pseudopodial prolongations of the *sarcode*, put forth through apertures in the shell, and capable, when retracted again, of coalescing with the general mass. In the case of the composite forms, he considered the entire animal to be made up of a series of segments which are essentially repetitions one of another, each possessing an independent vitality of its own*. These statements have been subsequently confirmed and rendered more precise by several other observers; their truth has been admitted by M. D'ORBIGNY, who, in all his recent works, has described the animals of the Foraminifera in accordance with them (though without any allusion to the fact, that he had himself previously laboured under an entire misconception of their character, and without any mention of the discoverer of their real nature); and they have been recently placed beyond all doubt, by the admirable researches of Professor SCHULTZE (*Op. cit.*).

It cannot but seem surprising, that notwithstanding the light thrown upon this inquiry by M. DUJARDIN in 1835, Professor EHRENBERG should in 1838 have announced to the Academy at Berlin, his conclusion, professedly based on observations of certain forms of these animals in their living state, that their true place in the animal kingdom is among the *Bryozoa*†. He described them as possessing a distinct alimentary canal, which extends from segment to segment; this, however, instead of being single, as in *Nonionina*, may (he tells us) be multiple, as in *Geoponus*; so that we must regard each segment of the latter, however apparently resembling the simple segment of the former, as in reality composed of several adhering bodies. In one instance (he affirms) he found the mouth surrounded by a plumose sensory and prehensile apparatus, like that of the *Flustræ* and *Halcyonellæ* (see *ultra*, ¶ 4.), but generally speaking he admits that this is altogether wanting, the mouth being a simple aperture. He saw minute extensile tentacula proceeding from all parts of the sieve-like shell, as described by DUJARDIN, and admitted their resemblance to the pseudopodia of *Difflugia*, &c., but he remarks, "the rest of their organization, which DUJARDIN has overlooked, removes them from the Infusoria, quite as far as from a chaotic primitive substance." Besides the alimentary canal, Professor EHRENBERG describes a yellowish-brown granular mass as accompanying and sometimes surrounding it up to the last of the spirals; this he considers as an ovary.

As I have reason to believe that Professor EHRENBERG stands quite alone in this opinion (if, indeed, he still maintains it), and that the real nature of the segments of sarcode and of their connecting threads, is no longer a matter of question among those Naturalists who have given their unprejudiced attention to the subject, I do not think it requisite to occupy either time or space with any further discussion of the question, and therefore dismiss it with this brief mention.

* See his "Histoire Naturelle des Infusoires," Paris, 1841; and Art. *Rhizopodes* in Dict. Univ. d'Hist. Nat., tom. xi. p. 115, Paris, 1848.

† See his Memoirs in the Transactions of the Royal Academy of Berlin, for 1839 and 1840; also TAYLOR's Scientific Memoirs, vol. iii. p. 319.

It may, however, be well here to remark *in limine*, that it obviously makes a most essential difference in our appreciation of the value of the characters afforded by the form, position, and multiplication of the apertures of communication between the chambers of the shell, whether we regard these as giving passage to an organ of such fundamental importance as an alimentary canal, or whether they merely serve for the connexion of the different segments by *stolons* of sarcode. For variations, which in the former case must be regarded as indicative of such essential differences, both in structure and function, as would rightly characterize distinct genera or even distinct families, may easily be admitted, on the latter view, to be of such comparatively trivial moment, as to rank no higher than specific characters, or perhaps even to be matters of individual difference. That the latter is the true view of the case, I have become completely assured in the course of my researches; and I shall hereafter be able to adduce some curious illustrations of it.

Turning now to the more recent History of research, I shall briefly notice those investigations which have done most towards the advance of our knowledge of the organization and physiology of the Foraminifera; the mere collection, description, and systematic arrangement of new forms, without any such advance, being no more a feature of progress, than is the building-up of an edifice, which must necessarily fall, through the insecurity of its foundation, before it shall have been completed.

The first series of these, made by Professor W. C. WILLIAMSON of Manchester, upon *Polystomella crispa**, not only established several important facts in regard to its minute structure, but may be regarded as having furnished the starting-point for all future investigations of a like kind. Among these facts were several that became of essential value to myself, in the inquiry on which I was engaged at the same time, in regard to the structure of *Nummulites*; and served to confirm the inferences which I had deduced from the other features of that important type, as to its participation in the characters of the Foraminifera generally. In the course of that inquiry I made the discovery†, not only of a most elaborate and previously-unsuspected structure in the shell itself, but also of a system of interseptal canals, which established a communication between the inner segments and the external surface, much more direct than that which they possess through the series of segments which form the outer turns of the spire. The existence of this system of canals has been verified, not merely in Nummulites by MM. D'ARCHIAC and HAIME (*Op. cit.*), but also in several recent types; thus Professor WILLIAMSON has detected it in *Amphistegina* and *Nonionina*‡, and more recently in *Faujasina*§ (which furnishes one of the most remarkable examples of it); whilst Mr. CARTER of Bombay has discovered it in *Operculina*||. My own inquiries, which have been carried-on with scarcely an intermission, from the time of my first

* Transactions of the Microscopical Society, 1st ser. vol. ii. p. 159.

† Quarterly Journal of the Geological Society of London, vol. vi. February 1850, p. 22.

‡ Transactions of the Microscopical Society, 1st series, vol. iii. p. 105.

§ Ibid. 2nd series, vol. i. p. 87.

|| Annals of Natural History, 2nd series, vol. x. p. 161.

discovery of this remarkable point of structure, have been specially directed to the determination of the extent to which it presents itself in the different sections of the group, and of its value as a distinctive character; and I think that I shall be able to show that it is a feature of the utmost importance, the presence of which marks an elevation of type, and its absence a corresponding degradation.

It is much to be regretted, that the recent investigations of Professor SCHULTZE should have been so entirely restricted to the structure of the *animal*, which can only be ascertained in a comparatively small number of cases; and that he should have failed so completely in the determination of the internal organization of the *shell**, which in a large proportion of instances is the only guide we possess to the nature of the being which formed it. The new classification which he proposes, whilst in many respects an improvement upon that of M. D'ORBIGNY, is essentially vitiated by this defect; and being in itself, therefore, just as provisional as that for which it is proposed as a substitute, can scarcely be expected to supersede it. My own researches I offer simply as *materials* to serve as a *basis for classification*; feeling assured that the time is not yet come, in which the superstructure can be erected with any prospect of permanent stability.

I shall commence with a minute analysis of one of the lowest types, *Orbitolites*; and propose to show hereafter, that *Orbiculina* and *Alveolina*, though ranked in a different order by M. D'ORBIGNY, are in reality closely allied to it; whilst a new genus (*Cycloclypeus*), which, in M. D'ORBIGNY's arrangement, would rank close to Orbitolites, is physiologically separated from it by the widest possible interval.

Genus ORBITOLITES.

I. *History*.

1. The Orbitolite has been chiefly known, until very recently, rather by its fossil, than by its existing forms. The abundant occurrence of its disks in the *Calcaire grossier* of the Paris basin, early attracted attention; but Orbitolites were not clearly distinguished by the older observers from Nummulites, and their true nature was entirely misunderstood. Thus we find them designated, often in association with Nummulites, under the title of *Umbilicus marinus* by PLANCUS (BIANCHI), who

* Professor SCHULTZE states (*op. cit.* p. 15) his inability to discover the canal-system above described; and as there is no question of his competency and accuracy as an observer, I can only impute his failure to his ignorance of the proper mode of preparing thin sections of these minute shells;—which consists in cementing them to a slip of glass by hardened Canada-balsam; grinding them down as far as may be desirable on one side; then loosening them by heat and turning them over, so that the flattened surface shall now be attached; and finally grinding down the other side, until the requisite degree of thinness shall have been attained; after which a drop of liquid Canada-balsam is laid upon the specimen, gentle warmth applied, and a cover of thin glass put on. Having myself thus prepared sections of *Faujasina*, which answer in every respect to the figures accompanying Professor WILLIAMSON's memoir above cited, I can bear the most explicit testimony to their exactness.

imagined them to be opercula of Ammonites*; of *Porpitæ nummulares* by STOBÆUS† and BROMELL‡, who seem to have regarded them as representing the disks of the existing Porpitæ; of *Helicites* and *Operculites* by GUETTARD §, who considered them as opercula of Gasteropods; of *Discolithes* by FORTIS ||, who supposed them to be skeletons of mollusks; of *Madreporites* by DELUC, and of *Milleporites* by FAUJAS DE ST. FOND, whose idea of their nature is sufficiently indicated by the names they assigned to them.

2. The genus *Orbitolites* seems first to have been erected, and distinctly separated from Nummulite, by LAMARCK, in the first edition of his 'Animaux sans Vertèbres,' its type being the *O. complanata* of the Paris basin. The following is his definition of the genus, which he ranks between Lunulites and Millepora, among his "Polypiers Foraminés":—"Polypiarium lapideum, liberum, orbiculare, planum seu concavum, utrinque vel margine porosum, nummulitem referens. Pori minimi, adamussim dispositi, conferti, interdum vix conspicui." These bodies, he says, are distinguished from Nummulites by the opening of their marginal pores, and by the absence of spiral arrangement in their minute chambers or cells. In his second edition (1816), he altered the name from *Orbitolites* to *Orbulites*; but the latter designation having been previously employed in Malacology, the first appellation has been restored by M. MILNE-EDWARDS in his posthumous edition of LAMARCK's work. Under one of the designations, *Orbitolites* or *Orbulites*, the genus has been recognized by SCHWEIGGER¶, BRONGNIART and CUVIER**, LAMOUROUX††, DESLONGCHAMPS‡‡, DEFRANCE§§, BLAINVILLE ||||, BRONN ¶¶, GOLDFUSS ***, MICHELIN †††, PICTET ‡‡‡, and DUJARDIN§§§; none of whom, however, have either given any account of its internal structure, or made any essential modification in the definition of the genus, which they all left in the place which LAMARCK had assigned to it.

3. The existence of more than one recent species of the same type was indicated or expressly mentioned by several of the foregoing writers. Thus FORTIS tells us

* De Conchis minus notis, 1739 (*fide* D'ARCHIAC et HAIME), and App. Phytol. F. Coll. 1764 (*fide* RUPERT JONES).

† Dissertatio epist. ad W. GROTHAUS de nummulo Brattenburgensi, 1732; Opera petrefactorum, 1752; Opusculis, p. 6 (*fide* D'ARCHIAC et HAIME).

‡ De Nummulo Brattenburgico, in Act. Litt. Suec., vol. ii. p. 50 (*fide* D'ARCHIAC et HAIME).

§ Mémoires sur différentes parties des Sciences et des Arts, 1770.

|| Mémoires pour servir a l'Hist. Nat. de l'Italie, 1816, vol. ii.; and Journal de Physique, 1801, vol. lii. p.106.

¶ Beobacht. auf Naturg. (1819), pl. 6.

** Ossemens Fossiles (1822), vol. ii. pt. 2. p. 270.

†† Expos. Méthod. des gen. des Polypiers (1821), p. 44. pl. 73.

‡‡ Encyclop. Méthod., Zooph. (1824), p. 584.

§§ Dictionn. des Sci. Nat. (1825), vol. xxxvi. pp. 294, 295.

|||| Manuel d'Actinologie (1830), p. 411. pl. 72.

¶¶ Lethæa Geognostica (1836-37), pl. 35.

*** Petrefacten (1826–33), vol. i. p. 41. pl. 12.

††† Icon. Zoophyt. (1845), p. 167. pl. 46.

‡‡‡ Traité Élément. de Paléontologie (1844–45), tom. iv.

§§§ Dict. Univ. d'Hist. Nat., tome ix. (1847), p. 162.

(*op. cit.*), on the authority of a voyager in the Indian seas, that living Discolites have been found there; and as there are no existing Nummulites, this statement probably refers to an Orbitolite. LAMARCK, in his second edition, describes, under the title of *O. marginalis*, a small form of Orbitolite, only 2 millims. (·08 inch) in diameter, found upon fuci, corallines, &c. in the European seas; this he speaks of as the only living example of the genus then known, and he defines it as *O. utrinque plana, margine poroso.* Besides this species, however, DEFRANCE (*loc. cit.*) mentions another, more closely resembling the *O. complanata* of the Paris basin, as existing on the shores of New Holland; and this seems the first clear indication of the body (afterwards found by MM. QUOY and GAIMARD in that locality, and erected by them into the distinct genus *Marginopora*), the structure of which constitutes the chief subject of my present communication. Both these existing forms are described by BLAINVILLE (*op. cit.*), the first from actual observation, the second on the information of MM. QUOY and GAIMARD, to the manuscript of whose 'Voyage de l'Astrolabe' (then unpublished) he refers as his authority. It is singular, however, that after an attentive search through the published "Zoology" of that work, I have not been able to find, either in the text or in the plates, any mention of *Marginopora* or of *Orbitolites*. Of the *Orbitolites marginalis*, M. DE BLAINVILLE says (*op. cit.* p. 412),—"Nous l'avons étudiée avec soin; et nous sommes presque convaincu que ces petits corps crétacés ne sont pas de véritables polypiers; mais bien quelque pièce intérieure, qui s'accroît par la circonference. Il est en effet évident, qu'il n'y a pas de cellules proprement dites, à moins qu'on ne veuille regarder comme telles les deux plans de locules qui occupent le bord, et qui n'offrent rien determiné. Tout le reste est couvert d'une légère croûte crétacée, qui ferme les anciens pores." I think it obvious, from this description, that it was founded on specimens resembling that in Plate VII. fig. 8, in which the marginal row of cells has been laid open above and below by accidental abrasion; and that the true marginal pores, opening *between* the protuberances formed by the cells (Plate V. fig. 1), were overlooked. The genus *Marginopora*, placed by M. DE BLAINVILLE in immediate sequence to Orbitolites, is thus characterized (*loc. cit.*):—"Animaux inconnus, contenus dans les cellules poriformes, excessivement petites, rondes, serrées, éparses dans les sinuosités, très fines et tortueuses, qui guillochent la circonference d'un polypier calcaire, libre, un peu irrégulier, discoïde, concave ou concentriquement strié en dessus comme en dessous, et plus épais sur les bords." The two surfaces, M. DE BLAINVILLE further tells us, only exhibit striæ of increase, without any trace of pores; but the turned-up edge is entirely riddled with very fine rounded pores, which are situated in the sinuosities of a very close but shallow engine-turning (guillochis). And when one of its surfaces is rubbed away, the disk is found to be formed of concentric canals, separated by partitions, and themselves divided into cells, thus recalling in some degree the structure of Orbitolites. Having myself had the opportunity of inspecting, by the kindness of M. VALENCIENNES, the specimens of Marginopora on which the foregoing

description was founded (these being now contained in the 'Musée d'Histoire Naturelle'), I have been able to compare them with my own; and finding that they correspond with the peculiar type of the latter, which is represented in Plate V. figs. 2 & 3, I have no hesitation in saying that in this description also the *true marginal pores,* represented in Plate V. fig. 6, have been overlooked; and that what are described by M. DE BLAINVILLE as pores, are nothing else than incomplete cells left open in the frilled edges which bound the marginal furrow above and below (see ¶ 25). A similar description has since been given by M. DUJARDIN *, who does not hesitate to regard the disk as a polypary, and to speak of the animals whereby it is formed, as polypes.

4. In the Memoir of Professor EHRENBERG already referred to, we find the genus *Orbitolites* for the first time associated with true Foraminifera, as a member of his class BRYOZOA, order *Polythalamia,* suborder *Polysomatia,* family *Asterodiscina,* wherein it is placed next to *Lunulites,* which undoubtedly belongs to the group of Bryozoa (Polyzoa) as now restricted. This family he characterizes as follows:—"Gemmis in eodem plano prodeuntibus, polypiaria plana, discoidea, formantibus, osculis distinctis post mortem apertis;" and it is by the last of these characters that he distinguishes it from the family *Soritidæ,* consisting of the two genera *Sorites* and *Amphisorus,* of which he says,—"Osculis contracto corpore, tanquam operculo duro clausis." If any faith whatever is to be placed in Professor EHRENBERG'S figures and descriptions, his *Sorites* is nothing else than LAMARCK'S *Orbulites marginalis*; whilst his *Amphisorus,* which differs from *Sorites* merely in having two layers of cells instead of one, is (as I shall hereafter show) the same type in a higher grade of development. I cannot conceal my astonishment, however, that so practised a microscopic observer should have entirely overlooked the real marginal openings between the cells; still more, that he should have described the entirely-closed cells of the surface as covered in by a moveable operculum, which merely shuts their orifices when the animal is contracted; and further, that, mistaking an accidental for a normal opening of some of the cells, he should have ventured to figure an eight-armed Bryozoon as issuing forth from one of them,—a phenomenon which, I do not hesitate to say, is entirely irreconcileable with our existing knowledge of the organization of the animal of which these disks are the skeletons.

5. The earlier publications of M. D'ORBIGNY on the subject of the Foraminifera do not include any notice of this genus; and neither in the systematic arrangement which he put forth in his article in the 'Dict. Univ. d'Hist. Nat.' tome v. (1844), nor in that contained in his 'Foram. Foss. de Vienne' (1846), is the order *Cyclostègues* recognized, which makes its appearance for the first time in his 'Cours Elémentaire de Paléontologie,' tome ii. (1852), between the Monostègues and the Helicostègues, with the following definition (p. 192):—"Animal composé de segments nombreux, placés en lignes circulaires. Coquille discoïdale, composée de loges, concentriques,

* Dict. Univ. d'Hist. Nat., tome vii. p. 777.

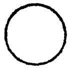
simples ou multiples; point de spirale." Under this order are ranged the genera *Cyclolina* (D'ORB. 1839), *Orbitolites* (LAMARCK, 1801, and *Marginopora*, QUOY et GAIMARD, 1836), *Orbitolina* (D'ORB. 1847), and *Orbitoides* (D'ORB. 1847). As I shall have occasion to show that the first three of these genera cannot be separated from each other by any valid distinctions, and that the greater number of the species ranked under them by M. D'ORBIGNY (see his 'Prodrome de Paléontologie Stratigraphique') really belong to the same specific type, I must here cite the generic characters which he assigns to them:—"*Cyclolina*, Coquille discoïdale, chaque loge percée de nombreux pores, faisant un circle entier autour des autres. L'espèce connue est de l'étage cénomanien.—*Orbitolites*, Coquille discoïdale, plane, égale, et encroûtée des deux côtés, pourvue de lignes concentriques. Loges nombreuses, par lignes irrégulières, transverses, visibles seulement au pourtour. Nous connaissons deux espèces; les premières, de l'étage suessonien; le maximum, dans les mers actuelles.—*Orbitolina*, Ce sont des Orbitolites à côtés inégaux; l'un, convexe, encroûté, à lignes concentriques; l'autre, concave, non encroûté, montrant des loges nombreuses, par lignes obliques sur le côté, au pourtour. Nous connaissons de ce genre perdu six espèces; les premières, de l'étage albien; les dernières, de l'étage senonien."

6. The first approach towards a more accurate knowledge of the real nature of the Orbitolite, through an examination of its internal structure, was made (I believe) by myself, in my Memoir "On the Microscopic Structure of *Nummulina*, *Orbitolites*, and *Orbitoides*," read before the Geological Society of London in November 1849, and published in its Quarterly Journal for February 1850. The place assigned to this genus in the system of M. D'ORBIGNY not having been at that time made public, and all other zoologists and palæontologists having ranked it in close approximation to *Lunulites* and other Polyparies of the Bryozoic (Polyzoic) kind, I entered upon the examination without the least suspicion that this organism was to be regarded in any other light; and that I was not undeceived in the course of it, may be attributed to the small number of specimens then placed at my disposal for the inquiry by my late friend Professor E. FORBES, and to the circumstance that these specimens were of the type that presents most resemblance to that of a Bryozoic polypary, and were all deficient in the central nucleus, which is the portion most indicative of their Foraminiferous nature. Nevertheless, the marked dissimilarity in structure which I found to exist between the disk of Orbitolite and the polypary of Lunulite or of any other undoubted members of the Bryozoic group, made me even then express myself doubtfully as to its title to be associated with them. In this Memoir, published two years previously to M. D'ORBIGNY's first announcement of the fact, I showed that the genus *Marginopora* must be abolished, since its sole representative is so closely allied in structure to the *Orbitolite* of the Paris basin, that no doubt of their generic identity can be entertained; the existing *M. vertebralis*, in fact, being only specifically distinguishable from the fossil *O. complanata*, by a difference in the form of its super-

ficial cells,—a character to which the more extended knowledge of this type has made it clear to me that not the slightest value is to be attached (see Sect. IV.). I showed that in one as in the other, the cells are normally closed-in over the whole surface; that the cells of the two surfaces are separated from each other by an intervening stratum, traversed by a set of round cells or passages of its own, with which each superficial cell communicates by two small apertures; and that the only real external orifices are the minute pores at the margin of the disk, which communicate, not directly with the cells of the superficial layers, but with the passages of the intermediate stratum.

7. A very important addition to this limited measure of information was made not long afterwards by Professor WILLIAMSON of Manchester, who, in a paper read before the Microscopical Society of London, June 12, 1850, and published in its Transactions (First Series, vol. iii. March 1851), first gave a minute description of the small recent *Orbitolites marginalis* of LAMARCK (which, however, he designated as *O. complanata*); contrasted it with that of a large recent Orbitolite from Tonga, shown by his description to be identical with the *Marginopora* of QUOY and GAIMARD, to my previous account of which he added some important particulars; and compared both with the well-known *Orbiculina adunca* of the Bahamas. His investigations led him to the conclusion, that these three forms should rank under the same genus *Orbiculina*; their most important structural features being common to all, whilst their differences are only of specific value. His *Orbiculina complanata* (really *Orbitolites marginalis*) he characterizes by its spiral commencement, and by its possessing only one layer of cells; his *O. Tonga* (Marginopora) he characterizes by its cyclical commencement, and by its possession of two superficial layers of cells, with an intermediate stratum, as I had previously pointed out; whilst the *O. adunca* he shows to commence on the spiral type, and to carry it on much further than his *O. complanata*, but finally to assume the cyclical, and then to correspond very closely with his *O. Tonga*. The title of the genus *Orbitolites* to a place in the group of Foraminifera, in near proximity to, if not in union with, *Orbiculina* (which had been ranked as such by M. D'ORBIGNY from an early period of his investigations), and the entire absence of any ground for ranking it among the Bryozoa (Polyzoa), were clearly established by Professor WILLIAMSON in this valuable memoir; and though I shall hereafter have occasion to show that some of his conclusions were erroneous, yet I regard them as fully justified at the time, by the information which the materials at his command afforded; my own means of correcting them being only supplied by the comparison I have been enabled to institute through a much wider range of specimens. And it is with great satisfaction that I am enabled to add, that, after a careful inspection of my preparations and drawings, Professor WILLIAMSON authorises me to express his full accordance in the results which I shall now proceed to detail.

8. As the value of such details cannot be truly estimated without some knowledge of the range of the observations from which they are derived, I think it right to state

in limine what are the opportunities I have enjoyed. The first and most important of these was afforded me by the great abundance of Orbitolites of various sizes, from o to , besides a vast multitude of fragments, in Mr. JUKES's Australian dredgings; besides which, Professor E. FORBES kindly put into my hands several specimens which had been taken in their living state by Mr. JUKES, from the marine plants to which they were attached. All these I have carefully examined under the microscope, so as to be able to make an exact determination of their external characters; and of a large proportion I have made microscopic sections in various directions, that I might assure myself of every particular respecting their internal structure. When I had nearly exhausted these sources of information, I found a new and most interesting series of specimens in Mr. CUMING's Philippine collection; and by the study of these I was enabled to test the validity of the conclusions, to which I had been led by the examination of the Australian forms. The kindness of various friends has further enabled me to examine specimens obtained from other widely-distant localities, such as different parts of the Indian, Southern, and Pacific Oceans, the Red Sea and the Ægean. And finally, I have had placed at my disposal, through the instrumentality of Professor QUEKETT, several Orbitolites of various sizes and ages, obtained on the shores of the Feejee Islands by the late Sir EVERARD HOME, in which the animal substance occupying the interior had been preserved by the immersion of the fresh specimens in spirits.

9. Besides the existing forms, I have examined a large number of specimens of *Fossil Orbitolites*, both from the Paris basin and from other localities; and I have instituted the same kind of minute comparison of these specimens, both with each other and with the recent types, that I had previously made among the diversified forms of the latter.

10. Postponing, until adequate means shall have been supplied by the details of their organization, to be presently given, the inquiry into the relationship of these different forms (Sect. VI.), we shall proceed in the first place to consider the general plan of organization of the Orbitolite, and then to study the variations to which this is subject.

II. *General Plan of Organization.*

11. In studying the organization of the Orbitolite, we shall have recourse, on the one hand, to the structural characters presented by the *Animal*, as displayed by the fleshy residuum left after the decalcification of specimens in which it has been preserved; and on the other, to the structural characters presented by the *Shell*, of which some are visible on its surface, and without any preparation, whilst others may be seen in thin specimens by transmitted light, but of which the greater part can only be brought into view by thin sections taken in various directions, especially *horizontal* or parallel to the surface, and *vertical* or perpendicular to the surface.

12. The state of preservation of the animal body of the Orbitolite, in the spirit-specimens which I have examined, is so complete, as to leave me no room for hesitation in affirming that it corresponds in every particular with the 'sarcode' which we have seen to have been first described by M. DUJARDIN, as constituting the bodies of many of the lowest organisms, and especially as the component of those of the Rhizopoda. A small portion of this substance, sufficiently magnified to exhibit its nearly homogeneous jelly-like aspect, with minute granules and somewhat larger particles scattered through it, is shown in Plate IV. fig. 2. Although it is so far decolorized in spirit-specimens as to present only a brownish hue, yet as specimens that have been gathered fresh and have been then dried, possess a reddish aspect, and as this is not due to the shelly substance, it may be presumed that the sarcode of the living Orbitolite has the same bright red colour as that of *Rotalia* and many other Foraminifera. The entire animal body (Plate IV. fig. 1) is composed of a numerous assemblage of minute segments, arranged at tolerably regular intervals in concentric zones around a sort of central 'nucleus;' the segments composing each zone being united with each other by a continuous annular 'stolon' or band of sarcode, and being connected with those of the adjoining zones by peduncles of the same material. I have not met with the least indication that the sarcode is contained within any *proper membrane*; and the absence of any such indication, notwithstanding the various manipulations to which I have subjected its segments, may be taken, I think, as strong negative evidence that it has no more existence in this animal, than it has in the species of Foraminifera which have been so well studied by M. DUJARDIN and Professor SCHULTZE. Nor is there the slightest trace of distinct *organs*, either in the mass of sarcode which forms the central nucleus, or in that which constitutes each one of the surrounding segments; and he would, I think, be a mere speculator, who should maintain the presence of a digestive cavity in any of these parts, or the existence of an intestinal canal in the peduncular threads which connect them together. The homogeneity of the component substance of the central nucleus, and of the entire assemblage of multiple segments, seems, indeed, to be conclusively established by the following facts:—In all the spirit-specimens which I have examined, the cavities of the outer zones are completely void, whilst those of the nucleus and of the inner zones are quite filled with their animal contents. This drawing-together of the soft body towards the centre, is evidenced also in many of the larger specimens which have been dried when collected in the living state, by the limitation of the red colour that indicates the presence of the sarcode, to the inner portion of the disk. In both cases it may be presumed that the animal matter has shrunk together, in the former through the corrugating action of the spirit, in the latter through desiccation. Now if the polypidom of a zoophyte be similarly treated, there is no such drawing together of the entire body, but each cell is found to contain the shrunk contents of its own polype-segment; and this difference seems to me to indicate a complete dissimilarity in the characters of the two organisms. For it is obvious that the sub-

stance of the peripheral segments of the Orbitolite-body can only be brought together towards the centre, through being completely unattached to the walls of the cavities which it occupies, and through having a form so alterable, as to be capable of being drawn in threads through the narrow connecting passages, and of then coalescing together again so perfectly, that the masses they form do not present the least trace of having been thus spun out. There is no known kind of animal texture, except *sarcode*, that is susceptible of this kind of alteration; and the evidence of it which I have adduced seems to me extremely valuable, not only as establishing the general nature of the animal body of the Orbitolite, but also as fully justifying the assumption, that, in the living state, the sarcode is projected in pseudopodia through the marginal apertures, and that alimentary particles are introduced by their instrumentality, as in other Foraminifera.

13. Turning from the animal body to the calcareous disks which enclose it, we find that, whether large or small, these are almost invariably circular, or nearly so; that they are usually nearly flat, any difference in thickness being generally in favour of the marginal portion; and that if, as sometimes happens, there is a slight central projection, this is formed by the nucleus alone. By these characters we may distinguish *Orbitolites* from *Orbiculina*; for although the discoidal forms of the latter so strongly resemble Orbitolites, that by the structure and arrangement of their marginal portion they could not be distinguished, yet they may always be discriminated by the knobby protuberance of their centre, which is occasioned by the mutual investment of the earlier whorls of the spiral in which they commence. The same entire absence, or very small size, of the central elevation, together with the uniformity or even slight increase of thickness towards the circumference, also helps us to separate *Orbitolites* from *Orbitoides*; the centre of the latter being always considerably elevated, and the thickness of its disk ordinarily diminishing gradually towards its margin *. Around the 'nucleus' which occupies the centre of the disk (Plate V. figs. 1, 6), are seen an indeterminate number of concentric zones of cells (*c*, *c*, *c*), the shape of which differs in different individuals (see Sect. IV.); these, although completely closed (unless laid open by abrasion), have their form

* I wish this statement to be understood with reference to the genus *Orbitoides*, as characterized by the structure which I have shown it to possess (Quart. Journ. of Geol. Soc., Feb. 1850), and not to the genus as defined by M. D'ORBIGNY (Cours Élémentaire de Paléontologie, tome ii. p. 194), who, notwithstanding that he has shown himself to be acquainted with my Memoir (by copying from it a figure of Nummulite), has not profited in any degree by my investigations, but has left the generic characters of *Orbitolites*, *Orbitolina* and *Orbitoides* in the state in which they might have been, and probably were, before that Memoir was published. The true distinction, however, has been fully recognized by M. D'ARCHIAC, who, in his 'Description des Animaux Fossiles du Groupe Nummulitique de l'Inde,' p. 349, has designated as *Orbitoides dispansa* and *Orbitoides Fortisi*, the bodies which, in the account of them he had previously given in the 'Mém. Soc. Géol. de France,' 2nd ser., vol. iii., he had designated as Orbitolites; thus correcting the error into which Mr. CARTER has fallen in his description of the same fossils, through reliance on M. D'ORBIGNY's insufficient and indeed erroneous characters of these genera.

distinctly indicated by the surface-markings. The only external orifices which communicate with these cells, are the minute pores (*d, d, d*) forming one or more ranges at the margin of the disk, each pore lying in a vertical furrow, between the projecting walls of two contiguous cells. In the smallest and thinnest disks (Plate V. fig. 1) we find but a single row, or sometimes two rows, of such pores; in disks somewhat thicker, there are three or four rows; and in the largest and thickest Orbitolites (Plate V. fig. 6), no fewer than ten or twelve such rows. This multiplication in the number of ranges of marginal orifices, indicates a like multiplication in the number of floors (so to speak), of which the disk is composed; and just as the total number of chambers in a building may be increased, either by extending its base over a larger area, or by additions to its number of storeys, so may an increase in the number of segments of which this animal is composed be provided for, either by the marginal addition of a new zone resembling the last, so that the diameter of the disk is alone augmented, or by an increase in the thickness of the newly-forming zone, so that it contains a larger number of superposed layers. The new zones, however, *never invest* or cover those which they surround, each being simply a continuation of the margin of the preceding; and in this respect the mode of growth of *Orbitolites* at every stage is pointedly distinguished from the early mode of growth of *Orbiculina,* just now specified as the cause of the protuberance of its centre.

14. The shelly *substance* of the calcareous disk, although firm, is by no means so dense and bony as that of the shells of many other Foraminifera of higher organization. It is apparently quite homogeneous, rarely presenting the least appearance of 'structure,' and this being probably fallacious; I refer to the punctated marking sometimes seen on the outer surface of the nucleus, which shows itself under the aspect represented in Plate VI. fig. 5, when the thin layer of shell which presents it is viewed by transmitted light. Although this appearance might be considered to indicate the existence of a cellular structure in the shell, yet I believe that such an inference would be fallacious; since I have not been able to detect the least trace of such a structure in the decalcified residuum, which, on the other hand, seems to me to be a substance as structureless as sarcode itself. Coupling these appearances with those which I have found to exist more distinctly in *Orbiculina,* I am disposed to interpret them as proceeding from minute *depressions* on the surface; and these are perhaps to be regarded as the rudiments of those minute closely-set apertures, which, in many Foraminifera, give passage to pseudopodial extensions of the sarcode from every part of their bodies.

15. In all the forms of Orbitolite that I have examined, the central *Nucleus* presents the same essential characters. When the interior of any disk, whether large or small, is laid open by a horizontal section passing through the central plane, the nucleus is seen to be occupied by a large cavity (Plate V. fig. 1) somewhat irregularly divided by a sinuous partition, which always, however, marks out a central

cell (*a*) of a somewhat pyriform shape, as distinct from the space (*b*, *b*) which surrounds it. The meaning of this feature is at once made apparent, by reference to the disposition of the sarcode which occupies the cavity of the nucleus; for we then see (Plate IV. fig. 5) that the large central pear-shaped cell is occupied by a mass (*a*) of corresponding shape, from the small extremity of which a peduncular process extends, that dilates again into a still larger mass (*b*, *b*) completely surrounding that from which it springs; the former may be conveniently designated as the 'central,' the latter as the 'circumambient' segment. In a *vertical* section of the disk, passing through the centre, such as that seen in Plate V. fig. 4, the nucleus seems to present *three* chambers; but this is simply due to the fact that such a section will traverse the circumambient cell twice, that is, will cut it through on both sides of the central cell. In the section represented in Plate V. fig. 9, there are but *two* central chambers; in this case the plane of division seems to have traversed the nucleus just where the neck of the central cell touches its margin, so that the circumambient cell is only on one side of it. If, on the other hand, the plane of division should happen not to pass through the central cell at all, so as to traverse the circumambient cell alone, a *single* broad cavity will present itself in the vertical section, as shown in Plate V. fig. 7. Frequently, however, it happens that the circumambient segment is partially subdivided on one side by an interposed partition (Plate VII. fig. 4); and then a vertical section will show *four* chambers, as is seen in Plate V. fig. 10, the central segment having a single portion of the circumambient segment on one side of it, and a double portion on the other. Some remarkable varieties in the size of the nucleus, and in the mode of its connexion with the surrounding parts, will be noticed hereafter (¶¶ 44–46, and 54, 55).

16. In describing the structure of the *Concentric Zones* which successively surround the nucleus, it will be requisite to make a distinction between the *simple* and *complex* types according to which the Orbitolite-disks may be generated; the former being characterized by the existence of only one layer or 'floor' of segments, the latter by the presence of *two or more* such layers. For although, as will subsequently appear, I can show, by a series of transitional gradations between these two types of structure, and by their occasional coexistence in the same *individual*, that they are not to be held to characterize distinct *genera* (as Professor Ehrenberg supposed), or even distinct *species* (as Professor Williamson has urged with more apparent reason), yet, when most characteristically displayed, they differ so much from one another, and each is so remarkably distinguished by features of its own, that it seems more advantageous to describe them separately in the first instance, and then to discuss their relationship to each other.

17. The *Simple* type is found to prevail in those minute Orbitolite-disks, which occur in greater or less abundance in sands and dredgings from almost every part of the globe, but which are particularly numerous in those of the Philippine shores. Their ordinary diameter is about ·05 of an inch, and they usually contain from ten

to fifteen concentric zones. The arrangement and connexions of these zones may be made out in the thinnest and most translucent specimens, by examining them by transmitted light, after mounting them in Canada balsam; this, however, gives such transparence to the thin shelly layer which is continuous over both surfaces, that it may escape notice (if not carefully looked for), so as to lead to the conclusion that the cells are open. Most specimens require to be somewhat reduced in thickness, by slightly grinding down one surface, to enable the arrangement of their interior to be distinctly made out; and this may be examined either by transmitted or by reflected light. Each zone thus seen in horizontal section (Plate V. fig. 1, *c, c*), consists of a circular set of small ovate cells, excavated, as it were, in the shelly substance of the disk, and communicating with each other laterally by passages which unite them together into a continuous annulus. The zone which immediately surrounds the nucleus is connected with it by passages which extend from the outer margin of the large circumambient segment to the several cells of which it is itself composed; and each zone communicates with the one on its exterior by similar passages, which usually extend, however, not from the *cells* of the inner zone to those of the outer, but from the *connecting passages* of the inner zone to the cells of the outer (Plate IV. figs. 8, 9); and thus it comes to pass, that the cells of each zone usually *alternate* with those of the zones that are internal and external to it. A vertical section of the disk, such as is shown in Plate V. figs. 4, 5, exhibits the same arrangement under a different aspect. The cells of the concentric zones are seen to be much higher than they are broad, so that they present a somewhat columnar form; the proportion of their height to their breadth, however, may vary greatly in different parts of the same disk, the former often increasing from the centre towards the periphery (fig. 4), whilst the latter remains constant, or nearly so; and the columns, instead of being straight, are generally more or less curved, and are sometimes bent in the middle at an obtuse angle (Plate V. fig. 7, *a, b*). The gradation which presents itself from one of these forms to the other, and their coexistence even in the same specimens, clearly proves that no value can be attached to the form and proportions of the cells, thus seen in a vertical section, as furnishing specific characters. In every perfect specimen, the columnar cells are seen to be closed at their two extremities by a thin shelly wall; and this is sometimes flat, sometimes more or less convex *.—The meaning of these arrangements is clearly seen, when we turn our attention to the structure of the animal (Plate IV. fig. 1). For the outer margin of

* In a large proportion of the specimens obtained from sands or dredging, the cells have been laid open by attrition; either throughout the surface of the disk, if it should be flat, or at its margin only, if it should be at all saucer-shaped. The constancy of this last character in a certain set of forms, resembling that represented in Plate VII. figs. 8, 10, might at first sight lead to the idea that they constitute a distinct specific type; but, as will hereafter appear, these plate-shaped disks cannot be separated by any definite line of demarcation from such as are quite plane; and in specimens of them which have not suffered attrition, the marginal cells are closed, like all the rest.

the circumambient segment of the nucleus is seen to give off a number of slender prolongations or 'stolons' of sarcode, which radiate from it to a short distance and then enlarge into columnar segments having a circular or ovoidal base, which are united with each other laterally by an annular 'stolon'; and from the portions of this 'stolon' which intervene between the segments of each annulus of sarcode, are given off the radiating 'stolons' that go to originate the next zone, the arrangement of whose parts is precisely similar to that just described.

18. In this manner, any number of concentric zones may be formed, which are exact repetitions of each other, except that the number of segments in the outer zones is greater than that of which the inner zones are composed. It does not increase, however, in the regular ratio of the respective diameters of the zones; for the cells of the outer zones, being usually both larger and more widely separated from each other than are those of the inner, are less numerous in proportion; thus in a specimen before me, there are twenty-eight cells in the innermost row and only forty-nine in the outermost, though the latter is more than twice the diameter of the former. The increase in the number of the segments is accomplished by the occasional *interpolation* of an additional segment, communicating directly with the one immediately interior to it, between the two segments which spring from the annular stolon on either side of the latter, as is shown in Plate IV. figs. 8, 9, *a*. Hence it is obvious that prolongations of sarcode giving origin to new segments, although ordinarily put forth rather by the connecting 'stolons' than by the segments themselves, may originate from any part of the annulus. This is shown still more forcibly by the occasional occurrence of irregularities, such as that represented in Plate VI. fig. 6.

19. The cells of the last-formed zone communicate with the exterior by the very same kind of radial passages, as in other instances communicate with the next zone; and the external orifices of these form the pores which present themselves at the margin of the disk (Plate V. fig. 1). Thus it is seen, on the one hand, how it happens that these pores are intermediate between the cells, instead of opening directly into them; and on the other, how each pore leads, by the divarication of its passage, into two cells, one on either side of it. When a new zone is formed, each pore opens into one of its cells; and this zone in its turn communicates with the exterior, through a new set of pores at its own margin. Each pore is often surrounded by a rather prominent annulus of shell (Plate V. fig. 1, *d*); and it is obvious that when the section passes through this, it will be indicated by a little 'beak' on either side of the entrance to the passage; such 'beaks' (which are of course repeated through the entire disk) are shown in their ordinary aspect in Plate IV. fig. 9, but they are frequently more prominent, as is shown in Plate VI. fig. 1.

20. In all cases in which the growth of the disk takes place with normal regularity, a *complete circular zone* is added at once. Exceptions to this regularity are rare, and they can be generally traced with probability to some accidental interruption. It can scarcely be doubted, I think, that when a new ring of cells is about to

be formed, the prolongations of sarcode issuing from the several pores of the preceding zone coalesce, so as to form a complete zone of segments and connecting stolons around the margin of the previously-formed disk; and that the deposit of calcareous matter forming the shelly walls of the cells and passages, takes place *upon*, or rather *in*, the superficial portion of this zone of sarcode. But I cannot find any evidence in the ordinary growth of the disk, that the sarcode extends itself over the surface of the portion previously formed; although occasional appearances will be hereafter described (¶ 53), that seem to indicate that it *may* do so.

21. It is a fact of much importance in the due appreciation of the relations of *Orbitolite* and its allied forms to other tribes of Foraminifera, that the calcareous partition which separates each cell of any one zone from its neighbours on either side, is not double, but *single*. And this is in great part the case, even with regard to the partitions that separate the cells of successive zones; the inner or central boundary of one being chiefly formed by the peripheral wall of the other. It is not easy even in thin *sections* to distinguish the boundary between the walls of one zone and those of another, so absolutely continuous do they appear to be. But it not unfrequently happens, that in *fracturing* these disks, their component zones come apart from each other, along their natural lines of junction, so as to disclose the real inner (or central) margin of the outer segment, which then presents a set of wide apertures, through which we look at once into its cells; thus proving their incomplete enclosure by proper walls on that side (Plate V. fig. 1, *ff*). Thus in the formation of each new zone, the calcareous envelope seems to be only generated where the sarcode is not already in contact with a solid wall.

22. There cannot be any reasonable doubt, that the number of concentric zones which any disk may present, is entirely determined by its stage of growth, and that it affords no basis whatever for specific distinction. Just as in the case of the concentric layers of wood in the stem of a tree, a minute nucleus, surrounded by only a single annulus of cells, may come in time to be the centre of a large disk consisting of many scores of concentric zones. Although, as already stated (¶ 17), most of the Orbitolites formed upon this simple type are of comparatively small size, yet there does not seem to be any definite limit to the multiplication of zones; for I possess specimens attaining ·15 of an inch in diameter, and consisting of about forty zones (much larger, therefore, than the younger zones of the complex type), in which there is no appearance of any departure from the original mode of growth. That comparatively few specimens, however, attain so large a size upon this simple type of structure, is due, I believe, to the circumstance that they early tend to develope themselves upon the more complex plan which I shall presently describe.

23. Although I have spoken of these disks as typically plane or nearly so (there being usually no great difference between the thickness of their central and that of their peripheral parts), yet it not unfrequently happens that the successive zones gradually increase in thickness from within outwards (as is shown in Plate V. fig. 5),

so that the height of the columnar segments progressively increases, and the entire disk becomes somewhat biconcave. Sometimes, again, without any alteration in the thickness of the several parts, the disk comes to assume, by the depression of its central portion, the shape of a plate, or that of a watch-glass, or (by the more complete upturning of its edges) that of a saucer. In any case in which either surface of the marginal zone is more exposed by its projection than those of the zones which it encloses, there will be a special liability to a laying-open of its cells (as shown in Plate VII. figs. 8, 10) if the disk should be subjected to attrition; and I believe that not only the recent species *O. marginalis*, but the fossil *O. macropora*, are nothing else than examples of this type, the figure of the latter given by GOLDFUSS * corresponding exactly with a form of it which I have frequently encountered. I have not met with any examples in this simple type, of that marginal thinning away as age increases, which is observable in many other Foraminifera.

24. From the simplest, it will be convenient to pass at once to the *most complex* type of structure presented by the Orbitolite, the existence of which is marked (as already noticed, ¶ 13) by a multiplication of the horizontal ranges of marginal pores. I have met with this form in specimens obtained by dredging, from the coast of Australia, from various parts of the Polynesian Archipelago, from the neighbourhood of the Philippine Islands, from the Red Sea, and from the Ægean; and as the sands of all these localities present the simpler type in great abundance, I am disposed to believe that the former is really not the less widely diffused than the latter, and would be discovered wherever it abounds, if properly searched for. The largest specimen in my possession, measuring seven-tenths of an inch in diameter, is from the coast of Australia, where these Orbitolites are so abundant at certain spots (as I learn from Mr. JUKES), that their entire disks and fragments, with fragments of Corallines (chiefly, I believe, the *Corallina palmata* of ELLIS), constitute the great mass of the dredgings. Among the Australian specimens, several attain a diameter of ·45 inch, and a considerable proportion as much as ·30 of an inch. Hence the Orbitolites of this type are among the largest forms of existing Foraminifera, being only surpassed, as far as I am aware, by the Cycloclypeus hereafter to be described. Of two specimens in my possession from the Feejee Islands, one measures ·63 inch, and the other ·53 inch in diameter; but the average of the Polynesian specimens, like that of the Philippine, Red Sea, and Ægean, seems to be considerably lower than that of the Australian, as their diameter seldom exceeds ·25 of an inch, and is usually not more than ·10 or ·12.

25. The disks formed on this plan, like the preceding, may be considered as typically circular, although they are seldom or never exactly so in reality. They may be considered, too, as typically flat, with a slight concavity in the central part, from which, however, the nucleus often projects; but, as will hereafter appear, there is no constant relation either between the thickness and the diameter of different specimens,

* Petrefacta, pl. 12. fig. 8.

or between the thickness of different parts of the same specimen, and the distance of these parts from its centre. The only remarkable departure from the ordinary form which I have met with, presents itself in certain Orbitolites from the Feejee Islands, of which several specimens in the Museum of the Royal College of Surgeons, and two in my own possession, exhibit a curious plication towards their margins; the degree of this departure varies so much, however, in different individuals, the plication being almost obsolete in some (see Plate V. figs. 2, 3), that it cannot be admitted to mark a specific diversity; and considering that these disks always grow on the surfaces of other bodies, it can scarcely, I think, be considered improbable, that the plications originate in the inflections of those surfaces *. These same specimens, moreover, also exhibit another curious abnormality; namely, the projection of the upper and lower edges of the margin, so that a groove is left between them, the projecting laminæ being thin and foliaceous, and their cells very irregularly arranged. This peculiarity, again, being far from uniform in its degree, and being altogether wanting in specimens which in other respects precisely resemble those with plicated and foliated margins, must be considered merely in the light of an accidental variety; but I cannot suggest any explanation of its occurrence, or of its limitation (so far as I am aware) to this particular locality.

26. The surface of the disk (Plate V. fig. 6) is marked out, as in the simpler type, by concentric zones of cells, the number of which bears a general (though not a constant) ratio to its diameter; these cells are usually somewhat rectangular in shape and sometimes approach a square, but are more commonly nearly twice as long in the line of the radius of the disk as they are in the transverse direction, their long sides being parallel to each other. We shall hereafter see, however, that the form of the superficial cells is very subject to variation, and that it may be very dissimilar even in different zones of the same disk (¶¶ 48–52). The pores at the margin of the disk are disposed, as in the simpler type, between the projections formed by the convexities of the cells; and each is usually surrounded by the projecting annulus formerly noticed (¶ 19). The disposition of these pores, however, is far from regular, as they seldom form rows that seem exactly continuous with each other, either horizontally or vertically; and the number of pores in each vertical row is by no means constant, even in different parts of the margin of the same disk †.

* I have elsewhere noticed the fact, that various species of *Orbitoides* are disposed to exhibit a like contortion; and that the well-marked *ephippial* shape which some specimens present, is nothing else than an accidental variety (see Quart. Journ. of Geol. Soc., vol. vi. pp. 34, 35).

† In following the description of the internal structure of this type of Orbitolite, it will be convenient for the reader to make frequent reference to the ideal representation which has been built up in Plate V. fig. 6, by the combination of materials furnished by a great number of preparations which are represented in separate figures accompanying the original Memoir in the Archives of the Royal Society; these last of course furnish the real authority for every point in the description, the ideal figure, however, serving to display the relation of different parts to each other in a manner that no single preparation would possibly admit.

27. The disks of this complex type are not distinguished from those of the simple type already described, by any difference in the structure of the Nucleus; and there is frequently nothing specially characteristic in the structure of the zones that immediately surround it. Each of the peripheral zones, however, consists of two *superficial* layers, an upper and a lower, and of an *intermediate* stratum;—these will now be described *seriatim*.

28. The *superficial* layers are formed of the (usually) oblong cells, whose contour is indicated by the surface-markings; when they are laid open horizontally, by rubbing away the thin shell which covers them in (Plate VI. fig. 3), it is seen that the floor of each cell has an aperture at either end; but no communication can be traced, either through the side-walls between the contiguous cells of the same zone, or through the end-walls, between the cells of successive zones. Moreover, there is no such alternating arrangement of the cells of successive zones, as we have seen to prevail in the simpler type (¶ 17); and they altogether seem to be quite independent one of another. When this superficial layer is examined in a vertical section having a radial direction (Plate VI. fig. 7), it is seen that the floors of its cells (*a*, *a*) are formed by the expanded summits (*d*, *d'*, *d''*) of the irregular septa, which separate from each other the columnar cells of the intermediate stratum (*c c c c*); and that the apertures at the two ends of the floor are the entrances to passages (*e*, *e'*, *e''*), which lead obliquely downwards (the passages on either side of the partition between two successive cells of the *superficial* layer inclining towards each other) towards these cavities. It is observable, moreover, that just at the point at which the contiguous passages meet each other, there is always a round aperture (*f*, *f'*, *f''*) in the partition (*g*, *g*) which divides the contiguous cells of each zone; and when, in a horizontal section, the superficial cells have been entirely ground away, so as to lay open the most superficial part of the intermediate stratum, this part is found to be traversed in each zone by a continuous circular canal (Plate VIII. fig. 3), with large rounded openings that lead into the columnar cells beneath. The meaning of this arrangement becomes obvious, when we examine the disposition of the animal substance which occupies these cavities; for we find, as might have been anticipated, that the superficial cells are filled with segments of sarcode of corresponding shape (Plate IV. figs. 4, 7, *aa*); and that whilst these have no direct connexion with one another, each of them is connected by means of fleshy peduncles with the annular stolons *bb* that run along its extremities; whilst from the under side of these annular stolons (fig. 4) descend the thick columns of sarcode (*cc*, *c'c'*), which occupy the columnar cells of the intermediate stratum. The absence of any essential dependence of the segments of the *superficial*, and of those of the *intermediate* strata upon each other, seems indicated by the fact that there is no constant numerical relation between them,—a circumstance which extremely perplexed me, until I had ascertained, by examination of the animal, that the passages (Plate VI. fig. 7, *e*, *e'*, *e''*) debouch, not (as I had at first supposed) into the columnar cavities, but into the annular canal,

which serves to bring the superficial and columnar segments of each zone into mutual communication.

29. As the description now given of the superficial layer applies equally to both surfaces, we may now proceed to the *intermediate* layer. When this is laid open by a horizontal section (Plate V. fig. 6), it is seen to consist of a series of concentric zones, the cells of which *alternate* with each other, like those of the simpler type (¶ 17.). The cells are usually circular (or nearly so) in form; but seem to differ considerably in size, even in different parts of the same zone. Their borders, however, very commonly present a funnel-like aspect; and thus we perceive that the diameter of the cavity is liable to vary, according to the part of it which the section happens to traverse,—a fact which becomes more obvious when *vertical* sections are examined; for it is then seen (Plate VI. figs. 7, 8, 9) that each columnar cell is narrowed by constrictions at intervals, so as to divide it imperfectly into a series of segments vertically superposed one upon another. The number of these segments varies according to the thickness of the disk; so that it is anything but constant, either in different individuals, or in different parts of the same. Moreover, it may be often observed that the columns neither always pass from end to end in a straight line, nor maintain a complete isolation from each other (Plate VIII. figs. 1, 2); an inosculation of two columns not being unfrequent, and more rarely a fusion of two columns into one. All these features of structure presented by the shell, are beautifully displayed by the animal (Plate IV. fig. 4); the columns of sarcode (*cc*, *c'c'*) exhibiting the imperfect transverse segmental division, the not unfrequent inosculation, and the occasional fusion, which we have seen to exist in the cavities which they occupy. At their upper and lower extremities, they unite with the horizontal bands (*bb'*, *bb'*), which pass continuously round, in each zone, between the intermediate and the superficial layers.

30. Save in the case of such accidental inosculations as those just noticed (which are indicated in vertical sections like that represented in Plate VI. fig. 7, by the irregularly disposed apertures *h*, *h*), no other lateral communication seems to exist between the contiguous cells of the same zone, than that which is established by the annular stolons just mentioned. The cells of the successive zones communicate with each other, however, as in the simple type previously described (¶ 17.); but with a curious modification; for whereas a horizontal section of the latter shows that each cell communicates with the *two* cells alternating with it in the interior zone (Plate V. fig. 1), a like section of the Orbitolite of complex type seems to show that such a connexion exists with only *one* cell of the interior zone, by a passage running obliquely from one to the other, and extending continuously through several successive zones (Plate V. fig. 6, *i*, *k*). I was long perplexed by the want of constancy in the direction of these passages; the very same section exhibiting opposite obliquities in contiguous parts (Plate VI. fig. 2). By the study of vertical sections, however, made tangentially instead of radially, so as to *cross* these connecting passages, I arrived at the explanation of this

apparent anomaly, which is simply as follows. Each columnar cell really communicates with the *two* alternating columnar cells in the next interior zone; but by two distinct passages, instead of by the divarication of one; and these passages are not upon the same plane, but those of different planes turn alternately to one side and to the other. This is well seen in the two tangential sections represented in Plate VI. figs. 8 & 9; of which 8 shows the back or central side of four contiguous columnar cells *aa'*, *bb'*, *cc'*, *dd'*, of the same zone, each of them perforated by a series of apertures, in which some degree of alternation is perceptible; whilst 9 shows the front or peripheral side of four other columnar cells, in which it is seen that, by the sinuosity of the partition, the apertures of any vertical row, even when in a line with each other, open alternately into the cells on the *right* and on the *left* of the septum; so that (*e.g.*) the passages extending backwards from the row of apertures in the columnar cell *bb'*, fig. 8, will debouch alternately in cells *aa'* and *bb'*, fig. 9, of the zone within. The same will of course be true of the pores which open on the margin, these being nothing else than the orifices of the inter-zonular passages just described, which, when another annulus is added, lead into its cells. This idea of the alternating direction of the inter-zonular passages, seemed to furnish the solution of the appearances presented in Plate VI. fig. 2; for, as the disks are seldom perfectly flat, the section which traverses, at one part of the disk, the set of passages running in one direction, will traverse the other set of passages, where, by the flexure of the disk, the plane of section is slightly altered in regard to it. All doubt, however, as to the validity of this explanation, was removed by the examination of the animal substance filling the vertical columns; for, as is shown in Plate IV. fig. 4, each column of sarcode in one zone (*cc*) does communicate with the two columns alternating with it in the next zone (*c'c'*) by two rows of peduncular stolons; and the peduncles which pass from each pair of contiguous columns, to the single column of the next zone, incline towards one another, so as to enter it nearly in the same vertical line, though in different horizontal planes.

31. That which has been already stated in regard to the partial deficiency of the inner wall in each of the concentric zones of the simple type (¶ 21.), holds good also in regard to the septa which divide the successive zones of the intermediate stratum in this more complex type; for the walls of the columnar cells close-in around them very imperfectly on their inner or central side, leaving large irregular vertical fissures (Plate VIII. fig. 1) which are applied to the vertical rows of orifices (Plate VIII. fig. 2) on the outer margin of the included zone.

32. The thickness of this intermediate stratum, and the number of vertical segments of which it consists, are found to vary considerably in different parts of the same disk; being usually least near the nucleus, and gradually augmenting in successive zones as their distance from the centre increases (Plate VI. fig. 7); or ceasing to augment at a certain point, so that the outer part of the disk is flat; or even diminishing again, so that the disk thins away towards its margin. It is specially worthy

of note, that whatever differences of this kind may exist, they are entirely due to the variable length of the columns of the *intermediate* stratum; the depth of the cells of the superficial layers being nearly constant, and no vertical multiplication of these ever taking place. The intermediate stratum, where it abuts on the nucleus, is often very thin; the annular stolons that run beneath the superficial layers being in such near proximity to each other, that the intervening column of sarcode is very short, and consists of only a single vertical segment, Plate V. fig. 12.

33. The foregoing description applies in every particular to those specimens only, which present the structure of this type of Orbitolite in its most regular and characteristic development; and the differences between this more complex form, and the simple form previously described, are such as at first sight to preclude the idea of their specific identity. Hence I am not in the least degree surprised, that Professor Williamson, by whom their respective plans of organization were first compared (*loc. cit.*), should have unhesitatingly regarded these two forms as specifically distinct. But when a large number of specimens of the more complex type are carefully examined and compared with each other, it becomes obvious that a vast amount of diversity in the arrangement of the cells of the shell, and of the segments of the animal, may present itself; and that one after another of the characters which at first seem most clearly marked and therefore most distinctive, may be shaded off (so to speak) in such a manner that a *complete transition is established*, sometimes even in a single disk, between the simple and complex types. Such a transition is exhibited by the specimen of which a vertical section is figured in Plate V. fig. 7; for it is obvious that the central portion of this disk (*a*—*b*) is so exactly conformable to the simpler type, that if this growth had stopped at the twenty-third zone, it would have undoubtedly been regarded as an unusually large example of that form. So many variations present themselves in the development of the different parts of the more complex type, that it will be desirable to describe them under a distinct head (Sect. IV.); and when these shall have been duly considered, I think that all doubt as to the specific identity of the simpler and more complex forms will be done away.

III. *Physiology.*

34. *Growth.*—Of the mode in which the *Nutritive* process is carried on in Orbitolites, our imperfect acquaintance with their living habits leaves us much in the dark; nevertheless it is fair to reason by analogy from a comparison of their structure with that of other Foraminifera whose habits of life are known; especially as this analogy is sufficiently complete in the present instance, to justify a tolerably firm reliance upon it, and as the results to which it would lead are in harmony with the facts of observation. All the Orbitolites, whether of the simple or complex type, which have been collected in the living state, have been found growing on the surface of Seaweeds or other marine plants (as *Zostera*), or of Zoophytes; it may therefore be fairly presumed, that such is their ordinary habitat; and hence it is scarcely conceivable

that their *attached* surface should ever be invested by sarcode. Moreover, several of the spirit-specimens which I have submitted to decalcification, have proved to be so closely invested by a covering of *vegetation*, chiefly composed of *Diatomaceæ*, *Desmidieæ*, and other minute Algæ, that I cannot suppose even the free surface of their disk to be ordinarily covered by sarcode*. The analogy of other Rhizopods, however, would lead us to suppose that the sarcode projects from the marginal pores under the form of *pseudopodia*, and that it is by the introduction of alimentary particles (chiefly minute forms of vegetation) through their means into the mass of sarcode from which they are put forth, that the fleshy body pervading the entire disk is nourished. For although there is nothing like a digestive cavity in any part of it, or an alimentary tube passing from one portion to another, still less any vascular communication between the segments, yet as the sarcode forms one soft homogeneous mass continuous throughout, the body as a whole will receive the benefit of any incorporation of new matter with its substance, in whatever situation this may be made. That organic particles small enough to pass through the marginal pores, are thus introduced into the chambers of the disk, is proved by the curious fact, that the residuum left after the decalcification of large and therefore aged disks, whose animal contents have not been preserved, consists almost entirely of an assemblage of remains of minute *Diatomaceæ*, *Desmidieæ*, &c., which have obviously been retained in the interior of their cavities, after the assimilation of the nutriment they were competent to afford.

35. The sarcode-body of the animal, growing at the expense of the nutriment thus appropriated, will gradually, it is probable, project itself through the marginal orifices, not merely in filamentous pseudopodia, but in quantity sufficient to form new segments on the outside of each pore; and these segments, extending themselves laterally, will come into mutual connexion, and will thus form a complete annulus. It may be presumed to be by the calcification of the surface of this beaded ring of sarcode, that the formation of the shelly zone is accomplished; and if the calcifying process commence on the segments, and extend from these along the surface of their connecting stolons, we can understand why the passages that are left for communication with the exterior, should arise from the intermediate divisions of the annular canal, instead of from the segments themselves.

36. The addition of new zones usually takes place with the same regularity in the complex as in the simple type of structure; but departures from this regularity, occasioned by a want of completeness of particular zones, are more frequent; and this is perhaps to be accounted for by the larger size of the disk, which will tend to produce a less intimate dependence of each part of the animal body upon every

* I have found such an investment also on several dried specimens; and until I had detached and examined this, I should have supposed from its aspect that it was the desiccated flesh of the animal. I have little doubt that the "greenish cuticle" described by Mr. CARTER as covering his *Operculina arabica* (Ann. of Nat. Hist. ser. 2. vol. x. pp. 168, 172) and supposed by MM. D'ARCHIAC and HAIME (*op. cit.* p. 52) to be specially concerned in the formation of the shell, is of the same nature.

other, and will thus favour the partial action of any cause (*e. g.* an excess of nutrient materials) which promotes a more rapid growth on one side than on the other. And this view is most remarkably borne out by the fact, which I shall more fully illustrate in a subsequent memoir, that in another example of this group*, which, though normally growing upon the cyclical type, possesses a greater degree of segmental independence, such irregularities occur far more frequently; so that, in fact, it is rare to meet with a disk whose increase has taken place with uniformity throughout.

37. *Reparation of Injuries.*—Looking at that vegetative repetition of parts which pre-eminently characterizes the body of the Orbitolite,—every one of the segments first budded-off from the nucleus, and subsequently from the margin of the pre-formed zones, being the precise repetition of every other,—it may be expected from the analogy of similar organisms, that every one of these parts should be equally capable, both of repairing injuries done to itself, and of maintaining an independent existence when detached from the mass to which it originally belonged. And although no opportunity has yet presented itself, of subjecting such a conclusion to the test of experiments devised for the purpose, yet *accident* has furnished the means of verifying it, to a degree that could scarcely have been anticipated. For in the course of my examination of the large collections which have been placed at my disposal, I have met with several specimens, in which it is evident that, after larger or smaller portions of the disk had been broken away, a new growth has taken place along the fractured edge. Various examples of this are shown in Plate VIII. In the first that I happened to meet with, which is represented in fig. 6, the injury is evidently very slight, being confined to the loss of a few rows near the edge of the disk, for something less than half its circumference (*a—b*). This injury had obviously been sustained previously to the formation of the last two zones; for these, whilst added to the uninjured part of the margin in the usual way, have followed the irregular contour of the broken edge; and whilst in the former case the cells present their normal conformity to those of the margin they invest, in the latter, the cells, while obviously continuous with the preceding, are quite unconformable to those of the fractured margin, as is shown on a larger scale in fig. 7. Hence it seems to me probable, that the growth of these two rows along the *fractured* edge, has taken place, not from that edge itself, but by an extension of the sarcode about to form the new circle of the *entire* edge, from the points *a* and *b*. In fig. 9 is seen an example of a similar kind, in which a much larger portion of the disk has been broken away, so as to leave only an irregular fragment, including its centre and about an eighth of its margin. Here seven rows of cells have been formed since the injury; and these, whilst produced conformably to those of the uninjured margin, present the most marked want of conformity to those of the fractured margin, which, nevertheless, they completely surround. A careful examination of this specimen, indeed, seems to me to leave little room for doubt, that the growth of the innermost, or what I may call the *reparative* zone of

* I refer to a genus hitherto undescribed, which I shall designate *Cycloclypeus*.

cells, took place, not from the broken edge, but from the margin of the unbroken; just as, to use a professional simile, an ulcerated surface 'skins-over' by an extension of the integument from its edges, not by the direct formation of skin upon the granulation-surface itself. All the six rows subsequently produced, are conformable to each other, and to the first or reparative row, from which they have obviously extended themselves after the normal manner. It is observable, however, that the breadth of these rows varies in different parts, being least where they invest the projecting portions of the fractured edge, and greatest where they sink into its hollows. And thus it comes to pass, that the irregularities left in the shape of the disk, by the loss of a large part of its substance, are gradually compensated, so as to restore it to a form much more nearly corresponding to its typical symmetry.

38. Even a very small fragment appears thus to serve as the nucleus for a new disk. In fig. 8 is shown an example of this kind, in which the tendency to the reproduction of the typical form, by the compensative reparation just described, is very curiously marked. This specimen also presents the following very curious feature, —that the new growth has taken place from the *inner* margin of the original fragment (*aa*), and not from its *outer* or growing margin, as in the cases previously noticed. Having carefully examined it in various modes, I cannot entertain the slightest doubt that such has been the case; for the cells of the first new zone, as well as those of all the zones subsequently produced, are so manifestly conformable to those of the thinner and older portion of the fragment, and are so unconformable to those of the thicker and newer margin, that it seems obvious that the sarcode must have extended itself from the former part, along the fractured edge on each side, and have then enveloped the margin which had been left entire. This may have more readily taken place in the present instance, because at the part (*aa*) the fracture seems to have followed the course of one of the zones, instead of passing, as at the sides of this fragment, and in the instances previously cited, in such a direction as to cut the zones transversely.

39. The preceding instance clearly proves, that connexion with the central nucleus is not in the least degree requisite for the continued growth of the peripheral parts; since these may be entirely detached from it, without any loss of vital activity. The same inference may be deduced from the examination of specimens, in which, the central portion of the disk having been broken-out, a growth of new zones seems to have taken place from without inwards, so as to fill up the void space thus left. In no other way can I account for the appearances presented by a specimen in my possession, in which the included portion is as evidently unconformable to that which surrounds it, as it is in the preceding case, but in which there is also an unfilled void, the shape of one part of which clearly indicates that it occupies the site of the original centre. The *included* portion, and not the peripheral, must therefore be the after-growth in this instance; and if a little more time had elapsed, the whole of the central vacuity would probably have been filled up by it.

40. In the specimen represented in fig. 5, the central portion appears to have been lost, with about a third of the peripheral; and the new growth seems to have taken place at the same time, from the inner margin *a a a* of the fragment, and from its *outer* margin *b b b*, the two growths becoming continuous with each other along the broken edges *ab*, *ab*. For although the zones that lie internally to *a a a* are conformable to those which surround them, yet there is a peculiar character about them (more apparent in the specimen than in the drawing) which indicates them to have been formed at a later period, and to have been contemporaneous with those which surround the zone *b b b*. Their actual continuity at the angles *a a* is unfortunately interrupted by an injury which the specimen seems subsequently to have received; yet its traces are sufficiently perceptible on one side, to justify the belief in its former existence.—The specimen of which fig. 4 is a delineation, seems to have been the subject of several minor fractures and reparations; but the course of its zones marks out an obvious separation between an earlier- and a later-formed portion, one having sprung from the other along the line *a b*. The incompleteness of the specimen, however, prevents me from coming to any certain conclusion, whether the small inner portion is here the older, the large outer portion having grown in the first instance from its margin *a b*, and having gradually extended itself around it; or whether the outer portion is the residue of an unusually excentric disk, which, having lost its nucleus and the zones immediately surrounding it, has filled up the central space with an extension from its innermost zone, which is consequently the newest portion of the whole.—It is interesting to find evidence in *fossil* specimens, that the same kind of reparation has taken place. Among the Orbitolites which I have examined from the Calcaire grossier of Paris, is a disk of which a large part had obviously been lost by fracture, but of which the original symmetry had been in great degree restored by a similar outgrowth from the zones formed from the uninjured margin, along the fractured edge.

41. This series of abnormal phenomena, then, not only confirms the conclusion that seemed fairly deducible from our previous examination of the normal mode of growth, with regard to the independent endowments of the component segments of the Orbitolite body, but also affords some additional information of much interest. For we see, in the first place, that the growth of the sarcode, and the addition of new parts, may take place in the direction of the centre, where a free edge is exposed at the *inner* margin of any zone, as well as in the peripheral direction from the normal *outer* margin. Secondly, the reparative *nisus* seems always to tend towards the production of a disk, whose shape shall approach the circular, whatever may be the form of the fragment which serves as its foundation; thus showing that, notwithstanding the repetition and independence of the separate parts of these organisms, each cluster, whether large or small, is an integer, having an archetypal symmetry to which it tends to conform,—thus strongly reminding us of the laws of crystallization. And thirdly, the plan by which this recurrence to the discoidal form is provided for,

seems partly to consist in the limitation of the new growth to the natural margins of the zones; no such growth taking place from the edge of a fracture which has crossed the zones transversely, although it may proceed from the remains of a zone which has been broken off by a fracture that partly follows its course.

42. *Question of Individuality.*—It has been frequently discussed, whether each of the composite forms of Foraminifera, such as Orbitolite or Nummulite, is to be regarded as a *single individual*, or as a *colony* or *cluster of individuals*. All occasion for this discussion would, I think, be removed by the adoption of philosophical views as to what really constitutes an individual, and as to the relationship between the parts which, having a common origin in one generative act, are multiplied by a process of gemmation. As I have elsewhere endeavoured to show*, the entire product of every generative act, whether developing itself into a body of high organization, distinguished by the structural differentiation of its parts, or evolving itself as an almost homogeneous aggregate of equal and similar segments, must be regarded as homologically the same; and the essential difference between the two, as living beings, lies in the *functional* relations of their respective parts. For whilst in the former there is so close an *interdependence* amongst them all, that no one can exist without the rest, and the life of the whole is (as it were) the *product* of the lives of the component parts, there may be in the latter such a mutual *independence*, that each part can continue to live, grow, and reproduce itself when separated from the rest, so that the life of the whole is (so to speak) but the *sum* of that of its components. Now the term 'individual,' being commonly applied to the entire organism in the first case, and to only a small segment of it, perhaps, in the second, is obviously inappropriate either to one or to the other, except in so far as it expresses the fact of independent existence. But the limits of such individuality as this cannot be strictly defined, and they even differ widely in animals whose general plan of structure is the same†. Hence in regard to the Foraminifera, as in regard to Zoophytes, Composite Acalephæ, &c., we are to regard the entire mass originating in a generative act, as a *single organism*; and the question in regard to the *functional independence* of its multiple segments, is one of degree in each particular type. Thus, as we have seen, this independence exists in the case of the *Orbitolite* to such a degree as to make each part entirely self-sustaining, and to prevent the existence of any definite limit to the growth of the whole; yet it is quite possible that in a form so much more elevated as *Nummulite*, there may be, as maintained by MM. D'ARCHIAC and HAIME (*op. cit.* p. 69), such a degree of mutual dependence among the segments, and of unity in their aggregate life, that the latter predominates sufficiently to limit the growth of the organism to a tolerably determinate size‡.

* Principles of Comparative Physiology, chap. xi. sect. 1.

† See also Mr. HUXLEY's observations on this subject, in Philosophical Transactions, 1851, pp. 578, 580.

‡ Whilst admitting the possibility of this view, I shall hereafter have occasion to question its correctness, since the evidence on which it is based appears to me by no means satisfactory. In fact, when I come to

43. *Reproduction.*—The mode of *Reproduction* of the Foraminifera generally, is at present involved in the deepest obscurity; and there is little probability that it will be fully elucidated by any other means, than continued observation of the animals in their living state, such as may probably be best carried out with regard to the species of our own seas by keeping them in Vivaria. In default of such observations, and as a guide to further inquiry, I think it as well to state what has fallen under my own notice. In many parts of the body of spirit-specimens of *Orbitolite*,—especially, but not solely, in the superficial cells,—I have found the sarcode broken-up as it were into little spherules, as represented in Plate IV. fig. 3; these spherules, however, do not seem to possess any peculiar investment, nor does their sarcode appear to have undergone any special change. Similar spherules are figured by Professor EHRENBERG (*op. cit.*) in several of the cells of his *Sorites orbiculus*; and Professor SCHULTZE has recently (*op. cit.* pp. 26, 27) described bodies which seem to be of the same kind, though more opake (probably through having a denser envelope), as frequently presenting themselves in certain chambers of *Rotaliæ*, or even throughout the entire series. I feel much inclined to believe that these bodies are *gemmules*, which, like the zoospores of the Algæ, are produced by a resolution of certain portions of the substance of the organism into independent particles, which, spontaneously detaching themselves, and escaping through the marginal pores of the disk, will go forth to lay the foundation for new disks elsewhere.

Besides these, however, I have more rarely met with certain other bodies, apparently imbedded in the sarcode, which may be either gemmules in a later stage, or may possibly be true *ova*; these, represented in Plate IV. fig. 11, seem to exhibit various stages of binary subdivision; and they present a deep-red colour, even in spirit-specimens. I can scarcely imagine that these can be vegetable organisms that have been introduced through the marginal pores; since they are much too large to pass through these, without a great alteration in form; and this would seem to be incompatible with the firmness of their envelope. At *g*, fig. 11, is represented, under the same magnifying power with the foregoing, an object which I have detected in one of my vertical sections of the shell, where it occupies one of the superficial cells, the cover of which is deficient. Now it is quite possible that this cell may have been accidentally abraded, and that the object in question may have found its way into it *ab externo*; its position and aspect, however, seem to me much more conformable to the idea, that it has been developed in the disk itself, and that it has burst through the lid of the cell by its own enlargement, in preparation for its final escape. And this view seems borne out by the fact, that I have frequently found a few cells open on different parts of the surface of disks which did not appear to have suffered any abrasion; as if the rupture of their lids had taken place as an ordinary

describe (in a future memoir) the structure and varieties of *Nonionina*, the nearest existing type to Nummulite, and in my belief generically identical with it, I shall have occasion to show, that there is not only no proof of the existence of such a limitation to its growth, but that there is strong evidence to the contrary.

phenomenon of their development, instead of being the result of accident.—I do not wish to attach any weight to the interpretations I have here offered; but I simply state the facts, and the explanations of them which have suggested themselves to my own mind; merely adding, what I hope to present in more detail at a future opportunity, that bodies resembling the first or primordial cell, in which Foraminifera of all forms originate, are not unfrequently met-with in the chambers of many other species.

IV. *Variations.*

44. *Variations in Size.*—We have already seen that diversities both in the *diameter* and in the *thickness* of the disk, arise directly from the degree in which the animal substance (whereon the skeleton is modelled) has extended itself either *horizontally* or *vertically*, so as to multiply either the number of concentric rings, or the number of the superposed segments of which each ring consists. This, however, is not the only source of variation in size; for a most extraordinary diversity presents itself in the dimensions of the individual components, by whose repetition the entire disk is made up. It is in the *nucleus* that I find this diversity most strongly marked, as will appear from a comparison of Plate VII. figs. 1—4, which exhibit parts of a gradational series of twelve, from the smallest to the largest forms I have met with, all of them accurately drawn, under the same magnifying power, from specimens in my possession*. The length of the entire nucleus of fig. 4 is about *seven* times that of the nucleus of fig. 1, and its breadth about *four* times as great; the area of the former is therefore about twenty-eight times that of the latter; and as it is also several times as thick, the whole of the cavity, which was occupied in the living state by animal substance, could scarcely have been less than a *hundred times as large* in the one as in the other. (Compare also figs. 5, 6, 10, 12, 13 of Plate IV.) There is not by any means the same amount of difference between the dimensions of the ordinary cells which are formed by concentric extensions of the nucleus; nevertheless, it will be seen by a glance at the figures just referred-to, that these also exhibit marked diversities in size, the largest cells being usually found to spring from the largest nuclei, and *vice versâ*. Moreover, the individual cells of the very same disk are occasionally found to differ amongst each other, as widely as do the cells of fig. 1 from those of fig. 4.

45. Similar differences present themselves in the *thickness* of individual cells; as is of course best seen in the simple type of Orbitolite, in which the augmentation of thickness is not produced by the vertical superposition of multiple segments. A remarkable example of this kind is presented in the comparison of figs. 4 and 5 of Plate V.; these being, like the figures in Plate VII., drawn under the same magnifying power. I possess a series of vertical sections of different individuals, in which the same gradual transition is seen from the thin to the thick, as I have just stated

* The entire series of figures is in the possession of the Royal Society.

to exist in regard to superficial area; and which also proves that the relative thickness of the central and of the peripheral portions is equally liable to variation.

46. It seems obvious, from the foregoing considerations, that neither the absolute nor the relative dimensions of the individual parts of these composite fabrics, can, any more than the dimensions of the entire disks, be taken as affording valid characters for the discrimination of species; and that such a wide range of variation exists among individuals, as would, if the extreme cases alone were known, seem fully to justify their separation under distinct specific designations. Thus, if the two extreme forms, figs. 1 and 4 (Plate VII.), had been the sole objects of comparison, most naturalists would undoubtedly have considered the strongly-marked difference in the size of their respective nuclei to entitle them to rank as separate species; and even if fig. 2 or fig. 3 had been brought into comparison with them, it might have been a question whether it should be associated with fig. 1 or with fig. 4, or should rank as a third species intermediate between them, or should be considered as a connecting link specifically identical with both, and therefore establishing their specific identity with each other. The comparison of the entire series must be felt to remove all ground for hesitation on this point, since it is manifestly impossible to draw a line across any part of it, which should divide it into two or any larger number of groups, respectively characterized by constant and well-marked differences in size. And here again, therefore, we have evidence of the great importance of bringing into comparison a sufficiently large number of forms, to enable us to determine in some degree the measure of individual variation.

47. *Variations in Shape.*—The very strong tendency which we have seen to prevail in the Orbitolite, not only to the maintenance of the circular type in the regular growth of the disk, but to its reproduction after accidental injuries, seems to prevent the occurrence of any considerable variation in its general form, except such as may be directly produced by external agencies. The circular sometimes gives place to an elliptical shape (Plate VII. fig. 4), especially in young specimens, whose form is more determined by that of the nucleus than is that of older individuals. And the occurrence of such a variety in the recent type, makes me indisposed to admit that ellipticity of shape can be in itself a sufficient basis for the specific differentiation of any fossil form of this genus. Of the marked differences in the general aspect of the disks, which may arise from differences in the relative proportions between their thickness and their diameter, and in the relative thickness of their central and peripheral portions, mention has been already made (¶¶ 17 and 32.). And the only other important departure from the typical shape which I have met with, either in the simple or in the more complex form, has been described under a former head (¶ 25.).

48. *Variations in the Form of the Superficial Cells, and in the Markings of the Surface.*—The appearances presented on minute observation by the surface of the Orbitolite, are so far from being uniform, that to any one whose eye had not become

familiarised with their variety by the examination of a considerable number of specimens, they would become sources of great perplexity. We have already seen that the cellular markings present two very distinct forms, the *rounded* (Plate V. fig. 1) and the *oblong* (Plate V. fig. 6); the first of these being specially characteristic of that simpler type of structure in which there is only a single layer of cells, but not being confined to it; whilst the second is peculiar to the complex type, in which there are two superficial layers, distinct from the intermediate stratum. Now the occasional coexistence of both these plans of structure in a single individual (¶ 33.), sufficiently proves that the diversity of the surface-markings to which they respectively give rise, cannot be regarded as a basis for specific distinction; and when these extremes of diversity are kept in view, it must be felt to be highly improbable that any modifications of either form should possess greater importance. That such modifications are mere individual varieties, is further evidenced by their *gradational* character, and by the fact that two or more of them may present themselves in the same disk. In my description of them, I shall limit myself to an account of those more remarkable and frequently-recurring varieties, which will serve, I think, as a key to any others that are likely to be met with.

49. Although each surface, in either of the two principal types, ordinarily shows a division into concentric zones, which are again transversely subdivided so as to mark the separation of the cells, yet sometimes the concentric zones are alone visible, and no transverse subdivision is indicated, save by the alternation of lights and shadows proceeding from a like alternation of solid substance and of hollow spaces beneath (Plate VII. fig. 7). This predominance of the concentric divisions, which gives a very distinctive aspect to the disks which exhibit it, is usually most apparent in individuals whose vertical section exhibits two planes of cells; and it has seemed to me to depend on the unusual freedom between the *lateral* communications, which I have noticed in certain individuals thus formed, so that the animal portion of each zone might be described as an annulus of sarcode, merely constricted at intervals. This peculiarly cyclical aspect of the surface (on whose occurrence in fossil specimens I believe the genus *Cyclolina* to have been founded, ¶ 5.) may pass into either of the principal types previously noticed; thus in fig. 14 we observe the concentric zones, though still very strongly marked, breaking up (so to speak) into bands of rounded cells with slightly convex covers; whilst in figs. 5, 6 they are subdivided by very definite transverse lines into cells of remarkable squareness, which still retain the original flatness of their surfaces.

50. On the other hand, the appearance of concentric division is sometimes almost entirely wanting; the surface of the disk exhibiting excentric circular markings, which resemble those of an engine-turned watch-case (Plate VII. fig. 8), and the boundaries of the cells being formed by the intersection of these with each other. This aspect, however, which seems due to an unusual freedom in the *oblique* communications between the cells in each zone and those alternating with them in the

contiguous zones on either side, insensibly passes into the ordinary type; and it is not uncommon to meet with disks, especially fossil, which exhibit in one part the engine-turned aspect (Plate VII. fig. 8), and in another (fig. 14) that of concentric zones transversely subdivided. Indeed I have sometimes found that the very same disk might be made to present either of these aspects, according to the manner in which the light is made to impinge upon it and is reflected from it.

51. Although the rounded or ovoidal form of the superficial divisions is specially characteristic of the simple type of Orbitolites, yet it is by no means restricted to this; being frequently met-with in the thicker disks of the more complex type, and being almost constant in the fossil forms that abound in the early Tertiaries. Its occurrence, however, may always (I believe) be considered as indicating an incomplete separation between the superficial cells and the columnar cells of the intermediate stratum (¶ 58.); so that the former present the shape of the latter, in place of the form which properly characterises them. The shape of the cell is sometimes marked out in unusual strength by the convexity of its lid or cover, as shown in Plate VII. fig. 15; and this feature is often so pronounced in the large fossil Orbitolites of the Paris basin, as to become visible to the naked eye. A very marked diversity in its degree, however, as well as in the size of the cells, is to be noticed in the contiguous zones of another specimen (Plate VII. fig. 16); whence it is obvious that the convexity is a mere accidental variation, and is a character of no value whatever as regards the differentiation of species. The relation of the rounded to the square or oblong cells is made evident by the occurrence of intermediate links of transition. Thus, from such *circular* cells as are delineated in Plate VII. fig. 9, the passage is easy, through those shown in figs. 8 and 14, to those of fig. 6 (which are drawn under about twice the magnifying power), and thence to the *square* cells of the inner part of the portion of the disk figured in Plate VII. fig. 13. This last figure illustrates the important fact, that while the cells in one part of the surface of the disk are *square*, others in close proximity with them may be *oblong*; thus conducting us to the extreme form of this type, represented in Plate VII. fig. 12.

52. The foregoing considerations seem to render it obvious, that the diversities in the *form* of the superficial cells do not afford any ground whatever for the establishment of a corresponding multiplicity of specific types, but that they must rank as individual variations to which there is scarcely any definite limit. If the originals of Plate VII. figs. 5, 7, 9, 12 and 15, had happened to have presented themselves to the Systematist without any of the connecting forms, he might have been pardoned for describing them as distinct species characterised by well-marked differences in the form and arrangement of their cells; but no such differentiation can be admitted in the face of the fact, that these are only extreme examples of variations, which show themselves in a minor degree between almost every two specimens brought into comparison, and even between the different parts of the same disk. Moreover, when it is borne in mind, that the animal basis on which the calcareous skeleton is moulded

is not a body of constant shape, provided with organs having a fixed relation one to another, but is a mass of almost homogeneous sarcode, which in the living state is continually undergoing changes of form, one part extending itself into pseudopodia, whilst another undergoes a corresponding contraction, a strong *à priori* improbability is seen to exist, that, in animals of such organization, the form of the component segments should possess that value as a specific character, which it can only derive from *constancy*.

53. Besides those regular markings of the surface, which correspond to the division of the interior into cells, a peculiar aspect is frequently given to it by the deposit of calcareous thickenings, which are sometimes irregular, but which occasionally present an approach to symmetry. The most remarkable example I have met with, of this kind of addition, is delineated in Plate VII. fig. 11, in which it will be seen that the deposit has taken place in radial lines disposed with a certain degree of regularity. But in fig. 10, which represents a specimen whose surface is far less altered by these deposits, no such symmetry presents itself; and other specimens in my possession exhibit the means between these extremes. Hence we are justified in pronouncing this peculiarity to result from an accidental outgrowth, which is so variable in its degree as not to afford the least basis for specific differentiation. It is worthy of note, however, that it presents itself far more frequently, and also in a far more characteristic manner, in the Orbitolites of the Philippine Seas, than in those of the Australian or of any other provinces; and this circumstance seems to render it probable, that the outgrowth is directly due to the influence of some external conditions, probably to an excess in the proportion of carbonate of lime in the waters inhabited by these particular specimens.

54. *Variations in Mode of Growth.*—Although the cyclical mode of growth, when once established, is subsequently maintained with great regularity, and although in what may be considered the typical form, it commences from the 'nucleus' itself, yet there are numerous instances in which the typical regularity is more or less widely departed-from, so that the early increase seems to take place after an altogether different plan. The most marked antithesis to that regularly concentric type of growth, in which a complete annulus of cells is formed around the large circumambient segment of the 'nucleus' (see ¶ 17, also Plate IV. fig. 5, and Plate VII. fig. 2), is presented by those forms in which this circumambient mass only gives origin to new cells at its extremity; these in their turn bud-forth others, which extend and multiply themselves laterally as well as in advance; and thus a kind of spiral is produced, which opens-out very rapidly, the lateral portions of its mouth tending to grow-round and embrace the nucleus. Thus, starting from the central globular mass, 1, of Plate IX. fig. 4, we see that the circumambient mass 2 2, which nearly surrounds it, gives origin at one of its extremities to a smaller mass, 3, from which bud-off two cells, 4 4, which again give origin to four cells, constituting the row 5 5. The cells of the next row, 6 6, are more numerous, but are themselves exceeded by

those of the zone 7 7, which not only surrounds it, but extends further backwards upon the nucleus. So the cells of the zones 8 8, 9 9, 10 10, 11 11, and 12 12, progressively increase in number, and each zone extends itself further back upon the nucleus, until those of the last of these zones nearly meet upon its yet unenclosed margin. The cells of zone 13 13 do actually meet there, so as to form a complete circle; and zones 14 14 and 15 15 are formed with cyclical regularity, as would be any other zones subsequently produced. A still more complete spiral, in which twenty-two zones (commencing with the central segment) succeed one another before the first complete annulus is formed, is shown in Plate IX. fig. 2. Both figs. 2 and 4 (the latter of which is diagrammatized) represent the central portions of large disks, whose peripheral portions grow on the regular cyclical plan.

55. Now if these two plans of growth—the one cyclical from the beginning, the other cyclical only after having been at first spiral—were constantly presented in well-marked contrast with each other, there would be good ground for considering them (as Professor Williamson has done*) to be characteristic of distinct specific types. But this idea cannot be entertained, when a large number of individuals are examined. For it then becomes apparent, that the number of cases in which the nucleus is surrounded on all sides by the same number of zones, indicating that the concentric mode of growth has prevailed from the very first, are very few; but that in by far the larger proportion of specimens, there is a slight excentricity of the nucleus, with a larger number of zones on one side than on the other, as in Plate IX. figs. 1, 3; indicating that the first-formed zones have been incomplete circles, owing to a restriction of the gemmation of the nucleus to one part of its periphery. This is shown extremely well by decalcified specimens of the animal, no two of which, in fact, precisely resemble one another as to the mode in which the first zone of segments originates in the nucleus. Thus in the specimen represented in Plate IV. fig. 1, of which the nucleus is represented on a larger scale in fig. 12, the circumambient segment of the nucleus gives off only *three* stolons, at the end most remote from its connexion with the central mass; and the first zone of segments is far from being entire, the cyclical type not being completely attained until two or three successive additions have been made. In fig. 13, *eight* stolons are given off from the nucleus; and from the half-zone which they form, an entire circle is next produced; thus affording a remarkable confirmation to the idea I have already suggested (¶ 37.), as to the capacity of a portion of a zone to give origin to a complete annulus, by the lateral extension of its bands of sarcode. In fig. 10, the nucleus gives off *eleven* stolons on one side, and there are indications of *three* or *four* on the other. In fig. 6, the stolons come forth from a still larger proportion of the periphery of the nucleus; the zone which first surrounds it, however, is still incomplete in some parts, though the succeeding zone forms an entire circle. Finally, in the specimen represented in fig. 5, which is almost the exact counterpart of the disk represented in Plate VII. fig. 2,

* Transactions of the Microscopical Society, 1st series, vol. iii. pp. 116, 119.

the stolons pullulate from the entire circumference of the nucleus, and the annular zones of segments are complete from the first. The greater the limitation of the power of gemmation to one side of the nucleus, and the larger the number of incomplete zones, the more will the early plan of growth approximate to the spiral type, such as is represented in Plate IX. figs. 2, 4. It is obvious that the existence of these intermediate gradations breaks-down that barrier between the extreme forms, which Professor WILLIAMSON had proposed to erect; and shows that in this, as in many other particulars, differential characters, which at first sight appeared to be perfectly satisfactory, lose all their force when carefully traced through a sufficiently extended series of specimens.

56. It is desirable to note, as bearing on the relations between *Orbitolites* and *Orbiculina*, that even in those forms of the first-named type, in which the *spiral* mode of early growth is most characteristically displayed, it never seems to proceed far beyond a single turn; and further, that the later portion of this whorl merely surrounds the earlier, and does not cover it; so that unless (as sometimes happens, Plate V. fig. 5) the nucleus should itself be thicker than the zones of cells which immediately surround it, there is no central protuberance. In *Orbiculina*, on the other hand, the early growth invariably takes place according to the spiral type; this type is always maintained, until several turns have been made; and the later whorls not only surround but cover-in the earlier, so as to give rise to the central knob or protuberance. Some general remarks, which I have to make on the combination of the helical and cyclical types of growth, bearing upon certain fundamental questions of classification, will be more appropriately introduced in a subsequent Memoir, after the structure of *Orbiculina* shall have been compared with that of *Orbitolites*.

57. It is not, however, in the early mode of development alone, that striking diversities present themselves; for numerous variations, some of them quite as remarkable, are seen in the course of the evolution of the several parts which are characteristic of the 'complex' type. Thus, in the first place, the intermediate stratum is sometimes entirely deficient in the zones immediately surrounding the nucleus; so that the upper and lower annuli of sarcode are represented by only a single band, as is indicated by the singleness of the aperture through which it passes. In the specimen figured in Plate V. fig. 9, we see this to be the case only with the *three* zones nearest the centre; in that represented in Plate V. fig. 10, the canal is single in the *five* inner zones; whilst in that represented in Plate V. fig. 7, the canal is single for the first *twenty-three* zones. Whenever the annular canal is single, the upper and lower superficial cells also become continuous, and form a series of columnar cells in every respect similar to those of the simpler type (compare Plate V. fig. 5 with the portion *a—b* of Plate V. fig. 7). If, then, the growth of either of these disks had been checked within the first zone in which its annular canal becomes double, it would have been accounted as belonging to the simpler type; and the wide variation which here shows itself, in regard to the distance from the nucleus at which the more com-

plex type begins to be assumed, sufficiently explains the fact already noticed (¶ 22.), that although the disks of the 'simple' type are for the most part of minute size, yet that the diameter of many of them exceeds that of the smaller disks of the complex type.

58. Even when an intermediate stratum is formed by the separation of the annular canals, the superficial cells are not always clearly marked-off from its columnar cavities; for instead of being separated by floors formed by the expanded summits of the zonal septa (¶ 28.), they sometimes open at once into the columnar cells of the intermediate substance, so as to be quite continuous with them. Of this we have an example in the three zones 6—8 of the specimen represented in Plate V. fig. 10, *b*—*c*. This continuity of the superficial cells with the intermediate columns, is sometimes maintained throughout the disk, so that in no part of it are the former clearly marked off from the latter; as is seen in the portion *b*—*d* of fig. 7, Plate V.; in which, however, the intermediate layer is much less regular than usual. This method of growth is so remarkably constant in the *Fossil* Orbitolites of the Eocene strata, whose intermediate layer is fully and very regularly developed (see Plate VI. figs. 10, 11, and Plate VIII. fig. 2), that it might be considered to be specifically characteristic of them, did we not occasionally find it to occur in certain zones of recent disks, which are elsewhere exactly conformable to what I have described as the regular type. Thus in the vertical section represented in Plate V. fig. 10, we see that whilst the superficial cells of the three zones *b*—*c* are continuous with the columns of the intermediate stratum, a change then occurs in the relative places of their zonal septa, so that the cells of the former come to be, as it were, detached from the columns of the latter, and to have floors formed by the summits of the partitions by which these are divided. It is, as already remarked (¶ 51.), where the superficial cells are continuous with the columnar cells of the intermediate substance, that they present the rounded or ovoidal shape, instead of the elongated straight-sided figure which is their characteristic form. And the former seems to give place to the latter, whenever the cells of the superficial layers are perfectly separated from those of the intermediate stratum, and are connected only with the annular passages.

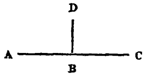
59. The intermediate stratum, again, may be altogether wanting, notwithstanding that the two superficial layers are separated from each other by a horizontal partition. In this case, each layer has its own annular canal; and its cells have sometimes such an arrangement as regards those of the other layer, that one of the connecting stolons from which each segment arises, will pass into the alternating cell of the upper layer, and the other into that of the lower. This arrangement may present itself as one of the modes of transition from the simpler to the more complex type, as is shown in Plate V. fig. 9; the columns being disposed to subdivide transversely when they attain a considerable length, and the annular canal becoming double; whilst in zones more distant from the centre, the two layers are separated by the interposition of the intermediate stratum. Sometimes, however, the disk continues to increase and attains

a considerable size on this duplex type; and its edge then presents two rows of rounded prominences with pores between them, those of the upper and lower rows alternating with each other, as is well seen in the vertical plate of the monstrosity represented in Plate IX. fig. 10. It is on a disk of this type that Professor EHRENBERG has founded his genus *Amphisorus*, which I cannot regard as even specifically distinct from the ordinary Orbitolite.

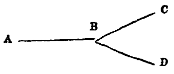
60. The next variety to be noticed, consists in a complete absence of regularity in the disposition of the columnar cells of the intermediate stratum, so that they present an assemblage of indefinitely-shaped passages, communicating with each other in various directions. This variety is chiefly interesting, as showing how little importance is to be attached to smaller deviations of the same kind. The most remarkable example of it which I have met-with, is represented in Plate V. fig. 7, *c—d*. In Plate V. figs. 11, 12 are represented two examples of irregularity in the disposition of the superficial and intermediate cells in the zones immediately surrounding the centre.

61. Lastly, I have to mention, that the septa dividing the contiguous cells of the same zone are occasionally deficient, so that the interior of the zone is a continuous circular passage, with only slight indications of the normal divisions. In such a case, it is obvious that the ring of sarcode must have been everywhere of nearly uniform thickness, showing no division either into horizontal or into vertical segments; and it may not be thought improbable that this is its first condition in every case, and that its segmental division is a subsequent process, so that the shelly investment, if formed previously to the segmentation, will have the character of incompleteness just described. I cannot help suspecting, that the peculiar groove around the margin of the Feejee specimens formerly noticed (¶ 25.), is referable to a still greater incompleteness of the production of the calcareous investment around the newly-forming zone.

62. *Monstrosities.*—Besides those departures from the normal type of growth, which have been described as variations or irregularities, there are certain others of rarer occurrence, which can only be regarded as '*monstrosities by excess*;' consisting in the production of one or more incomplete secondary disks by outgrowth from the first. In the specimen represented in Plate IX. fig. 8, the secondary disk forms a half-circle BD, of about the same diameter with the primary AC, and is superposed vertically upon the latter, the plane of junction passing through its centre. In the specimens represented in figs. 7 and 9, the secondary disk is relatively smaller, extending only from the centre to the margin of the primary,



but still meeting it nearly at right angles.—In the specimen of which two different aspects are shown in figs. 5, 6, it would seem difficult to say which is the primary and which the secondary disk, and it would be more correct to describe the entire structure as consisting of a single half-disk AB and of two half-disks BC and BD, meeting each other at an acute angle



CBD, neither of them being in the same plane with the single half-disk, but both of them meeting it at similarly obtuse angles ABC and ABD. The opening of one of the

obtuse angles is shown in fig. 6; and that of the acute in fig. 5; and in each view it is seen, that the divergence takes place in a plane which passes through the common centre of all three.—The specimen delineated in fig. 10 exhibits a multiple outgrowth of a nature resembling that shown in figs. 8 and 9. For from the surface of the disk there rises a triradiate crest, formed by three vertical plates meeting one another at nearly equal angles, but all of them nearly perpendicular to the plane on which they rest. It is a very remarkable feature in this specimen, however, that the line in which the three vertical planes meet, is traceable at its base to the nucleus of the horizontal disk; so that they all bear the same relation to it, as does the single outgrowth in the instances previously cited. Hence we may attribute all these monstrosities to an excess of productive power in the sarcode of the original nucleus, which has put forth its first extension, not merely in the horizontal, but also in the perpendicular direction; the whole subsequent development of these outgrowths taking place after the normal plan, from the foundation thus laid.—It is interesting to remark, that the presence of such outgrowths as those now described, is far more frequent in certain localities than it is in others. Among some hundreds of specimens which I have examined from the coast of Australia, I have only met with those represented in figs. 7, 8, 9, and two or three others; the remarkable specimen delineated in figs. 5, 6, occurred with another less peculiar among a comparatively small number of Orbitolites collected by Mr. Cuming in the Philippine Seas; but in a small collection which I have inspected from the Ægean Sea, the monstrosities of this kind (of which fig. 10 was the most remarkable) were so numerous, that I think I am scarcely wrong in asserting that one specimen out of every three or four presented some excess*. Among the *fossil* Orbitolites of the Paris basin, the presence of a completely-semicircular vertical plate is not at all uncommon.

63. There may be some doubt in the first instance, as to the light in which we are to regard the specimen represented in Plate VIII. fig. 10; whether as a 'monstrosity by excess,' or as the product of the fusion of two individuals: but I think this will be removed by a closer examination. For it is obvious, that the smaller disk, which is surrounded by the outer zones of the larger one, has been developed from a nucleus of its own; and this nucleus does not appear to have any direct connexion with the periphery, still less with the centre, of the larger disk: on the other hand, when we consider the circumstances under which Orbitolites grow (¶ 34.), it is very easy to understand, that the smaller and younger individual, having attached itself in too near proximity to the larger and older one, should become imbedded therein (so to speak) by the extension of the newly-forming zones of the latter around its margin.

* This is by no means a solitary case of the prevalence of monstrosities in particular localities. The collection of Mr. Bean of Scarborough contains a number of curiously-distorted specimens of the common *Planorbis marginatus*, which have all been collected in one brook. Their peculiarities are by no means repetitions of each other; and I am disposed, therefore, to regard them rather as resulting from the influence of external conditions, than as accidental varieties hereditarily propagated.

In fact, the manner in which the outer zones of the larger disk envelope the smaller, precisely corresponds with that in which we have seen the new zones originating from the uninjured margin of a mutilated specimen, to extend themselves along its fractured edges (¶ 37.).

V. *Of the Essential Characters of Orbitolites, and of its relations to other Types of Structure.*

64. If, now, we seek to determine the essential characters of that type of organization which is known under the designation *Orbitolites*, we find them to lie in the presence of *a series of annuli of sarcode* (and of corresponding *passages* in the shelly disk) arranged concentrically round a nucleus; each zone in the simpler type containing but a *single* annulus, so constricted at intervals as to form a series of somewhat columnar segments (occupying the cells of the shelly disk), connected with each other by narrow bands of sarcode; whilst in the more complex type each zone contains two such annuli, including between them a portion of its series of columnar segments, so as to constitute an *intermediate* stratum, distinct from the *superficial* portions. In either case, the segments of successive zones freely communicate with each other by *radiating bands of sarcode* (also leaving *passages* in the shelly disk), whose normal direction is such as to connect each segment with the two segments that alternate with it in each of the adjacent zones.

65. Now the addition of new zones, each similar to the last, is a simple matter of *growth*; but the passage from the simpler to the more complex plan marks an advance in *development*; and this advance essentially consists (here as elsewhere) in a progressive *differentiation of parts*. When, with the vertical extension of the columnar segments, the annular canal subdivides itself into two, the communications between the successive zones no longer come-off, as before, from the annular canal, but from the intermediate portions of the columnar cells; and instead of the two diverging passages from each cell being in the same plane, they lie in different planes, alternating with each other vertically. Up to this point, we observe little else than a multiplication of parts vertically, as well as horizontally, and a separation of connexions that were previously confluent. But in the highest stage of development, we find a marked alteration in plan; for those portions of the columnar segments, which lie between the two annular canals of each zone and the two surfaces of the disk, become completely differentiated from the portions that occupy the intermediate stratum, so as to form a peculiar set of *superficial cells*; and these are so equally connected with *two* zones, as to make it impossible to say that they belong specially to either.

66. Now we have seen that *development* may be checked, while *growth* continues, at any period of its progress; so that we find Orbitolites growing to a considerable size upon the very simplest plan,—others still larger formed upon the duplex plan,—the largest yet known (fossilized in the Paris basin) developed upon the multiple plan

without separation of the superficial cells,—while the most complete, in regard alike to multiplication and to differentiation of parts, are only found among the disks at present existing; and it is interesting to observe, that some of them present this highest grade of development, while as yet of comparatively minute size. I am not acquainted with any other Animal body, in which so wide a range of developmental variation normally exists. The lower classes of the Vegetable Kingdom, however, especially the group of Fungi, afford abundant examples of it *.

67. The extreme freedom with which all the cavities of the shell mutually communicate, is a very marked feature in the structure of this type; and shows that the several parts of the animal body are far more closely connected into one whole, than they are in most of the other Foraminifera whose general plan of conformation is more or less analogous. Indeed, if we were to imagine a discoidal mass of sarcode to be traversed by a reticulated calcareous skeleton, somewhat resembling that open areolar texture which forms the *shell* of the Echinida, and this network to possess something of that regularity of the disposition of its successively formed parts, which is presented to us in the *spines* of the Echinida, we should have no unapt representation of the relation of the shelly disk of the Orbitolite to the animal which it envelopes. There are certain Sponges which have a reticulated skeleton composed of mineral matter disposed in a mode not altogether dissimilar, whilst the constitution of their soft bodies is essentially the same. And a remarkable connecting link between Orbitolites and Sponges, seems to be presented to us in the curious *Thalassicolla* discovered by Mr. HUXLEY †. The relations of Orbitolite to other Foraminifera have already been partly touched-on, and will become more clear hereafter, when the types which most approximate to it shall have been themselves described.

VI. *Of the Species of Orbitolites.*

68. It only remains to inquire, whether the diversities which have been described as existing among Orbitolites, afford any ground for assuming the existence of more than a single species. With regard to the *recent* forms, with which, so far as they are at present known, I have made myself fully acquainted, I can speak confidently; since, as I have demonstrated, the Orbitolite with a *single* stratum of cells (*O. marginalis* of LAMARCK, *Sorites* of EHRENBERG), that with a *double* stratum (*Amphisorus* of EHRENBERG), and that with *multiple* strata (*Marginopora* of QUOY and GAIMARD, *Orbiculina Tonga* of Professor WILLIAMSON), are fundamentally the same forms, developed in three different modes.

69. Of the identity of all the *fossil* species with the foregoing, I cannot speak with the same confidence; since there are some of which I can only judge from figures. Into the structure of that which is best known, however, and which has been commonly accounted the type of the genus, viz. the *Orbitolites complanata* of the Paris

* See especially the recent Memoirs of M. TULASNE, in the 'Annales des Sciences Naturelles.'

† Annals of Natural History, 2nd series, vol. viii. p. 433.

basin, I have made investigations scarcely less minute and extended than into that of the recent forms; and I have come to the conclusion, that it cannot be specially distinguished from the large Australian Orbitolite, to which it bears a very obvious general conformity. It is true that it differs from the typical forms of the latter in two important features of structure, which are, however, mutually connected;—namely, the direct continuity of the cells of the superficial layers with the columnar cells of the intermediate layers; and the rounded or ovoidal form of the superficial cells, which (as already stated) these always possess, as in the simpler type, unless they are disconnected with the columnar cells, and communicate only with the annular stolons (see ¶ 58.). But since this very peculiarity does present itself in certain existing individuals, whose development seems to have taken place upon a lower type, and since it occasionally shows itself in the course of the passage from the simplest to the most complex type, in such as ultimately attain the latter, there appears to me no room for questioning the specific identity of the *O. complanata* with the Australian forms, notwithstanding that I have never met, among the numerous specimens which I have examined of the former, with those elongated parallel-walled superficial cells, which constitute the most distinctive feature in the latter. It may be well, moreover, to bear in mind the remark I have already made, respecting the local prevalence of particular varieties of form; since there is nothing more strange in the incompleteness of the type of development presented by the Paris-basin Orbitolite, than in that tendency to excessive development, which gives rise to the numerous monstrosities that are presented by the Ægean specimens (¶ 62.), or in those radial deposits on the surface, which are so common among the Philippine forms (¶ 53.). My belief in the specific identity of this fossil with the recent types has been strongly confirmed by the circumstance, that among the Paris-basin forms I have found a minute specimen, which corresponds in every respect with the simple type of the existing species.

70. Of the other fossil species cited by LAMARCK, the *O. macropora* of the Maestricht beds, judging from the figure given of it by GOLDFUSS, is nothing else than an Orbitolite of simple type, whose marginal cells have been laid open by attrition both above and below, as in Plate VII. figs. 8, 10. The *O. concava* and *O. pileolus* of LAMARCK are not distinguished in his definition by any other character than that drawn from *form*, which we have seen to be so variable as to be quite insufficient as a distinctive feature. It is quite possible, moreover, that they may belong to another type, nothing being said in the description of them, either of concentric lines, or of pores. If, as I believe, the *O. concava* of LAMARCK (figured by MICHELIN in his 'Icon. Zoophyt.,' pl. 7. fig. 9) be identical with the *O. conica* of M. D'ARCHIAC, I feel certain (from careful examination of its imperfectly-preserved internal structure) that, whatever it may prove to be, it is *not* an Orbitolite. So again, the *O. lenticulata* of LAMARCK, judging by the figure given of it by LAMOUROUX *, is *not* an Orbitolite,

* Polypiers, pl. 72. figs. 13, 16.

but probably a Lunulite. Of the species subsequently described by other authors, I entertain no doubt that the *O. disculus* of M. LEYMERIE *, as well as in all probability the *O. plana* of M. D'ARCHIAC, is nothing else than a variety of *O. complanata*, more especially as its young is said to be like the *O. macropora* of Maestricht. The *O. gensacica, O. secans*, and *O. socialis* of M. LEYMERIE, are all undoubtedly *Orbitoides*, and all belong, I believe, to the same species; though on this point I could not speak positively, without an examination of their internal structure. So, again, the *O. mamillata, O. Fortisii* (*O. gigantea* of D'ORBIGNY), *O. papyracea, O. stellata, O. sella* and *O. radians* (*O. radiata* of D'ORBIGNY) of M. D'ARCHIAC † are all probably Orbitoides, as he has himself subsequently recognized in regard at least to some of them ‡. The *O. elliptica* of MICHELIN, so far as I can judge from the figure he gives of it (pl. 71, fig. 11), is certainly not entitled to rank as a distinct species, its elliptic form being utterly insufficient to separate it (¶ 47.). And as I have already pointed out (¶ 49.), unless some distinctive character be furnished by the internal conformation of the bodies which have been ranked in the genus *Cyclolina* by M. D'ORBIGNY, this also should take rank merely as a variety of Orbitolites §.

VII. *Concluding Remarks.*

71. It might be asked with some show of reason, what good purpose can possibly be answered by such a minute and prolix description of a type of animal structure, so mean and insignificant as that which has been occupying our attention. To such a question I would reply—First, that I hold it to be a worthy labour to learn, and to place within reach of others, *everything* that *can* be learned respecting *any* form of Organized Being; that such a complete acquaintance is the great desideratum in every department of Biological Science; and that no works have ever exercised so beneficial an influence on its progress, as those admirable Monographs of single species, which, by thoroughly elucidating their structure and physiological history, have served as a basis for all subsequent inquiry into the nature of the Plants or Animals formed on a like plan:—Secondly, that such an inquiry can scarcely be otherwise than of peculiar utility, as relating to a tribe of Animals whose nature and history are almost as unknown to us now, as those of Polypes were to the Naturalists of a century ago, when TREMBLEY wrote his immortal 'Mémoires pour servir à l'Histoire d'un Genre de Polypes d'eau douce;' what little *is* known respecting them, being of a nature to mark them as distinct from every other type of living beings

* Mém. Géol. Soc. de France, 2 sér., tom. v. pp. 190, 191.

† *Ibid.* tom. ii. p. 178, and 2 sér., tom. iii. p. 4.

‡ Description des Animaux Fossiles du groupe Nummulitique de l'Inde, p. 350.

§ For a recent description of a form of this reputed genus, which occurs in contiguity with typical Orbitolites, and which seems to me to correspond in every respect with those recent specimens of which Plate VII. fig. 14 exhibits the external aspect, see Mr. CARTER's Memoir on the Fossil Foraminifera of Scinde, in 'Ann. of Nat. Hist.,' 2nd ser., vol. xi. p. 174.

with which we have a tolerable acquaintance:—and Thirdly, that certain *general principles* evolve themselves as results of these investigations, which are quite as applicable to every other department of Biological Science, as they are to the single case of the *Orbitolite*.

72. It has been shown that a very wide range of variation exists among *Orbitolites*, not merely as regards *external form*, but also as to *plan of development*; and not merely as to the shape and aspect of the *entire organism*, but also with respect to the size and configuration of its *component parts*. It would have been easy, by selecting only *the most divergent types*, from amongst the whole series of specimens which I have examined, to prefer an apparently substantial claim on behalf of these to be accounted as so many *distinct species*; and I could thus have easily created an almost indefinite number of such species. But after having classified the specimens which could be arranged around these types, a large proportion would yet have remained, either presenting characters *intermediate* between those of two or more of them, or actually *combining* those characters in different parts of their fabric; thus showing that no lines of demarcation can be drawn across any part of the series, that shall definitely separate it into any number of groups, each characterized by features entirely peculiar to itself. Thus, then, we see that the real relationship of the different types to each other, can only be determined by *the careful comparison of a very large number of individuals obtained from as many different sources as possible*;—a process which is too frequently neglected by Systematists, many of whom erect species, and even genera, without the least care to determine, by any such process, the real value of the distinctions by whieh they characterize them.

73. The right mode of proceeding in every other department of Natural History, must be that which has thus been proved to be the only reliable method in this; and I venture, therefore, to lay down the following general Canons, deduced from the results of the preceding investigation; which will, I think, be found accordant with the experience of all soundly-judging Naturalists, although they have not yet, so far as I am aware, received a formal expression.

74. To become fully acquainted with the Natural History of any Species, it is requisite;—(1) to study not only its external conformation, but also its *internal organization*; by which alone can the value of superficial resemblances or differences be duly estimated; (2) to trace out its entire *developmental history*, so that the true relationship of individuals in different stages of evolution may be appreciated; (3) to compare together a *large number of specimens* taken from any one locality, in order to ascertain what is the range of its variation under nearly identical conditions; (4) to search out its whole *Geographical distribution*, and to bring into mutual comparison large numbers of specimens from the remotest regions, as well as from all the intermediate areas over which it spreads, in order to determine the range of its variation under the most diverse conditions; and (5) to follow the same course of comparison throughout its *Geological distribution*, still relying only on the informa-

tion afforded by large numbers of specimens, collected (so far as may be possible) from different formations, and from different geographical areas. Until the whole of this process shall have been carefully and systematically gone through, no limitation of a species by a definition of any kind, can be regarded in any other light than as *a provisional means of marking-out the existence of a particular type of structure*, whose relationship to other types must be a matter of further investigation.

75. Let me subjoin such "pregnant instances," as shall prove the importance of each of the foregoing principles, from the result of the violation or neglect of it:—(1) so long as external conformation was alone regarded, and no account was taken of *internal organization*, the Nautiloid Foraminifera were placed among Cephalopods, and the Coralloid forms among Polypifera; to neither of which classes have they any kind of relationship; (2) so long as *developmental history* was unstudied, the Hydroid Zoophytes and the Medusoid Acalephæ were considered as entirely disconnected groups, belonging to two different Zoological classes, instead of (as in reality) different states of the very same organisms; (3) so long as reliance is placed on the comparison of a *few individual specimens* only, without any account being taken of the intermediate forms by which the more divergent types may be connected, so long are species multiplied to a most unwarrantable excess, as is found to be the case in almost every department of Zoology and Botany by those who devote themselves to a more extended comparison; thus, nineteen species have been made from the common *Potatoe*, and many more from the *Solanum nigrum*; so, multitudes of species have been instituted in various genera of Californian shells, by the late Mr. C. B. ADAMS, whose identity is established by a more extended comparison of individuals (as will be shown in a Report which is being prepared at the request of the British Association, by my brother, the Rev. P. P. CARPENTER); in fact, wherever this test is conscientiously applied, its effect is to *reduce* the number of reputed species, sometimes in a most remarkable degree*. (4) In like manner it has been by comparing only a small number of specimens from remote geographical provinces, and by neglecting the intermediate varieties that present themselves even among sufficiently large collections from these, still more among specimens collected from intervening regions, that not only numerous errors of detail have been committed, but general doctrines have been propounded, which the advance of Science has proved to be utterly untenable. As an example of the former kind, may be cited the facts mentioned by Dr. J. D. HOOKER (*op. cit.*), that of the New Zealand varieties of *Oxalis corniculata*, one of the most widely-diffused and most variable Flowering plants in the world, no less than seven or eight species have been made, neither of them supposed to be identical with any belonging to the European Flora; whilst *Pteris*

* I am most glad to find my views on this point in accordance with those of Dr. JOSEPH D. HOOKER (see his 'Introductory Essay on the Flora of New Zealand,' § 2), who has been led to the conviction, that instead of affirming the existence of 100,000 species of known Plants, we ought not to reckon more than half that number.

aquilina has a different name in almost every country in the world. It has been through reliance on such ignorant determinations, most of them proceeding on the notion that "the plants (or animals) of newly-discovered, isolated, or little-visited localities must necessarily be new," that the doctrine of the universal distinctness of the species of the New World from those of the Old, and of those of the Southern from those of the Northern Hemisphere, has attained a very wide currency amongst Naturalists, and is still obstinately persisted-in by some, in defiance of ample evidence to the contrary*. (5) Lastly, not only has the limitation of the comparison among Fossil types, to a small number of individuals, led to the excessive multiplication of species in the forms that are furnished by the same strata; but the same habit of relying on minute differences, without attention to osculant characters, has given rise to that disposition to regard the species of successive formations as necessarily different, which is introducing the greatest confusion into geological and palæontological determinations of every kind. How an extended comparison of individual forms tends not only to reduce the number of reputed species, but to establish the continuity of the same specific types from one stratum to another, will be remarkably seen when the laborious researches of Dr. WRIGHT of Cheltenham on the *Cidarites* of the Liassic and Oolitic formations shall have been made public.

76. Another general consideration of some interest, appears to me naturally to connect itself with the foregoing history,—namely, that the lower the general plan of organization of any being (that is, the greater the prevalence of 'vegetative' or 'irrelative' repetition in its different parts), the more is that plan liable to be modified by slight differences in external conditions, and the wider, therefore, may we expect its range of variation to be, if it be disposed to vary at all. In some instances, it is true, there appears (as in many higher forms of organization) to be an absolute incapacity for any such variation; and a limitation of the geographical and geological distribution of the species results from its want of power to exist under any great diversity of external conditions. But when the same general type of organization is found to prevail extensively both in space and in time, it may, I think, be safely regarded as probable, that that type has within itself the power of accommodation to a considerable diversity of external conditions; and hence that in the comparison of individuals, differences of conformation should be considered as of less account towards the establishment of specific distinctions, than they are when there is an obvious restriction of the type to a limited Geographical area or a particular Geological epoch.

77. In the foregoing communication, I have thought it right not only to make known the *results* of my researches, but so to develope my *plan of investigation*, that the value of those results may be duly estimated. In the memoirs which I trust to be

* "Thus as long ago as 1814, Mr. ROBERT BROWN gave a list of 150 European plants common to Australia and Europe. The identity of many of these has repeatedly been called in question, but almost invariably erroneously; added to which, more modern collectors have greatly increased the list."—HOOKER, *op. cit.* p. 18.

enabled to forward from time to time, regarding those other typical forms of Foraminifera of which I have made a special study, it is not my intention to do more than state the *results*; hoping that they may be understood to have been attained by a method of inquiry as closely resembling that which I have here followed through its details, as the circumstances of each case may have admitted. I would have it borne in mind throughout, that, as has been admirably remarked by one of the most accomplished Botanists of our time, "the Naturalist who has the true interest of science at heart, not only feels that the thrusting of an uncalled-for synonym into the nomenclature of science is an exposure of his own ignorance, and deserves censure, but that a wider range of knowledge and a greater depth of study are required, to prove those dissimilar forms to be identical, which any superficial observer can separate by words and a name" (Dr. J. D. HOOKER, *op. cit.* p. 14, note).

EXPLANATION OF THE PLATES.

PLATE IV.

Structure of the Animal *of Orbitolite.*

Fig. 1. Entire Animal, from a small and simple disk, the shell having been removed by maceration in acid; in the peripheral portion the segments of sarcode are wanting, and the structureless residuum of the shell is alone seen:—magnified 40 diameters.

Fig. 2. Appearance of a portion of Sarcode, highly magnified:—180 diam.

Fig. 3. Portion of the body of one of the more complex forms (resembling fig. 4), in which the sarcode has broken up into little spheres (gemmules?);—*a, a*, superficial segments; *b, b*, annular band:—180 diam.

Fig. 4. Portion of the body of one of the more complex forms, as seen in *vertical* section;—*aa*, *a'a'*, upper and lower rows of superficial cells, each cell connected, at its two extremities, with the annular bands *bb* and *b'b'* of *two* zones; from these annular bands spring the columnar segments *cc*, *c'c'*, those of the same zone occasionally passing into each other, and communicating with those of the next zone by oblique peduncles alternately passing towards one side and the other:—150 diam. (N.B. This figure is somewhat *ideal*, being made-up from several preparations; but for every point which it represents, these preparations give warranty.)

Fig. 5. Nucleus and first two annular zones, exhibiting the typical conformation;—*a*, the central segment; *bb*, the circumambient segment, from the entire margin of which are given off peduncles of sarcode, which give origin to the first annular zone:—84 diam.

Fig. 6. Nucleus and first two annular zones of another disk, showing a deficiency in the connexions of the first annular zone with the circumambient segment *bb*, for about a third of the circumference of the latter, between the points *cc*:—84 diam.

Fig. 7. Portion of one of the *surfaces* of an animal of complex type (as in fig. 4):—*aa, aa*, rows of superficial cells, connected at their two extremities with the annular bands *bb, bb*:—150 diam.

Fig. 8. Portion of fig. 1, enlarged, to show the ordinary mode in which the segments of each zone are connected by peduncles with the *annular band* of the preceding zone, so as to alternate with its segments; showing also the occasional interpolation of additional segments, *a, a*, whose peduncles come off from the *segments* of the preceding zone:—90 diam.

Fig. 9. Portion of a section of the shelly disk enlarged, to show the corresponding appearances it presents; each cell being ordinarily connected by a radial passage with the *annular canal* of the preceding zone, and thus with the two cells alternating with itself; but cells *a, a*, being sometimes interpolated, which open directly into the *cells* of the preceding zone:—90 diam.

Fig. 10. Central and circumambient segments (*a* and *bb*) of a large disk, showing the origin of the segments of the first annular zone, *c, c*, from less than half the circumference of the nucleus:—84 diam.

Fig. 11. Peculiar bodies (ova?) found in the substance of the sarcode in different parts, showing successive stages, *a, b, c, d, e, f*, of binary subdivision; *g*, other bodies of somewhat larger size, found in one of the superficial cells of a vertical section:—130 diam.

Fig. 12. Central portion of fig. 1, enlarged to the same scale as figs. 5, 6, 10; showing the central segment *a*, the circumambient segment *bb*, and the origin of the first annular zone in *three* peduncles proceeding from the end of the latter:—84 diam.

Fig. 13. Central portion of another disk, showing the origin of the first annular zone by *eight* peduncles from the circumambient cell:—84 diam.

PLATE V.

Structure of the Calcareous Disks *of Orbitolite.*

Fig. 1. Ideal Representation of a Disk of the Simple Type, the details of the different parts made-up from actual specimens; showing the natural surface, with the markings of the cells; the natural margin, with the single row of pores between the protuberances of the cells;—a portion of the interior, as displayed by a *horizontal* section, showing the central cell *a*, the circumambient cell *b, b*, the concentric zones of cells *c, c*, with the annular passages which connect together the cells of the same zone, and the radiating passages

which extend from the annular passages of each zone to the cells of the next, and, in the outermost zone, to the pores *d, d*;—another portion as displayed by a *vertical* section *e, e*, in a *radial* direction, which lays open the columnar cells, but passes through the intercellular partitions, in alternate zones; —and another portion *ff*, as displayed by a fracture in the course of one of the zones, laying open the entrances to the cells from its inner or central side.

Figs. 2, 3. Two large recent disks from the Feejee islands, plicated towards the margin, but one much more so than the other, and with a projection of the upper and lower edges, so as to leave a deep marginal furrow:—enlarged 2 diam.

Figs. 4, 5. Vertical sections of two disks, drawn under the same magnifying power, showing the marked difference in the size and proportions of their parts. In each disk we see the central cell, with the circumambient cell laid open on either side of it; and the cells of successive zones, with their communications:—30 diam.

Fig. 6. Ideal Representation of a Disk of the Complex Type; the details of the different parts made-up from actual specimens:—*a*, central cell; *b*, circumambient cell; *c, c*, concentric zones of oblong superficial cells, some of them laid open; *d, d*, marginal pores, forming several rows; *d''*, *d'''*, corresponding pores of inner zones, once marginal, but now connecting them with surrounding zones; *e, e*, vertical section in a radial direction, showing the zones nearest the centre to be made up of simple columnar cells, but those of the remainder of the disk to be composed of two superficial layers and of an intermediate stratum; *f, f*, floors of the superficial cells, with an aperture at each end of every one; *g, g*, annular canals, running beneath these floors, with the large apertures leading to the columnar cells of the intermediate stratum; *g'*, the same canals near the other surface of the disk; *g''*, the same canals laid open through the plane at which they give off the two passages into the superficial cells; *g'''*, the same canals, as cut transversely by a vertical section; *h*, passage of the horizontal section through the intermediate stratum, showing the summits of its columnar cells about to enter the annular canals; *i, i, i*, and *k, k, k*, passage of the horizontal section through two different planes of the intermediate stratum, showing the connexion between the columnar cells of successive zones, by oblique passages running in different directions; *l*, portion immediately surrounding the nucleus, formed upon the simple type, as in fig. 1.

Fig. 7. Vertical section, taken in a radial direction, of a recent disk incompletely developed on the complex type; showing at *a*, a single chamber of the cavity of the nucleus, the section having traversed the circumambient cell; from *a* to *b*, including twenty-three zones, the disk developed upon the simple

type; from *b* to *c* the annular canal double, and an irregular intermediate stratum interposed, from which, however, the superficial cells are not completely differentiated; and from *c* to *d* the imperfect separation of the superficial cells from the intermediate layer, and the extreme irregularity of the latter:—50 diam.

Fig. 8. Tangential section, near the margin of the same disk, showing the same peculiarities:—50 diam.

Fig. 9. Central portion of a disk, showing the nuclear cavity divided into two (the section having traversed the central cell and one side of the circumambient); the formation of the first three zones (*a*—*b*) on the simple type; the imperfect separation of the superficial cells from the intermediate stratum in the next two zones (*b*—*c*); and the regular development of the remainder (*c*—*d*) on the complex type:—50 diam.

Fig. 10. Vertical section of the central portion of a disk of complex type; showing *four* cavities in the nucleus; the first five zones (*a*—*b*) constructed upon the simple type, with a single annular canal, but an incipient separation manifesting itself between the superficial cells and the intermediate stratum; in the next three zones (*b*—*c*) the annular canal double, but the superficial cells still partly continuous with the columnar; the outer part (*c*—*d*) framed according to the regular complex type:—48 diam.

Fig. 11. Vertical section of a disk of complex type, showing an unusual development of vertically superposed cells immediately around, and even partly covering, the nucleus, from which they arise by four layers of passages; a progressive diminution in the thickness of the disk, as far as the fifth zone (*b*), with a gradual approximation towards the regular type, on which the remainder (*b*—*c*) is developed:—48 diam.

Fig. 12. Vertical section of a disk of complex type, showing commencement of its first zone by a single layer of passages (*a*) from the nucleus; the presence of two annular canals, with the absence of separation of the superficial cells, in the first zone (*b*); the incomplete separation of the superficial cells from the intermediate columnar portion in the second zone (*c*); and the normal conformation of the remainder (*c*—*d*):—48 diam.

PLATE VI.

Fig. 1. Portion of a disk of simple type, from the immediate neighbourhood of the nucleus, showing the beak-like projections at the sides of the passages very remarkably developed:—100 diam.

Fig. 2. Horizontal section of the intermediate layer, passing (in consequence of the flexure of the disk) through two different planes, and thus showing the two different directions of the oblique passages which connect the colum-

nar cells of different zones; also showing a marked difference in the size of adjacent cells:—100 diam.

Fig. 3. Horizontal section through one of the superficial layers, showing the elongated form of its cells, and the aperture at each end of their floor:—100 diam.

Fig. 4. Nucleus and surrounding zones of the excentrically-developed disk shown in Plate IX. fig. 3; the beak-like projections at each side of the pores opening into the new zones (shown in fig. 1) are here generally deficient:—100 diam.

Fig. 5. Appearance of the thin wall covering-in the central cell of a large nucleus, as seen under a high magnifying power, showing the quasi-cellular markings:—100 diam.

Fig. 6. Horizontal section through the intermediate layer of a disk of complex type, showing an unusual irregularity in the communications of the cells:—100 diam.

Fig. 7. Radial section of a recent disk, showing the complex development on the normal type; *aa*, *bb*, upper and lower superficial layers; *c*, *c*, *c*, *c*, intermemediate stratum; *dd*, $d'd'$, $d''d''$, summits of partitions between successive zones, forming the floors of the superficial cells; *e*, e', e'', oblique passages through the floors of the cells of the superficial layer, leading towards the annular canals; *f*, f', f'', f''', annular canals of four zones, near the lower surface of the disk; *gg*, partitions between the adjacent cells of the same zones; *hh*, perforations in these, through which the columnar cells inosculate with each other:—100 diam.

Fig. 8. Tangential section of a fossil disk of complex type, showing four columnar cells, aa', bb', cc', dd', of the same zone, divided by sinuous partitions, with the orifices leading into the columnar cells of the next interior zone:—100 diam.

Fig. 9. Similar tangential section, showing four cells of the next interior zone, aa', bb', cc', dd', alternating with the preceding, and the entrance of the passages of the same vertical row, alternately into one and the other of the cells on the two sides of each of the sinuous vertical partitions:—100 diam.

Fig. 10. Radial section of a fossil disk, showing the incompleteness of the separation of the superficial cells from the intermediate layer, and the irregularity in the arrangement of its cells, with their numerous lateral inosculations (compare Plate V. fig. 7, *b*—*d*):—48 diam.

Fig. 11. Radial section of a thicker fossil disk, showing a more regular arrangement of the columnar cells of the intermediate layer (still, however, with numerous passages for lateral inosculation), and the complete continuity of the superficial cells with these:—48 diam.

PLATE VII.

Figs. 1—4. A series of specimens, drawn under the same magnifying power (35 diam.), illustrating the variations which present themselves in regard to the *Size* of the nucleus.

The remaining figures represent various appearances exhibited by the *Surfaces* of Disks, simple and complex:—all magnified 35 diam., except fig. 6, which is magnified 70 diam.

Figs. 5—7. Portions of surfaces of simple disks, in which the concentric circles are strongly marked, and the transverse divisions of the cells are comparatively obscure.

Fig. 8. Simple disk, slightly concave, the marginal cells laid open by abrasion, the surface marked rather by 'engine-turned' or excentric, than by concentric circles.

Fig. 9. Portion of the surface of a complex disk, having small, round, and flattened superficial cells.

Fig. 10. Simple disk (from Australia), slightly concave, the marginal cells laid open by abrasion, the surface irregularly thickened by calcareous deposits.

Fig. 11. Flattened simple disk (from the Philippines), with abundant superficial deposits, arranged with considerable regularity in a radiating direction.

Fig. 12. Portion of the surface of a large complex disk, with very narrow oblong superficial cells.

Fig. 13. Portion of the surface of a complex disk, showing a marked difference in the proportions of the straight-sided cells, even in adjacent zones.

Fig. 14. Surface of simple disk (*Cyclolina*? of D'Orbigny), with an excentric nucleus, surrounded by strongly-marked concentric circles.

Fig. 15. Portion of the surface of a simple disk, showing unusually large and protuberant ovoidal cells.

Fig. 16. Portion of the surface of a large fossil disk of complex type, showing varieties in the size and form of the superficial cells.

PLATE VIII.

Fig. 1. Inner surface of a zone of a thick fossil disk, showing a tolerably regular arrangement of the columnar cells, the segmental constrictions of these, and the fissures leading into those of the next row:—35 diam.

Fig. 2. Outer surface of a similar zone, showing the continuity of the cells of the superficial layer with the vertical columns of the intermediate (the latter not unfrequently dividing, however, at their extremities, so as to form two cells at the end of each column), and the pores leading to the oblique

passages, directed alternately to one side and to the other, which lead to the cavities of the columnar cells :—35 diam.

Fig. 3. Horizontal section of the intermediate layer, immediately beneath the floors of the superficial cells, laying open the annular canals, with the summits of the columnar cells :—100 diam.

Figs. 4—9. Various examples of the *Reparation* of disks after fracture, and of the growth of new disks from detached fragments. For description, see ¶ 37—40 of Memoir. Fig. 6 is magnified 6 diam.; the rest 35 diam.

Fig. 10. Monstrosity formed by the inclusion of a young disk within the outer zones of an older one, that seems to have been brought into contact with it by the progressive increase of its own diameter :—25 diam.

PLATE IX.

Figs. 1—4. Various departures from the typical mode of concentric development, presented in their early state, both by simple and complex disks; some of them even passing towards a spiral mode of evolution. Figs. 1 and 3 are magnified 35 diam.; Figs. 2 and 4 are enlarged 90 diam.

The remaining figures show various forms of *Monstrosity*, produced by excess of growth from the nucleus.

Fig. 5, 6. Two views of a disk from the Philippine shores, having a single plane on one side, and two planes, meeting at an acute angle, on the other :—6 diam.

Figs. 7, 8, 9. Small Australian disks, with vertical crests :—35 diam.

Fig. 10. Disk from the Ægean Sea, with a tri-radiate crest :—35 diam.

XI. *Further Researches on the Polarity of the Diamagnetic Force.* By JOHN TYNDALL, *F.R.S., Membre de la Société Hollandaise des Sciences; Foreign Member of the Physical Society of Berlin, and Professor of Natural Philosophy in the Royal Institution.*

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INTRODUCTION.

A YEAR ago I placed before the Royal Society the results of an investigation "On the Nature of the Force by which Bodies are repelled from the Poles of a Magnet*." The simultaneous exhibition of attraction and repulsion in the case of magnetized iron is the fact on which the idea of the polarity of this substance is founded; and it resulted from the investigation referred to, that a corresponding duality of action was manifested by bismuth. In those experiments the bismuth was the moveable object upon which fixed magnets were caused to act, and from the deflection of the bismuth its polarity was inferred. But, inasmuch as the action is reciprocal, we ought also to obtain evidence of diamagnetic polarity by reversing the conditions of experiment; by making the magnet the moveable object, and inferring from its deflection the polarity of the mass which produces the deflection. This experiment would be complementary to those described in the communication referred to, and existing circumstances invested the experiment with a great degree of interest and importance.

In fact an experiment similar to that here indicated was made by Professor W. WEBER, previous to my investigation, and the result was such as to satisfy its author of the reverse polarity of diamagnetic bodies. I will not here enter into a minute description of the instrument and mode of experiment by which this result was obtained; for the instrument made use of in the present inquiry being simply a refinement of that made use of by M. WEBER, its explanation will embrace the explanation of his apparatus. For the general comprehension of the criticisms to which M. WEBER's results have been subjected, it is necessary, however, to remark, that in his experiments a bismuth bar, within a vertical spiral of copper wire, through which an electric current was transmitted, was caused to act upon a steel magnet freely suspended without the spiral. When the two ends of the bar of bismuth were permitted to act successively upon the suspended magnet, a motion of the latter was observed, which indicated that the bismuth bar was polar, and that its polarity was the reverse of that of iron.

Notwithstanding the acknowledged eminence of M. WEBER as an experimenter,

* Philosophical Transactions, 1854.

this result failed to produce general conviction. Mr. FARADAY, in his paper "On the polar or other condition of diamagnetic bodies*," had shown that results quite similar to those obtained by M. WEBER, in his first investigation with bismuth, were obtained in a greatly exalted degree, with gold, silver and copper; the effect being one of induction, and not one due to diamagnetic polarity. He by no means asserted that his results had the same origin as those obtained by M. WEBER; but as the latter philosopher had made no mention of the source of error which Mr. FARADAY's experiments rendered manifest, it was natural to suppose that it had been overlooked, and the observed action attributed to a wrong cause. In an article published in his 'Massbestimmungen' in 1852, M. WEBER, however, with reference to this point, writes as follows:—"I will remark that the article transferred from the Reports of the Society of Sciences of Saxony to POGGENDORFF's 'Annalen' was only a preliminary notice of my investigation, the special discussion of which was reserved for a subsequent communication. It will be sufficient to state here, that in the experiments referred to I sought to eliminate the inductive action by suitable combinations; but it is certainly far better to set aside this action altogether, as has been done in the experiments described in the present memoir."

One conviction grew and strengthened throughout these discussions—this, namely, that in experiments on diamagnetic polarity great caution is required to separate the pure effects of diamagnetism from those of ordinary induction. With reference to even the most recent experiments of M. WEBER—those to which he has referred at the conclusion of the above citation—it is strongly urged that there is no assurance that the separation referred to has been effected. In those experiments, as already stated, a cylinder of bismuth was suspended within a vertical helix of covered copper wire, and the action of the cylinder upon a magnet suspended opposite to the centre or neutral point of the helix was observed. To increase the action, the position of the cylinder was changed at each termination of the minute swing of the magnet, the amplitude of the oscillations being thus increased, and the effect rendered more sensible to the eye. Now, it is urged, there is every reason to believe that in these motions of a metallic mass within an excited helix induced currents will be developed, which, acting upon the magnet, will produce the motions observed. The failure indeed to demonstrate the existence of diamagnetic polarity by other means has, in the case of some investigators, converted this belief into a certainty.

Among the number whom M. WEBER's experiments have failed to convince, M. MATTEUCCI occupies a prominent place. With reference to the question before us this philosopher writes as follows†:—"In reading the description of the experiments of M. WEBER, we are struck on beholding the effects produced by moving the bismuth when there is no current in the spiral. Although the direction of oscillation in this latter case is opposed to that observed when the spiral is active, still

* Experimental Researches, 2640, Philosophical Transactions, 1850, p. 171.

† Cours Spécial sur l'Induction, p. 206.

the fact excites doubts as to the truth of the conclusions which have been drawn from these experiments*. *To deduce rigorously the demonstration of diamagnetic polarity, it would be necessary to substitute for the bismuth, masses formed of insulated fragments of the metal*†, *to vary the dimensions of the cylinder, and above all, to compare the effects thus obtained with those which would probably be obtained with cylinders of copper and silver in a state of purity.*

"We are obliged to make the same remarks on another series of experiments which this physicist has made to obtain anew, by the effects of induction, the proof of diamagnetic polarity. It is astonishing that after having sought to neutralize the development of induced currents in the moving cylinders of bismuth, by means of a very ingenious disposition of the spiral—it is astonishing, I repeat, that no attempt was made to prove by preliminary essays with metals possessing a higher conductibility than bismuth, that the same end could be obtained. I cannot leave you ignorant that the doubts which I have ventured to advance against the experiments of M. Weber are supported by the negative result which I have obtained in endeavouring to excite diamagnetic polarity in bismuth by the discharge of the Leyden jar."

It will be seen in the following pages that the conditions laid down by M. Matteucci for the rigorous demonstration of diamagnetic polarity are more than fulfilled.

The conclusions of M. Weber find a still more strenuous opponent in his countryman Professor v. Feilitzsch, who has repeated Weber's experiments, obtained his results, but who denies the validity of his inferences. M. v. Feilitzsch argues that in the experiments referred to it is impossible to shut out ordinary induction, and for the rigorous proof of diamagnetic polarity proposes the following conditions‡. "To render the experiment free from the action of induced currents two ways are open. The currents can be so guided that they shall mutually neutralize each other's action upon the magnet, or the induced currents can be *completely got rid of* by using, instead of a diamagnetic *conductor*, a diamagnetic *insulator*." To test the question, M. v. Feilitzsch resorted to the latter method: instead of cylinders of bismuth he made use of cylinders of wax, and also of a prism of heavy glass, but in neither case was he able to detect the slightest action upon the magnet. "However the motions of the prism might be varied, it was not possible either to cause the motionless magnet to oscillate, or to bring the magnet from a state of oscillation to one of rest."

* It is not my place to account for the effect here referred to. I may however remark, that there appears to be no difficulty in referring it to the ordinary action of a diamagnetic body upon a magnet. It is the result which Brugmann published upwards of half a century ago; the peculiar form of this result in one of the series of experiments quoted by M. Weber must, I think, be regarded as purely accidental.—J. T.

† Also in page 204:—"Il fallait donc, pour prouver si l'influence d'un corps diamagnétique produit sur un aimant une variation de sens contraire à celle développée dans le fer doux, opérer avec ce corps *privé de conductibilité*."

‡ Poggendorff's Annalen, xcii. 377.

M. v. Feilitzsch pushes his experiments further, and finds that when the bismuth is *motionless* within its spiral, the position of the magnet is just the same as when the bismuth is entirely withdrawn; hence his final conclusion, that the deflection of the magnet in Weber's experiments is due to induced currents, which are excited in the bismuth by its mechanical motion up and down within the spiral.

These divergent opinions upon a question of such vital bearing upon the general theory of magnetic phenomena, naturally excited in me the desire to make myself acquainted with the exact value of M. Weber's experiments. The most direct way of accomplishing this I considered to be, to operate with an instrument similar to that made use of by Weber himself; I therefore resolved to write to the constructor of his apparatus, but previous to doing so the thought occurred to me of writing to M. Weber, to inquire whether his further reflections on the subject had suggested to him any desirable modification of his first instrument. In reply to my question he undertook to devise for me an apparatus, surpassing in delicacy any hitherto made use of. The design of M. Weber was ably carried out by M. Leyser of Leipzig, and with the instrument thus placed in my possession I have been able, not only to verify the experiments of M. Weber, but to satisfy the severest conditions proposed by those who saw in the results of these experiments the effects of ordinary induction.

DESCRIPTION OF APPARATUS.

A sketch of the instrument made use of in the present investigation is given in fig. 2. BO, B′O′ is the outline of a rectangular box, the front of which is removed so as to show the apparatus within. The back of the box is prolonged, and terminates in two semicircular projections, which have apertures at H and H′. Stout bolts of brass, which have been made fast in solid masonry, pass through these apertures, and the instrument, being secured to the bolts by screws and washers, is supported in a vertical position, being free from all disturbance save such as affects the foundations of the Royal Institution. All the arrangements presented to the eye in fig. 2 are made fast to the back of the box, but are unconnected with the front, so as to permit of the removal of the latter. WW′ are two boxwood wheels with grooved peripheries, which permit of motion being transferred from one wheel to the other by means of a string *ss′*. Attached to this string are two cylinders, *mn*, *op*, of the body to be examined: in some cases the cylinders are perforated longitudinally, the string passes through the perforation, and the cylinders are supported by knots on the string. HE, H′E′ are two helices of copper wire overspun with silk, and wound round two brass reels, the upper ends of which protrude from H to G, and from H′ to G′. The internal diameter of each helix is 0·8 of an inch, and its external diameter about 1·3 inch; the length from H to E is 19 inches, and the centres of the helices are 4 inches apart; the diameters of the wheels WW′ being also 4 inches. The cross bar GG′ is of brass, and through its centre passes the screw R, from which depends a number of silk

fibres which support an astatic arrangement of two magnets, the front one of which, SN, is shown in the figure. An enlarged section of the instrument through the system of magnets is shown in fig. 4. The magnets are connected by a brass cross-piece, in which is the point of suspension P; and the position of the helices is shown

Fig. 1.

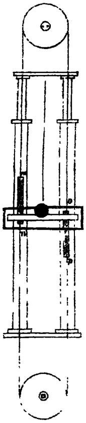


Fig. 2.

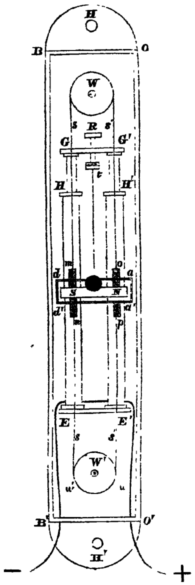


Fig. 3.

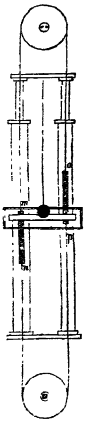
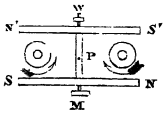


Fig. 4.

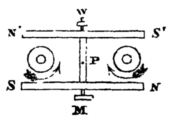


to be between the magnets. It will be seen that the astatic system is a horizontal one, and not vertical, as in the ordinary galvanometer. The black circle in front of the magnet SN, fig. 2, is a mirror, which is shown in section at M, fig. 4; to balance the weight of this mirror, and adjust the magnets in a horizontal position, a brass washer, W, is caused to move along a screw, until a point is attained at which its weight has brought both the magnets into the same horizontal plane. There is also another adjustment, which permits of the magnets being brought closer together or separated more widely asunder. The motions of the compound magnet are observed by means of a distant scale and telescope, according to the method applied to the magnetometer of Gauss. The rectangle *da*, *d'a'*, fig. 2, is the outline of a copper damper, which, owing to the currents induced in it by the motion of the magnet, soon brings the latter to rest, and thus expedites experiment.

It is well known that one end of a magnet attracts, while the other end repels the

same pole of a magnetic needle; and that between both there is a neutral point which neither attracts nor repels. The same is the case with the helices HE, H'E'; so that when a current is sent through them, if the astatic magnet be exactly opposite the neutral point, it is unaffected by the helices. This is scarcely attainable in practice; a slight residual action remains which draws the magnets against the helices; but this is very easily neutralized by disposing an external portion of the circuit so as to act upon the magnets in a direction opposed to that of the residual action. Here then we have a pair of spirals which, when excited, do not act upon the magnets, and which therefore permit us to examine the pure action of any body capable of magnetic excitement placed within them. In the experiments to be described, it was always arranged that the current flowed in opposite directions through the two spirals; so that if the bodies within them were polar, the two upper ends of these bodies should be poles of opposite names, and consequently the two lower ends opposed also. Supposing now our two cylinders to occupy the central position indicated in fig. 2: even if the cylinders became polar through the action of the surrounding current, the magnets, being opposite to the neutral points of the cylinders, would experience no action from the latter. But suppose the wheel W' to be so turned that the two cylinders are brought into the position shown in fig. 1, the upper end *o* of *op* and the lower end *n* of *mn* will act simultaneously upon the suspended magnets. For the sake of illustration, let us suppose the ends *o* and *n* to be both north poles, and that the section, fig. 4, is taken when the bars are in the position shown in fig. 1. The right-hand pole *o* will attract S' and repel N, which attraction and repulsion sum themselves together to produce a deflection of the system of magnets. On the other hand, the left-hand pole *n*, being also north, will attract S and repel N', which two effects also sum themselves to produce a deflection in the same direction as the former two. Hence, not only is the action of terrestrial magnetism annulled by this arrangement, but the moving force due to the reciprocal action of the magnets and the bodies within the helices is increased fourfold. By turning the wheel in the other direction we bring the cylinders into the position shown in fig. 3, and thus may study the action of the ends *m* and *p* upon the magnets.

Fig. 4.



By means of the screw R the magnets can be raised or lowered; and at the end, *t*, of the screw is a small torsion circle which can be turned independently of the screw; by means of the latter the suspending fibre can be twisted or untwisted without altering the level of the magnets.

The front of the box is attached by means of brass hasps, and opposite to the mirror M a small plate of glass is introduced, through which the mirror is observed; the magnets within the box being thus effectually protected from the disturbances of the external air. A small handle to turn the wheel W' accompanied the instrument from its maker; but in the experiments, I used, instead of it, a key attached to the end of a rod 10 feet long; with this rod in my right hand, and the telescope

and scale before me, the experiments were completely under my own control. Finally, the course of the current through the helices was as follows:—Proceeding from the platinum pole of the battery it entered the box along the wire *w*, fig. 2, which passed through the bottom of the latter; thence through the helix to H′, returning to E′; thence to the second helix, returning to E, from which it passed along the wire *w*′ to the zinc pole of the battery. A commutator was introduced in the circuit, so that the direction of the current thus indicated could be reversed at pleasure.

EXPERIMENTS.—DEPORTMENT OF DIAMAGNETIC BODIES.

A pair of cylinders of chemically pure bismuth, 3 inches long and 0·7 of an inch in diameter, accompanied the instrument from Germany. These were first tested, commencing with a battery of one cell of Grove. Matters being as sketched in fig. 2, when the current circulated in the helices and the magnet had come to rest, the cross wire of the telescope cut the number 482 on the scale. Turning the wheel W′ so as to bring the cylinders into the position fig. 1, the magnet moved promptly, and after some oscillations took up a new position of equilibrium; the cross wire of the telescope then cut the figure 468 on the scale. Reversing the motion so as to place the cylinders again central, the former position 482 was assumed; and on turning further in the same direction, so as to place the cylinders as in fig. 3, the position of equilibrium of the magnet was at the number 493. Hence by bringing the two ends *n* and *o* to bear upon the astatic magnet, the motion was from greater to smaller numbers, the position of rest being then fourteen divisions less than when the bars were central. By bringing the ends *m* and *p* to bear upon the magnet, the motion was from smaller to greater numbers, the position of rest being eleven divisions more than when the bars were central.

As the positions here referred to will be the subject of frequent reference, for the sake of convenience I will call the position of the cylinders sketched in fig. 1, Position 1; that sketched in fig. 2, Position 2; and that sketched in fig. 3, Position 3. The results which we have just described, tabulated with reference to these terms, would then stand thus:—

I.

Bismuth cylinders.

length 3 inches.
diameter 0·7.

Position 1.	468
Position 2.	482
Position 3.	493

In changing therefore from position 1 to position 3, a deflection corresponding to twenty-five divisions of the scale was produced.

Wishing to place myself beyond the possibility of illusion as regards the fact of deflection, I repeated the experiment with successive batteries of two, three and four cells. The following are the results:—

II.

	2 cells.	3 cells.	4 cells.
Position 1.	450	439	425
Position 2.	462	450	437
Position 3.	473	462	448

In all the cases cited we observe the same result. From position 2 to position 1 the motion is from larger to smaller numbers; while from position 2 to position 3 the motion is from smaller to larger numbers.

It may at first sight appear strange that the amount of the deflection did not increase with the battery power; the reason, in part, is that the magnet, when the current circulated, was held in a position free from the spirals, by forces emanating partly from the latter and partly from a portion of the external circuit. When the current increased, the magnetization of the bismuth increased also, but so did the force which held the magnets in their position of equilibrium. To remove them from this position, a greater amount of force was necessary than when only the residual action of a feeble current held them there. This fact, coupled with the circumstance that less heat was developed, and less disturbance caused by air currents, when a feeble battery was used, induced me for some time to experiment with a battery of two cells. Subsequent experience however enabled me to change this for five cells with advantage.

Notwithstanding the improbability of the argument, still it may be urged that these experiments do not prove beyond a doubt that the bismuth cylinders produce the motion of the magnets in virtue of their excitement by the voltaic current; for it is not certain that these cylinders would not produce the same motion wholly independent of the current. Something of this kind has occurred to M. Leyser* in his experiments, and why not here?

In answer to this I reply, that if the case be as here suggested, the motion of the magnet will not be changed when the current surrounding the bismuth cylinders flows in the opposite direction. Here is the experiment.

III.

Position 1.	764
Position 2.	742
Position 3.	704

We observe here that in passing from position 2 to position 1 the motion is from smaller to larger numbers; while in passing from position 2 to position 3 the motion

* Scientific Memoirs, New Series, vol. i. page 184.

is from larger to smaller numbers. This is the opposite result to that obtained when the current flowed in the opposite direction; and it proves that the polarity of the bismuth cylinders depends upon the direction of the current, changing as the latter changes. It was pleasant to observe the prompt and steady march of the magnet as the cylinders were shifted in the helices. When the magnets, operated on by the bars of bismuth, were moving in any direction, by bringing the two opposite ends of the bismuth bars into action, the motion could be promptly checked; the magnets could be brought to rest, or their movement converted into one in the opposite direction.

I may add to the above a series of results obtained some days subsequently in the presence of Professors Faraday, De la Rive and Marcet.

IV.

Bismuth cylinders.

Position 1.	670
Position 2.	650
Position 3.	630

The difference between positions 1 and 3 amounts here to forty divisions of the scale; subsequent experience enabled me to make it still greater.

It was found by experiment, that when the motion was from lower to higher numbers it denoted that the poles NN′, fig. 4, were repelled from the spirals, and the poles SS′ attracted towards them. When, on the contrary, the motion was from larger to smaller numbers, it indicated that the poles NN′ were attracted and the poles SS′ repelled. In the position fig. 1, therefore, of Tables III. and IV. the poles NN′ were repelled by the ends *no* of the bismuth cylinders, and the poles SS′ attracted, while in the position fig. 3, the poles NN′ were attracted by the ends *mp*, and the poles SS′ repelled; the ends *n* and *o*, therefore, acted as two north poles, while the ends *m* and *p* acted as two south poles. Now the direction of the current in the experiments recorded in the two tables referred to was that shown by the arrows in fig. 4. Standing in front of the instrument, the direction in the adjacent face of the spiral H′E′ was from right to left, while it was from left to right in HE. From this we may infer that the polarity of the bismuth cylinders was the reverse of that which would be excited in cylinders of iron under the same circumstances. The assertion however shall be transferred from the domain of deduction to that of fact before we conclude.

Let us now urge against these experiments all that ever has been urged by the opponents of diamagnetic polarity. The bismuth cylinders are metallic conductors, and in moving them through the spirals induced currents more or less powerful may be excited in these conductors. The motion observed may not, after all, be due to diamagnetic polarity, but to the currents thus excited. I reply, that in all cases the number set down marks the *permanent* position of rest of the magnets. Were the action due to induced currents, these currents, being momentary, could only impart

a *shock* to the magnets, which, on the disappearance of the currents, would return to their original position. But the deflection is permanent, and is therefore due to an enduring cause. In his paper on "Supposed Diamagnetic Polarity," Mr. FARADAY rightly observes,—"If the polarity exists, it must be in the particles, and for the time permanent, and therefore distinguishable from the momentary polarity of the mass due to induced temporary currents, and it must also be distinguishable from ordinary magnetic polarity by its contrary direction." These are the precise characteristics of the force made manifest by the experiments now under consideration.

Further, the strength of induced currents depends on the conducting power of the mass in which they are formed. Expressing the conducting power of bismuth by the number 1·8, that of copper would be expressed by 73·6*, the conductivity of the latter being therefore forty times that of the former. Hence the demand made by the opponents of diamagnetic polarity, to have the experiments repeated with cylinders of copper; for if the effect be due to induced currents, they will show themselves in copper in a greatly increased degree. The following is the result of a series of experiments made with two copper cylinders, of the same dimensions as the bismuth ones already described.

V.

Cylinders of Copper.

Position 1.	754
Position 2.	754
Position 3.	755

Now if the effects obtained with bismuth were due to induced currents, we should have the same effects forty times multiplied in the case of copper, in place of which we have scarcely any sensible effect at all.

Bismuth is the only substance which has hitherto produced an action in experiments of this nature; another illustration, however, is furnished by the metal antimony, which possesses a greater conductive power, but a less diamagnetic power than bismuth. The following results were obtained with this substance:—

VI.

Cylinders of Antimony.

length 3 inches.
diameter 0·7.

	Current direct†.	Current reversed‡.
Position 1.	693	244
Position 2.	688	252
Position 3.	683	261

* Philosophical Magazine, Series 4, vol. vii. p. 37.
† As in III. and IV. ‡ As in I. and II.

On comparing these numbers with those already obtained with bismuth, we observe that for like positions the actions of both metals are alike in direction. We further observe that the results are determined, not by the relative conductive powers of the two metals, but by their relative diamagnetic powers. If the former were the determining cause, we should have greater deflections than with the bismuth, which is not the case; if the latter, we should have less deflections, which is the case.

The third and severest condition proposed by those who object to the experiments of M. WEBER is to substitute insulators for conductors. I call this condition severe for the following reasons:—according to the experiments of FARADAY*, when bismuth and sulphur are submitted to the same magnetic force, the repulsion of the former being expressed by the number 1968, that of the latter will be expressed by 118. Hence an action which, with the means hitherto used, was difficult of detection in the case of bismuth, must wholly escape observation in the case of sulphur, the intensity of whose excitement is nearly twenty times less. The same remarks apply, in a great measure, to all other insulators.

But the admirable apparatus made use of in this investigation has enabled me to satisfy this condition also. To Mr. FARADAY I am indebted for the loan of two prisms of the self-same heavy glass with which he made the discovery of diamagnetism. The bismuth cylinders were withdrawn from the helices and the prisms of glass put in their places. It was now necessary to have a perfectly steady magnet, the expected result being so small as to be readily masked by, or confounded with, a motion arising from some extraneous disturbance. The feeble warmth developed in the helices by a current from two cells I found able to create air currents of sufficient power to defeat all attempts to obtain the pure action of the prisms. To break up these currents I stuffed all unfilled spaces of the box with old newspapers, and found the expedient to answer perfectly. With a fresh battery, which delivered a constant stream throughout the duration of an experiment, the magnet was admirably steady†, and under these favourable conditions the following results were obtained:—

VII.

Prisms of Heavy Glass.

length 3 inches.
width 0·6.
depth 0·5.

	Current direct.
Position 1.	664
Position 2.	662
Position 3.	660

* Proceedings of the Royal Institution, 1853, p. 5.

† It was necessary however to select a portion of the day when Albemarle Street was free from cabs and carriages, as the shaking of the entire building, by the rolling of these vehicles, rendered the magnets unsteady.

Thus in passing from position 1 to 3, or *vice versâ*, a permanent deflection corresponding to four divisions of the scale was produced. By raising or lowering the respective prisms at the proper moments the amplitude of the oscillations could be considerably augmented, and, when at a maximum, could be speedily extinguished by reversing the motions of the prisms. In six different series of experiments made with this substance the same invariable result was obtained. It will be observed that the deflections are in all cases in the same direction as those produced by bismuth under the same circumstances.

The following results were afterwards obtained with the same prisms in the presence of M. De la Rive; the current was "direct."

VIII.

Position 1.	652
Position 2.	650
Position 3.	648

On the negative result arrived at with this substance, it will be remembered that M. v. Feilitzsch bases one of his arguments against the conclusions of M. Weber.

Calcareous spar was next submitted to experiment. Two cylinders of the transparent crystal were prepared and examined in the manner already described. The results are as follows:—

IX.

Cylinders of Calcareous Spar.

length 3 inches.
diameter 0·7.

	Current direct.
Position 1.	699·5.
Position 2.	698·5.
Position 3.	697·5.

Here, as in the other cases, the deflection was permanent, and could be augmented by the suitable raising or lowering of the respective cylinders. The action is small, but perfectly certain. The magnet was steady and moved promptly and invariably in the directions indicated by the numbers. It will be observed that the deflections are the same in kind as those produced by bismuth.

The intrusion of other employments compelled me to postpone the continuation of these experiments for several weeks. On taking up the subject again, my first care was to assure myself that the instrument retained its sensibility. Since the experiments last recorded it had been transported over several hundred miles of railway, and hence the possibility of a disturbance of its power. The following experiments,

while they corroborate the former ones, show that the instrument retained its power and delicacy unimpaired:—

X.

Bismuth cylinders.

	Current direct.	Current reversed.
Position 1.	612	264
Position 2.	572	230
Position 3.	526	200

The deflections, it will be observed, are the same in kind as before; but by improved manipulation the effect is augmented. In passing from position 1 to 3 we have here a deflection amounting in one case to 64, and in the other to 86 divisions of the scale.

To Mr. Noble I am indebted for two cylinders of pure statuary marble; the examination of these gave the following results:—

XI.

Cylinders of Statuary Marble.

length 4 inches.
diameter 0·7.

	Current direct.	Current reversed.
Position 1.	601	215
Position 2.	598	218
Position 3.	596	220

Here, in passing from position 1 to 3, we have a permanent deflection corresponding to five divisions of the scale. As in all other cases, the impulsion of the magnet might be augmented by changing the position of the cylinders at the limit of each swing. The deflections are the same in kind as those produced by bismuth, which ought to be the case, as marble is diamagnetic.

An upright iron stove influenced by the earth's magnetism becomes a magnet with its bottom a north and its top a south pole. Doubtless, though in an immensely feebler degree, every erect marble statue is a true diamagnet, with its head a north pole and its feet a south pole. The same is certainly true of man as he stands upon the earth's surface, for all the tissues of the human body are diamagnetic.

A pair of cylinders of phosphorus enclosed in thin glass tubes were next examined.

XII.

Cylinders of Phosphorus.

length 3·5 inches.
diameter 0·63.

	Current direct.		Current reversed.
	Series I.	Series II.	
Position 1.	620	670	224
Position 2.	618	668	226
Position 3.	616	666	228

The change of the bars from position 1 to 3 is in this case accompanied by permanent deflection corresponding to four divisions of the scale. The deflection indicates the polarity of diamagnetic bodies. The magnet was remarkably steady during these experiments, and the consequent clearness and sharpness of the result pleasant to observe.

XIII.

Cylinders of Sulphur.

length 6 inches.
diameter 0·7.

	Current direct.	Current reversed.
Position 1.	658·5	222
Position 2.	657	223·5
Position 3.	655·5	225·5

XIV.

Cylinders of Nitre.

length 3·5 inches.
diameter 0·7.

	Current direct.	Current reversed.
Position 1.	648·5	263
Position 3.	647	265

Finally, as regards solid diamagnetics, a series of experiments was made with wax; this also being one of the substances whose negative deportment is urged by M. v. Feilitzsch against M. Weber.

XV.

Cylinders of Wax.

length 4 inches.
diameter 0·7.

	Current direct.	Current reversed.
Position 1.	624·5	240
Position 3.	623	241

The action is very small, but it is nevertheless perfectly certain. The argument founded on the negative deportment of this substance must therefore give way.

When we consider the feebleness of the action with so delicate a means of examination, the failure of M. v. FEILITZSCH to obtain the effect, with an instrument constructed by himself, will not excite surprise.

Thus in the case of seven insulating bodies the existence of diamagnetic polarity has been proved; the list might be augmented without difficulty, but sufficient, I trust, has been done to remove the scruples of those who saw in M. WEBER's results an action produced by induced currents.

A portion of the subject hitherto untouched, but one of great interest, has reference to the polar condition of liquid bodies while under magnetic influence. The first liquid examined was distilled water; it was enclosed in thin glass tubes, corked at the ends, and by means of a loop passing round the cork the tubes were attached to the string passing round the wheels WW'. Previous to using, the corks were carefully cleansed, so that any impurity contracted in cutting, or by contact with ferruginous matters, was completely removed. The following are the results obtained with this liquid:—

XVI.

Cylinders of Distilled Water.

length 4 inches.
diameter 0·65.

	Current direct.	Current reversed.
Position 1.	605	246
Position 2.	603	248
Position 3.	601	250

The experiment was many times repeated, but always with the same result; indeed the polarity of the liquid mass is as safely established as that of iron. Pure water is diamagnetic, and the deflections produced by it are the same as those of all the other diamagnetic bodies submitted to examination.

From the position which it occupies in Mr. FARADAY's list*, I had also some hopes of proving the polarity of sulphide of carbon. The following results were obtained:—

XVII.

Cylinders of Bisulphide of Carbon.

length 4 inches.
diameter 0·65.

	Current direct.	Current reversed.
Position 1.	631	210
Position 2.	629	213
Position 3.	626	216

* Proceedings of the Royal Institution, 1853, p. 5.

As in the case of distilled water, we observe a deflection in one direction when the current is "direct" and in the other when it is "reversed," the action in the first case, in passing from position 1 to 3, amounting to five and in the latter case to six divisions of the scale. The polarity exhibited is that of diamagnetic bodies.

DEPORTMENT OF MAGNETIC BODIES.

Thus far we have confined our examination to diamagnetic substances; turn we now to the deportment of magnetic bodies when submitted to the same conditions of experiment. Here we must select the substances suitable for examination, for all are not so. Cylinders of iron, for example, of the same size as our diamagnetic cylinders, would, through the intensity of their action, quite derange the apparatus; so that we are obliged to have recourse to bodies of smaller size or of feebler magnetic capacity. Besides, the remarks of writers on this subject render it of importance to examine whether bodies through which the magnetic constituents are very sparingly distributed present a veritable polarity the same as that exhibited by iron itself.

Slate rock usually contains from eight to ten per cent. of oxide of iron, and a fragment of the substance presented to the single pole of an electro-magnet is attracted by the pole. A cylinder of slate from the Penrhyn quarries near Bangor was first examined. It was not found necessary to increase the effect by using two cylinders, and the single one used was suspended in the right-hand helix H′E′. The deportment of the substance was as follows:—

XVIII.

Cylinder of Penrhyn Slate.

length 4 inches.
diameter 0·7.

	Current direct.	Current reversed.
Position 1.	620	280
Position 2.	647	240
Position 3.	667	198

Comparing these deflections with those obtained with diamagnetic bodies, we see that they are in the opposite direction. With the direct current a change from position 1 to 3 is followed, in the case of diamagnetic bodies, by a motion from higher to lower numbers; while in the present instance the motion is from lower numbers to higher. In the former case the north poles of the astatic magnet are attracted, in the latter they are repelled. We also see that a *direct* current acting on diamagnetic bodies produces the same deflection as a *reverse* current on magnetic ones. Thus, as we promised at a former page, the opposite polarities of diamagnetic and magnetic bodies are transferred from the region of deduction to that of fact.

XIX.

Cylinder of Caermarthen Slate.
length 4 inches.
diameter 0·7.

	Current direct.	Current reversed.
Position 1.	664	300
Position 2.	690	235
Position 3.	720	185

The deflections in this case are also indicative of magnetic polarity.

These two cylinders were so taken from the rock that the axis of each lay in the plane of cleavage. The following experiments, made with a cylinder of the same size, show the capability of a rock of this structure to be magnetized across the planes of cleavage.

XX.

Cylinder of Slate: axis of cylinder perpendicular to cleavage.

	Current direct.	Current reversed.
Position 1.	655	240
Position 2.	678	205
Position 3.	695	192

Chloride of iron was next examined: the substance, in powder, was enclosed in a single glass tube, which was attached to the string passing round the wheels WW′ of the instrument.

XXI.

Cylinder of Chloride of Iron.
length 3·8 inches.
diameter 0·5.

	Current direct.	Current reversed.
Position 1.	185	990
Position 2.	——	230
Position 3.	990	185

The deflection here indicates ordinary magnetic polarity. The action was very powerful. When swiftly moving in any direction, a change in the position of the cylinder instantly checked the magnet in its course, brought it to rest, or drove it forcibly in the opposite direction. The numbers 185 and 990 mark indeed the utmost limit between which it was possible for the magnet to move; here it rested against the helices.

Two glass tubes were filled with red oxide of iron and examined. The action of the poles of these cylinders upon the magnets was so strong, as to efface, by the velocity imparted to the magnets, all distinct impression of the numbers on the scale.

By changing the position of the tubes within the helices, the magnets could be driven violently through the field of view, or could be held rigidly against the respective helices. As in all other cases, the centre of the cylinders were neutral points, and the two ends of each were poles of opposite qualities. The polarity was of course the same as that of iron.

A small quantity of iron filings were kneaded thoroughly in wax, and a cylinder formed from the mass. Its deportment was also very violent, and its polarity was of course just as clear and pronounced as that of a solid cylinder of iron could possibly be.

Sulphate of iron was next examined: the crystallized substance was enclosed in two glass tubes and tested in the usual manner.

XXII.

Cylinders of Sulphate of Iron.

length 4·5 inches.
diameter 0·7.

	Current direct.	Current reversed.
Position 1.	510	510
Position 2.	600	370
Position 3.	700	220

The red ferroprussiate of potassa is a magnetic salt; with this substance the following results were obtained:—

XXIII.

Cylinders of red Ferroprussiate of Potassa.

length 4·5 inches.
diameter 0·65.

	Current direct.	Current reversed.
Position 1.	610	250
Position 2.	630	220
Position 3.	655	197

In this case also the crystallized salt was enclosed in glass tubes.

Two glass tubes were next filled with carbonate of iron in the state of powder: the following are the results:—

XXIV.

Cylinders of Carbonate of Iron.

length 4 inches.
diameter 0·5.

	Current direct.
Position 1.	185
Position 2.	620
Position 3.	740

In all these cases the deflections show that the bodies are polar after the manner of iron.

As the complement of the experiments made with diamagnetic liquids, we now pass on to the examination of the polarity of magnetic liquids. A concentrated solution of sulphate of iron was enclosed in two glass tubes and submitted to examination.

XXV.

Sulphate of Iron solution in tubes.

length 4 inches.
diameter 0·65.

	Current direct.
Position 1.	548
Position 2.	600
Position 3.	648.

A solution of muriate of nickel, examined in the same manner, gave the following results:—

XXVI.

Muriate of Nickel solution in tubes.

length 3·6 inches.
diameter 0·65.

	Current direct.	Current reversed.
Position 1.	605	224
Position 2.	632	200
Position 3.	650	185

A solution of muriate of cobalt yielded as follows:—

XXVII.

Muriate of Cobalt solution in tubes.

length 3·6 inches.
diameter 0·65.

	Current direct.	Current reversed.
Position 1.	630	262
Position 2.	645	235
Position 3.	660	202

In all these cases we have ample evidence of a polar action the reverse of that exhibited by diamagnetic liquids. These, I believe, are the first experiments on which the action of either liquid magnets or liquid diamagnets upon a suspended steel magnet has been exhibited.

Thus far then the following substances have been submitted to examination :—

Diamagnetic bodies.	Magnetic bodies.
Bismuth.	Penrhyn slate.
Antimony.	Caermarthen slate.
Heavy glass.	Slate perpendicular to cleavage.
Calcareous spar.	Chloride of iron.
Statuary marble.	Sulphate of iron.
Phosphorus.	Carbonate of iron.
Sulphur.	Ferrocyanide of potassium.
Nitre.	Oxide of iron.
Wax.	Iron filings.
Liquids.	Liquids.
Distilled water.	Sulphate of iron.
Bisulphide of carbon.	Muriate of nickel.
	Muriate of cobalt.

Every substance in each of these lists has been proved to be polar under magnetic influence, the polarity of the diamagnetic bodies being invariably opposed to that of the magnetic ones.

In his investigation on the supposed polarity of diamagnetic bodies, Mr. Faraday made use of a core of sixpenny pieces, and obtained with it the results he sought. Wishing to add the testimony of silver as a good conductor to that of copper, two cylinders were formed of sixpenny pieces, covered with paper and submitted to experiment. The following are the results obtained :—

XXVIII.

Silver cylinders (sixpenny pieces).

	Current direct.
Position 1.	724
Position 2.	774
Position 3.	804

The action here was prompt and energetic, strongly contrasted with the neutrality of copper; but the deflection was permanent, and could not therefore be the result of induced currents. Further, it was a deflection which shows magnetic polarity, whereas pure silver is feebly diamagnetic. The cylinders were removed and examined between the poles of an electro-magnet; they proved to be magnetic.

On observing this deportment of the silver, I tried the copper cylinders once more. The results with a direct current were,—

XXIX.

Position 1.	766
Position 2.	767
Position 3.	768

Here almost the same neutrality as before is evidenced.

Deeming that the magnetism of the cores of silver was due to magnetic impurity attaching itself to the paper which covered them, a number of fourpenny pieces were procured, washed in ammonia and water, and enclosed in thin glass tubes. The following were the results :—

XXX.

Silver cylinders (fourpenny pieces).

	Current direct.
Position 1.	490
Position 2.	565
Position 3.	660

Here also we have a very considerable action indicative of magnetic polarity. On examining the cylinders between the poles of an electro-magnet, they were found decidedly magnetic. This, therefore, appears to be the common character of our silver coins. The tubes which contained the pieces were sensibly neutral.

Knowing the difficulty of demonstrating the existence of diamagnetic polarity in ordinary insulators, M. Matteucci suggested that insulated fragments of bismuth ought to be employed, the insulation being effected by a coat of lac or resin. I constructed a pair of cylinders in accordance with the suggestion of M. Matteucci. The following are the results they yielded with a direct current :—

XXXI.

Position 1.	730
Position 2.	750
Position 3.	768

Here we have a very marked action, but the polarity indicated is magnetic polarity. On subsequent examination, the cylinders proved to be magnetic. This was due to impurities attaching themselves to the resin.

But the resin may be done away with and the powdered metal still rendered an insulator. This thought was suggested to me by an experiment of Mr. Faraday, which I will here describe. Referring to certain effects obtained in his investigations on supposed diamagnetic polarity, he writes thus :—" If the effect were produced by induced currents in the mass, division of the mass would stop these currents, and so alter the effect; whereas, if produced by *a true diamagnetic polarity*, division of the mass would not affect the polarity seriously or in its essential nature. Some copper filings were therefore digested for a few days in dilute sulphuric acid to remove any adhering iron, then well washed and dried, and afterwards warmed and stirred in the air, until it was seen by the orange colour that a very thin film of oxide had formed upon them; they were finally introduced into a glass tube and employed as a core. It produced no effect whatever, but was as inactive as bismuth." (Exper. Resear. 2658.)

Now when bismuth is powdered and exposed to the action of the air, it very soon becomes tarnished, even without heating. A quantity of such powder was prepared, and its conducting power for electricity tested. The clean ends of two copper wires proceeding from a battery of GROVE were immersed in the powder; but though the wires were brought as near as possible to each other, short of contact, not the slightest action was observed upon a galvanometer placed in the circuit. When the wires touched, the needle of the galvanometer flew violently aside, thus proving that the current was there, but that the powder was unable to conduct it. Two glass tubes were filled with the powder and submitted to experiment. The following results were obtained:—

XXXII.

Cylinders of Bismuth Powder.

length 3 inches.
diameter 0·7.

	Current direct.	Current reversed.
Position 1.	640	230
Position 2.	625	245
Position 3.	596	260

These deflections are the same in kind as those obtained with the cylinders of massive bismuth. This experiment responds perfectly to the conditions proposed by Mr. FARADAY. We have here no cessation of action. The division of the mass does not affect the result seriously or in its essential nature, and hence the deportment exhibits the characteristics of "a true diamagnetic polarity."

In summing up the results of his inquiry on this subject, Mr. FARADAY writes thus:—"Finally, I am obliged to say that I can find no experimental evidence to support the hypothetical view of diamagnetic polarity, either in my own experiments, or in the repetition of those of WEBER, REICH and others. .... It appears to me also, that, as magnetic polarity conferred by iron or nickel in small quantity, and in unfavourable states, is far more easily indicated by its effects upon an astatic needle, or by pointing between the poles of a strong horseshoe magnet, than by any such arrangement as mine or WEBER'S or REICH'S, so *diamagnetic polarity would be much more easily distinguished in the same way*." I was struck, on reading this passage, to find how accurately the surmise has been fulfilled by the instrument with which the foregoing experiments were made. In illustration of the powers of this instrument, as compared with that made use of by Mr. FARADAY, I may be permitted to quote the following result from his paper on supposed diamagnetic polarity, so often referred to:—"A thin glass tube, 5½ inches by three-quarters of an inch, was filled with a saturated solution of protosulphate of iron, and employed as an experimental core; the velocity given to the machine at this and all average times was such as to cause five or six approaches and withdrawals of the core in one second;

yet the solution produced no sensible indication on the galvanometer." Referring to Table XXV., it will be seen that the instrument made use of in the present inquiry has given with a solution of protosulphate of iron a deflection amounting to no less than one hundred divisions of the scale. Mr. FARADAY proceeds:—"A tube filled with small crystals of protosulphate of iron caused the needle to move about 2°. . . . Red oxide of iron produced the least possible effect." In the experiments recorded in the foregoing pages, the crystallized sulphate of iron gave a deflection of nearly two hundred divisions of the scale, while the red oxide gave a deflection as wide as the helices would permit, which corresponds to about eight hundred divisions of the scale. The correctness of Mr. FARADAY's statement regarding the inferiority of the means first devised to investigate this subject, is thus strikingly illustrated. It might be added, that red ferroprussiate of potash and other substances, which have given us powerful effects, produced no sensible impression in experiments made with the other instrument.

Thus have we seen the objections raised against diamagnetic polarity fall away one by one, and a body of evidence accumulated in its favour, which places it among the most firmly established truths of science. This I cannot help thinking is mainly to be attributed to the bold and sincere questioning of the principle when it seemed questionable. The cause of science is more truly served, even by the denial of what may be a truth, than by the indolent acceptance of it on insufficient grounds. Such denials drive us to a deeper communion with Nature, and, as in the present instance, compel us through severe and laborious inquiry to strive after certainty, instead of resting satisfied, as we are prone to do, with mere probable conjecture.

Royal Institution,
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XII. *On Axes of Elasticity and Crystalline Forms.*

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§ 1. *General Definition of Axes of Elasticity.*

As originally understood, the term "Axes of Elasticity" was applied to the intersections of three orthogonal planes at a given point of an elastic medium, with respect to each of which planes the molecular actions causing elasticity were conceived to be symmetrical.

If the elasticity of solids arose either wholly from the mutual attractions and repulsions of centres of force, such attractions and repulsions being functions of the mutual distances of those centres, or partly from such mutual actions, and partly from an elasticity like that of a fluid, resisting change of volume only, it is easy to prove that there would be three such orthogonal planes of symmetry of molecular action in every homogeneous solid.

But there is now no doubt that the elastic forces in solid bodies are not such as can be analysed into fluid elasticity and mutual attractions between centres simply; and though there are, as will presently be shown, orthogonal planes of symmetry for certain kinds of elastic forces, those planes are not necessarily the same for all kinds of elastic forces in a given solid.

The term "*Axes of Elasticity,*" therefore, may now be taken in a more extended sense, to signify *all directions, with respect to which certain kinds of elastic forces are symmetrical*; or speaking algebraically, *directions for which certain functions of the coefficients of elasticity are null or infinite.*

The theory of Axes and Coefficients of Elasticity is specially connected with that branch of the Calculus of Forms which relates to linear transformations, and which has recently been so greatly advanced by the researches of Mr. Sylvester, Mr. Cayley, and Mr. Boole. In such applications of that Calculus as occur in this paper, the nomenclature of Mr. Sylvester is followed *; and by the adoption of the "*Umbral Notation*" of that author, immense advantages are gained in conciseness and simplicity †.

* See Cambridge and Dublin Mathematical Journal, vol. vii.; and Philosophical Transactions, 1853.

† See the Note at the end of the paper.

2. *Strains, Stresses, Potential Energy, and Coefficients of Elasticity.*

In this paper, the word "*Strain*" will be used to denote the change of volume and figure constituting the deviation of a molecule of a solid from that condition which it preserves when free from the action of external forces; and the word "*Stress*" will be used to denote the force, or combination of forces, which such a molecule exerts in tending to recover its free condition, and which, for a state of equilibrium, is equal and opposite to the combination of external forces applied to it.

In framing a nomenclature for quantities connected with the theory of elasticity, θλίψις is adopted to denote *strain*, and τάσις to denote *stress*.

It is well known that the condition of *strain* at any given point in the interior of a molecule may be completely expressed by means of the following six *elementary strains*, in which ξ, η, ζ are the components of the molecular displacement parallel to three rectangular axes x, y, z.

$$\text{Elongations} \quad \ldots \quad \frac{d\xi}{dx}=\alpha; \quad \frac{d\eta}{dy}=\beta; \quad \frac{d\zeta}{dz}=\gamma;$$

$$\text{Distortions} \quad \ldots \quad \frac{d\zeta}{dy}+\frac{d\eta}{dz}=\lambda; \quad \frac{d\xi}{dz}+\frac{d\zeta}{dx}=\mu; \quad \frac{d\eta}{dx}+\frac{d\xi}{dy}=\nu.$$

It is also well known that the condition of *stress* at a given point may be completely expressed, relatively to the three rectangular coordinate planes, by means of six *elementary stresses*, viz.—

$$\text{Normal Pressures} \quad \ldots \quad P_1, \quad P_2, \quad P_3,$$
$$\text{Tangential Pressures} \quad \ldots \quad Q_1, \quad Q_2, \quad Q_3;$$

these quantities being estimated in units of force per unit of surface.

Let each elementary stress be integrated with respect to the elementary strain which it tends directly to diminish, *from* the actual amount of that strain, *to* the condition of freedom; the sum of the integrals is the *Potential* Energy of Elasticity of the molecule $dxdydz$, expressed in units of work per unit of volume; viz.—

$$\begin{aligned} U=&\int_\alpha^0 P_1 d\alpha+\int_\beta^0 P_2 d\beta+\int_\gamma^0 P_3 d\gamma \\ &+\int_\lambda^0 Q_1 d\lambda+\int_\mu^0 Q_2 d\mu+\int_\nu^0 Q_3 d\nu \quad \ldots \ldots \quad (1.) \end{aligned}$$

The condition that the function U shall have the same value, in what order soever the variations of the different elementary strains take place, amounts to supposing, that no transformation of energy of the kind well distinguished by Professor THOMSON as *frictional* or *irreversible* takes place during such variations; in other words, that the substance is *perfectly elastic*.

Each of the elementary stresses being sensibly a linear function of the six elementary strains, the Potential Energy of Elasticity is, as Mr. GREEN first showed, a function

of those strains of the second degree, having twenty-one constant coefficients, which are the coefficients of elasticity of the body, and will in this paper be called the *Tasinomic Coefficients*; that is to say, adopting Mr. GREEN's notation for such coefficients,—

$$\begin{aligned}U=(\alpha^2)\frac{\alpha^2}{2}+(\beta^2)\frac{\beta^2}{2}+(\gamma^2)\frac{\gamma^2}{2}+(\lambda^2)\frac{\lambda^2}{2}+(\mu^2)\frac{\mu^2}{2}+(\nu^2)\frac{\nu^2}{2}&\\
+(\beta\gamma)\beta\gamma+(\gamma\alpha)\gamma\alpha+(\alpha\beta)\alpha\beta&\\
+(\mu\nu)\mu\nu+(\nu\lambda)\nu\lambda+(\lambda\mu)\lambda\mu&\\
+(\alpha\lambda)\alpha\lambda+(\beta\mu)\beta\mu+(\gamma\nu)\gamma\nu&\\
+(\beta\lambda)\beta\lambda+(\gamma\mu)\gamma\mu+(\alpha\nu)\alpha\nu&\\
+(\gamma\lambda)\gamma\lambda+(\alpha\mu)\alpha\mu+(\beta\nu)\beta\nu. \quad \ldots \ldots \quad &(2.)\end{aligned}$$

From a theorem of Mr. SYLVESTER it follows, that every such function as U is reducible by linear transformations to the sum of six positive squares, each multiplied by a coefficient. The nature and meaning of this reduction have been discussed by Professor WILLIAM THOMSON.

The following classification of the Tasinomic Coefficients will be used in the sequel:—

	Designation of Coefficients.	Elasticities.	Symbols.		
Orthotatic	Euthytatic . . .	Direct or Longitudinal .	(α^2)	(β^2)	(γ^2)
Orthotatic	Platytatic . . .	Lateral	$(\beta\gamma)$	$(\gamma\alpha)$	$(\alpha\beta)$
Orthotatic	Goniotatic . . .	Rigidities	(λ^2)	(μ^2)	(ν^2)
Plagiotatic		Unsymmetrical . . .	$(\mu\nu)$, &c. &c.		

The twenty-one equations of transformation by which the values of these coefficients, being known for any one set of orthogonal axes, are found for any other, are founded on the following principles.

It is well known, that for rectangular transformations, the operations

$$\frac{d}{dx}, \quad \frac{d}{dy}, \quad \frac{d}{dz}$$

are respectively covariant with

$$x, \quad y, \quad z,$$

from which it is easily deduced, that because the displacements

$$\xi, \quad \eta, \quad \zeta$$

are respectively covariant with

$$x, \quad y, \quad z,$$

therefore the elementary strains,

$$\alpha, \quad \beta, \quad \gamma, \quad \lambda, \quad \mu, \quad \nu$$

the operations,

$$\frac{d}{d\alpha}, \quad \frac{d}{d\beta}, \quad \frac{d}{d\gamma}, \quad 2\frac{d}{d\lambda}, \quad 2\frac{d}{d\mu}, \quad 2\frac{d}{d\nu}$$

and the strains

$$P_1, \quad P_2, \quad P_3, \quad 2Q_1, \quad 2Q_2, \quad 2Q_3$$

must be respectively covariant with the squares and products,

$$x^2, \quad y^2, \quad z^2, \quad 2yz, \quad 2zx, \quad 2xy.$$

3. *Thlipsimetric and Tasimetric Surfaces and Invariants.*

Isotropic functions of the elementary strains and stresses, which may be called respectively *Thlipsimetric* and *Tasimetric Invariants*, are easily deduced from the principle, that the strains may be represented by the coefficients of the following *Thlipsimetric Surface*,

$$\alpha x^2+\beta y^2+\gamma z^2+\lambda yz+\mu zx+\nu xy=1, \quad \ldots \ldots \ldots \quad (3.)$$

and the stresses by the coefficients of the *Tasimetric Surface*,

$$P_1x^2+P_2y^2+P_3z^2+2Q_1yz+2Q_2zx+2Q_3xy=1. \quad \ldots \ldots \quad (4.)$$

These surfaces, and others deduced from them, have been fully discussed by M. CAUCHY and M. LAMÉ.

The invariants in question may all be deduced from the following pair of contragredient matrices;—

$$\begin{array}{ccc} \text{For Strains.} & & \text{For Stresses.} \\ (5.)\left\{\begin{matrix} \alpha & \frac{\nu}{2} & \frac{\mu}{2} \\[6pt] \frac{\nu}{2} & \beta & \frac{\lambda}{2} \\[6pt] \frac{\mu}{2} & \frac{\lambda}{2} & \gamma \end{matrix}\right. & \qquad & \left.\begin{matrix} P_1 & Q_3 & Q_2 \\[6pt] Q_3 & P_2 & Q_1 \\[6pt] Q_2 & Q_1 & P_3 \end{matrix}\right\}(5\text{ A.}) \end{array}$$

The following are the primitive thlipsimetric invariants, from which an indefinite number of others may be deduced by involution, multiplication, addition, and subtraction:—

$$\left.\begin{aligned} &\alpha+\beta+\gamma=\theta_1 \text{ (the cubic dilatation)};\\ &\beta\gamma+\gamma\alpha+\alpha\beta-\tfrac{1}{4}(\lambda^2+\mu^2+\nu^2)=\theta_2;\\ &\alpha\beta\gamma+\tfrac{1}{4}\lambda\mu\nu-\tfrac{1}{4}(\alpha\lambda^2+\beta\mu^2+\gamma\nu^2)=\theta_3. \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (6.)$$

The Potential Energy U is what Mr. SYLVESTER calls a "Universal Mixed Concomitant," its value being

$$U=-\tfrac{1}{2}(P_1\alpha+P_2\beta+P_3\gamma+Q_1\lambda+Q_2\mu+Q_3\nu). \quad \ldots \ldots \quad (7.)$$

4. *Tasinomic Functions, Surfaces, and Umbræ.*

If, in any isotropic function of the coordinates and the elementary strains, there be substituted for each square or product of elementary strains, that Tasinomic Coefficient which is covariant with it, the result will be an Isotropic Function of the Coordinates and Tasinomic Coefficients, called a *Tasinomic Function.*

The following Table of Covariants is readily deduced from the principles stated at the end of § 2 :—

$$\left.\begin{array}{llllllll}
\text{Covariant}\begin{cases}\text{Squares of Strains} & \alpha^2, & \beta^2, & \gamma^2, & \lambda^2, & \mu^2, & \nu^2, \\ \text{Tasinomic Coefficients} & (\alpha^2), & (\beta^2), & (\gamma^2), & 4(\lambda^2), & 4(\mu^2), & 4(\nu^2) ;\end{cases} \\
\text{Covariant}\begin{cases}\text{Products of Strains} & \beta\gamma, & \gamma\alpha, & \alpha\beta, & \mu\nu, & \nu\lambda, & \lambda\mu, \\ \text{Tasinomic Coefficients} & (\beta\gamma), & (\gamma\alpha), & (\alpha\beta), & 4(\mu\nu), & 4(\nu\lambda), & 4(\lambda\mu),\end{cases} \\
\begin{array}{lllllllll} \alpha\lambda, & \alpha\mu, & \alpha\nu, & \beta\lambda, & \beta\mu, & \beta\nu, & \gamma\lambda, & \gamma\mu, & \gamma\nu, \\ 2(\alpha\lambda), & 2(\alpha\mu), & 2(\alpha\nu), & 2(\beta\lambda), & 2(\beta\mu), & 2(\beta\nu), & 2(\gamma\lambda), & 2(\gamma\mu), & 2(\gamma\nu). \end{array}
\end{array}\right\} \quad (8.)$$

Each Tasinomic Function being equated to a constant, forms the equation of a *Tasinomic Surface*; and on the geometrical properties of such surfaces depend many of the laws of coefficients and Axes of Elasticity.

A convenient and expeditious mode of forming Tasinomic Functions is obtained by the aid of an *Umbral Notation* analogous to that introduced by Mr. SYLVESTER in the Calculus of Forms.

Let each Tasinomic Coefficient be regarded as compounded of two *Tasinomic Umbræ,* those umbræ being expressed by the following notation :

$$(\alpha), \quad (\beta), \quad (\gamma), \quad (\lambda), \quad (\mu), \quad (\nu) ;$$

then the following equation, deduced from that of the Thlipsimetric Surface (3), by substituting umbræ for elementary strains according to the following Table of Covariance,

Strains . .	α,	β,	γ,	λ,	μ,	ν,
Umbræ . .	(α),	(β),	(γ),	$2(\lambda)$,	$2(\mu)$,	$2(\nu)$,

is the equation of the *Tasinomic Umbral Ellipsoid,* from which, by elimination, multiplication, involution, addition, subtraction, and differentiation, various Tasinomic Functions may be deduced,

$$(\alpha)x^2+(\beta)y^2+(\gamma)z^2+2(\lambda)yz+2(\mu)zx+2(\nu)xy=(\phi)=1. \quad . \quad . \quad . \quad . \quad (8\text{ A.})$$

5. *Tasinomic Invariants and Spheres.*

Tasinomic Invariants are constant Isotropic functions of the Tasinomic Coefficients, which are deduced, either by substitution from Thlipsimetric Invariants, or directly from the *Umbral Matrix,*

$$\left.\begin{matrix} (\alpha) & (\nu) & (\mu) \\ (\nu) & (\beta) & (\lambda) \\ (\mu) & (\lambda) & (\gamma) \end{matrix}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.)$$

The following invariant is umbral of the first order :—

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)\cdot(\phi)=(\alpha)+(\beta)+(\gamma)=(\theta_1). \quad . \quad . \quad . \quad . \quad (9a.)$$

Invariants of the second order in Umbræ are real quantities of the first order, viz.—

$$(\alpha^2)+(\beta^2)+(\gamma^2)+2(\beta\gamma)+2(\gamma\alpha)+2(\alpha\beta)=(\theta_1)^2 \text{ (the \textit{cubic elasticity})}$$
$$(\beta\gamma)+(\gamma\alpha)+(\alpha\beta)-(\lambda^2)-(\mu^2)-(\nu^2)=(\theta_2)$$
$$(\alpha^2)+(\beta^2)+(\gamma^2)+2(\lambda^2)+2(\mu^2)+2(\nu^2)=(\theta_1)^2-2(\theta_2). \quad . \quad . \quad . \quad . \quad . \quad (10.)$$

The equation of a *Tasinomic Sphere* is formed by multiplying a Tasinomic Invariant by

$$x^2+y^2+z^2,$$

or any power of that quantity, and equating the result to a constant.

6. *Of Two Tasinomic Ellipsoids, and their Axes, Orthotatic and Heterotatic.*

The equations of two Ellipsoids with tasinomic coefficients are derived from that of the Umbral Ellipsoid (8A.), in one case, by multiplying each term by the Umbral Invariant (θ_1), and in the other, by substituting for each Umbra in the function (ϕ), the contravariant component of the *Inverse* to the Umbral Matrix (9.). The results are as follows:—

ORTHOTATIC ELLIPSOID.

$$(\theta_1)\times(\phi)=\{(\alpha^2)+(\alpha\beta)+(\gamma\alpha)\}x^2+\{(\beta^2)+(\beta\gamma)+(\gamma\alpha)\}y^2$$
$$+\{(\gamma^2)+(\gamma\alpha)+(\beta\gamma)\}z^2$$
$$+2\{(\alpha\lambda)+(\beta\lambda)+(\gamma\lambda)\}yz+2\{(\alpha\mu)+(\beta\mu)+(\gamma\mu)\}zx+2\{(\alpha\nu)+(\beta\nu)+(\gamma\nu)\}xy=1. \quad (11.)$$

HETEROTATIC ELLIPSOID.

$$\{(\beta\gamma)-(\lambda^2)\}x^2+\{(\gamma\alpha)-(\mu^2)\}y^2+\{(\alpha\beta)-(\nu^2)\}z^2$$
$$+2\{(\mu\nu)-(\alpha\lambda)\}yz+2\{(\nu\lambda)-(\beta\mu)\}zx+2\{(\lambda\mu)-(\gamma\nu)\}xy=1. \quad . \quad . \quad . \quad (12.)$$

The three *Orthotatic Axes* are three rectangular directions for which the following sums of Plagiotatic Coefficients are null:—

$$(\alpha\lambda)+(\beta\lambda)+(\gamma\lambda)=0\,; \quad (\alpha\mu)+(\beta\mu)+(\gamma\mu)=0\,; \quad (\alpha\nu)+(\beta\nu)+(\gamma\nu)=0. \quad . \quad . \quad (13.)$$

It was proved by Mr. HAUGHTON, in a paper published in the Transactions of the Royal Irish Academy, vol. iii. part 2, that there are three rectangular directions having this property in a solid whose elasticity arises solely from the mutual actions of physical points, and which has but fifteen independent coefficients of elasticity. The present investigation shows that there are three such axes at each point of every solid, independently of all hypothesis. The physical meaning of this result is expressed by the following

THEOREM AS TO ORTHOTATIC AXES.

At each point of an elastic solid, there is one position in which a cubical molecule may be cut out, such, that a uniform dilatation or condensation of that molecule by equal elongations or equal compressions of its three dimensions, shall produce no tangential stress on the faces of the molecule.

The properties of the *Heterotatic Axes* are expressed by the following equations:—

$$(\mu\nu)-(\alpha\lambda)=0\,;\quad (\nu\lambda)-(\beta\mu)=0\,;\quad (\lambda\mu)-(\gamma\nu)=0\,;\quad .\ .\ .\ .\ \ (14.)$$

or by the following

THEOREM AS TO HETEROTATIC AXES.

At each point of an elastic solid, there is one position in which a cubical molecule may be cut out, such, that if there be a distortion of that molecule round x (x being any one of its three axes) *and an equal distortion round* y (y being either of its other two axes), *the normal stress on the faces normal to* x *arising from the distortion round* x, *shall be equal to the tangential stress round* z *arising from the distortion round* y.

The six coefficients of the Heterotatic Ellipsoid may be called the *Heterotatic Differences.* For a solid whose elasticity is wholly due to the mutual attractions and repulsions of physical points, each of those differences is necessarily null; therefore they represent a part of the elasticity which is necessarily irreducible to such attractions and repulsions. There is reason to believe that part at least of the elasticity of every substance is of this kind.

If this part of the elasticity of a solid be, as suggested in a series of papers in the Cambridge and Dublin Mathematical Journal for 1851–52, a species of *fluid elasticity*, resisting change of volume only, the solid may be said to be *Heterotatically Isotropic.* The equations (14.) will be fulfilled for all directions of axes, and also the following equations:—

$$(\beta\gamma)-(\lambda^2)=(\gamma\alpha)-(\mu^2)=(\alpha\beta)-(\nu^2)\,;\quad .\ .\ .\ .\ .\ .\ .\ \ (15.)$$

that is to say, the excess of the Platytatic above the Goniotatic Coefficient will be the same in every plane.

In a substance *Orthotatically Isotropic*, the equations (13.) are fulfilled for all directions, and also the following:—

$$(\alpha^2)+(\alpha\beta)+(\gamma\alpha)=(\beta^2)+(\beta\gamma)+(\alpha\beta)=(\gamma^2)+(\gamma\alpha)+(\beta\gamma),\quad .\ .\ .\ \ (16.)$$

that is to say, a uniform compression in all directions produces a uniform normal stress in all directions, and no tangential stress.

The equations (16.) may be reduced to the following form:—

$$(\alpha^2)-(\beta\gamma)=(\beta^2)-(\gamma\alpha)=(\gamma^2)-(\alpha\beta).\quad .\ .\ .\ .\ .\ .\ .\ \ (17.)$$

In a substance which is at once *Orthotatically* and *Heterotatically isotropic*, there may still be eleven independent quantities amongst the tasinomic coefficients, viz.—

$$\left.\begin{array}{ll}\text{Three Euthytatic Coefficients,} & (\alpha^2),\ (\beta^2),\ (\gamma^2),\\ \text{The isotropic excess}\ .\ .\ .\ . & (\alpha^2)-(\beta\gamma),\\ \text{The isotropic excess}\ .\ .\ .\ . & (\beta\gamma)-(\lambda^2),\\ \text{Six Plagiotatic Coefficients}\ \ . & (\beta\lambda),\ (\gamma\lambda),\ (\gamma\mu),\ (\alpha\mu),\ (\alpha\nu),\ (\beta\nu)\end{array}\right\}\ .\ .\quad (18.)$$

Such a substance may therefore be far from being completely isotropic with respect to elasticity.

7. *Biquadratic Tasinomic Surface. Homotatic Coefficients. Euthytatic Axes defined.*

If the equation (8A.) of the Umbral Ellipsoid be squared, there is obtained the following equation of a *Biquadratic Tasinomic Surface*:—

$$
\begin{aligned}
(\phi)^2 = {} & (\alpha^2)x^4+(\beta^2)y^4+(\gamma^2)z^4 \\
& +2\{(\beta\gamma)+2(\lambda^2)\}y^2z^2+2\{(\gamma\alpha)+2(\mu^2)\}z^2x^2+2\{(\alpha\beta)+2(\nu^2)\}x^2y^2 \\
& +4\{2(\mu\nu)+(\alpha\lambda)\}x^2yz+4\{2(\nu\lambda)+(\beta\mu)\}xy^2z+4\{2(\lambda\mu)+(\gamma\nu)\}xyz^2 \\
& +4(\beta\lambda)y^3z+4(\gamma\lambda)yz^3+4(\gamma\mu)z^3x+4(\alpha\mu)zx^3+4(\alpha\nu)x^3y+4(\beta\nu)xy^3=1. \quad (19.)
\end{aligned}
$$

The fifteen coefficients of this surface (which will be called the *Homotatic Coefficients*) are covariant respectively with the fifteen biquadratic powers and products of the coordinates, with proper numerical factors.

It is obvious, that when the fifteen Homotatic Coefficients, and the six Heterotatic Differences, are known for any set of Orthogonal Axes, the twenty-one tasinomic coefficients are completely determined.

Mr. HAUGHTON, in the paper previously referred to, discovered the biquadratic surface for a solid constituted of centres of force. It is here shown to exist for all solids, independently of hypotheses.

Those diameters of the Biquadratic Surface which are normal to that surface, are *axes of maximum and minimum direct elasticity*, and have also this property, that a direct elongation along one of them produces, on a plane perpendicular to it, a normal stress, and no tangential stress; so that they may be called *Euthytatic Axes.* Though such axes sometimes form Orthogonal Systems, their complete investigation requires the use of oblique coordinates, and is therefore deferred till after the eighteenth section of this paper, which relates to such coordinates.

8. *Orthogonal Axes of the Biquadratic Surface. Metatatic Axes, Orthogonal and Diagonal.*

By rectangular linear transformations, it is always possible to make three of the terms with odd exponents, or three functions of such terms, vanish from the equation of the Biquadratic Surface. Thus are ascertained sets of Orthogonal Axes having special properties.

To exemplify this, let the rectangular transformation be such as to make the following functions vanish:—

$$\{(\beta\lambda)-(\gamma\lambda)\}(y^2-z^2)yz\,; \quad \{(\gamma\mu)-(\alpha\mu)\}(z^2-x^2)zx\,; \quad \{(\alpha\nu)-(\beta\nu)\}(x^2-y^2)xy.$$

A cubical molecule having its faces normal to the axes fulfilling this condition has the following property:—*if there be a linear elongation along* y, *and an equal linear compression along* z (or *vice versâ*), *no tangential stress will result round* x *on planes normal to* y *and* z; and similarly of other pairs of axes.

This set of axes may be called the *Orthogonal* or *Principal Metatatic Axes,* and their planes, *Metatatic Planes.*

Let the suffix 1 designate coordinates and coefficients referred to these axes. Let Oy, Oz be any new pair of orthogonal axes in the plane y_1z_1. Then since $(\beta\lambda)-(\gamma\lambda)$ is covariant with $(y^2-z^2)yz$, it follows that

$$(\beta\lambda)-(\gamma\lambda)=\{2(\beta\gamma)_1+4(\lambda^2)_1-(\beta^2)_1-(\gamma^2)_1\}\cdot\frac{\sin 4\omega}{4} \quad . \quad . \quad . \quad . \quad (20.)$$

$$(\text{where } \omega=\angle y_1\text{O}y),$$

a quantity which is $=0$ for all values of ω which are multiples of 45°. There are of course similar equations for the other metatatic planes. Hence it appears that *in each of the three Metatatic Planes there is a pair of Diagonal Metatatic Axes, bisecting the right angles formed by the Principal Metatatic Axes.*

Each pair of diagonal axes is metatatic for that plane only in which it is situated.

Thus there are in all *nine* metatatic axes, three orthogonal axes, and three pairs of diagonal axes. The diagonal axes are normal to the faces of a regular rhombic dodecahedron.

Let Oy, Oz be a pair of rectangular axes in *any plane whatsoever*; Oy', Oz' any other pair of rectangular axes in the same plane; and let

$$\angle y\text{O}y'=\omega';$$

then

$$(\beta\lambda)'-(\gamma\lambda)'=\{2(\beta\gamma)+4(\lambda^2)-(\beta^2)-(\gamma^2)\}\frac{\sin 4\omega'}{4}+\{(\beta\lambda)-(\gamma\lambda)\}\cos 4\omega', \quad . \quad (21.)$$

a quantity which is null for eight values of ω', differing from each other by multiples of 45°. Hence, *in each plane in an elastic solid, there is a system of two pairs of axes metatatic for that plane and forming with each other eight equal angles of* 45°.

In equation (21.), make

$$\omega'=-\omega$$

$$(\beta\lambda)'-(\gamma\lambda)'=(\beta\lambda)_1-(\gamma\lambda)_1=0;$$

then from equations (20.) and (21.), it is easily seen that

$$2(\beta\gamma)+4(\lambda^2)-(\beta^2)-(\gamma^2)=\{2(\beta\gamma)_1+4(\lambda^2)_1-(\beta^2)_1-(\gamma^2)_1\}\,.\cos 4\omega. \quad . \quad . \quad (22.)$$

The trigonometrical factor $\cos 4\omega$ is $+1$ for all values of ω which are even multiples of 45°, -1 for all odd multiples of 45°, and $=0$ for all odd multiples of $22\frac{1}{2}$°. Hence, in every plane in an elastic solid, the quantity (22.), which may be called the *Metatatic Difference*, is a maximum for one of the two pairs of Metatatic Axes, a minimum of equal amount and negative sign for the other, and null for the eight intermediate directions.

9. *Of Metatatic Isotropy.*

A solid is *Metatatically Isotropic*, when if a cubical molecule, cut out in any position whatsoever, undergo simultaneously an elongation along one axis, and an equal and opposite linear compression along another axis, no tangential stress will result on the faces of that molecule.

For such a substance, the metatatic differences must be null for all sets of axes, viz.—

$$\left.\begin{array}{l} 2(\beta\gamma)+4(\lambda^2)-(\beta^2)-(\gamma^2)=0\,; \\ 2(\gamma\alpha)+4(\mu^2)-(\gamma^2)-(\alpha^2)=0\,; \\ 2(\alpha\beta)+4(\nu^2)-(\alpha^2)-(\beta^2)=0. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23.)$$

In a paper in the Cambridge and Dublin Mathematical Journal, vol. vi., this theorem was alleged of all Homogeneous solids, it having been, in fact, tacitly taken for granted, that Homogeneity involves Metatatic Isotropy, as above defined.

10. *Of Orthotatic Symmetry.*

If it be taken for granted that symmetrical action with respect to a certain set of axes, between the parts of a body under one kind of strain, involves symmetrical action with respect to the same axes under all kinds of strains, then one and the same set of orthogonal axes will be at once Orthotatic, Heterotatic, Metatatic, and Euthytatic, and for them the whole twelve plagiotatic coefficients will vanish at once, and the independent tasinomic coefficients be reduced to the nine Orthotatic Coefficients enumerated in Article 2. As long as the rigidity of solid bodies was ascribed wholly to mutual attractions and repulsions between centres of force, it is difficult to see how, with respect to homogeneous substances, the above assumption could be avoided. It is probable that there exist substances for which it is true. Such substances may be said to be *Orthotatically Symmetrical.*

Orthotatic Symmetry requires that the equation (19.) of the Biquadratic surface should be reducible by rectangular transformations to its first six terms, and that the axes so found should also be those of the Heterotatic Ellipsoid. The conditions which must be fulfilled in order that a Biquadratic function of three variables may be reducible by rectangular transformations to its first six terms, have been investigated by Mr. Boole*.

11. *Of Cybotatic Symmetry.*

Let a substance be conceived which is not only Orthotatically Symmetrical, but for which the three kinds of Orthotatic Coefficients are equal for the three orthotatic axes, viz.—

$$(\alpha^2)=(\beta^2)=(\gamma^2)\,; \quad (\beta\gamma)=(\gamma\alpha)=(\alpha\beta)\,; \quad (\lambda^2)=(\mu^2)=(\nu^2)\,. \quad . \quad . \quad . \quad (24.)$$

Then for such a substance the Metatatic Difference may be expressed by

$$2(\beta\gamma)+4(\lambda^2)-2(\alpha^2)\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25.)$$

and if the body be not Metatatically Isotropic, this difference will have equal maxima and minima for the three Orthogonal Axes, normal to the faces of a cube, and conversely, equal minima or maxima for the six diagonal axes, normal to the faces of a regular rhombic dodecahedron.

* Cambridge and Dublin Mathematical Journal, vol. vi.

Symmetry of this kind may be called *Cybotatic*, from its analogy to that of crystals of the Tessular System.

12. *Of Pantatic Isotropy.*

When a body fulfils the conditions of Cybotatic Symmetry, and at the same time those of Metatatic Isotropy, it is completely isotropic with respect to Elasticity, or Pantatically Isotropic. It has but three tasinomic coefficients, viz. the Euthytatic, Platytatic, and Goniotatic coefficients, which are equal for all sets of axes, and are connected by the following equation, expressing the condition of Metatatic Isotropy:

$$(\alpha^2)=(\beta\gamma)+2(\lambda^2). \quad \ldots \ldots \ldots \quad (26.)$$

The properties of such bodies have been fully investigated by various authors.

13. *Of Thlipsinomic Coefficients.*

If the six elementary strains α, &c. at a given point in an elastic solid, be expressed as linear functions of the six elementary stresses P_1, &c., these expressions will contain twenty-one coefficients of compressibility, extensibility, and pliability, which are the second differential coefficients of the potential energy of elasticity with respect to the six elementary stresses; that energy being represented as follows:—

$$\begin{aligned} U=&(a^2)\frac{P_1^2}{2}+(b^2)\frac{P_2^2}{2}+(c^2)\cdot\frac{P_3^2}{2}+(l^2)\cdot\frac{Q_1^2}{2}+(m^2)\frac{Q_2^2}{2}+(n^2)\frac{Q_3^2}{2} \\ &+(bc)P_2P_3+(ca)P_3P_1+(ab)P_1P_2+(mn)Q_2Q_3+(nl)Q_3Q_1+(lm)Q_1Q_2 \\ &+\{(al)\,P_1+(bl)\,P_2+(cl)\,P_3\}Q_1 \\ &+\{(am)P_1+(bm)P_2+(cm)P_3\}Q_2 \\ &+\{(an)\,P_1+(bn)\,P_2+(cn)\,P_3\}Q_3. \quad \ldots \ldots \ldots \quad (27.) \end{aligned}$$

The twenty-one coefficients in the above equation may be comprehended under the general term *Thlipsinomic*, and classified as follows:—

Designations of Coefficients.		Properties expressed by them.	Symbols.		
Orthothliptic	Euthythliptic .	Longitudinal Extensibilities	(a^2),	(b^2),	(c^2),
	Platythliptic .	Lateral Extensibilities . .	(bc),	(ca),	(ab),
	Goniothliptic .	Pliabilities	(l^2),	(m^2),	(n^2),
Plagiothliptic		Unsymmetrical Pliabilities .	(mn), &c. &c.		

14. *Of Thlipsinomic Transformations, Umbræ, Surfaces, and Invariants.*

The equations of transformation of the Thlipsinomic Coefficients are easily deduced from the principle, that the operations

$$\frac{d}{dP_1},\quad \frac{d}{dP_2},\quad \frac{d}{dP_3},\quad \frac{d}{dQ_1},\quad \frac{d}{dQ_2},\quad \frac{d}{dQ_3}$$

are respectively covariant with

$$P_1,\quad P_2,\quad P_3,\quad 2Q_1,\quad 2Q_2,\quad 2Q_3,$$

and these with

$$x^2,\quad y^2,\quad z^2,\quad 2yz,\quad 2zx,\quad 2xy.$$

We may regard the Thlipsinomic Coefficients, like the Tasinomic Coefficients, as binary compounds of the following six *Umbræ*,

$$(a),\quad (b),\quad (c),\quad (l),\quad (m),\quad (n),$$

which being respectively substituted for

$$P_1,\quad P_2,\quad P_3,\quad 2Q_1,\quad 2Q_2,\quad 2Q_3$$

in the equation of the Tasimetric Surface (4.), produce the following equation of the *Umbral Thlipsinomic Ellipsoid*,

$$(a)x^2+(b)y^2+(c)z^2+(l)yz+(m)zx+(n)xy=1, \quad \ldots \ldots \quad (28.)$$

from which, by involution, multiplication, and other operations exactly analogous to those performed on the Umbral Tasinomic Ellipsoid, there may be deduced the equations of *Thlipsinomic Surfaces* exactly corresponding to the Tasinomic Surfaces already described; while, from the Umbral Matrix,

$$\left.\begin{matrix} (a) & \frac{1}{2}(n) & \frac{1}{2}(m) \\ \frac{1}{2}(n) & (b) & \frac{1}{2}(l) \\ \frac{1}{2}(m) & \frac{1}{2}(l) & (c) \end{matrix}\right\} \quad \ldots \ldots \ldots \ldots \quad (29.)$$

may be formed *Thlipsinomic Invariants* corresponding to the Tasinomic Invariants.

Hence it appears, that every function of the Tasinomic Coefficients is converted into a function of the Thlipsinomic Coefficients with analogous properties, by the substitution of Thlipsinomic for Tasinomic Umbræ according to the following table:—

Tasinomic Umbræ	(α),	(β),	(γ),	(λ),	(μ),	(ν),
Thlipsinomic Umbræ	(a),	(b),	(c),	$\frac{1}{2}(l)$,	$\frac{1}{2}(m)$,	$\frac{1}{2}(n)$.

Amongst the Thlipsinomic Invariants may be distinguished the *Cubic Compressibility*, which is formed by squaring the umbral invariant $(a)+(b)+(c)$, and has the following value:

$$(a^2)+(b^2)+(c^2)+2(bc)+2(ca)+2(ab).$$

15. *Thlipsinomic and Tasinomic Contragredient Systems.*

Let the following square matrices be formed with the Tasinomic and Thlipsinomic Coefficients respectively:—

$$(30.)\left\{\begin{matrix} (\alpha^2) & (\alpha\beta) & (\gamma\alpha) & (\alpha\lambda) & (\alpha\mu) & (\alpha\nu) \\ (\alpha\beta) & (\beta^2) & (\beta\gamma) & (\beta\lambda) & (\beta\mu) & (\beta\nu) \\ (\gamma\alpha) & (\beta\gamma) & (\gamma^2) & (\gamma\lambda) & (\gamma\mu) & (\gamma\nu) \\ (\alpha\lambda) & (\beta\lambda) & (\gamma\lambda) & (\lambda^2) & (\lambda\mu) & (\nu\lambda) \\ (\alpha\mu) & (\beta\mu) & (\gamma\mu) & (\lambda\mu) & (\mu^2) & (\mu\nu) \\ (\alpha\nu) & (\beta\nu) & (\gamma\nu) & (\nu\lambda) & (\mu\nu) & (\nu^2) \end{matrix}\right\| \begin{matrix} (a^2) & (ab) & (ca) & (al) & (am) & (an) \\ (ab) & (b^2) & (bc) & (bl) & (bm) & (bn) \\ (ca) & (bc) & (c^2) & (cl) & (cm) & (cn) \\ (al) & (bl) & (cl) & (l^2) & (lm) & (nl) \\ (am) & (bm) & (cm) & (lm) & (m^2) & (mn) \\ (an) & (bn) & (cn) & (nl) & (mn) & (n^2) \end{matrix}\right\} \quad (31.)$$

Then will these matrices be mutually *inverse*, the two systems of coefficients arrayed in them, with their respective systems of functions, mutually *contragredient*, and

each coefficient or function belonging to one system *contravariant* to the corresponding coefficient or function belonging to the other system.

The values of the coefficients in either of those matrices are expressed in terms of those in the other matrix, in Mr. SYLVESTER's umbral notation, by twenty-one equations, of which the following are examples:—

$$\left.\begin{aligned}(a^2)&=\left|\begin{matrix}(\beta), & (\gamma), & (\lambda), & (\mu), & (\nu)\\ (\beta), & (\gamma), & (\lambda), & (\mu), & (\nu)\end{matrix}\right| \div \left|\begin{matrix}(\alpha), & (\beta), & (\gamma), & (\lambda), & (\mu), & (\nu)\\ (\alpha), & (\beta), & (\gamma), & (\lambda), & (\mu), & (\nu)\end{matrix}\right|;\\ (ab)&=\left|\begin{matrix}(\beta), & (\gamma), & (\lambda), & (\mu), & (\nu)\\ (\alpha), & (\gamma), & (\lambda), & (\mu), & (\nu)\end{matrix}\right| \div \left|\begin{matrix}(\alpha), & (\beta), & (\gamma), & (\lambda), & (\mu), & (\nu)\\ (\alpha), & (\beta), & (\gamma), & (\lambda), & (\mu), & (\nu)\end{matrix}\right|;\end{aligned}\right\} \quad (32.)$$

16. *Of Thlipsinomic Axes.*

If, under given conditions, any symmetrical system or function of the constituents of one of the above matrices be null, then under the same conditions will the contravariant system or function of the constituents of the inverse matrix be null or infinite. Therefore *Systems of Thlipsinomic Axes coincide with the corresponding systems of Tasinomic Axes.*

17. *Platythliptic Coefficients are negative.*

It may be observed as a matter of fact, that in consequence of the largeness of the Euthytatic Coefficients (α^2), (β^2), (γ^2), as compared with the other Tasinomic Coefficients, the Platythliptic Coefficients (bc), (ca), (ab) are generally, if not always, negative.

To illustrate this, the case of Pantatic Isotropy may be taken, for which the two matrices have the following forms:—

$$\left.\begin{matrix}(\alpha^2) & (\beta\gamma) & (\beta\gamma) & 0 & 0 & 0\\ (\beta\gamma) & (\alpha^2) & (\beta\gamma) & 0 & 0 & 0\\ (\beta\gamma) & (\beta\gamma) & (\alpha^2) & 0 & 0 & 0\\ 0 & 0 & 0 & (\lambda^2) & 0 & 0\\ 0 & 0 & 0 & 0 & (\lambda^2) & 0\\ 0 & 0 & 0 & 0 & 0 & (\lambda^2)\end{matrix}\;\right\|\;\begin{matrix}(a^2) & (bc) & (bc) & 0 & 0 & 0\\ (bc) & (a^2) & (bc) & 0 & 0 & 0\\ (bc) & (bc) & (a^2) & 0 & 0 & 0\\ 0 & 0 & 0 & (l^2) & 0 & 0\\ 0 & 0 & 0 & 0 & (l^2) & 0\\ 0 & 0 & 0 & 0 & 0 & (l^2)\end{matrix}\left.\vphantom{\begin{matrix}1\\1\\1\\1\\1\\1\end{matrix}}\right\} \quad (33.)$$

from which it is easily seen that the sole Platythliptic coefficient has the following value:

$$(bc)=\frac{-(\beta\gamma)}{(\alpha^2)^2+(\alpha^2)(\beta\gamma)-2(\beta\gamma)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33\text{A.})$$

The denominator of this fraction is always positive so long as (α^2) exceeds $(\beta\gamma)$; a condition invariably fulfilled by solid bodies, and, in fact, necessary to their existence.

18. *Of Oblique Coordinates and Contraordinates.*

As there are, in the relations between two systems of oblique coordinates, or between a system of oblique coordinates and a system of rectangular coordinates, six independent constants of transformation, it is possible, by referring the equation of the Biquadratic Surface (19.) to Oblique Coordinates, to make the six terms vanish which contain the cubes of the coordinates.

The conception of the physical meaning of such a transformation is much facilitated by the employment of a system of three auxiliary variables, which will be designated as *Contraordinates.*

The relations between coordinates and contraordinates are as follows:—

Through an origin O let any three axes pass, right or oblique. Let R be any point, and let

$$\overline{\mathrm{OR}}=r.$$

Through R draw three planes, parallel respectively to the three coordinate planes, and intersecting the axes respectively in the points X, Y, Z. Also, on OR, as a diameter, describe a sphere, intersecting the axes respectively in U, V, W. Then will

$$\mathrm{OX}=x, \quad \mathrm{OY}=y, \quad \mathrm{OZ}=z$$

be the *coordinates* of R, as usual, and

$$\mathrm{OU}=u, \quad \mathrm{OV}=v, \quad \mathrm{OW}=w$$

its *contraordinates,* being, in fact, the projections of OR on the three axes.

For rectangular axes, coordinates and contraordinates are identical.

Coordinates and Contraordinates are connected by the following equation:—

$$r^2=ux+vy+wz \quad \ldots\ldots\ldots\ldots \quad (34.)$$

In the language of Mr. SYLVESTER, a system of Coordinates and the concomitant system of Contraordinates are mutually *Contragredient*; and the square of the radius-vector is their *universal mixed concomitant.*

Let the cosines of the angles made by the axes with each other be denoted as follows:—

$$\cos yOz=c_1; \quad \cos zOx=c_2; \quad \cos xOy=c_3;$$

then the contraordinates of a given point are the following functions of the coordinates:—

$$\left.\begin{array}{l} u=x+c_3y+c_2z \\ v=c_3x+y+c_1z \\ w=c_2x+c_1y+z \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (35.)$$

Also let

$$\left.\begin{array}{l} \begin{vmatrix} 1, & c_3, & c_2 \\ c_3, & 1, & c_1 \\ c_2, & c_1, & 1 \end{vmatrix}=1-c_1^2-c_2^2-c_3^2+2c_1c_2c_3=\mathrm{C}; \\ \dfrac{1-c_1^2}{\mathrm{C}}=h_1; \quad \dfrac{1-c_2^2}{\mathrm{C}}=h_2; \quad \dfrac{1-c_3^2}{\mathrm{C}}=h_3; \\ \dfrac{c_1-c_2c_3}{\mathrm{C}}=k_1; \quad \dfrac{c_2-c_3c_1}{\mathrm{C}}=k_2; \quad \dfrac{c_3-c_1c_2}{\mathrm{C}}=k_3; \end{array}\right\}$$

then the coordinates are the following linear functions of the contraordinates:—

$$\left.\begin{array}{l} x=\ \ h_1u-k_3v-k_2w\,; \\ y=-k_3u+h_2v-k_1w\,; \\ z=-k_2u-k_1v+h_3w. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36.)$$

Also,

$$r^2=x^2+y^2+z^2+2c_1yz+2c_2zx+2c_3xy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37.)$$

$$=h_1u^2+h_2v^2+h_3w^2-2k_1vw-2k_2wu-2k_3uv. \quad . \quad . \quad . \quad . \quad (37\text{A.})$$

Differentiations with respect to the contraordinates are obviously covariant with the coordinates, and *vice versâ*; that is to say,

$$\left.\begin{array}{lcccccc} \text{the operations} & \frac{d}{dx}, & \frac{d}{dy}, & \frac{d}{dz}, & \frac{d}{du}, & \frac{d}{dv}, & \frac{d}{dw} \\ \text{are respectively covariant with} & u, & v, & w, & x, & y, & z. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38.)$$

By making substitutions according to the above law of covariance in the equations (34.), (37.), (37A.), three equivalent symbols of operation are obtained, which, being applied to isotropic functions of the second degree, produce invariants of the first degree.

19. *Of Molecular Displacements and Strains as referred to Oblique Axes.*

If the displacement of a particle from its free position be resolved into three components, ξ, η, ζ, parallel respectively to three oblique axes, Ox, Oy, Oz, those components are evidently covariant respectively with the coordinates x, y, z.

It is now necessary to find a method of expressing the strain at any particle in an elastic solid by a system of six elementary strains, which shall be covariant respectively with the squares and doubled-products of these oblique coordinates. This condition is fulfilled by considering the elementary strains as being constituted by the variations of the components of the molecular displacement with respect to the distances of the strained particle from three planes passing through the origin, and normal respectively to the three axes; that is to say, with respect to the *contraordinates* of the particle, as expressed in the following equations:—

$$\left.\begin{array}{ll} \text{Elongations} \quad . \quad . \quad . & \alpha=\frac{d\xi}{du}\,; \quad \beta=\frac{d\eta}{dv}\,; \quad \gamma=\frac{d\zeta}{dw}\,; \\ \text{Quasi-Distortions} \ . & \lambda=\frac{d\zeta}{dv}+\frac{d\eta}{dw}\,; \quad \mu=\frac{d\xi}{dw}+\frac{d\zeta}{du}\,; \quad \nu=\frac{d\eta}{du}+\frac{d\xi}{dv} \end{array}\right\} \quad . \quad . \quad . \quad . \quad (39.)$$

The six elementary strains, as above defined, are obviously covariant with the squares and doubled-products of the coordinates, according to the following table:—

$$\left.\begin{array}{cccccc} \alpha, & \beta, & \gamma, & \lambda, & \mu, & \nu, \\ x^2, & y^2, & z^2, & 2yz, & 2zx, & 2xy. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40.)$$

20. *Of Stresses, as referred to Oblique Axes.*

It is next required to express the stress at any particle of an elastic solid by means of a system of six elementary stresses which shall be contragredient to the system of six elementary strains defined in the preceding article. This is accomplished in the following manner.

It is known that the total stress at any point may be resolved into three normal stresses on the three principal planes of the tasimetric surface. Let the direction and sign of any one of those three *principal* stresses be represented by those of a line OR, and its magnitude, as reduced to unity of area of the plane normal to that direction, by the square of that line,

$$\overline{\mathrm{OR}}^2 = r^2.$$

Let u, v, w be the contraordinates of R, as referred to the oblique axes OX, OY, OZ. Then will the stresses on unity of area of planes normal to those axes, in the direction OR, be represented respectively by

$$ur, \quad vr, \quad wr.$$

Let the *Elementary Stresses* be defined to be, *the projections on the three axes of coordinates, of the total stresses on unity of area of the three pairs of faces of a parallelopiped, normal to the three axes respectively*:—then, if we take S to denote the summation of three terms arising from the three principal stresses, the elementary stresses will be expressed as follows:—

$$\left.\begin{array}{ll}
\text{Normal Stresses on the faces normal to} & \\
& \begin{array}{ccc} x, & y, & z, \\ \mathrm{P}_1 = \mathrm{S}.u^2; & \mathrm{P}_2 = \mathrm{S}.v^2; & \mathrm{P}_3 = \mathrm{S}.w^2; \end{array} \\
\text{Oblique Stresses on the faces normal to} & \begin{array}{cccccc} y & z & z & x & x & y \end{array} \\
\text{in the directions} & \begin{array}{ccc} \underbrace{z \qquad y} & \underbrace{x \qquad z} & \underbrace{y \qquad x} \\ \mathrm{Q}_1 = \mathrm{S}.vw; & \mathrm{Q}_2 = \mathrm{S}.wu; & \mathrm{Q}_3 = \mathrm{S}.uv. \end{array}
\end{array}\right\} \quad . \quad . \quad . \quad (41.)$$

These expressions fulfil the condition of making the elementary stresses

$$\mathrm{P}_1, \quad \mathrm{P}_2, \quad \mathrm{P}_3, \quad \mathrm{Q}_1, \quad \mathrm{Q}_2, \quad \mathrm{Q}_3$$

contravariant respectively to the elementary strains

$$\alpha, \quad \beta, \quad \gamma, \quad \lambda, \quad \mu, \quad \nu,$$

so that for oblique axes, as for rectangular axes, the potential energy of elasticity is represented by

$$\mathrm{U} = -\tfrac{1}{2}(\mathrm{P}_1\alpha + \mathrm{P}_2\beta + \mathrm{P}_3\gamma + \mathrm{Q}_1\lambda + \mathrm{Q}_2\mu + \mathrm{Q}_3\nu),$$

the universal concomitant; and may be expressed either by a homogeneous quadratic function of the six elementary strains (as in equation 2), with twenty-one tasinomic coefficients, or by a homogeneous quadratic function of the six elementary stresses,

as in equation (27.), with twenty-one thlipsinomic coefficients, forming a system contragredient to that of the tasinomic coefficients.

21. *Of Tasinomic and Thlipsinomic Umbræ for Oblique Axes.*

The *tasinomic* coefficients for oblique axes may be regarded as compounded of Umbræ

$$(\alpha),\quad (\beta),\quad (\gamma),\quad (\lambda),\quad (\mu),\quad (\nu),$$

contravariant respectively to the elementary strains

$$\alpha,\quad \beta,\quad \gamma,\quad \tfrac{1}{2}\lambda,\quad \tfrac{1}{2}\mu,\quad \tfrac{1}{2}\nu,$$

and consequently *covariant* with the squares and products of the *contraordinates*

$$u^2,\quad v^2,\quad w^2,\quad vw,\quad wu,\quad uv;$$

and the *thlipsinomic* coefficients for Oblique Axes may be regarded as compounded of Umbræ

$$(a),\quad (b),\quad (c),\quad (l),\quad (m),\quad (n),$$

contravariant respectively to the stresses

$$P_1,\quad P_2,\quad P_3,\quad 2Q_1,\quad 2Q_2,\quad 2Q_3,$$

and consequently *covariant* with the squares and products of the *coordinates*

$$x^2,\quad y^2,\quad z^2,\quad 2yz,\quad 2zx,\quad 2xy.$$

22. *Of the Biquadratic Surface, and of Principal Euthytatic Axes.*

For oblique as well as for rectangular axes of coordinates, the characteristic function of the Biquadratic Tasinomic Surface is represented by equation (19.); and the fifteen Homotatic Coefficients are covariant respectively with suitable multiples of the fifteen biquadratic powers and products of the contraordinates.

If by linear transformations a system of three axes, oblique or rectangular, be found which reduce the characteristic function of the Biquadratic Surface to the canonical form, consisting of not more than nine terms, viz.—

$$\begin{aligned}(\phi)^2 &= (\alpha^2)x^4+(\beta^2)y^4+(\gamma^2)z^4\\ &+2\{(\beta\gamma)+2(\lambda^2)\}y^2z^2+2\{(\gamma\alpha)+2(\mu^2)\}z^2x^2+2\{(\alpha\beta)+2(\nu^2)\}x^2y^2\\ &+4\{2(\mu\nu)+(\alpha\lambda)\}x^2yz+4\{2(\nu\lambda)+(\beta\mu)\}xy^2z+4\{2(\lambda\mu)+(\gamma\nu)\}xyz^2=1;\end{aligned} \qquad (42.)$$

then for that system of axes, the following six Plagiotatic Coefficients are null,

$$(\beta\lambda)=0;\quad (\gamma\lambda)=0;\quad (\gamma\mu)=0;\quad (\alpha\mu)=0;\quad (\alpha\nu)=0;\quad (\beta\nu)=0; \qquad (43.)$$

and each of those axes is EUTHYTATIC, according to the definition in § 7, that is to say, is a direction of maximum or minimum direct Elasticity (absolute or relative), and also a direction in which a direct elongation or compression produces a simply normal stress.

There are necessarily *three* Euthytatic Axes at least in every solid, viz. the three *Principal Euthytatic Axes* as above described, which are normal to the faces of a

Hexahedron, right or oblique as the case may be; but in special cases of symmetry there are *additional* or *secondary* euthytatic axes, of which examples will now be given.

23. *Of Rhombic and Hexagonal Symmetry.*

When a solid has three oblique principal euthytatic axes making equal angles with each other round an axis of symmetry, and having equal systems of Homotatic Coefficients corresponding to them, viz.—

$$\left.\begin{array}{l}(\alpha^2)=(\beta^2)=(\gamma^2)\,;\quad (\beta\gamma)+2(\lambda^2)=(\gamma\alpha)+2(\mu^2)=(\alpha\beta)+2(\nu^2)\\ 2(\mu\nu)+(\alpha\lambda)=2(\nu\lambda)+(\beta\mu)=2(\lambda\mu)+(\gamma\nu)\quad .\quad .\quad .\quad .\quad .\end{array}\right\}\quad .\quad .\quad .\quad (43\text{ A.})$$

it may be said to possess Rhombic Symmetry, because the three oblique axes are normal to the faces of one Rhombohedron, and to the edges of another belonging to the same series, crystallographically speaking. It is evident in this case, that the Axis of Symmetry must be a *fourth Euthytatic Axis*.

In the limiting case, when the three oblique axes make with each other equal angles of 120°, they lie in the same plane, normal to the axis of symmetry, and are normal to the faces of one hexagonal prism, and the edges of another.

Let Oy_1 denote the longitudinal axis of symmetry of the prism; Oz_1 any one of the three transverse axes perpendicular to Oy_1. The equation of a section of the Biquadratic surface by the *Plane of Hexagonal Symmetry* y_1z_1, is as follows:—

$$(\beta^2)_1y_1^4+(\gamma^2)_1z_1^4+2\{(\beta\gamma)_1+2(\lambda^2)_1\}y_1^2z_1^2=1.\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad (44.)$$

The equation of the same section, referred to any other pair of orthogonal axes Oy, Oz, in the plane of y_1z_1, is as follows:—

$$(\beta^2).y^4+(\gamma^2).z^4+2\{(\beta\gamma)+2(\lambda^2)\}y^2z^2+4\{(\beta\lambda)y^2+(\gamma\lambda)z^2\}yz=1.\quad .\quad .\quad (44\text{ A.})$$

From considerations of symmetry, it is evident that the coefficient $(\beta\nu)$ must be null for every direction of the axis Oy in the plane of y_1z_1; consequently, every direction Oy in that plane, for which $(\beta\lambda)=0$, is an Euthytatic Axis.

To ascertain whether, and under what conditions, there are other Euthytatic Axes in the planes of hexagonal symmetry besides the longitudinal and transverse axes, it is to be considered, that for rectangular coordinates $(\beta\lambda)$ is covariant with y^3z; hence, let

$$\angle y_1Oy=\omega,$$

then $$(\beta\lambda)=\frac{\sin 2\omega}{4}\cdot\Big[\{2(\beta\gamma)_1+4(\lambda^2)_1-(\beta^2)_1-(\gamma^2)_1\}\cos 2\omega-(\beta^2)_1+(\gamma^2)_1\Big]\quad .\quad (45.)$$

The first factor of the above expression is null for the longitudinal and transverse axes only. The conditions of there being additional euthytatic axes in the plane y_1z_1 is, that the second factor shall vanish; that is to say, that

$$\cos 2\omega=\frac{(\beta^2)_1-(\gamma^2)_1}{2(\beta\gamma)_1+4(\lambda^2)_1-(\beta^2)_1-(\gamma^2)_1}\cdot\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad (46.)$$

and that the value of ω which makes it vanish shall neither be 0° nor 90°; that is to say, that the second member of the above equation (46.) shall lie between +1 and −1; in which case the equation is satisfied by equal values of ω with opposite signs. Hence are deduced the following theorems, which are stated in such a form as to be applicable to planes of symmetry, whether hexagonal or otherwise.

If, in any plane of tasinomic symmetry containing a pair of Orthogonal Euthytatic Axes, the difference of the Euthytatic coefficients for these axes be equal to or greater than the Metatatic Difference, there are no additional euthytatic axes in that plane.

If, on the other hand, the difference of such Euthytatic coefficients be less than the metatatic difference, there are, in such plane of symmetry, a pair of additional euthytatic axes making with each other a pair of angles bisected by the orthogonal euthytatic axes.

2ω is the angle bisected by the axis Oy_1.

In the case of Hexagonal Symmetry, the additional axes thus found are normal to the faces of one pyramidal dodecahedron, and the edges of another.

24. *Of Orthorhombic Symmetry.*

Let a solid have one of the three principal euthytatic axes, Oz_1, normal to the other two, Ox_1, Oy_1; let the last two be oblique to each other, and have equal sets of homotatic coefficients, viz.—

$$(\alpha^2)_1=(\beta^2)_1;\quad (\beta\gamma)_1+2(\lambda^2)_1=(\gamma\alpha)_1+2(\mu^2)_1;\quad 2(\mu\nu)+(\alpha\lambda)=2(\nu\lambda)+(\beta\mu), \quad (47.)$$

then that solid may be said to have *Orthorhombic Symmetry,* its principal euthytatic axes being normal to the faces of a right rhombic prism.

The existence or non-existence, and the position, of a pair of additional euthytatic axes in the longitudinal planes of y_1z_1, z_1x_1, is to be determined as in the preceding article. When such axes exist, they are normal to the faces of an *Octahedron with a Rhombic Base.*

25. *Of Orthogonal Symmetry.*

If the three principal Euthytatic Axes be orthogonal, they are normal to the faces of a *right rectangular or square prism,* and to the edges of a *right rhombic or square prism.* The existence or non-existence, and position, of a pair of additional euthytatic axes in each of the principal planes of such a solid, are determined as in article (23.).

If there be a pair of such additional axes in each of the three principal planes, they are normal to the faces of an *irregular Rhombic Dodecahedron,* and to the edges of a *Rhombic Octahedron.*

If there be a pair of such additional axes in two of the three principal planes, those axes are normal to the faces of an *Octahedron with a Rectangular or square base,* and to the edges of an *Octahedron with a Rhombic or square base.*

If there be a pair of such additional axes in one of the planes of orthotatic symmetry only, those axes are normal to the lateral faces of a *Right Rhombic Prism.*

26. *Of Cyboïd Symmetry.*

The case of *Cyboïd Symmetry* is that in which the Homotatic Coefficients are equal for three Orthogonal Axes, viz.—

$$(\alpha^2)=(\beta^2)=(\gamma^2)\,;\quad (\beta\gamma)+2(\lambda^2)=(\gamma\alpha)+2(\mu^2)=(\alpha\beta)+2(\nu^2)\,;$$
$$2(\mu\nu)+(\alpha\lambda)=2(\nu\lambda)+(\beta\mu)=2(\lambda\mu)+(\gamma\nu)=0. \quad \ldots\ldots \quad (48.)$$

In this case, the Principal Metatatic Axes coincide with the Principal Euthytatic Axes, which are normal to the faces of a cube; the Diagonal Metatatic Axes, normal to the faces of a regular Rhombic Dodecahedron, are Euthytatic also; and there are, besides, four additional euthytatic axes symmetrically situated between the first nine, and normal to the faces of a regular octahedron, making in all *thirteen euthytatic axes.*

27. *Of Monaxal Isotropy.*

Monaxal Isotropy denotes the case in which the homotatic coefficients are completely isotropic round one axis only. In this case, the principal euthytatic axes are, the axis of isotropy, and every direction perpendicular to it; and when there are additional axes, determined as in the preceding articles, they are normal to the surface of a cone.

28. *Of Complete Isotropy.*

In the case of Complete Isotropy of the Homotatic coefficients, every direction is a euthytatic axis.

29. *Probable Relations between Euthytatic Axes and Crystalline Forms.*

In the preceding articles it has been shown, what must be the nature of the relations between the fifteen homotatic coefficients, for various solids, having systems of euthytatic axes normal to the faces and edges of the several *Primitive Forms* known in Crystallography.

It is probable that the normals to *Planes of Cleavage* are Euthytatic Axes of Minimum Elasticity.

It may also be considered probable, that in some cases, especially in the Tessular System, which corresponds to Cyboïd Symmetry, and in the case of the pyramidal summits of crystals of the Rhombohedral System, Euthytatic Axes correspond to symmetrical summits of crystalline forms. In the icositetrahedral crystals of leucite and analcime, and the tetracontaoctahedral crystals of diamond, there are twenty-six symmetrical summits, one pair corresponding to each of the thirteen axes of cyboïd symmetry.

The following is a synoptical table of the various possible systems of euthytatic axes, arranged according to their degrees and kinds of symmetry, and of the crystalline forms to the faces and edges of which such systems of axes are respectively normal.

Systems of Euthytatic Axes.	Crystalline Forms. Faces.	Edges.
I. Asymmetry.	**Tetarto-prismatic System.**	
1. Three unequal Oblique Axes	Oblique Hexahedron.	
II. Symmetry about one plane.	**Hemiprismatic System.**	
2. Two unequal oblique axes, and one rectangular axis	Right Rhomboidal Prism	Oblique Rhombic Prism.
3. Two equal and one unequal oblique axis	Oblique Rhombic Prism	Right Rhomboidal Prism.
III. Rhombic and Hexagonal Symmetry.	**Rhombohedral System.**	
4. Three equi-oblique principal axes round one axis of symmetry	Rhombohedron	Rhombohedron.
5. Three equi-oblique principal axes in one plane, normal to axis of symmetry	Hexagonal Prism	Hexagonal Prism.
6. Three pairs of secondary axes in planes of symmetry.	Pyramidal Dodecahedron	Pyramidal Dodecahedron.
IV. Orthorhombic Symmetry.	**Prismatic and Pyramidal Systems.**	
7. Two equal oblique transverse axes normal to one longitudinal axis	Right Rhombic Prism	Rectangular Prism.
8. Two pairs of secondary axes in longitudinal planes	Octahedron with Rhombic Base	Octahedron with Rectangular Base.
V. Orthogonal Symmetry.		
9. Three orthogonal axes, not all equal	Rectangular and Square Prisms	Right Rhombic and square prisms.
10. Three pairs of secondary axes in principal planes	Irregular Rhombic Dodecahedron	Octahedron with Rhombic Base and Rectangular Prism.
11. Two pairs of secondary axes	Octahedron with square or rectangular base	Octahedron with square or Rhombic Base.
Same with 7. One pair of secondary axes	Right Rhombic Prism	Rectangular Prism.
VI. Cyboïd Symmetry.	**Tessular System.**	
12. Three equal Orthogonal Axes	Cube.	
13. Six Diagonal Axes	Regular Rhombic Dodecahedron	Cube and regular Octahedron.
14. Four symmetrical intermediate axes	Regular Octahedron	Rhombic Dodecahedron.
VII. Monaxal Isotropy.		
15. One Axis of Isotropy	Isotropic Laminæ.	
16. Innumerable Transverse Axes	Isotropic Fibres.	
17. Innumerable Equi-Oblique Axes	Conical Cleavage.	
VIII. Complete Isotropy.		
18. Innumerable Axes of Isotropy	Amorphism.	

30. *Mutual Independence of the Euthytatic and Heterotatic Axes, and of the Homotatic and Heterotatic Coefficients.*

The fifteen Homotatic Coefficients of the Biquadratic Surface, on which the Euthytatic Axes depend, and the six Heterotatic Differences, coefficients of the Heterotatic Ellipsoid, constitute twenty-one independent quantities; so that the Euthytatic Axes may possess any kind or degree of symmetry or asymmetry, and the Heterotatic Axes any other kind or degree, in the same solid.

Hence if it be true that crystalline form depends on the arrangement of Euthytatic Axes, it follows that two substances may be exactly alike in crystalline form, and yet differ materially in the laws of their elasticity, owing to differences in their respective Heterotatic Coefficients.

It may be observed, however, that this complete independence of those two systems of axes and coefficients is *mathematical* only; and that their physical dependence or independence is a question for experiment.

31. *On Real and Alleged Differences between the Laws of the Elasticity of Solids, and those of the Luminiferous Force.*

For every conceivable system of tasinomic coefficients in a solid, the *plane of polarization* of a wave of distortion is that which includes the direction of the molecular vibration and the direction of its propagation, being, in fact, the plane of distortion.

On the other hand, it appears to be impossible to avoid concluding, from the laws of the Diffraction of Polarized Light, as discovered by Professor STOKES, and from those of the more minute phenomena of the reflexion of light, as investigated theoretically by M. CAUCHY and experimentally by M. JAMIN, that in plane-polarized light the plane of polarization is perpendicular to the direction of vibration, or rather (to avoid hypothetical language) to the direction of some physical phenomenon whose laws of communication are to a certain extent analogous to those of a vibratory movement.

This constitutes an essential difference between the laws of the Elastic Forces in a solid, and those of the luminiferous force.

In order to frame, in connexion with the wave-theory of light, a mechanical hypothesis which should take that difference into account, it has been proposed to consider the *elasticity* of the luminiferous medium to be the same in all substances, and for all directions, or *Pantatically Isotropic*, and to ascribe the various retardations of light to variations in the *inertia* of the mass moved in luminiferous waves, in different substances, and for different directions of motion*.

Another essential difference between the laws of Solid Elasticity and those of the

* Philosophical Magazine, June 1851, December 1853.

luminiferous force is, that under no conceivable system of tasinomic coefficients in a homogeneous solid, would the plane of distortion in a wave be rotated continuously round the direction of propagation.

Much has been written, both recently and in former times, concerning an alleged difficulty in the theories of waves, both of sound and of light, arising from the physical impossibility of the actual divergence of waves from, or their convergence to, a mathematical point. This impossibility must be admitted; but the supposed difficulty to which it gives rise in the theories of waves is completely overcome in Mr. STOKES's paper on the Dynamical Theory of Diffraction*, in which that author proves, that waves spreading from a focal space, or origin of disturbance, of finite magnitude, and of any figure, sensibly agree in all respects with waves spreading from an imaginary focal point, so soon as they have attained a distance from the focal space, which is large as compared with the dimensions of that space; so that the equations of the propagation of waves spreading from imaginary focal points may be applied without sensible error to all those cases of actual waves to which it is usual to apply them.

The physical impossibility of focal points applies to light independently of all hypotheses; for at such points the intensity would be infinite. It appears to be worthy of consideration, whether this impossibility may not be connected with the appearance of spurious disks of fixed stars in the foci of telescopes.

32. *On the Action of Crystals on Light.*

If we set aside those actions on light to which there is nothing analogous in the phenomena of the elasticity of homogeneous solids, the laws of the refractive action of a crystal on light are in general of a more symmetrical kind, or depend on fewer quantities than those of its elasticity.

Thus, the elasticity of a homogeneous solid depends on twenty-one quantities; its crystalline form, on fifteen (the Homotatic Coefficients), while its refractive action on homogeneous light in most cases is expressible by means of the magnitudes and directions of the Orthogonal axes of FRESNEL's Wave-Surface, making in all six quantities. Crystals which possess only Rhombic or Hexagonal Symmetry in their Euthytatic Axes, are usually Monaxally Isotropic in their action on light; while crystals which possess only Cyboïd Symmetry in their euthytatic axes, are completely isotropic in their action on light.

From these remarks, however, there are exceptions, as in the case of the extraordinary optical properties discovered by Sir DAVID BREWSTER in Analcime, which, in its refraction as well as in its form, is Cyboïdally Symmetrical without being Isotropic.

* Cambridge Transactions, vol. ix. part 1.

Glasgow, June 1855.

Note referred to at page 261.

On Sylvestrian Umbræ.

Without attempting to enter into the abstract theory of the Umbral Method, it may here be useful to explain the particular case of its application which is employed in this paper.

Let U be a quantity having an absolute value, constant or variable (such, for example, as any physical magnitude), and $u, v, \ldots$ &c. a set of quantities, m in number, such that U is of them a homogeneous rational function of the nth degree. There are an indefinite number of possible sets of m quantities satisfying this condition; and the quantities of each set are related to those of each other set by m equations of the first degree, called equations of *linear transformation.* Let

$$u_1, \; v_1, \ldots\ldots.$$

$$u_2, \; v_2, \ldots\ldots.$$

be two such sets.

Let $C_{a,b,\ldots\ldots}$ denote the coefficient of $u^a v^b \ldots.$ in the development of

$$(u+v+\ldots.)^n,$$

and let

$$U=\Sigma\{C_{a,b,\ldots}A_{1,a,b,\ldots}u_1^{\,a}v_1^{\,b}\ldots.\}$$

$$=\Sigma\{C_{a,b,\ldots}A_{2,a,b,\ldots}u_2^{\,a}v_2^{\,b}\ldots.\}.$$

The two sets of coefficients A_1, A_2 are connected by linear equations of transformation, the investigation of which is much facilitated by the following process.

Let two sets, each of m symbols, α_1, β_1, &c. ... α_2, β_2, ... &c. be assumed, such that

$$\alpha_1 u_1+\beta_1 v_1+\ldots..=\alpha_2 u_2+\beta_2 v_2+\ldots..$$

and that, consequently,

$$(\alpha_1 u_1+\beta_1 v_1+\ldots.)^n=\Sigma\{C_{a,b,\ldots.}\alpha_1^{\,a}\beta_1^{\,b}\ldots.u_1^{\,a}v_1^{\,b}\ldots.\}$$

$$=(\alpha_2 u_2+\beta_2 v_2+\ldots.)^n=\Sigma\{C_{a,b,\ldots.}\alpha_2^{\,a}\beta_2^{\,b}\ldots.u_2^{\,a}v_2^{\,b}\ldots.\}.$$

Then if the m equations of transformation between the two sets of symbols α_1, $\beta_1\ldots$ and α_2, $\beta_2\ldots$ be formed, and if from them be deduced the equations between the two sets of products $\alpha_1^{\,a}\beta_1^{\,b}\ldots.$, and $\alpha_2^{\,a}\beta_2^{\,b}\ldots.$, &c., and if, in the latter system of equations, there be substituted for each product $\alpha^a\beta^b\ldots..$ the corresponding coefficient $A_{a,b,\ldots..}$, the result will be the system of equations sought. Also, if any function of the products $\alpha^a\beta^b\ldots.$ be *invariant* (*i. e.* a function, whose value, like that of the original function U, is not altered by the transformation), the corresponding function of the coefficients A will be invariant.

The symbols α, β, &c., with reference to their relation to the coefficients A, are called *umbræ*; that is, *factors of symbols, whose equations of transformation are simi-*

lar to those of the coefficients A. In the *umbral notation,* umbræ are usually distinguished from symbols denoting actual quantities by being enclosed in brackets thus:

$$(\alpha),\ (\beta),\ \&c.\ \ldots\ldots$$

and each coefficient A is represented by enclosing in brackets that product of umbræ with which it is *covariant*; thus:

$$A_{a,b,\ldots\ldots}=(\alpha^a\beta^b\ldots\ldots).$$

The Umbral Notation is applied to abbreviate the expression of determinants in a manner of which the following are examples:—

$$\begin{vmatrix} \alpha, & \beta, & \gamma, & \&c. \\ \alpha, & \beta, & \gamma, & \&c. \end{vmatrix} \text{ denotes } \boxed{\begin{matrix} (\alpha^2) & (\alpha\beta) & (\alpha\gamma) & \&c. \\ (\alpha\beta) & (\beta^2) & (\beta\gamma) & \&c. \\ (\alpha\gamma) & (\beta\gamma) & (\gamma^2) & \&c. \\ \&c. & \&c. & \&c. & \&c. \end{matrix}}$$

$$\begin{vmatrix} \alpha, & \gamma, & \delta, & \&c. \\ \beta, & \gamma, & \delta, & \&c. \end{vmatrix} \text{ denotes } \boxed{\begin{matrix} (\alpha\beta) & (\beta\gamma) & (\beta\delta) & \&c. \\ (\alpha\gamma) & (\gamma^2) & (\gamma\delta) & \&c. \\ (\alpha\delta) & (\gamma\delta) & (\delta^2) & \&c. \\ \&c. & \&c. & \&c. & \&c. \end{matrix}}$$

February 24, 1856.

XIII. *Introductory Research on the Induction of Magnetism by Electrical Currents.*

By J. P. Joule, *F.R.S., Corr. Mem. R.A., Turin, Hon. Mem. of the Philosophical Society, Cambridge, &c.*

Received June 21,—Read June 21, 1855.

THE researches of Jacobi and Lenz led them some years ago to the announcement as a law, that when two bars of iron of different diameters but equal to one another in length and surrounded with coils of wire of the same length carry equal streams of electricity, the magnetism developed in the bars is proportional to their respective diameters. Experiments which I made about the same time threw doubts on my mind as to the general accuracy of the above proposition, for I found that the magnetism induced in straight bars of a variety of dimensions varying from $\frac{1}{8}$ to 1 inch in diameter, and from 7 inches to one yard in length, was nearly proportional to the length of the wire and the intensity of the current it conveyed, irrespectively of the shape or magnitude of the bars. The valuable experimental researches which have recently been made by Weber, Robinson, Müller, Dub and others, refer chiefly to the attraction of the keeper or submagnet, and are not calculated to confirm or disprove either of the above propositions; and the correct view is probably that of Professor Thomson, who considers both of them as corollaries (applying to the particular conditions under which the experiments were made) of the general law, that "similar bars of different dimensions, similarly rolled with lengths of wire proportional to the squares of their linear dimensions and carrying equal currents, cause equal forces at points similarly situated with reference to them*." I have been induced to undertake some further experiments with an endeavour to elucidate the subject, and also to open the way to the investigation of the molecular changes which occur during magnetization.

I procured four iron bars one yard long and of the respective diameters $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{2}$ and 1 inch, their weights being 1736, 3802, 14560, and 55060 grs. Each bar was wound with 56 feet of copper wire $\frac{1}{40}$th of an inch in diameter covered with silk, the number of convolutions being 1020, 712, 388, and 207 respectively. The smallest bar was closely covered throughout its entire length, but on account of the larger surface of the other bars the coils had to be distributed upon them as evenly as possible. Four other bars were also procured of the same diameters as the above. They were however twice as long, weighing 3500, 7624, 29944, and 108574 grs., and were wrapped with double the length of wire, forming 2060, 1435, 768, and 418 convolutions respectively.

* Letter to the author.

To measure the electrical currents, I employed a galvanometer of tangents, the needle of which, half an inch long, carried a glass index over a divided circle 6 inches in diameter. This instrument was furnished with a coil of sixteen circumvolutions of 1 foot diameter, which could be exchanged for a single circle of 1 foot diameter when the intensities to be measured were very considerable. It was ascertained by experiment that the tangent of deflection by the former coil was exactly sixteen times that of the latter when the same intensity of current was employed. For convenience sake I have reduced all the observations to the latter standard; the unit current being therefore that which, passing through a circle 1 foot in diameter, is able to deflect the needle through 45°.

The amount of magnetism induced in a bar was ascertained by placing it vertically with its lower end at a distance of 6 or 12 inches from a magnetized needle $\frac{3}{16}$ths of an inch long and $\frac{1}{40}$th of an inch in diameter, suspended by a filament of silk, and having a fine glass index traversing over a graduated circle 6 inches in diameter. The force of torsion of the filament was found to be so trifling, that the tangents of the deflections of the needle could be taken as representing, without sensible error, the magnetism of the bar. Observations with so small a needle were made with great facility, the pointer moving steadily up to and attaining a new angle of deflection in eight or ten seconds after the electrical circuit was completed, the resistance of the air to the motion of the pointer being such as to prevent the smallest degree of oscillation. This resistance, however, of the air, so useful in bringing the needle speedily to rest, renders it necessary to guard carefully against any irregularity of the temperature of the case in which it is enclosed. A ray of sun-light would speedily occasion a deflection of several degrees*; and I found that the heat of the hand held over a part of the thick glass case 45° in advance of the pointer was sufficient, after penetration through the glass, to produce a current of air causing a steady deflection of no less than 30°, a deflection which subsided with extreme regularity and great slowness after the hand was removed. I would suggest that this circumstance points to the means of constructing a new and exceedingly sensible thermometer which would be valuable in many researches, particularly those on the conduction of heat.

Previously to employing electric currents, I made some experiments simply with a view to ascertain the inductive power of the earth's magnetism on the bars; and in which the action on the suspended needle was observed both at the distance of 12 and 6 inches, in order to determine the influence of distance for the convenience of future reductions. Having noticed the deflection produced by any bar, it was reversed and the observation repeated, the sum of the tangents of deflection showing the total effect produced on the magnetism of the bar by its reversion. I may here remark, that both ends of the pointer of the needle were invariably observed, though to save unnecessary detail the tangent of the mean is only given.

* Dr. TYNDALL has drawn attention to the importance of guarding against these effects of heat on a delicately poised needle. Philosophical Magazine, 4th series, vol. iii. p. 127.

Effect of Reversal of Bars two yards long.

Diameter of bar.	Sum of tangents of deflection. At 6 inches distance.	At 12 inches distance.
$\frac{1}{6}$ inch	·0450	·0088
$\frac{1}{4}$ inch	·0850	·0300
$\frac{1}{2}$ inch	·5912	·1922
1 inch	1·3910	·4598

The magnetism induced in the smaller bars appears to be nearly proportional to the square of the diameter, as might have been anticipated. The ratio of the attraction at 6 inches to that at 12 inches is 2·98.

Effect of Reversal of Bars one yard long.

Diameter of bar.	Sum of tangents of deflection. At 6 inches distance.	At 12 inches distance.
$\frac{1}{6}$ inch	·0480	·0138
$\frac{1}{4}$ inch	·1260	·0384
$\frac{1}{2}$ inch	·4926	·1430
1 inch	1·0380	·3084

The magnetism induced in the smaller bars of the above set is nearly proportional to the square of the diameter; the greater amount of discrepancy arising in all probability from the inferior length of the bars compared with those of the last set. The ratio of the attractions at the two distances is as 3·39 to unity.

In the following experiments on the induction of magnetism in the above bars by electrical currents, the method employed was,—1st, to observe the magnetism of a bar under the influence of the current; 2nd, that left permanently developed; 3rd, to observe the magnetism when the current was reversed; and 4th, the magnetism remaining after the current was the second time cut off. The difference between the first and third observations gives the entire change in the magnetism of the bar consequent on the reversal of the current; the difference between the second and fourth gives the entire permanent change, or as I may term it for convenience, the *magnetic set.*

The results were obtained by using currents of four degrees of intensity, in the first two of which the needle was at 6 inches distance, in the last two at 12 inches. The latter results are reduced to the action at 6 inches distance by employing the data arrived at from the foregoing experiments.

TABLE I.

Attraction, at 6 inches, of bars one yard long wrapped with 56 feet of wire.

Diameter of bar.	Intensity of current.	Total change of magnetism by reversal of current.	Magnetic set.	Total change minus magnetic set.	Set divided by square of current.	Total change minus set, divided by current.
$\frac{1}{6}$ inch	·0044	·0164	·0014	·0150	72·31	3·409
	·0197	·1012	·0266	·0746	68·54	3·787
	·0417	·3020	·1085	·1935	62·40	4·640
	·1450	2·7747	1·7036	1·0711	81·03	7·387
$\frac{1}{4}$ inch	·0041	·0364	·0038	·0326	226·05	7·951
	·0197	·2336	·0628	·1708	161·82	8·670
	·0414	·8798	·4085	·4713	238·34	11·384
	·1446	8·2871	4·9179	3·3692	235·20	23·300
$\frac{1}{2}$ inch	·0045	·0857	·0113	·0744	558·02	16·533
	·0194	·4573	·0882	·3691	234·35	19·026
	·0419	1·2162	·3207	·8955	182·67	21·372
	·1460	8·6948	2·7628	5·9320	129·61	40·630
inch	·0045	·1017	·0128	·0889	632·10	19·755
	·0195	·5089	·0817	·4272	214·86	21·908
	·0416	1·0935	·1377	·9558	79·57	22·976
	·1404	5·6858	1·0248	4·6610	51·99	33·198
1	2	3	4	5	6	7

Although the covered wire was fine and wound close to the iron, it could not be expected to act with exactly equal advantage in the bars of small as of large diameter, chiefly on account of the circuit taken by the wire being, relatively to the circumference of the bar, greater in the small than in the large bars. In comparing the results together, it should therefore be borne in mind, that those obtained with the bar of $\frac{1}{6}$th of an inch diameter are somewhat diminished from the above circumstance.

A very cursory inspection of the results convinced me that the *magnetic set* followed a very different law from that which regulated the magnetic action under the influence of the current. I have therefore subtracted the former from the latter in the 5th column of the Table. Even after this separation has been effected, it will be seen from column 7 that the magnetic action over and above the set increases with considerably greater rapidity than the intensity of the current, a result which is I believe owing to a portion of the set actually existing during the action of the current being destroyed on the breaking of the circuit. It will be remarked, on inspecting column 6, that the set of the bars of $\frac{1}{6}$ and $\frac{1}{4}$ of an inch diameter increases nearly in proportion to the square of the current, but that with the thicker bars the ratio is diminished; so that, although the set of the bars of small diameter is greater than that of the large bars when a current of powerful intensity is employed, the reverse takes place when a weak stream is used. From the 7th column it may be

gathered that the magnetism induced by an equal current, increasing at first nearly with the section of the bars, becomes ultimately almost independent of their thickness, the attractions of the half-inch and inch bars being almost exactly equal to one another.

TABLE II.

Attraction, at 6 inches, of bars two yards long wrapped with 112 feet of wire.

Diameter of bar.	Intensity of current.	Total change of magnetism by reversal of current.	Magnetic set.	Total change minus magnetic set.	Set divided by square of current.	Total change minus set, divided by current.
$\frac{1}{8}$ inch	·0042	·0150	·0009	·0141	51·02	3·357
	·0160	·0826	·0190	·0636	74·22	3·975
	·0281	·1440	·0410	·1030	51·92	3·665
	·0988	1·6531	1·0030	·6501	102·75	6·580
$\frac{1}{4}$ inch	·0042	·0451	·0037	·0414	209·75	9·857
	·0167	·2555	·0513	·2042	183·94	12·227
	·0297	·6227	·2392	·3835	271·17	12·912
	·1048	6·5007	4·3887	2·1120	399·59	20·152
$\frac{1}{2}$ inch	·0044	·0937	·0095	·0842	490·70	19·136
	·0192	·5275	·0870	·4405	236·00	22·943
	·0386	1·2243	·2597	·9646	174·30	24·990
	·1338	10·6557	4·9784	5·6773	278·08	42·429
inch	·0043	·1280	·0128	·1152	692·27	26·791
	·0178	·6088	·0822	·5266	259·44	29·584
	·0316	1·0440	·1833	·8607	183·56	27·237
	·1154	6·1017	1·6200	4·4817	121·65	38·836
1	2	3	4	5	6	7

An inspection of the above results, obtained from bars of double length wrapped with twice the length of wire, leads to conclusions similar to those we drew from Table I.

It appeared to me a matter of very great importance to investigate more closely the laws which regulate the *magnetic set*, and to determine with certainty whether the proportionality between the set and the square of the current, leading as it inevitably would to the better understanding of the nature of the molecular changes which occur in a magnetized bar, existed, and to what modifications it was subject. Seeing, therefore, that the supposed law began to fail when the thicker bars were employed, in which the mutual action of the particles distributed over a large section would naturally tend to counteract the magnetic induction developed on the exterior surface, I constructed two straight electro-magnets, one of an iron wire one yard long and $\frac{1}{25\cdot6}$ of an inch in diameter, the other of an iron wire one yard long and $\frac{1}{17\cdot2}$ of an inch in diameter. The former was wrapped with a single layer of covered copper wire $\frac{1}{40}$th of an inch in diameter and 21 feet long, the latter similarly with wire 27 feet long. The attractions of these wire electro-magnets were ascertained at

XIV. *Account of Pendulum Experiments undertaken in the Harton Colliery, for the purpose of determining the Mean Density of the Earth.*

By G. B. AIRY, *Esq., Astronomer Royal.*

Received December 26, 1855,—Read January 24 and 31, 1856.

SECTION I.—*Introductory and Historical.*

1. IN the spring of the year 1826, the idea occurred to me of attempting a determination of the Mean Density of the Earth by means of pendulum-experiments at the top and the bottom of a deep mine; and rough preliminary calculations, which seemed to show that the effect of accidental errors of observation on the ultimate result would probably be less than in the methods which up to that time had been used, confirmed me in the wish to try it. The nature of these preliminary calculations was nearly the following.

2. Conceive the earth to be a sphere of radius R and mean density D, surrounded by a spherical shell of thickness c and density d, so that the radius of the external surface is $R+c$. As the attraction of the spherical shell upon a point at or within its inner surface is nothing, the attraction at the confines of the inner sphere and shell is represented by $\frac{4\pi}{3}\cdot\frac{R^3D}{R^2}=\frac{4\pi}{3}RD$; and the attraction at the external surface of the shell is represented by $\frac{4\pi}{3}\cdot\frac{R^3D+(\overline{R+c}^3-R^3)d}{(R+c)^2}$, which to the first power of $\frac{c}{R}$ is $\frac{4\pi}{3}RD\left(1-\frac{2c}{R}+\frac{3c}{R}\cdot\frac{d}{D}\right)$. Calling the gravity at those two points respectively G and g, $\frac{g}{G}=1-\frac{2c}{R}+\frac{3c}{R}\cdot\frac{d}{D}$; from which $\frac{d}{D}=\frac{R}{3c}\cdot\frac{g}{G}-\left(\frac{R}{3c}-\frac{2}{3}\right)$. Considering the fractions $\frac{d}{D}$ and $\frac{g}{G}$ as the only quantities in this equation liable to sensible numerical error, $\delta\left(\frac{d}{D}\right)=\frac{R}{3c}\,\delta\left(\frac{g}{G}\right)$. As the density of the shell, or d, may be ascertained by actual examination, $\delta\left(\frac{d}{D}\right)=-\frac{d}{D}\cdot\frac{\delta D}{D}$; and therefore $\frac{\delta D}{D}=-\frac{D}{d}\cdot\frac{R}{3c}\,\delta\,\frac{g}{G}$.

3. In order to give a numerical value to the error of result, suppose that $\frac{D}{d}=2$ nearly (a proportion rather higher than MASKELYNE'S); that $\frac{R}{c}=16000$ (which supposes the thickness of the shell, or the depth of the mine, to be a quarter of a mile nearly); and that $\delta\left(\frac{g}{G}\right)=\frac{1}{432000}$ (which corresponds to an error of 0·1 per day in the

vibrations of a seconds' pendulum). Then $\frac{\delta D}{D} = -2 \cdot \frac{16000}{3} \cdot \frac{1}{432000} = -\frac{2}{81}$: or the mean density would be determined with an error not exceeding $\frac{1}{40}$th part of the whole. This error, I apprehend, is far less than those to which CAVENDISH's experiments were liable (REICH's and BAILY's experiments had not been made at the date of this investigation, but I do not except them from the same remark). It is also quite as small as that to which the mere astronomical determination in the Schehallien experiment was subject. When other elements of calculation are examined, as the general simplicity in the form of the ground for the mine-experiment and the complexity for the mountain-experiment, the accurate knowledge of the geology for the mine-experiment and the obscurity for the mountain-experiment, the difference becomes still more striking in favour of the mine-experiment as compared with that on Schehallien.

4. In a subsequent section, means will be given for enabling the reader to judge whether an accuracy like 0·1 vibration per day in the difference of the rates of pendulums above and below, in the mine-experiment, has really been obtained. Methods will also be indicated for computing the corrections depending on the irregularities of the earth's surface. I advert to these at present only to have the opportunity of explaining that the form of computation exhibited above is not final, but is merely intended for a preliminary calculation, showing the antecedent plausibility of the experiment.

5. Upon communicating my views to Mr. (now Dr.) WHEWELL (then, like myself, a resident Fellow of Trinity College, Cambridge), I found him entirely disposed to join me in undertaking the experiment. My first idea had been, to ascertain the rate of a clock at the top and the bottom of a mine; and the locality which first occurred to me was the Ecton Mine in Staffordshire. But it was soon settled, on discussion with Mr. WHEWELL, that detached pendulums would be preferable to clock pendulums; and, the Ecton Mine having proved on examination to be ill suited to our purpose, the Dolcoath Mine near Camborne in Cornwall was selected (partly at the suggestion of JOHN TAYLOR, Esq.). The Royal Society and the Board of Longitude most liberally lent to us invariable pendulums, clocks, and other apparatus, sufficient for the simultaneous observation of a pendulum above and one below; and the Board of Admiralty lent us box and pocket chronometers, to be used principally for the comparison of the upper and lower clocks.

6. About the end of May 1826 we proceeded to Dolcoath, assisted by the friendly introductions of the late DAVIES GILBERT, Esq. and of Dr. PARIS; and we received from the resident authorities of the mine every possible assistance to our experiment; and from the late Lord DE DUNSTANVILLE, E. W. W. PENDARVES, Esq., and other gentlemen of the country the most hospitable attention to our personal comforts. And, in spite of the labours and misfortunes of the Cornish enterprises, I do not doubt that they are regarded by my companion (as well as by myself) as among the most

interesting in our lives. Stations were provided for us at the surface and near the lowest part of the mine; the latter being at a short distance from the South Shaft or Harriet Shaft, near the junction of the granite and the killas, in the 180-fathom level (the depth being measured from the adit for the discharge of the water pumped from the mine), or nearly 1200 feet below the surface. This place, as is usual in the Cornish mines, could only be reached by ladders.

7. Our intention was, to compare the vibrations of a detached pendulum in each station with the vibrations of a clock pendulum, in the manner which has acquired so much celebrity from the labours of KATER and SABINE, and to compare the two clocks by means of chronometers carried on the person of an attendant. One set of observations, extending over several hours (including if necessary more than one series of coincidences), was to be observed each day. When this should be carried far enough, the pendulums were to be reversed and the observations were to be repeated, or the two pendulums were to be compared, as might seem best. After overcoming some difficulties, and with very great personal labour, we found on computing approximately the results that the chronometer-comparisons were not trustworthy. We resumed the work, with a modification of the method of using the chronometers which promised to render the results more accurate. We were raising the lower pendulum up the South Shaft for the purpose of interchanging the two pendulums, when (from causes of which we are yet ignorant) the straw in which the pendulum-box was packed took fire, the lashings were burnt away, and the pendulum with some other apparatus fell to the bottom. This terminated our operations of 1826.

8. In the summer of 1828 we again attempted the experiment, in the same localities, and with the same general instrumental means. Our personal powers however were far greater than in 1826. We had now the assistance of the Rev. R. SHEEPSHANKS and of two junior observers (my brother Mr., now the Rev. W. AIRY, and the late Rev. S. JACKSON). Our plan, principally at the instance of Mr. SHEEPSHANKS, was so modified as to admit of incessant observations being made, day and night, for several days consecutively; and this arrangement greatly diminished the injurious effects of chronometer-errors. A new difficulty now presented itself in the irregular and varying form of the pendulums' knife-edges. After tedious experiments on these, which seemed at last likely to be successful, our labours were suddenly stopped by the occurrence of a "fall" in the mine. The lodes or metalliferous veins in the Cornish mines are usually bounded by nearly parallel planes inclined perhaps 30° or 40° to the vertical, and the removal of the vein-stuff (even when, as in this case, the vacuity has been filled up as much as circumstances permit) endangers the falling of the rock which is on the upper side of the lode. On this occasion, the general fall (in consequence of the precautions above described) did not exceed a few inches; but large masses of rock were detached from their places, and interrupted the working of the pumps; and our lower station was soon flooded by the rise of the water. And thus for the second time our attempts were defeated.

9. Many years passed on before sufficient leisure or sufficient motive for again trying the experiment presented itself to me. The only great improvement in the application of the pendulum to the measure of gravity was BESSEL's discovery, reduced to a practical form (as regarded the English construction of the pendulum) by Colonel SABINE, of the necessity of an increased correction for the pendulum's buoyancy in the atmosphere. I had, however, opportunities of observing the difficulties inherent in the CAVENDISH experiment, from witnessing the repetition of that experiment by my late friend Mr. F. BAILY. At length, in the year 1854, a new power was placed in my hands. The galvanic system was established at the Royal Observatory, and in the familiarity which we now possessed with telegraphic applications I perceived that the difficulty of comparing the upper and lower clocks would be almost entirely removed. The coal-mines of the Durham coal-field had been worked much deeper, and the facility of access to these mines would materially diminish the labour of the experiment; while the intimate acquaintance with the geological character of the country possessed by the coal-owners, and the general regularity of the beds, would give great confidence in the ultimate calculations of the attraction of the mass of matter principally affecting the experiment. After a lapse of twenty-six years from the last attempt in Cornwall, I therefore seriously took up the subject again, and proceeded personally to examine the fitness of the coal-mines of Durham for the experiment.

10. Assisted by the introductions of DAVID LIETCH, Esq., M.D., and by the local knowledge of JAMES MATHER, Esq., I had little difficulty in fixing on a mine. The deepest mines in the Durham coal-field are near the coast. The deepest of all is the Monk Wearmouth Colliery; but it is so close to the sea (its workings in fact extending far under the sea), that it seemed probable that more of disadvantage would be introduced by the complication of the elements of final computations than of advantage by its extreme depth. The next in depth, I believe, is the Harton Pit, at the distance of somewhat more than two miles from South Shields, and about the same distance from the coast. The general circumstances of form of surface, &c. are as favourable as can usually be found. Its depth is reputed to be 1260 feet. On making known to WILLIAM ANDERSON, Esq., the principal owner of the coal-mine, my wish to try the experiment in that place, I was at once assured that every assistance should be given to me. In company with C. W. ANDERSON, Esq. and with G. W. ARKLEY, Esq. (local viewer and superintendent of the mine-works) in 1854, August 5, I examined the buildings on the surface and the workings underground in the "Bensham seam," and stations for the pendulums were speedily chosen. The two stations were exactly in the same vertical. The upper station was a stable near the Mine Office. The lower station was a wide part of a gallery (now disused), less than 200 yards from the bottom of the shaft: in going from the bottom of the shaft to the station, one-half (roughly speaking) of the way passed along one of the tram-ways of the mine, and the other half was along the disused gallery; and at this distance the sound of

the coal-trams, &c. on the tram-way was inaudible. The Harton shaft is the down-cast air-shaft for a very extensive series of workings, of which the upcast shaft is at the St. Hilda pit in the town of South Shields; and the lower pendulum station, from its proximity to the Harton pit, was constantly and agreeably ventilated with pure fresh air.

The owners of the mine immediately proceeded, at their own expense, to make the following preparations for the pendulums.

11. In the stable for the upper station, an additional brick wall was built round three sides of the space (about 16 feet square) intended for the pendulum room, and an additional substantial ceiling was constructed above it; and an anteroom was constructed on the fourth side; so that the room was in fact surrounded by double walls and ceiling. The immediate entrance from the external air was into the anteroom; and from the anteroom there were doors into the pendulum room and into a narrow room at one side. [In this narrow room were afterwards placed the galvanic batteries and journeyman-clock, and the lamp for illuminating the pendulum disk. The lamp for illuminating the face of the principal clock was in the anteroom, giving its light through a hole in the dividing wall; and the observer was stationed in the anteroom, his observing telescope being fixed in a hole in the wall.] From the gas-works of the mine, gas was led to a gas-stove in the anteroom, and also to a writing-lamp. In order to make the support of the pendulum firm, the soft earth was removed to the depth of 3 feet (at which level the hard clay was reached, which extends about 90 feet lower to the first bed of rock), and the cavity was filled with ashlar stones united with mortar at the joints: the surface was paved level with flagstones and bricks.

12. In the lower station, some upright shores which had been placed in the space assigned for the pendulum room were removed, and inclined rafters were substituted. The rock was cut level, and the bricks and flagstones were laid immediately on it. Brick walls were built enclosing a trapezoidal room nearly representing a square of 16 feet, with an anteroom, and a third room; and level ceilings were constructed above them.

13. For the galvanic communication between the two stations, two wires covered with gutta percha were led underground from the stable to the mouth of the pit; and were then carried down to the bottom of the pit, being supported in the following manner. The shaft is divided from top to bottom by a wooden partition or brattice. At every 100 feet in the descent, a wooden peg about 6 inches long and 2 inches in diameter was fixed in the brattice. The galvanic wires were wrapped round this peg, and were pinched fast by a piece of wood which was screwed to the peg. In this manner the wires were held perfectly steady, and no part was strained by an excessive weight hanging upon them. On reaching the bottom of the shaft, the wires were carried horizontally along the top of the galleries, protected by boards of wood, till they entered the lower station. The labour of Telegraph Engineers, which was

required for fixing these wires and for other adjustments of the galvanic apparatus, was contributed gratuitously by the Electric Telegraph Company; the work being done under the direction of their resident agent at Newcastle, Mr. MATHEWS.

14. The local arrangements being thus brought to a very satisfactory state, I had next to provide the proper apparatus. On my request, through Colonel SABINE, Treasurer of the Royal Society, the Council of the Society immediately placed at my disposal the instruments which they possessed and which applied to my wants. These were two invariable pendulums, both by THOMAS JONES (distinguished by the marks "1821" and "No. 8" respectively), which were preserved in the Kew Observatory; iron stands for the pendulums, accompanied by wooden stands for the comparison clocks, preserved in the Magnetic Office at Woolwich; and a clock by SHELTON (which I had formerly used in Cornwall, and which still bore my paper marks upon its pendulum). To these I added a clock by EARNSHAW (from the east dome of the Royal Observatory); a journeyman-clock (from the Royal Observatory) to be used in signal-transmission; two galvanometers, similar to the ordinary speaking-needle of the galvanic telegraph; galvanic batteries; barometers, thermometers, &c. Every instrument was brought, for trial, to the Royal Observatory.

15. Each iron stand consists of a horizontal triangle of iron bars for base; two inclined iron bars in a vertical plane for front, their tops being screwed to the sides of an iron box-frame which supports the agate-frame on which the knife-edge vibrates; and one inclined bent iron bar behind, screwed to the back of the same box-frame. On erecting one of the iron stands at the Royal Observatory, it was found impossible to pass the upper brass block of the pendulums (to which the steel knife-edge is screwed) through the holes of the box-frame. It was necessary therefore to enlarge the holes by filing; and as this weakened the front of the box-frame, the following apparatus was introduced in order to strengthen it. A small hexagonal iron frame was prepared, consisting of a horizontal base nearly equal in length to the interior measure of the box-frame, two vertical sides nearly half the height of the box-frame, two inclined sides, and a horizontal top about one-third as long as the bottom. Screws were tapped through the horizontal base. This apparatus was placed in the front opening of the box-frame; and, when the screws were driven, the short horizontal top was forced up to the upper bar of the box-frame so as to give it all desirable firmness. As the weakened stand, furnished with this apparatus, was evidently firmer than the other stand, a similar apparatus was provided for the other stand.

16. The agate-planes, and the knife-edges (which were much injured by rust), were reground and polished by Mr. SIMMS with the utmost care; the knife-edges being finished to an angle of about 120°. On mounting them at the Royal Observatory, it was found that they touched the agate-planes only in a few points; and the singularity of the mode of contact reminded me so strongly of the contact in Cornwall in 1828, that I do not doubt that one pendulum was the same and affected by the same faults. A few interchanges, &c. of pendulums showed that the fault was not in the agate-

planes, but in the knife-edges. Mr. SIMMS on examination found that, when the attaching screws were relaxed, the bearing on the agate-planes was continuous and perfect. It was evident therefore that the fault was in the surface of the brass blocks which carry the knife-edges. On filing these it was found easy to bend the knife-edge into any form. A surface was at length given to the brass which made the bearing of the knife-edges upon the agates absolutely perfect, as far as the eye could discover.

The upper and lower pendulums, when mounted in their proper stations, vibrated in parallel planes, as nearly as possible in the direction of magnetic East and West.

The graduated arcs for measuring the extent of vibration were divided to inches, with continuous numeration from one end; and were placed behind the pendulum tails.

The thermometers (two for each pendulum) were suspended in front, at about $\frac{1}{4}$ and $\frac{3}{4}$ the distances from the knife-edge to the bob. Their indications were found to be all sensibly accordant.

17. In fitting up the comparison-clocks, a small alteration was made which proved exceedingly convenient. The illuminated disk (to be concealed by the tail of the invariable pendulum) was an inclined section of a small cylindrical block attached by a central screw to the bob of the clock pendulum. The inclined surface was covered with gold-leaf. A hole (covered with glass) was made in each side of the clock-case, and through either of these the light of the illuminating lamp was thrown upon the gold-leaf. By shifting the lamp and turning the small cylindrical block, a brilliant light was reflected to the observing-telescope, the lamp being always in a distant position and in a lateral direction. Moreover, by slightly turning the clock stand in azimuth, the apparent breadth of the inclined disk was altered, and it could thus be adapted to disappearance behind the pendulum-tail.

The adjustible aperture through which the disk was seen, and which was covered by the pendulum-tail in its quiescent position, was in front of the clock-case.

18. By the side of each clock-face a galvanometer was fixed. The galvanic wires were led to and from the terminals of the galvanometer, not immediately, but through the intermediation of a circuit-breaker; so that the observer could at any time interrupt the current.

19. The journeyman-clock was thus fitted up. Two wires were led into it (one from the galvanic battery, and the other in continuation of the course of the same wire from the journeyman to the next comparison-clock), terminating within the journeyman in a pair of springs which performed the duty of circuit-breaker. Upon the minute-wheel of the journeyman were four pins, which, as the wheel revolved, pressed the two springs together, thus completing the circuit (in that part) at every 15^s of the journeyman's time.

20. The battery was the ordinary sand-battery, of 24 cells. The batteries and journeyman were in the side-room of the upper station.

The wire from one end of the battery was led to one terminal of the journeyman. From the other terminal of the journeyman a wire was led to the circuit-breaker in connexion with the galvanometer of the upper clock. From the other terminal of the galvanometer a wire was led down the mine-shaft to the circuit-breaker in connexion with the galvanometer of the lower clock. From the other terminal of this lower galvanometer a wire was led up the mine-shaft to the other pole of the battery. Thus when both the upper and the lower circuit-breakers completed contact (and at no other time), the journeyman-clock made the circuit complete and sent a current through both the galvanometers at every 15^s of the journeyman's time.

20*. Plate XI. contains views of the pendulum-apparatus nearly in the state in which it was used in the upper station. The principal diagram is a front view of the apparatus as mounted at Greenwich, taken with the camera lucida, and may be trusted for general accuracy. The iron bars of the pendulum-stand are about $1\frac{1}{4}$ inch square. The stand of the clock does not touch the pendulum-stand in any part. The hexagonal frame introduced into the box-frame is conspicuous in this view. The battery is not the same which was used at the mine. The mounting at the lower station was exactly similar, wanting only the journeyman-clock and the battery. The frame with agate-planes (represented on a larger scale), which is planted on the top of the box-frame, is supported on three screw-feet: the screw-stalks are perforated; two are cut with an internal screw-thread, and long screws are passed upwards through smooth holes in the box-frame, and act in these internal screw-threads and draw them firmly down: the perforation in the third is smooth, and the long screw which passes down through it acts in a screw-thread cut in the box-frame. At the sides of the blocks which carry the agate-planes are notched brass plates turning upon pins, connected by a stouter piece of brass beyond the pins, through which a screw passes that acts in the solid block below; by driving this screw the notches are raised, and engage with the ends of the pendulum-knife-edge, and lift it off the agates.

21. The system of observations which I proposed was the following. One of the invariable pendulums was to be mounted at the upper station and the other at the lower station, and the two pendulums were to be observed simultaneously by two observers. The "Swings" or series of vibrations were to follow each other incessantly, day and night, with no more interruption than would be required for observing the galvanic signals, reading thermometers, and making petty adjustments. Six Swings (each occupying in the gross four hours) were to be taken in each day. At concerted minutes of time before the first and after the last Swing, and between the end of each preceding Swing and the beginning of that which followed, galvanic signals were to be observed. Several "Coincidences" of the vibrations of the detached pendulum with the clock pendulum were to be observed in KATER's manner at the beginning and end of each Swing. This system was to be maintained during the whole efficient time of one week: then the pendulums with their agate-planes and

thermometers, but with no other apparatus, were to be interchanged, and a week's observations were to be made in this state. The pendulums were to be interchanged a second time and a third time, so that, in the whole, four series of observations would be taken: but I thought it probable that less than a week would suffice for each of these latter series.

22. On considering the amount of labour required for carrying out this plan, and considering also that it was not in my power to take an active part, I judged that six observers would be necessary. With the sanction of the Lords Commissioners of the Admiralty, I appointed Mr. EDWIN DUNKIN and Mr. WILLIAM ELLIS, Assistants of the Royal Observatory, as two of the observers. With the permission of M. J. JOHNSON, Esq., I was enabled to solicit and to obtain the services of Mr. NORMAN POGSON, Assistant of the Oxford Observatory; Professor CHALLIS consented to my asking the aid of Mr. G. S. CRISWICK, Assistant of the Cambridge Observatory; by permission of Professor CHEVALLIER, Mr. GEORGE RÜMKER, Astronomer of the Durham Observatory, joined the party; and finally I was enabled, by the kindness of R. C. CARRINGTON, Esq., to avail myself of the services of Mr. G. H. SIMMONDS, Assistant of the Red Hill Observatory. Mr. DUNKIN superintended the party, and, during the observations, controlled the local operations of every kind. Under the admirable management of Mr. DUNKIN, and the zealous and orderly assistance of all the gentlemen whom I have named, the work went on with the most perfect regularity. I cannot express how much I am indebted to the hearty cooperation of every individual Assistant during the whole course of the operations from the beginning to the end.

Lodgings were provided for the party in the town of South Shields. Their comfort, during their singular occupation, as well as my own on the occasions of my visits, were greatly increased by the uniform kindness of the authorities of the mine, and by the hospitable attentions of ROBERT INGHAM, Esq., M.P., JOHN ROBINSON, Esq., Mayor of South Shields, JAMES MATHER, Esq., J. C. STEVENSON, Esq., of the Jarrow Chemical Works, and other gentlemen resident in the town or neighbourhood. Attendants in the mine were selected and placed under the command of the observers by the owners of the mine.

23. I may now give a brief journal of the operations.

1854. August 5.—I examined the mine, and stations were selected.

Instruments were procured, repaired, and sent to the Harton Mine; and observers were collected.

September 26.—I went to South Shields.

September 27.—I erected the clocks and pendulum at the upper station.

September 28.—I erected the clocks and pendulum at the lower station. In the evening, four of the observers arrived.

September 29.—In the morning, the remaining observers arrived. All were employed in adjusting the pendulums, &c. and in the practice of observations.

1854. September 30.—I returned to Greenwich, leaving all in the charge of Mr. DUNKIN.

October 2.—Observations commenced in the morning with Swing 1. Pendulum 1821 above, No. 8 below. Observers for the day, Mr. DUNKIN (above), Mr. ELLIS (below), from the beginning of Swing 1 to the beginning of Swing 3.

Observers for the night, Mr. POGSON and Mr. RÜMKER, from the end of Swing 3 to the beginning of Swing 6.

October 3.—Observers for the day, Mr. CRISWICK and Mr. SIMMONDS, from the end of Swing 6 to the beginning of Swing 9: for the night, Mr. ELLIS and Mr. POGSON, from the end of Swing 9 to the beginning of Swing 12.

October 4.—Observers for the day, Mr. RÜMKER and Mr. CRISWICK, from the end of Swing 12 to the beginning of Swing 15: for the night, Mr. ELLIS and Mr. SIMMONDS, from the end of Swing 15 to the beginning of Swing 18.

October 5.—Observers for the day, Mr. CRISWICK and Mr. POGSON, from the end of Swing 18 to the beginning of Swing 21: for the night, Mr. SIMMONDS and Mr. RÜMKER, from the end of Swing 21 to the beginning of Swing 24.

October 6.—Observers for the day, Mr. DUNKIN and Mr. ELLIS, from the end of Swing 24 to the end of Swing 26. The First Series was closed at the end of Swing 26.

October 7.—Mr. DUNKIN, with the assistance of Mr. ELLIS and Mr. POGSON, interchanged the pendulums.

October 9.—The Second Series commenced with Swing 27. Pendulum No. 8 above, 1821 below.

Observers for the day, Mr. SIMMONDS and Mr. CRISWICK, from the beginning of Swing 27 to the beginning of Swing 29: for the night, Mr. ELLIS and Mr. DUNKIN, from the end of Swing 29 to the beginning of Swing 32.

October 10.—Observers for the day, Mr. RÜMKER and Mr. POGSON, from the end of Swing 32 to the beginning of Swing 35: for the night, Mr. CRISWICK and Mr. SIMMONDS, from the end of Swing 35 to the beginning of Swing 38.

October 11.—Observers for the day, Mr. DUNKIN and Mr. ELLIS, from the end of Swing 38 to the beginning of Swing 41: for the night, Mr. RÜMKER and Mr. POGSON, from the end of Swing 41 to the beginning of Swing 44.

October 12.—Observers for the day, Mr. SIMMONDS and Mr. CRISWICK, from the end of Swing 44 to the beginning of Swing 47: for

the night, Mr. Dunkin and Mr. Ellis, from the end of Swing 47 to the beginning of Swing 50.

1854. October 13.—Observers for the day, Mr. Pogson and Mr. Rümker, from the end of Swing 50 to the end of Swing 52. The Second Series was closed at the end of Swing 52.

October 14.—Mr. Dunkin, with the assistance of Mr. Ellis and Mr. Pogson, interchanged the pendulums.

October 16.—The Third Series commenced with Swing 53. Pendulum 1821 above, No. 8 below. Observers for the day, Mr. Criswick and Mr. Simmonds, from the beginning of Swing 53 to the beginning of Swing 55: for the night, Mr. Rümker and Mr. Pogson, from the end of Swing 55 to the beginning of Swing 58.

October 17.—Observers for the day, Mr. Ellis and Mr. Dunkin, from the end of Swing 58 to the beginning of Swing 61: for the night, Mr. Simmonds and Mr. Criswick, from the end of Swing 61 to the beginning of Swing 64.

October 18.—Observers for the day, Mr. Pogson and Mr. Rümker, from the end of Swing 64 to the beginning of Swing 67; of which the end was observed by Mr. Dunkin and Mr. Ellis. The Third Series was closed at the end of Swing 67. Mr. Dunkin and Mr. Ellis interchanged the pendulums at night.

October 19.—The Fourth Series commenced with Swing 68, begun by Mr. Dunkin and Mr. Ellis. Pendulum No. 8 above, 1821 below. Observers for the day, Mr. Criswick and Mr. Simmonds, from the end of Swing 68 to the beginning of Swing 71: for the night, Mr. Rümker and Mr. Pogson, from the end of Swing 71 to the beginning of Swing 74.

October 20.—Observers for the day, Mr. Dunkin and Mr. Ellis, from the end of Swing 74 to the beginning of Swing 77: for the night, Mr. Simmonds and Mr. Criswick, from the end of Swing 77 to the beginning of Swing 80.

October 21.—Observers for the day, Mr. Pogson and Mr. Rümker, from the end of Swing 80 to the beginning of Swing 82: the end of Swing 82 was observed by Mr. Dunkin and Mr. Pogson. The Fourth Series and the whole operation closed with Swing 82.

October 22.—I arrived from Greenwich.

October 23.—I inspected the instruments in position, and dismounted them.

October 24.—The instruments were mounted in the Central Hall of South

Shields, and I explained the nature of the observations to an assembly of the residents of South Shields and its neighbourhood.

1854. October 25.—The instruments were finally dismounted, and packed up for return to Greenwich; and the party dispersed.

24. In terminating this general history of the operation, it is proper perhaps that I should briefly allude to the sources from which the expenses were defrayed.

The Royal Society, in granting the loan of their pendulums and clock, not only enabled me to undertake the operation with promptitude and with the security of using trustworthy instruments, but also removed one of the most serious causes of expense. I have already alluded to the extensive works constructed by the owners of the Harton Colliery, and which they made entirely at their own expense; and to the liberality of the Electric Telegraph Company, who gratuitously gave me their important assistance in the galvanic connexions. In transmitting the instruments from the wharfs at Newcastle to South Shields, Messrs. STEVENSON and Co., of the Jarrow Chemical Works, gave the use of their river-craft. The Mayor and Corporation of South Shields, unsolicited, caused the surveys to be made which were needed for computation of the attraction of the ground. Some parts of the apparatus (galvanometers, wires, stoves, &c.) were so evidently applicable to the prospective wants of the Royal Observatory, that they were not considered as a charge on the Experiment. The Lords Commissioners of the Admiralty, on my laying before them my proposals, contributed £100, which defrayed the greater part of the miscellaneous expenses. The residual charges were borne by myself.

SECTION II.—*Comparisons of the Upper and Lower Clocks.*

25. In the ordinary state, the galvanic circuit was interrupted at both circuit-breakers. At the pre-arranged minute for commencing signals, as nearly as possible, each of the observers completed the connexion, and then gave his attention to the galvanometer. The movement of the needle, at intervals of 15^s nearly, assured each observer that the other was prepared to observe; and every observation of the clock-time of the movement of the needle was then efficient for comparison of the two clocks. Only, it was necessary to know precisely, when the time of examining the written record of the time-signals should arrive, *which* were the *corresponding* observations of the same galvanic current. For this purpose, the following instructions were given. After observing the clock-times of a few signals, the upper observer interrupted the circuit during one signal only, and then restored the connexion. The lower observer, after waiting one or two minutes, did the same thing. After another short delay, the upper observer interrupted the circuit during two signals. The lower observer, after a short delay, did the same. Then the observations were continued till the upper observer was satisfied with the number, after which he definitively

broke the circuit; the lower observer then broke circuit; and both prepared to observe Coincidences of the pendulums. On examining the interruptions of signals, no difficulty was found in confronting the corresponding observations. In one instance only (between Swings 16 and 17) was the comparison totally lost, in consequence of one of the needles sticking fast. The observation of the time of the start of the needle is not very delicate; and this part of the operation would be made much more exact, by causing each clock to register its own seconds upon a common revolving barrel, in the manner of the American transits.

26. The observations on each side which had no corresponding observations on the other side being struck out, the means of all the times of the remaining observations in each group were taken, and the difference of means was formed. As a check, the differences of the times by the two clocks for each individual signal were formed, and the mean of the differences was taken. In this manner there were formed, for every group, mean corresponding times of SHELTON (the upper clock) and EARNSHAW (the lower clock); the differences of these, from group to group, gave the corresponding changes of clock-indications by the two clocks during the same period of absolute time; and the quotient $\frac{\text{EARNSHAW's change of indication}}{\text{SHELTON's change of indication}}$ gave the apparent ratio of rates. The logarithm of this ratio was formed, by means of CALLET's logarithms, to eight decimals.

27. A very cursory examination of these ratios showed that there was a considerable personal equation in the observation of the galvanometer-signals. Though (from the nature of the combination to be hereafter described) this scarcely produces an appreciable effect on the ultimate result, I thought it desirable to ascertain approximately its magnitude, and to apply the corresponding correction. I proceeded as follows:—In every instance in which the signal-observations at the beginning and at the end of a Swing were made by different observers, I compared the logarithms of apparent rates of $\frac{\text{EARNSHAW}}{\text{SHELTON}}$ during that swing with the logarithms for the preceding and following swings (in three instances, however, with only the preceding swing); and formed the excess of the logarithm of the intermediate swing above the mean of the preceding and following logarithms. This numerical excess may be compared with a symbolical formula, in which the symbols represent, not the actual error in time committed by each observer, but the logarithm (in 8-figure units) of the influence of his error on rates deduced from comparisons at 4-hour intervals. Twenty-seven equations were thus formed. By the method of minimum squares, these were reduced to six, of which (from the nature of the case) one was unnecessary, their sum being identically equal to zero. Assuming D (Mr. DUNKIN's error) $=0$, the others are found by solution of the equations. Though the equations are not very favourable, they suffice for giving very good corrections, and they show in particular that Mr. SIMMONDS recorded his times too early by nearly half a second. The following

Table exhibits this work; the fundamental numbers being obtained from a table which is to follow:—

No. of Swing.	Order of Observers. Above.	Order of Observers. Below.	Symbol for the Errors of Logarithmic Rate $\frac{\text{Earnshaw}}{\text{Shelton}}$ produced by each observer's error in the estimation of time.	Errors inferred from the computed Logarithmic Rates.	No. of Swing.	Order of Observers. Above.	Order of Observers. Below.	Symbol for the Errors of Logarithmic Rate $\frac{\text{Earnshaw}}{\text{Shelton}}$ produced by each observer's error in the estimation of time.	Errors inferred from the computed Logarithmic Rates.
3.	D P	E R	D+R−P−E	− 277	47.	S D	C E	S+E−D−C	− 946
6.	P C	R S	P+S−C−R	− 45	50.	D P	E R	D+R−P−E	+ 135
9.	C E	S P	C+P−E−S	+ 134	55.	C R	S P	C+P−R−S	+2850
12.	E R	P C	E+C−R−P	−1957	58.	R E	P D	R+D−E−P	− 381
15.	R E	C S	R+S−E−C	+ 512	61.	E S	D C	E+C−S−D	− 254
18.	E C	S P	E+P−C−S	+2257	64.	S P	C R	S+R−P−C	−1621
21.	C S	P R	C+R−S−P	+1125	67.	P D	R E	P+E−D−R	+ 443
24.	S D	R E	S+E−D−R	+ 177	68.	D C	E S	D+S−C−E	−1079
29.	S E	C D	S+D−E−C	− 933	71.	C R	S P	C+P−R−S	+1156
32.	E R	D P	E+P−R−D	+ 360	74.	R D	P E	R+E−D−P	−1135
35.	R C	P S	R+S−C−P	−1581	77.	D S	E C	D+C−S−E	+1083
38.	C D	S E	C+E−D−S	+ 747	80.	S P	C R	S+R−P−C	−1706
41.	D R	E P	D+P−R−E	+ 956	82.	P D	R P	2P−D−R	− 465
44.	R S	P C	R+C−S−P	+ 531					

Equations.

$$
\begin{array}{r}
18.C-2.D+2.E+P-4.R-15.S=+10557 \\
-2.C+15.D-14.E-5.P+5.R+S=+577 \\
+2.C-14.D+18.E+2.P-6.R-2.S=-458 \\
C-5.D+2.E+22.P-16.R-4.S=+14048 \\
-4.C+5.D-6.E-16.P+19.R+2.S=-7873 \\
-15.C+D-2.E-4.P+2.R+18.S=-16851.
\end{array}
$$

Approximate Solution, assuming $D=0$

$$
\begin{aligned}
C &= -\ 720\\
D &= \quad 0\\
E &= -\ 240\\
P &= +\ 180\\
R &= -\ 330\\
S &= -1500
\end{aligned}
$$

These are applied, with signs changed, to correct the computed Logarithmic Rates in the Swings which are included in the last Table: and thus are formed the "Corrected Logarithmic Rates" in the Table which follows next.

28. It is impossible to give here in detail the whole of the observations of Galvanic Signals. The whole number of efficient signals observed at each station was 2454 (or 4908 observations of signals in all). It will be sufficient to give here the mean of each group (which, from the check described above, is extremely certain), and the calculations founded on those means.

It will be remarked that the pendulums of the clocks were altered at the times of interchanging the detached pendulums; that is, between Swings 26 and 27, between 52 and 53, and between 67 and 68.

No. of Swing.	Approximate Time (Astronomical Reckoning).	Number of Signals.	Mean of Times by SHELTON.	Mean of Times by EARNSHAW.	Interval by SHELTON.	Interval by EARNSHAW.	Rate $\frac{\text{EARNSHAW}}{\text{SHELTON}}$	Logarithm of Rate $\frac{\text{EARNSHAW}}{\text{SHELTON}}$	Corrected Logarithm of Rate $\frac{\text{EARNSHAW}}{\text{SHELTON}}$
	Oct. h		h m s	h m s	h m s	h m s			
	1. 23	22	3 19 36·505	21 23 28·764					
1					4 0 23·100	4 0 38·722	1·0010831	0·00047012	
	2. 3	21	7 19 59·605	1 24 7·486					
2					3 58 21·652	3 58 37·400	1·0011011	0·00047793	
	2. 7	21	11 18 21·257	5 22 44·886					
3					4 45 27·829	4 45 46·421	1·0010855	0·00047117	0·00047387
	2. 11	29	16 3 49·086	10 8 31·307					
4					4 17 6·532	4 17 23·234	1·0010827	0·00046995	
	2. 16	17	20 20 55·618	14 25 54·541					
5					3 13 21·898	3 13 34·795	1·0011116	0·00048249	
	2. 19	25	23 34 17·516	17 39 29·336					
6					3 49 42·503	3 49 57·654	1·0010994	0·00047720	0·00047990
	2. 23	31	3 24 0·019	21 29 26·990					
7					3 55 2·071	3 55 17·433	1·0010893	0·00047282	
	3. 3	21	7 19 2·090	1 24 44·423					
8					4 2 41·510	4 2 57·445	1·0010944	0·00047503	
	3. 7	25	11 21 43·600	5 27 41·868					
9					4 31 5·786	4 31 23·591	1·0010947	0·00047516	0·00046316
	3. 11	22	15 52 49·386	9 59 5·459					
10					3 27 49·747	3 28 3·324	1·0010888	0·00047260	
	3. 15	24	19 20 39·133	13 27 8·783					
11					3 59 47·292	4 0 3·188	1·0011049	0·00047959	
	3. 19	24	23 20 26·425	17 27 11·971					
12					4 3 30·416	4 3 46·029	1·0010686	0·00046384	0·00047194
	3. 23	17	3 23 56·841	21 30 58·000					
13					3 58 21·005	3 58 37·058	1·0011225	0·00048723	
	4. 3	24	7 22 17·846	1 29 35·058					
14					5 14 58·735	5 15 19·267	1·0010865	0·00047160	
	4. 8	16	12 37 16·581	6 44 54·325					
15					2 40 24·234	2 40 34·859	1·0011040	0·00047920	0·00048790
	4. 11	13	15 17 40·815	9 25 29·184					
16 17	....	...	....	....	8 46 49·620	8 47 24·325	1·0010979	0·00047655	
	4. 20	31	0 4 30·435	18 12 53·509					
18					3 34 0·822	3 34 15·481	1·0011416	0·00049550	0·00047390
	4. 23	21	3 38 31·257	21 47 8·990					
19					3 44 11·953	3 44 26·496	1·0010812	0·00046930	
	5. 3	21	7 22 43·210	1 31 35·486					
20					3 58 15·444	3 58 31·268	1·0011071	0·00048054	
	5. 7	24	11 20 58·654	5 30 6·754					
21					4 3 26·760	4 3 43·156	1·0011225	0·00048723	0·00048453
	5. 11	29	15 24 25·414	9 33 49·910					
22					3 56 39·589	3 56 55·010	1·0010861	0·00047142	
	5. 15	30	19 21 5·003	13 30 44·920					
23					3 58 32·060	3 58 47·569	1·0010836	0·00047034	
	5. 19	19	23 19 37·063	17 29 32·489					
24					4 0 58·264	4 1 14·066	1·0010930	0 00047442	0·00048852
	5. 23	22	3 20 35·327	21 30 46·555					
25					3 59 49·427	4 0 5·170	1·0010942	0·00047495	
	6. 3	24	7 20 24·754	1 30 51·725					
26					4 0 15·008	4 0 30·854	1·0010993	0·00047716	
	6. 7	24	11 20 39·762	5 31 22·579					

Table (continued).

No. of Swing.	Approximate Time (Astronomical Reckoning).	Number of Signals.	Mean of Times by Shelton.	Mean of Times by Earnshaw.	Interval by Shelton.	Interval by Earnshaw.	Rate $\frac{\text{Earnshaw}}{\text{Shelton}}$.	Logarithm of Rate $\frac{\text{Earnshaw}}{\text{Shelton}}$.	Corrected Logarithm of Rate $\frac{\text{Earnshaw}}{\text{Shelton}}$.
	Oct. h		h m s	h m s	h m s	h m s			
27	8. 23	49	3 14 11·153	21 37 30·535	3 51 17·182	3 51 37·539	1·0014670	0·00063664	
28	9. 3	43	7 5 28·335	1 29 8·074	3 58 30·751	3 58 51·994	1·0014844	0·00064419	
29	9. 7	28	11 3 59·086	5 28 0·068	4 6 14·677	4 6 36·178	1·0014553	0·00063157	0·00063697
30	9. 11	24	15 10 13·763	9 34 36·246	4 0 41·885	4 1 3·098	1·0014690	0·00063751	
31	9. 15	25	19 10 55·648	13 35 39·344	3 52 14·148	3 52 34·667	1·0014726	0·00063907	
32	9. 19	27	23 3 9·796	17 28 14·011	4 18 25·029	4 18 47·889	1·0014744	0·00063985	0·00063715
33	9. 23	24	3 21 34·825	21 47 1·900	3 59 34·514	3 59 55·495	1·0014596	0·00063344	
34	10. 3	23	7 21 9·339	1 46 57·395	3 43 1·815	3 43 21·572	1·0014765	0 00064076	
35	10. 7	24	11 4 11·154	5 30 18·967	4 1 33·412	4 1 54·153	1·0014311	0·00062107	0·00063397
36	10. 11	39	15 5 44·566	9 32 13·120	3 58 38·771	3 58 59·655	1·0014586	0·00063300	
37	10. 15	32	19 4 23·337	13 31 12·775	3 59 55·890	4 0 16·912	1·0014603	0·00063374	
38	10. 19	39	23 4 19·227	17 31 29·687	4 15 19·657	4 15 42·268	1·0014760	0·00064054	0·00063514
39	10. 23	38	3 19 38·884	21 47 11·955	3 44 44·120	3 45 3·768	1·0014572	0·00063240	
40	11. 3	26	7 4 23·004	1 32 15·723	3 59 52·573	4 0 13·338	1·0014427	0·00062610	
41	11. 7	26	11 4 15·577	5 32 29·061	4 17 53·278	4 18 16·016	1·0014696	0·00063777	0·00063027
42	11. 11	22	15 22 8·855	9 50 45·077	3 46 55·016	3 47 14·790	1·0014524	0·00063031	
43	11. 15	24	19 9 3·871	13 37 59·867	3 55 13·879	3 55 34·324	1·0014486	0·00062866	
44	11. 19	22	23 4 17·750	17 33 34·191	4 15 16·791	4 15 39·072	1·0014547	0·00063131	0·00062861
45	11. 23	46	3 19 34·541	21 49 13·263	3 45 19·636	3 45 39·054	1·0014363	0·00062333	
46	12. 3	35	7 4 54·177	1 34 52·317	3 58 57·703	3 59 18·356	1·0014405	0·00062515	
47	12. 7	30	11 3 51·880	5 34 10·673	4 10 12·288	4 10 33·581	1·0014184	0·00061556	0·00062576
48	12. 11	28	15 14 4·168	9 44 44·254	3 53 8·084	3 53 28·225	1·0014399	0·00062489	
49	12. 15	29	19 7 12·252	13 38 12·479	3 57 45·889	3 58 6·269	1·0014286	0·00061999	
50	12. 19	27	23 4 58·141	17 36 18·748	4 22 20·477	4 22 42·963	1·0014286	0·00061999	0·00062269
51	12. 23	27	3 27 18·618	21 59 1·711	3 36 44·350	3 37 2·817	1·0014201	0·00061630	
52	13. 3	25	7 4 2·968	1 36 4·528	3 58 50·329	3 59 10·661	1·0014188	0·00061574	
	13. 7	29	11 2 53·297	5 35 15·189					
53	15. 23	104	2 51 48·661	22 3 1·492	3 32 42·619	3 32 54·360	1·0009200	0·00039937	
54	16. 3	25	6 24 31·280	1 35 55·852	4 0 3·781	4 0 16·936	1·0009134	0·00039650	
55	16. 7	26	10 24 35·061	5 36 12·788	4 2 14·546	4 2 28·746	1·0009770	0·00042410	0·00041120
56	16. 11	29	14 26 49·607	9 38 41·534	4 17 55·132	4 18 9·202	1·0009092	0·00039469	
57	16. 15	28	18 44 44·739	13 56 50·736	3 38 20·977	3 38 32·960	1·0009148	0·00039712	
58	16. 19	25	22 23 5·716	17 35 23·696	4 2 2·672	4 2 15·792	1·0009034	0·00039216	0·00039486
59	16. 23	25	2 25 8·388	21 37 39·488	3 58 57·599	3 59 10·638	1·0009095	0·00039482	
60	17. 3	23	6 24 5·987	1 36 50·126	4 0 55·113	4 1 8·229	1·0009074	0·00039390	
61	17. 7	29	10 25 1·100	5 37 58·355	4 1 48·703	4 2 1·816	1·0009039	0·00039238	0·00038698
62	17. 11	31	14 26 49·803	9 40 0·171	3 59 31·125	3 59 44·233	1·0009121	0·00039594	
63	17. 15	29	18 26 20·928	13 39 44·404	4 0 54·254	4 1 7·117	1·0008899	0·00038631	
64	17. 19	34	22 27 15·182	17 40 51·521	4 17 49·235	4 18 2·410	1·0008518	0·00036977	0·00038267
65	17. 23	29	2 45 4·417	21 58 53·931	3 42 13·263	3 42 25·106	1·0008884	0·00038565	
66	18. 3	30	6 27 17·680	1 41 19·037	3 58 48·764	3 59 1·081	1·0008597	0·00037320	
67	18. 7	27	10 26 6·444	5 40 20·118	4 3 7·643	4 3 20·333	1·0008699	0·00037763	0·00037493
	18. 11	45	14 29 14·087	9 43 40·451					
68	18. 19	55	21 34 41·845	17 32 42·435	4 1 58·198	4 2 16·440	1·0012565	0·00054535	0·00055075
69	18. 23	28	1 36 40 043	21 34 58·875	4 0 38·893	4 0 57·393	1·0012814	0·00055614	
70	19. 3	25	5 37 18·936	1 35 56·268	4 0 17·451	4 0 35·535	1·0012544	0·00054444	
71	19. 7	31	9 37 36·387	5 36 31·803	4 4 20·858	4 4 39·775	1·0012903	0·00056001	0·00054711
72	19. 11	56	13 41 57·245	9 41 11·578	3 54 17·299	3 54 35·192	1·0012729	0·00055246	
73	19. 15	27	17 36 14·544	13 35 46·770	4 1 31·831	4 1 50·051	1·0012573	0·00054570	
74	19. 19	28	21 37 46·375	17 37 36·821	4 11 36·610	4 11 55·090	1·0012241	0·00053129	0·00053879
75	19. 23	27	1 49 22·985	21 49 31·911	3 48 54·229	3 49 11·303	1·0012432	0·00053958	
76	20. 3	28	5 38 17·214	1 38 43·214	3 57 35·469	3 57 53·148	1·0012402	0·00053828	
77	20. 7	24	9 35 52·683	5 36 36·362	4 15 21·064	4 15 40·318	1·0012567	0·00054543	0·00053523
78	20. 11	30	13 51 13·747	9 52 16·680	3 47 1·059	3 47 17·720	1·0012232	0·00053091	
79	20. 15	31	17 38 14·806	13 39 34·400	3 58 48·581	3 59 6·177	1·0012281	0·00053302	
80	20. 19	30	21 37 3·387	17 38 40·577	4 5 56·270	4 6 13·720	1·0011826	0·00051330	0·00052620
81	20. 23	30	1 42 59·657	21 44 54·297	3 52 31·475	3 52 48·435	1·0012158	0·00052770	
82	21. 3	28	5 35 31·132	1 37 42·732	4 11 15·522	4 11 33·690	1·0012051	0·00052305	0·00051615
	21. 7	56	9 46 46·654	5 49 16·422					

Section III.—*General System of observing the Pendulums and of reducing the Observations.*

29. Before describing the observations, &c., I will remark that the pendulums above and below were mounted in exactly the same manner. Each angle of the iron stand, in each station, rested on a single brick; and great care was taken by Mr. Dunkin that the bearing of these bricks should be perfectly solid. It was also a subject of Mr. Dunkin's special attention to make the supplementary hexagonal iron frame (which I have described in article 15) quite firm; and, above all, to fix firmly at the beginning of each series and to examine carefully at the end of each series the frame carrying the agate-planes. In every instance these were found perfectly firm in their attachment. At every interchange of pendulums, Mr. Dunkin carefully oiled and wiped the knife-edges and their agate-planes. At the beginning of every swing, the observer raised the knife-edge from contact with the agate-plane, by the screw-lifting apparatus, and then lowered it gently to a definite line of bearing.

30. The pendulum being observed in Kater's manner, by using the concealment of the bright disk on the clock-pendulum-bob behind the tail of the detached pendulum, when passing the aperture through which alone the disk can be seen, as indication of the Coincidence of the two pendulums in the times of passing their respective quiescent points; and supposing that there are trifling errors in the adjustments of position; it is seen in practice, or is shown by a very simple investigation, that the disk will first disappear in passing from one side (suppose the right side) towards the centre, then will disappear in passing from the left side (after which, if the errors of adjustment are in the proper direction, it will be invisible during several vibrations), then it will reappear on the left side, and will finally reappear on the right side. Either the mean of the times of the first and fourth phenomenon, or the mean of the second and third, or the mean of all four, may be used as the true time of Coincidence; and it was left to the discretion of the observers to adopt which they preferred. They chose, in every case, to observe the first and fourth only. This amounts in fact to using only one side of the aperture.

It is necessary for the success of this observation that the arc of the detached pendulum be less than that of the clock pendulum; in fact it was always much less. It is indifferent whether the detached pendulum vibrate quicker or slower than the clock pendulum; in fact it always vibrated slower.

31. Several Coincidences were always observed at the beginning of a Swing, and several at the end. These gave the Interval of Coincidences nearly enough to enable me to fix upon the number of intervening Coincidences. In general, some of the observed Coincidences were rejected, so that from two to five Coincidences were retained at the beginning, and a number at the end so corresponding that the difference of their means would represent an integral number of Intervals (thus there might be 3 at the beginning and 5 at the end, or 4 at the beginning and 2 at the

end). The mean of the first retained and the last retained being taken, the difference of means divided by the number of Intervals gave a very exact value of Mean Interval.

In the Interval, the detached pendulum lost two vibrations on the clock pendulum. If then the times of the first and last mean be taken *from the clock-face,* and if n be the number of seconds, referred to the clock, in the Mean Interval, the rate of the detached pendulum on the clock pendulum will be $\frac{n-2}{n}$. The following Tables contain all the values of $\log \frac{n-2}{n}$ which are required here. For estimating the effect of any error in the Interval, it should be remembered that the logarithm of $\frac{86401}{86400}$, or of a rate of one second daily, is 500 in the last figures of 8-figure logarithms, nearly.

n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.
293·0	9·99702537	300·0	9·99709501	376·0	9·99768376	392·0	9·99777854	405·0	9·99785003
·1	2639	·1	9598	·1	8438	·1	7911	·1	5056
·2	2740	·2	9695	·2	8499	·2	7967	·2	5109
·3	2842	·3	9792	·3	8561	·3	8024	·3	5162
·4	2943	·4	9889	·4	8622	·4	8080	·4	5215
·5	3045	·5	9·99709986	·5	8684	·5	8137	·5	5268
·6	3146	·6	9·99710083	·6	8746	·6	8194	·6	5321
·7	3248	·7	0179	·7	8807	·7	8251	·7	5374
·8	3349	·8	0276	·8	8869	·8	8307	·8	5428
·9	3451	·9	0372	·9	8930	·9	8364	·9	5481
294·0	3552	301·0	0469	377·0	8992	393·0	8421	406·0	5534
·1	36[illegible]3	·1	0565			·1	8477	·1	5587
·2	3754	·2	0662	389·0	9·99776137	·2	8534	·2	5639
·3	3856	·3	0758	·1	6194	·3	8590	·3	5692
·4	3957	·4	0855	·2	6252	·4	8647	·4	5744
·5	4058	·5	0951	·3	6309	·5	8703	·5	5797
·6	4158	·6	1047	·4	6367	·6	8759	·6	5850
·7	4259	·7	1143	·5	6424	·7	8816	·7	5903
·8	4359	·8	1239	·6	6482	·8	8872	·8	5955
·9	4460	·9	1335	·7	6539	·9	8929	·9	6008
295·0	4560	302·0	1431	·8	6597	394·0	8985	407·0	6061
				·9	6654			·1	6114
298·0	9·99707545	374·0	9·99767134	390·0	6712	403·0	9·99783932	·2	6166
·1	7643	·1	7196	·1	6769	·1	3986	·3	6219
·2	7741	·2	7258	·2	6827	·2	4040	·4	6271
·3	7840	·3	7321	·3	6884	·3	4094	·5	6324
·4	7938	·4	7383	·4	6942	·4	4148	·6	6377
·5	8036	·5	7445	·5	6999	·5	4202	·7	6429
·6	8134	·6	7507	·6	7056	·6	4255	·8	6482
·7	8232	·7	7569	·7	7113	·7	4308	·9	6534
·8	8330	·8	7632	·8	7170	·8	4362	408·0	6587
·9	8428	·9	7694	·9	7227	·9	4415	·1	6639
299·0	8526	375·0	7756	391·0	7284	404·0	4468	·2	6692
·1	8624	·1	7818	·1	7341	·1	4522	·3	6744
·2	8722	·2	7880	·2	7398	·2	4575	·4	6797
·3	8819	·3	7942	·3	7456	·3	4629	·5	6849
·4	8917	·4	8004	·4	7513	·4	4682	·6	6901
·5	9015	·5	8066	·5	7570	·5	4736	·7	6953
·6	9112	·6	8128	·6	7627	·6	4789	·8	7006
·7	9209	·7	8190	·7	7684	·7	4843	·9	7058
·8	9307	·8	8252	·8	7740	·8	4896	409·0	7110
·9	9404	·9	8314	·9	7797	·9	4950		

TABLE (continued).

n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.
488·0	9·99821645	489·3	9·99822120	490·6	9·99822591	491·9	9·99823062	493·2	9·99823529
·1	1682	·4	2156	·7	2628	492·0	3098	·3	3565
·2	1718	·5	2193	·8	2664	·1	3134	·4	3601
·3	1755	·6	2229	·9	2701	·2	3170	·5	3637
·4	1791	·7	2265	491·0	2737	·3	3206	·6	3673
·5	1828	·8	2302	·1	2773	·4	3242	·7	3708
·6	1864	·9	2338	·2	2809	·5	3278	·8	3744
·7	1901	490·0	2374	·3	2845	·6	3314	·9	3779
·8	1937	·1	2410	·4	2881	·7	3350	494·0	3815
·9	1974	·2	2446	·5	2917	·8	3385		
489·0	2010	·3	2483	·6	2953	·9	3421		
·1	2047	·4	2519	·7	2989	493·0	3457		
·2	2083	·5	2555	·8	3026	·1	3493		

32. The number given by this table is the logarithm of the mean Rate of the Detached Pendulum upon the Clock Pendulum under the actual circumstances of observation. It is next required to investigate the correction to this logarithm depending on the extent of the arc of vibration; one of the conditions of the data of the problem being, that the arc is observed only at the beginning and the end of the Swing.

In the first place, to compute the correction to the logarithm, supposing the arc of vibration constant. Let the whole arc of vibration, as seen upon the scale of inches, be I. For the pendulum 1821, suppose the scale to be placed 1 inch behind the pendulum; and for the pendulum 8, 1·8 inch behind the pendulum. And suppose the distance of the object-glass of the observing telescope to be 100 inches. Then the real whole arc of vibration is $\mathrm{I}\times\frac{100}{101}$ for pendulum 1821 and $\mathrm{I}\times\frac{100}{101\cdot8}$ for pendulum 8. The lengths of the two pendulums, from the knife-edge to the indicating point of the tail, are respectively 60·7 and 60·2 inches. Hence the proportion of the real whole arc of vibration to the length of the pendulum is $\mathrm{I}\times\frac{100}{101\times60\cdot7}$ for pendulum 1821 and $\mathrm{I}\times\frac{100}{101\cdot8\times60\cdot2}$ for pendulum 8; in which expressions the factors of I are sensibly the same. Call this proportion C. Then the number of vibrations observed is to be multiplied by $1+\frac{\mathrm{C}^2}{64}$ or $1+\mathrm{I}^2\times\frac{1}{64}\left(\frac{100}{101\times60\cdot7}\right)^2$. In the first instance therefore we require the logarithm of $1+\mathrm{I}^2\times\frac{1}{64}\left(\frac{100}{101\times60\cdot7}\right)^2$ for values of I not exceeding 2·5. The following Table contains the numbers required, in units of the last figures of 8-figure logarithms.

I.	Log.	I.	Log.	I.	Log.	I.	Log.	I.	Log.
0·1	2	0·6	65	1·1	218	1·6	462	2·1	796
0·2	7	0·7	88	1·2	260	1·7	522	2·2	874
0·3	16	0·8	116	1·3	305	1·8	585	2·3	955
0·4	29	0·9	146	1·4	353	1·9	652	2·4	1040
0·5	45	1·0	180	1·5	405	2·0	722	2·5	1128

Next, it is necessary to determine experimentally the law, or rather the numerical succession of values, in the diminution of the arc of vibration. For this purpose, observations were made at Greenwich on the extent of the whole arc in successive half-hours. The following Table contains (with sufficient approximation) the range of arc through each half-hour, the middle arc on which the correction may be supposed to depend, and the logarithmic correction as taken from the last table.

Range of Arc through half-hour.	Middle Arc.	Log. Correction.	Range of Arc through half-hour.	Middle Arc.	Log. Correction.	Range of Arc through half-hour.	Middle Arc.	Log. Correction.
2·35—1·97	2·14	831	0·71—0·59	0·64	81	0·22—0·19	0·20	7
1·97—1·64	1·79	579	0·59—0·49	0·54	52	0·19—0·16	0·17	6
1·64—1·38	1·50	402	0·49—0·41	0·45	36	0·16—0·13	0·14	4
1·38—1·16	1·26	287	0·41—0·34	0·37	26	0·13—0·10	0·11	3
1·16—0·98	1·06	205	0·34—0·29	0·31	18	0·10—0·08	0·09	2
0·98—0·83	0·90	146	0·29—0·25	0·27	13			
0·83—0·71	0·76	103	0·25—0·22	0·23	10			

Suppose now that we wished to find the logarithmic correction for a Swing whose first arc was 1·97 and whose last arc was 0·98. This Swing extends over four equal intervals of time, for which the log. corrections are respectively 579, 402, 287, 205. The log. correction therefore applicable to the whole time will be the mean of these four corrections, or 368. In a similar manner, we may obtain the correction with any other beginning and concluding arcs among those in the table above, and thus the next table is formed.

Logarithmic Correction for the whole Swing.

Concluding Arc.	Commencing Arc. 2·35	1·97	1·64	1·38	1·16	0·98	0·83	0·71	0·59	0·49	0·41	0·34	0·29	0·25	0·22	0·19	0·16	0·13	0·10
1·97	831																		
1·64	705	579																	
1·38	604	491	402																
1·16	525	423	345	287															
0·98	461	368	298	246	205														
0·83	408	324	260	213	176	146													
0·71	365	287	229	185	151	125	103												
0·59	329	258	204	164	134	110	92	81											
0·49	298	232	182	146	117	96	78	67	52										
0·41	272	210	164	130	104	84	63	56	44	36									
0·34	250	192	149	117	93	74	60	49	38	31	26								
0·29	231	176	136	106	83	66	53	43	33	27	22	18							
0·25	214	162	124	97	76	59	47	38	29	23	19	16	13						
0·22	199	151	115	89	69	54	42	34	26	21	17	14	12	10					
0·19	186	140	107	82	63	49	38	30	23	18	15	12	10	9	7				
0·16	175	131	99	76	59	45	35	28	21	17	13	11	9	8	6	6			
0·13	165	123	93	71	54	42	32	25	19	15	12	10	8	7	6	5	4		
0·10	156	116	87	66	51	38	30	23	17	14	11	9	7	6	5	4	4	3	
0·08	148	110	82	62	48	36	28	22	16	13	10	8	6	5	4	4	3	3	2

It was only necessary in fact to use so much of this table as is included between 2·35 and 1·38 for Commencing Arc, and between 0·98 and 0·34 for Concluding Arc. Between these limits, a skeleton table was prepared for every 0·01 in each argu-

ment, and was filled up by interpolation as far as was required, and no further. It is unnecessary to give it here, as all the essentials are contained in the table which has just been exhibited.

The application of this number to the logarithm of the mean Rate of the Detached Pendulum upon the Clock Pendulum, under the actual circumstances, gave the logarithm of mean Rate of the Detached Pendulum upon the Clock Pendulum, supposing the arc of vibration of the Detached Pendulum to have been indefinitely small.

The Commencing and Concluding Arcs which were used as Arguments for the table were those corresponding to the means of the Coincidences retained at the beginning and at the end of the Swing.

33. The next correction is that depending on the temperature of the Pendulum. On considering the slight discordance in the coefficients of expansion found by different experimenters, as well as the difficulty of exactly identifying the quality of the metal on which they experimented, it appeared to me best to adopt the result of Colonel SABINE (Experiments*, page 202—207), both because the method of experimenting was precisely the same as the method of using the pendulum in these operations, and because there can scarcely be a doubt that the metal was similar, as nearly as is possible in different bars. A small correction is required to Colonel SABINE's results, because at the time of his drawing the conclusion as to the effect of temperature, the ancient erroneous computation for the effect of buoyancy was still in use. Adopting his multiplier 1·655 of the correction computed for mere statical buoyancy, as applicable to pendulums of the same form as those used in these experiments †, the corrections to the numbers in the "Experiments" are as follows:—

Page 202, *for* 6·25 *read* 10·34; which gives for Pendulum 3, 86166·49.
Page 203, *for* 6·2 *read* 10·26; which gives for Pendulum 4, 86174·99.
Page 205, *for* 5·65 *read* 9·35; which gives for Pendulum 3, 86149·61.
Page 206, *for* 5·7 *read* 9·43; which gives for Pendulum 4, 86159·35.
Page 207, the results for 1° FAHRENHEIT will be respectively 0·4318 and 0·4300. The mean 0·4309 corresponds to 86166 vibrations in one day.

Hence, to reduce the vibrations observed at the temperature t° of FAHRENHEIT to the vibrations which would have been observed in air of the same density at temperature 50°, the number of observed vibrations must be multiplied by $\frac{86166+(t-50)\times 0\cdot4309}{86166}$, or by $1+(t-50)\times 0\cdot00000501$. I do not at present form the logarithm of this quantity, as there will be another term depending on temperature, introduced by the consideration of the buoyancy-correction, which will be combined with this.

* "An Account of Experiments to determine the Figure of the Earth by means of the Pendulum vibrating seconds in different Latitudes, as well as on various other subjects of Philosophical Inquiry. By EDWARD SABINE, &c. London, 1825."

† Philosophical Transactions, 1829.

34. The next correction is that for the density of the atmosphere. If we adopt Sir GEORGE SHUCKBURGH's elements (which are abundantly accurate for this purpose), the barometer-reading being B (expressed in English inches) at the temperature t° of FAHRENHEIT, its reading at temperature 53° would be

$$\frac{\mathrm{B}}{29{\cdot}27}(29{\cdot}27-\overline{t-53}\times 0{\cdot}002615)=\mathrm{B}(1-\overline{t-53}\times 0{\cdot}0000896).$$

The proportion of the weight of air in this state to that of air at barometer-reading $29^{\mathrm{in.}}{\cdot}27$, thermometer 53° (Sir GEORGE SHUCKBURGH's standard elements), will be

$$\frac{\mathrm{B}(1-\overline{t-53}\times 0{\cdot}0000896)}{29{\cdot}27}\times\left(1+\frac{53-t}{480}\right)=\frac{\mathrm{B}}{29{\cdot}27}(1-\overline{t-53}\times 0{\cdot}002173)$$
$$=\frac{\mathrm{B}}{29{\cdot}27}(1{\cdot}006519-\overline{t-50}\times 0{\cdot}002173).$$

With the elements $29^{\mathrm{in.}}{\cdot}27$ and 53°, the weight of air is $\frac{1}{836}$ that of water; and, with KATER's specific gravity 8·469, the weight of air is $\frac{1}{836}\times\frac{1}{8{\cdot}469}$ that of the pendulum; the effect of this on the vibrations of the pendulum, adopting Colonel SABINE's factor 1·655 of the statical buoyancy, will be to diminish them by the part

$$\frac{\mathrm{B}\times 1{\cdot}655}{1672\times 8{\cdot}469\times 29{\cdot}27}\times(1{\cdot}006519-\overline{t-50}\times 0{\cdot}002173).$$

In the small term multiplied by $t-50$, we may consider $\frac{\mathrm{B}}{29{\cdot}27}=\frac{26}{25}$. Then the diminution of the number of vibrations will be

$$\frac{\mathrm{B}\times 1{\cdot}655\times 1{\cdot}006519}{1672\times 8{\cdot}469\times 29{\cdot}27}-\overline{t-50}\cdot\frac{0{\cdot}00226\times 1{\cdot}655}{1672\times 8{\cdot}469}, \text{ or } \mathrm{B}\times 0{\cdot}000004019-\overline{t-50}\times 0{\cdot}00000026.$$

In order to correct the number of vibrations observed, so as to produce the number of vibrations which would take place in vacuum, we must multiply the number observed by

$$1+\mathrm{B}\times 0{\cdot}000004019-\overline{t-50}\times 0{\cdot}00000026.$$

35. Combining this factor with the factor depending on the temperature of the pendulum, or $1+\overline{t-50}\times 0{\cdot}00000501$, the complete factor is

$$1+\mathrm{B}\times 0{\cdot}000004019+\overline{t-50}\times 0{\cdot}00000475,$$

or

$$(1+\mathrm{B}\times 0{\cdot}000004019)\times(1+\overline{t-50}\times 0{\cdot}00000475).$$

The logarithms of these factors are $\mathrm{B}\times 0{\cdot}0000017453$ and $\overline{t-50}\times 0{\cdot}000002063$. The following Tables will suffice for the examination of the corrections in the succeeding Section.

Thermometer.	Correction to Log. Rate.	Thermometer.	Correction to Log. Rate.
50°·0	0·00000000	60°·0	0·00002063
·5	103	·5	2166
51·0	207	61·0	2269
·5	310	·5	2372
52·0	413	62·0	2475
·5	516	·5	2578
53·0	619	63·0	2681
·5	722	·5	2784
54·0	825	64·0	2887
·5	928	·5	2990
55·0	1032	65·0	3094
·5	1135	·5	3197
56·0	1238	66·0	3300
·5	1341	·5	3403
57·0	1444	67·0	3506
·5	1547	·5	3609
58·0	1650	68·0	3712
·5	1753	·5	3815
59·0	1856	69·0	3918
·5	1960	·5	4022
60·0	2063	70·0	4125

Barometer.	Correction to Log. Rate.
in.	
29·0	0·00005061
·2	5096
·4	5131
·6	5166
·8	5201
30·0	5236
·2	5271
·4	5306
·6	5340
·8	5375
31·0	5410
·2	5445
·4	5480
·6	5514
·8	5549
32·0	5584

The "Corrected Log. Rate of Pendulum upon Clock," or Log. Rate supposing that the temperature of the Pendulum is 50°, and that it vibrates in vacuum, in an indefinitely small arc, is found by adding together the logarithm of $\frac{n-2}{2}$, the correction for arc, the correction for thermometer, and the correction for barometer, all taken from the tables above.

Section IV.—*Abstract of the Pendulum Observations at the Upper Station.*

36. The remarks in the preceding Sections explain all the essential points in the following Table. It is only necessary to add, that the temperature of the Barometer (which was suspended in the Anteroom, very near to the Stove) was higher than that of the Pendulum; and a small correction, never exceeding −0·075, has been applied to its reading.

First Series. Pendulum 1821 above.

No. of Swing.	Number of Coincidences.		Number of Intervals.	Mean Interval.	Arc of Vibration.		Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on SHELTON.
	Beginning.	End.			Beginning.	End.			
				s			°	in.	
1	3	3	22	493·182	1·91	0·58	57·81	29·930	9·99830603
2	3	3	21	492·341	1·75	0·60	58·78	29·856	466
3	3	3	28	491·923	1·90	0·44	58·81	29·729	284
4	3	2	23	491·399	1·85	0·59	59·38	29·537	209
5	3	2	17	491·711	1·79	0·67	58·84	29·503	211
6	3	3	22	492·470	1·88	0·63	58·30	29·658	409
7	3	3	22	492·553	1·71	0·59	58·26	29·763	410
8	3	3	22	492·932	1·64	0·50	57·54	29·834	379
9	2	2	27	493·352	1·77	0·42	56·88	29·896	406
10	3	3	20	493·450	1·83	0·66	56·79	29·895	488
11	3	3	24	493·167	1·98	0·55	57·64	29·861	556
12	3	2	24	492·719	2·13	0·60	57·00	29·813	295
13	3	3	22	493·235	1·92	0·61	56·98	29·700	421
14	3	3	23	492·391	2·09	0·55	57·44	29·553	201
15	3	3	20	492·759	1·88	0·62	57·89	29·488	394
16	3	3	22	492·507	1·99	0·61	58·25	29·395	379
17	3	3	22	492·371	1·88	0·59	58·61	29·298	363
18	3	3	21	492·126	1·91	0·71	58·55	29·218	282
19	3	3	20	492·625	1·88	0·65	58·78	29·243	495
20	3	3	23	493·189	1·85	0·58	58·10	29·272	541
21	3	3	23	493·217	1·87	0·63	57·94	29·318	542
22	4	3	20	493·775	1·70	0·57	57·55	29·366	628
23	3	3	21	493·722	1·91	0·58	57·13	29·438	569
24	3	3	24	493·882	2·02	0·56	56·55	29·537	539
25	3	3	22	493·879	1·95	0·63	56·41	29·622	529
26	3	3	24	493·764	2·03	0·56	56·44	29·702	504

Second Series. Pendulum 8 above.

No. of Swing.	Coincidences Beginning.	Coincidences End.	Number of Intervals.	Mean Interval.	Arc Beginning.	Arc End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on SHELTON.
27	4	4	27	376·519	1·96	0·60	54·18	29·699	9·99775000
28	4	4	29	376·082	1·97	0·66	55·71	29·725	5069
29	4	4	33	375·803	2·03	0·54	56·13	29·770	4974
30	4	4	28	375·647	1·85	0·62	56·58	29·820	4965
31	3	3	31	375·650	2·00	0·62	56·53	29·864	4990
32	4	4	33	375·534	2·03	0·52	56·45	29·892	4887
33	3	3	29	375·511	1·93	0·61	58·05	29·916	5211
34	3	2	31	375·191	2·00	0·63	59·26	29·908	5279
35	3	3	32	375·162	1·77	0·51	59·84	29·869	5306
36	4	4	28	374·938	1·82	0·61	60·01	29·808	5223
37	4	4	29	375·172	2·00	0·62	59·48	29·878	5303
38	4	4	33	375·424	1·90	0·64	57·95	30·024	5157
39	4	4	29	375·397	1·98	0·68	57·68	30·134	5127
40	4	4	31	375·665	1·93	0·60	57·76	30·235	5299
41	3	2	35	375·807	2·03	0·54	57·04	30·358	5262
42	3	3	30	376·428	1·90	0·51	56·43	30·445	5505
43	4	4	30	376·692	1·81	0·54	55·36	30·518	5454
44	4	4	32	376·902	1·95	0·53	55·23	30·567	5583
45	3	3	27	376·729	2·07	0·73	55·30	30·561	5566
46	4	4	30	376·458	1·98	0·63	55·84	30·557	5466
47	4	4	34	376·507	1·99	0·57	55·95	30·535	5502
48	4	4	30	376·425	1·84	0·57	56·56	30·493	5545
49	4	4	30	376·308	1·96	0·62	56·61	30·430	5504
50	4	4	34	376·158	2·03	0·55	56·78	30·399	5437
51	4	4	28	376·304	1·88	0·64	56·86	30·368	5533
52	2	3	33	376·126	1·91	0·52	57·38	30·343	5503

Third Series. Pendulum 1821 above.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Interval.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on SHELTON.
				s			°	in.	
53	3	3	19	489·000	1·99	0·71	53·05	29·916	9·99828152
54	3	3	23	488·775	2·00	0·60	53·30	29·811	8078
55	3	3	24	488·805	1·91	0·53	53·59	29·721	8100
56	3	3	26	489·397	1·76	0·46	53·51	29·638	8244
57	3	3	21	489·222	1·91	0·62	53·39	29·556	8204
58	3	3	25	489·480	2·00	0·53	52·84	29·542	8175
59	3	3	22	489·311	1·81	0·60	52·75	29·529	8080
60	3	3	23	488·848	1·98	0·59	53·18	29·499	8020
61	3	3	23	488·717	2·05	0·65	52·99	29·454	7954
62	3	3	22	489·038	1·72	0·60	53·26	29·493	8063
63	3	3	22	489·303	1·94	0·59	53·10	29·544	8171
64	3	3	27	489·568	2·00	0·48	52·75	29·599	8186
65	3	3	22	489·409	1·97	0·67	52·73	29·633	8172
66	3	3	25	489·553	2·03	0·58	52·59	29·649	8188
67	3	3	25	489·826	1·81	0·54	51·85	29·662	8090

Fourth Series. Pendulum 8 above.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Interval.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on SHELTON.
68	5	5	29	375·879	2·00	0·61	51·53	29·662	9·99774063
69	5	5	29	376·117	1·81	0·58	51·25	29·608	4102
70	5	5	29	375·855	1·80	0·60	51·65	29·481	4004
71	5	5	29	375·517	1·88	0·63	52·11	29·333	3885
72	5	5	28	375·804	1·74	0·57	53·28	29·279	4258
73	5	5	29	375·700	1·71	0·56	53·28	29·250	4179
74	3	3	34	375·480	1·87	0·53	53·03	29·245	4010
75	5	5	26	375·208	1·75	0·68	53·43	29·250	3938
76	5	5	28	374·893	2·02	0·74	53·46	29·294	3824
77	4	4	32	374·953	1·91	0·59	53·39	29·381	3802
78	5	5	26	375·273	1·90	0·74	52·95	29·447	3956
79	5	5	28	375·332	1·98	0·72	52·65	29·502	3950
80	4	4	33	375·493	1·91	0·50	52·56	29·561	3976
81	5	5	28	375·236	1·92	0·68	53·66	29·559	4089
82	5	5	30	375·213	1·85	0·59	54·08	29·538	4122

SECTION V.—*Abstract of the Pendulum Observations at the Lower Station.*

37. No correction was applied to the immediate readings of the instruments, except that the Barometer-reading was increased by 0$^{\text{in.}}$·016, which was required to make its indications correspond to those of the upper Barometer.

First Series. Pendulum 8 below.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Intervals.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on EARNSHAW.
				s.			°	in.	
1	3	5	31	293·542	2·01	0·73	63·55	31·356	9·99711658
2	5	5	34	293·600	2·03	0·67	63·71	31·261	718
3	4	2	64	293·891	1·97	0·36	63·70	31·155	902
4	4	5	24	293·369	1·96	0·97	63·61	30·900	475
5	5	4	30	293·434	1·78	0·67	63·60	30·897	420
6	4	4	35	293·314	2·00	0·68	63·90	31·079	433
7	4	4	35	293·375	1·55	0·51	63·92	31·205	409
8	4	4	36	293·125	1·93	0·84	64·16	31·274	361
9	4	3	46	293·266	1·92	0·49	63·94	31·337	374
10	5	3	33	293·276	1·77	0·73	63·55	31·372	344
11	4	4	40	293·281	1·74	0·53	63·50	31·327	280
12	3	3	39	293·385	1·82	0·58	63·43	31·234	379
13	5	5	36	293·261	1·76	0·61	63·44	31·068	222
14	5	5	38	293·253	1·82	0·59	63·33	30·940	176
15	5	3	39	293·275	1·85	0·63	63·45	30·880	227
16	4	4	37	293·216	2·00	0·73	63·44	30·807	202
17	4	4	39	293·250	1·98	0·69	63·46	30·701	209
18	4	4	36	293·337	1·96	0·72	63·39	30·628	273
19	3	3	39	293·350	1·93	0·66	63·49	30·627	288
20	4	4	39	293·324	1·78	0·58	63·43	30·663	211
21	4	4	38	293·349	1·90	0·52	63·05	30·713	172
22	5	3	38	293·283	1·89	0·63	63·31	30·752	191
23	5	5	37	293·376	1·90	0·60	63·48	30·824	328
24	5	3	42	293·502	1·83	0·49	63·64	30·938	470
25	4	4	39	293·474	2·00	0·60	63·20	31·022	419
26	5	5	40	293·425	1·93	0·61	63·16	31·132	370

Second Series. Pendulum 1821 below.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Intervals.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on EARNSHAW.
27	4	4	26	389·510	1·95	0·69	62·91	31·118	9·99784802
28	4	4	29	389·452	2·15	0·65	63·14	31·139	4849
29	3	3	32	389·370	2·17	0·65	63·44	31·212	4879
30	4	4	27	389·509	1·95	0·70	63·25	31·260	4898
31	4	4	27	389·644	1·94	0·74	63·06	31·283	4951
32	4	4	32	389·992	1·80	0·51	63·13	31·313	5091
33	4	4	31	390·012	1·97	0·59	63·31	31·310	5186
34	4	4	28	390·210	1·87	0·64	63·31	31·288	5291
35	4	4	30	390·333	1·88	0·56	63·51	31·255	5380
36	4	4	29	390·314	1·99	0·67	63·66	31·195	5434
37	4	4	28	390·602	1·81	0·59	63·64	31·261	5557
38	3	3	33	390·692	2·01	0·51	63·60	31·454	5647
39	3	3	29	390·885	1·97	0·65	63·53	31·571	5790
40	3	3	32	391·172	1·82	0·55	63·50	31·694	5921
41	4	4	34	391·397	1·91	0·54	63·48	31·799	6076
42	4	4	26	391·615	1·76	0·69	63·31	31·902	6194
43	4	4	30	391·800	1·84	0·63	63·28	31·966	6302
44	3	3	34	392·069	1·99	0·56	63·16	32·019	6449
45	4	4	25	392·125	2·01	0·76	63·54	32·014	6615
46	4	4	29	392·422	1·88	0·59	63·30	31·992	6660
47	3	3	31	392·554	2·07	0·64	63·29	31·983	6778
48	4	4	28	392·705	1·96	0·75	62·84	31·943	6774
49	4	4	29	392·927	2·03	0·75	62·70	31·894	6877
50	4	3	33	393·396	1·77	0·48	63·15	31·831	7109
51	4	4	26	393·755	1·71	0·59	63·63	31·794	7421
52	4	4	29	394 013	1·64	0·59	63·43	31·763	7509

Third Series. Pendulum 8 below.

No. of Swing.	Number of Coincidences.		Number of Intervals.	Mean Intervals.	Arc of Vibration.		Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on EARNSHAW.
	Beginning.	End.			Beginning.	End.			
				s			°	in.	
53	5	5	28	298·521	1·96	0·90	62·05	31·335	9·99716353
54	5	5	37	298·819	1·89	0·67	61·85	31·229	6506
55	4	4	42	298·929	2·05	0·63	61·84	31·135	6615
56	4	4	45	299·144	1·92	0·54	61·68	31·049	6732
57	4	5	34	299·342	1·83	0·68	61·61	30·987	6919
58	3	3	40	299·479	2·05	0·67	61·81	30·971	7129
59	5	5	38	299·661	1·89	0·66	61·75	30·953	7259
60	5	5	38	299·832	1·98	0·67	61·65	30·922	7416
61	4	4	40	299·950	2·08	0·65	61·71	30·883	7553
62	5	5	36	300·150	1·92	0·65	62·04	30·907	7788
63	5	5	36	300·300	1·89	0·65	62·08	30·970	7946
64	4	4	43	300·468	1·97	0·59	62·30	31·021	8162
65	5	5	35	300·723	1·84	0·68	62·53	31·050	8472
66	5	5	37	301·449	1·91	0·67	62·80	31·084	9234
67	3	3	42	301·730	2·11	0·61	62·64	31·108	9497

Fourth Series. Pendulum 1821 below.

No. of Swing.	Coincidences Beginning.	Coincidences End.	Number of Intervals.	Mean Intervals.	Arc Beginning.	Arc End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on EARNSHAW.
68	4	4	29	403·647	1·98	0·63	61·55	31·119	9·99792364
69	5	5	25	403·892	2·08	0·79	61·73	31·045	2585
70	4	4	27	404·338	1·89	0·59	61·81	30·893	2722
71	3	3	30	404·817	1·95	0·57	61·64	30·746	2922
72	5	5	25	405·256	1·68	0·63	61·66	30·680	3117
73	5	5	27	405·578	1·79	0·64	61·65	30·652	3300
74	4	4	31	405·984	1·80	0·52	61·75	30·651	3512
75	4	2	28	406·281	1·81	0·58	61·88	30·661	3712
76	4	4	29	406·677	1·81	0·54	61·96	30·703	3937
77	3	3	32	407·141	1·91	0·49	62·00	30·796	4208
78	5	5	25	407·372	1·85	0·67	62·19	30·868	4414
79	5	5	26	407·835	1·80	0·60	62·20	30·916	4645
80	3	3	30	408·133	2·13	0·64	62·28	30·982	4897
81	5	5	27	408·511	1·91	0·64	62·73	30·969	5146
82	4	4	31	409·032	2·07	0·59	62·31	30·909	5338

SECTION VI.—*Computation of Logarithmic Rate of Lower Pendulum upon Upper Pendulum; combination of individual results; and conclusion on the proportion of gravity at the Lower Station to gravity at the Upper Station.*

38. The quantity now to be found is $\text{Log.}\,\dfrac{\text{Rate of Lower Pendulum}}{\text{Rate of Upper Pendulum}}$. This is

$$= \text{Log.}\,\frac{\text{Rate of Lower Pendulum}}{\text{Rate of EARNSHAW}} + \text{Log.}\,\frac{\text{Rate of EARNSHAW}}{\text{Rate of SHELTON}} - \text{Log.}\,\frac{\text{Rate of Upper Pendulum}}{\text{Rate of SHELTON}}.$$

The first of these quantities is given, for every Swing, in Section V.; the second in Section II.; and the third in Section IV. Thus the following numbers are formed.

No. of Swing.	Log. Rate $\frac{\text{Lower Pendulum}}{\text{Upper Pendulum}}$.	No. of Swing.	Log. Rate $\frac{\text{Lower Pendulum}}{\text{Upper Pendulum}}$.	No. of Swing.	Log. Rate $\frac{\text{Lower Pendulum}}{\text{Upper Pendulum}}$.	No. of Swing.	Log. Rate $\frac{\text{Lower Pendulum}}{\text{Upper Pendulum}}$.
First Series. Log. Rate of Pendulum 8 below on Pendulum 1821 above.							
1	9·99928067	8	9·99928485	15	9·99929623	22	9·99927705
2	9045	9	7284	16	8478	23	7793
3	9005	10	8116	17	8501	24	9783
4	8261	11	8683	18	8381	25	8385
5	9458	12	8278	19	7723	26	8582
6	9014	13	9524	20	8724		
7	8281	14	8135	21	9083		
Second Series. Log. Rate of Pendulum 1821 below on Pendulum 8 above.							
27	0·00073466	34	0·00074088	41	0·00073841	48	0·00073718
28	4199	35	3471	42	3720	49	3371
29	3602	36	3511	43	3714	50	3941
30	3684	37	3628	44	3727	51	3518
31	3868	38	4004	45	3382	52	3580
32	3919	39	3903	46	3709		
33	3319	40	3232	47	3852		
Third Series. Log. Rate of Pendulum 8 below on Pendulum 1821 above.							
53	9·99928138	57	9·99928427	61	9·99928297	65	9·99928865
54	8078	58	8440	62	9319	66	8366
55	9635	59	8661	63	8406	67	8900
56	7957	60	8786	64	8243		
Fourth Series. Log. Rate of Pendulum 1821 below on Pendulum 8 above.							
68	0·00073376	72	0·00074105	76	0·00073941	80	0·00073541
69	4097	73	3691	77	3929	81	3827
70	3162	74	3381	78	3549	82	2831
71	3748	75	3732	79	3997		

39. On tracing the irregularities in these numbers to their sources, it will be seen that they arise almost entirely from the irregularities in the comparisons of the clocks. The rate of each pendulum upon its clock is either so constant, or changes by such uniform degrees in the same direction, that there is every reason to presume on the extreme steadiness both of the detached pendulums and of the clocks.

Remarking then that, when a large variable error is combined with a small variable error, the magnitude of the probable error in the combination is scarcely affected by the small error, we may treat these irregularities of result as if they were entirely due to irregularities of comparison; and we have now to investigate the rule to be followed in combining the special results, supposed to be erroneous from that cause only, in order to obtain a final result whose probable error shall be the smallest possible. It is evident that we are not to give equal weights to the

different results, for we should thus neglect all comparisons except the first and the last.

40. Let the comparisons be numbered

$$0,\ 1,\ 2,\ 3,\ 4,\ \ldots\ldots\ldots\ \overline{n-1},\ n,$$

and let the swings be numbered

$$1,\ 2,\ 3,\ 4,\ \ldots\ldots\ldots\ n.$$

Let the *actual* errors of comparison, estimated by their effect on a four-hours' rate, in units of the last figure of 8-figure logarithms, be

$$E_0,\ E_1,\ E_2,\ E_3,\ E_4,\ \ldots\ldots\ldots\ E_{n-1},\ E_n,$$

(where E_{16} is supposed to be taken as if the comparison, which was really omitted, agreed with that produced by interpolation between the two neighbouring comparisons,) and let the *probable* errors of comparison be

$$e_0,\ e_1,\ e_2,\ e_3,\ e_4,\ \ldots\ldots\ldots\ e_{n-1},\ e_n,$$

and let the weights for the results of the separate Swings be

$$w_1,\ w_2,\ w_3,\ w_4,\ \ldots\ldots\ldots\ w_n.$$

Then the errors in the results of the separate Swings, produced by the errors of comparisons, are

$$(E_1-E_0),\ (E_2-E_1),\ (E_3-E_2),\ \ldots\ldots\ldots\ (E_n-E_{n-1});$$

and, combining these with the weights

$$w_1,\ w_2,\ w_3,\ \ldots\ldots\ldots\ w_n,$$

the *actual* error of the final result will be

$$\frac{w_1(E_1-E_0)+w_2(E_2-E_1)+w_3(E_3-E_2)+\ldots\ldots+w_n(E_n-E_{n-1})}{w_1+w_2+w_3+\ldots\ldots\ldots+w_n}$$

$$=\frac{-w_1E_0+(w_1-w_2)E_1+(w_2-w_3)E_2+(w_3-w_4)E_3\ldots\ldots+w_nE_n}{w_1+w_2+w_3+\ldots\ldots\ldots+w_n}.$$

Hence, by the well-known rules of the Calculus of Probabilities, the square of the *probable* error of the final result will be

$$\frac{w_1^2.e_0^2+(w_1-w_2)^2.e_1^2+(w_2-w_3)^2e_2^2+(w_3-w_4)^2e_3^2+\ldots\ldots+w_n^2.e_n^2}{(w_1+w_2+w_3+\ldots\ldots\ldots+w_n)^2}.$$

And if, for simplicity, we suppose all the comparisons equally good, so that for e_0, e_1, e_2, &c. we may put e, the expression for the square of the *probable* error of the final result becomes

$$e^2\times\frac{w_1^2+(w_1-w_2)^2+(w_2-w_3)^2+\ldots\ldots+w_n^2}{(w_1+w_2+.w_3+\ldots+w_n)^2}:$$

and we have now to determine values of w_1, w_2, &c., which will make this square of the final probable error a minimum.

For convenience, call this fraction $e^2 \times \frac{N}{S^2}$.

Differentiate with respect to w_1, and make the differential coefficient $=0$;

$$S \times (2w_1 - w_2) - N = 0, \text{ or } 2w_1 - w_2 = \frac{N}{S}.$$

Differentiate with respect to w_2, and make the differential coefficient $=0$;

$$S \times (2w_2 - w_1 - w_3) - N = 0, \text{ or } 2w_2 - w_1 - w_3 = \frac{N}{S}$$

$$\text{Similarly } 2w_3 - w_2 - w_4 = \frac{N}{S}$$

$$\cdots\cdots$$

$$2w_{n-1} - w_{n-2} - w_n = \frac{N}{S}$$

$$2w_n - w_{n-1} \qquad = \frac{N}{S}.$$

Let $\frac{N}{S} = 2b$, the value of b being at present unknown or perhaps arbitrary. Then

$$\begin{aligned}
w_1 &= w_1 &&= w_1 \\
w_2 &= 2w_1 - 2b &&= 2w_1 - 2b \\
w_3 &= 2w_2 - w_1 - 2b &&= 3w_1 - 6b \\
w_4 &= 2w_3 - w_2 - 2b &&= 4w_1 - 12b \\
&\cdots\cdots \\
w_{n-1} &= &&(n-1)w_1 - (n-1)(n-2)b \\
w_n &= &&nw_1 - n(n-1)b.
\end{aligned}$$

Substituting the two last in the equation $2w_n - w_{n-1} = 2b$, we obtain

$$w_1 = nb,$$

and, substituting this in the other expressions,

$$\begin{aligned}
w_2 &= (2n - 2)b \\
w_3 &= (3n - 6)b \\
w_4 &= (4n - 12)b \\
w_5 &= (5n - 20)b \\
&\&c.
\end{aligned}$$

of which the law is evident. The second difference is constant, and $= -2b$.

Substituting these in the expressions for N and S, we find (after all reductions)

$$N = \frac{b^2}{3} . n . \overline{n+1} . \overline{n+2}$$

$$S = \frac{b}{6} . n . \overline{n+1} . \overline{n+2},$$

and the equation $\frac{N}{S} = 2b$ becomes identical. Therefore b is arbitrary. For conve-

nience, make $b=1$. Then the weights for the results of the successive Swings are, n, $2n-2$, $3n-6$, $4n-12$, &c.

The square of the probable error of the final result was found $=e^2\times\frac{N}{S^2}$. Substituting, this becomes $e^2\times\frac{12}{n.\overline{n+1}.\overline{n+2}}$; or the probable error $=e\times\sqrt{\frac{12}{n.\overline{n+1}.\overline{n+2}}}$.

41. It will be instructive to contrast this result with the result obtained on two other suppositions.

First: suppose that the Swings had been continuous, but that there had been no intermediate comparisons of clocks. The probable error of the first comparison being e, and that of the last comparison being also e, the probable error in their combination by subtraction will be $e\sqrt{2}$; and as this applies to n Swings, the probable error on the mean $=\frac{1}{n}e\sqrt{2}=e\sqrt{\frac{2}{n^2}}$. Comparing this with the probable error found above, it appears that the intermediate comparisons have diminished the probable error in the proportion expressed by the fraction $\sqrt{\frac{6n}{\overline{n+1}.\overline{n+2}}}$. When $n=26$, this fraction is $\sqrt{\frac{13}{63}}$, or the weight of the result is increased nearly five-fold by the intermediate comparisons. When $n=15$, the fraction is $\sqrt{\frac{45}{136}}$, or the weight is increased three-fold.

Second: suppose that the Swings had been discontinuous. The probable error in each Swing, found by combining its first and last comparison, would have been $e\sqrt{2}$; and, as the different Swings are strictly independent, the probable error on the mean of all would have been $e\sqrt{\frac{2}{n}}$. Comparing this with our probable error above, it appears that our system has diminished the probable error in the proportion $\sqrt{\frac{6}{\overline{n+1}.\overline{n+2}}}$. When $n=26$, this fraction is $\sqrt{\frac{1}{126}}$, or the weight of the result is increased 126-fold by our system. When $n=15$, the fraction is $\sqrt{\frac{3}{136}}$. or the weight in increased 45-fold.

These contrasts will suffice to show the great advantage of a system of continuous Swings with intermediate comparisons such as has been employed in this experiment. I cannot quit this subject without repeating that my first impression on the advantage of such a system was derived from the representations of Mr. SHEEPSHANKS*, on occasion of the experiments of 1828.

42. In the First and Second Series, $n=26$, and the successive weights are 26, 50, 72, 92, 110, 126, 140, 152, 162, 170, 176, 180, 182, 182, 180, ... 50, 26. In the Third and Fourth Series, $n=15$, and the successive weights are 15, 28, 39, 48, 55,

* Since I commenced drawing up this paper, my valued friend has been snatched away by death; a victim, I believe, to his labours gratuitously undertaken for the formation of the National Standard of Length.

60, 63, 64, 63, ... 15. Combining the separate results by these weights, we obtain the following mean results:—

First Series.

Log. Rate of Pendulum 8 below upon Pendulum 1821 above
$=9{\cdot}99928536$.

Second Series.

Log. Rate of Pendulum 1821 below upon Pendulum 8 above
$=0{\cdot}00073691$.

Third Series.

Log. Rate of Pendulum 8 below upon Pendulum 1821 above
$=9{\cdot}99928584$.

Fourth Series.

Log. Rate of Pendulum 1821 below upon Pendulum 8 above
$=0{\cdot}00073715$.

43. For ascertaining the probable error e I have used the following process.

Let E_0, E_1, E_2, &c., as before, be the actual errors of comparison, estimated by the effect which they produce on a 4-hours' rate, in the 8th decimal place of logarithms; let R_1, R_2, &c. be the successive individual results for the rate of the lower pendulum on the upper pendulum; and let A be the adopted value of that rate. Now if A were rigorously correct, we should have the following equations:—

$$\begin{aligned}
E_0 &= E_0 \\
E_1 &= E_0+R_1-A=E_0+R_1-A \\
E_2 &= E_1+R_2-A=E_0+(R_1+R_2)-2A \\
E_3 &= E_2+R_3-A=E_0+(R_1+R_2+R_3)-3A \\
&\&c.
\end{aligned}$$

and, adding all for the First Series, which terminates with R_{26} and E_{26},

$$E_0+E_1+\ \&c.\ +E_{26}=27E_0+26R_1+25R_2+\ \&c.\ +R_{26}-\frac{26{\cdot}25}{2}A$$

E_0 is yet undetermined. Now the Theory of Probabilities which we have used requires that the chances of positive and negative errors be equal, and therefore that (subject to the irregularities of chance) E_0+E_1+ &c. $+E_{26}=0$. This gives

$$E_0=\frac{26{\cdot}25}{2{\cdot}27}A-\frac{1}{27}(26R_1+25R_2+\ \&c.\ +R_{26});$$

and, substituting this for E_0 in the different expressions above, E_1, E_2, &c. will be

formed. Squaring each, forming $\frac{1}{27}$th part of the sum of squares, and multiplying its square root by 0·6745, the probable error is obtained.

The quantity thus obtained is however a little too great. For, the number which we have found for E_0 contains $\frac{26\cdot25}{2\cdot27}$ A or 12A nearly; and as the probable error of A is about $\frac{1}{40}e$, the probable error of 12A is about $\frac{3}{10}e$; and therefore we have on the right side of the equation an aggregate of terms whose probable error is $\sqrt{e^2+\frac{9}{100}e^2}$ or $e\left(1+\frac{1}{22}\right)$ nearly. The same is true for E_{26} and those near it. But for E_{13} the factor of A is 0. Thus it will easily be seen that the quantity which we obtain is really $e\left(1+\frac{1}{66}\right)$ nearly. The correction scarcely deserves notice.

44. In this manner the following (uncorrected) values of E_0, E_1, &c. are found; arranged with reference to the Swings to which they relate. It will be remembered that the number 300 represents an error of $0^{s}\cdot1$ in absolute time, very nearly.

First Series.			Second Series.			Third Series.			Fourth Series.		
No. of Swing.	Error of Comparison.		No. of Swing.	Error of Comparison.		No. of Swing.	Error of Comparison.		No. of Swing.	Error of Comparison.	
	+	−		+	−		+	−		+	−
...	....	454	...	....	276	...	476		...	211	
1	....	923	27	....	501	53	30		68	....	128
2	....	414	28	7		54	....	476	69	254	
3	55		29	....	82	55	575		70	....	299
4	....	220	30	....	89	56	....	52	71	....	266
5	702		31	88		57	....	209	72	124	
6	1180		32	316		58	....	353	73	100	
7	925		33	....	56	59	....	276	74	....	234
8	874		34	341		60	....	74	75	....	217
9	....	378	35	121		61	....	361	76	9	
10	....	798	36	....	59	62	374		77	223	
11	....	651	37	....	122	63	196		78	57	
12	....	909	38	191		64	....	145	79	339	
13	79		39	403		65	136		80	165	
14	....	322	40	....	56	66	....	82	81	277	
15	765		41	94		67	234		82	....	607
16	707		42	123							
17	672		43	146							
18	517		44	182							
19	....	296	45	....	127						
20	....	108	46	....	109						
21	439		47	52							
22	....	392	48	79							
23	....	1135	49	....	241						
24	112		50	9							
25	....	39	51	....	164						
26	7		52	....	275						

From these are found the values of the probable error of a single comparison, treating the four series separately:

In Series 1, $e = \pm\ 420$.
In Series 2, $e = \pm\ 135$.
In Series 3, $e = \pm\ 203$.
In Series 4, $e = \pm\ 173$.

And hence the probable errors and weights of the mean results are:—

For First Series, probable error $= \pm\ 10{\cdot}4$, weight $= \frac{93}{10000}$.

For Second Series, probable error $= \pm\ 3{\cdot}3$, weight $= \frac{899}{10000}$.

For Third Series, probable error $= \pm\ 11{\cdot}0$, weight $= \frac{82}{10000}$.

For Fourth Series, probable error $= \pm\ 9{\cdot}4$, weight $= \frac{114}{10000}$.

45. Combining the results of the First and Third Series, with the weights just found, and still adopting as unit, in the probable error, the unity of the 8th decimal of logarithms,

Log. Rate of Pendulum 8 below upon Pendulum 1821 above
$= 9{\cdot}99928558 \pm 7{\cdot}5$.

Combining the results of the Second and Fourth Series,

Log. Rate of Pendulum 1821 below upon Pendulum 8 above
$= 0{\cdot}00073694 \pm 3{\cdot}1$.

And, remarking that Log. $\frac{\text{Gravity below}}{\text{Gravity above}}$ is the sum of these logarithms, we have finally

$$\text{Log.}\ \frac{\text{Gravity below}}{\text{Gravity above}} = 0{\cdot}00002252 \pm 8{\cdot}2,$$

$$\text{or}\quad \frac{\text{Gravity below}}{\text{Gravity above}} = 1{\cdot}00005185 \pm 0{\cdot}00000019;$$

or we may otherwise express it,

Gravity below is greater than Gravity above by $\frac{1}{19286}$ part, with an uncertainty of $\frac{1}{270}$ part of the excess.

The acceleration of a seconds' pendulum below is $2^{s}{\cdot}24$ per day, with an uncertainty of less than $0^{s}{\cdot}01$.

46. But it is to be remarked that this estimate of the amount of uncertainty is obtained from the amount of uncertainty in each mean as deduced separately from the discordances of the individual results in the group contributing to that mean. In comparing the means, we shall see reason to suppose that the ultimate uncertainty is greater. Thus, though the probable errors of the means of Series 1 and 3 are 10·4 and 11·0, the difference of the means is 48: though the probable errors of the means of Series 2 and 4 are 3·3 and 9·4, the difference of the mean is 24. It is likely therefore that some cause of irregularity has occurred, special to each series. The most

probable cause is some trifling error or unsteadiness in the manner of fixing the agate-plane-frames upon the iron stands. It is not a change in either of the pendulums after the Second Series, inasmuch as the value of the mean is altered the same way under the alternation of position of the pendulums. Whatever the cause may have been, the effect is extremely small. There appears to be no reason for altering the concluded ratio of Gravity below to Gravity above. The probable error stated in the last paragraph may be doubled, but I think that there is no sufficient ground for trebling it. The amount of the uncertainty, so increased, is insignificant for the purposes of this experiment.

47. There remains, however, a serious question whether there may have been any difference in the circumstances of the upper and lower pendulums, not included in the corrections applied, which can produce an effect similar to that of a change of gravity. The first point to be considered is, the instability of the mountings. The importance of a very firm foundation was perfectly understood by the able practical men by whom the ground-work was arranged, and particularly by Mr. ARKLEY; and I conceive all was done which it was possible to do, to make the floors solid. The form of the iron stands is particularly well adapted to firmness. Any tendency to lateral or other movement is counteracted by the endwise resistance of strong straight iron bars. I had at first intended to interchange the iron stands in the middle of the operation, but upon contemplating the mechanical firmness ensured by the plan of their construction, and the exact similarity of the two stands in every respect, I gave up this design; being fully convinced that there might be risk of instability in a change, but that there could scarcely be any sensible absolute instability and (as I believe) no sensible relative instability, in the stands as they were planted. The stands were supported in the same manner at both stations.

The mere determination of the relative rates of the detached pendulum and the clock pendulum, by the method of coincidences, is accurate almost beyond conception. I do not see how it is possible that an error of $0^{s}\cdot01$ per day can enter from this cause.

The observers were so evenly interchanged in the upper and lower stations, that no personal peculiarity in the method of reading the thermometers or of taking any other observations can have produced a sensible effect. The following Table shows the aggregate number of turns taken by each:—

Initials of observer's name	C	D	E	P	R	S.
Number of turns above	5	5	4	4	5	5.
Number of turns below	5	2	5	6	5	5.

48. I will now point out the only cause from which, in my judgment, any perceptible error can arise. It was my intention that the temperature of the upper station should be brought, as near as could practically be done, to that of the lower station. In the first week, however, Mr. DUNKIN was seized with a sudden and severe illness,

and the transmission of the detailed observations to me was in consequence delayed. An inequality of temperature, which began accidentally, was thus allowed to exist too long to admit of correction. On comparing the mean of all the upper temperatures with the mean of all below, I find that the lower station was warmer than the upper by $7^{\circ}\cdot13$ FAHR. (the upper and lower means being $55^{\circ}\cdot75$ and $62^{\circ}\cdot88$). Therefore, if the adopted coefficient of thermal correction is erroneous, the result of these experiments will be erroneous by the amount of that error on a range of temperature of $7^{\circ}\cdot13$. There may yet be opportunity of verifying the coefficient of temperature-correction.

SECTION VII.—*Measure of the difference of level of the two Pendulum-Stations; survey of the neighbouring country; and computation of the difference of attraction of the Earth's shell on the two pendulums, upon an assumed value of density.*

49. For measuring the depth of the mine, Mr. SIMMS furnished an iron wire 100 feet long, with an attached scale at each end. After the operation, this wire was returned to Mr. SIMMS, and was found to have preserved its length sensibly unaltered. It was used by Mr. ARKLEY to measure the depth, in the following manner. Mr. ARKLEY placed himself on the top of the "Cage" in which the coal-trams are conveyed from the bottom of the shaft to the top, and, when it was lowered 100 feet, an assistant attached himself to the strong wire-rope on which the Cage is suspended. The assistant carried the upper end of the measuring-wire, and in the first instance held it level with the top of the shaft (the steam-engine used for raising or lowering the Cage being stopped); Mr. ARKLEY noted the position of the lower end of the wire (to which a small stretching weight was attached), and fixed a small nail in the brattice opposite to that lower end. Then the Cage was lowered 100 feet; the assistant held the top of the wire to the nail so fixed, and Mr. ARKLEY drove a second nail in the brattice, opposite the lower end; and so on to the bottom; the last fraction of 100 feet being measured with a measuring tape. After this, the levels of the pendulum-stations were referred to the top and bottom of the shaft by the ordinary surveyors' levelling operations.

The following is an abstract of the result:—

	ft.	in.
Depth of the shaft	1263	6
Rise of the floor of the mine from the point of measurement at the bottom of the shaft to the floor of the lower pendulum-station	4	$9\frac{3}{4}$
Fall of the surface from the point of measurement at the top of the shaft to the floor of the upper pendulum-station	2	$7\frac{1}{4}$
Distance between the floors of the upper and lower pendulum-stations .	1256	1

50. For computing the attractions which determine the difference of gravity at the

upper and lower stations; if we refer to the simple investigation in article 2, it will appear desirable to conduct the calculation which is to apply to the earth's irregular form in such a manner as to preserve the characteristics of that simpler investigation as closely as possible. These characteristics are,—

1. That a shell may be traced, whose inner surface passes through the lower station, and whose attraction at that lower station is $=0$.
2. That the attraction of the same shell at the upper station is the same as if its matter were collected at the centre of the earth.
3. That the attraction of the inclosed nucleus follows the same law, in reference to the difference at the upper and lower stations, as if all its matter were collected at the centre of the earth.

And we are to find how nearly we can approach to these circumstances on the supposition that the earth's constitution is irregular, both in the neighbourhood of Harton and in distant regions.

51. Now if there are sensible irregularities near the upper station, it will be impossible to satisfy the first and second conditions at the same time. For, the demonstration of the evanescence of shell-attraction at the lower station rests upon this: that if chords be drawn through the point L, Plate XI. fig. 1, included within the solid angle $a\mathrm{L}b$, as $c\mathrm{LFC}$, the portion FC must be equal to Lc; and therefore, if in the surface ab (which may be a very minute field) there be an elevation or depression, there must be a corresponding elevation or depression over the whole AB (which will be, in extent, a large continent); and this will disturb the second condition. It will be better, therefore, in the first instance, to give no attention to the local irregularities near the upper station; to assume that the surface there is spherical; to find with this assumption how we can satisfy the three conditions; and afterwards to make allowance for the effect of the irregularities near the upper station.

52. In fig. 2, then, conceive that for some distance on each side of L (say twenty or thirty times the depth of L) the external surface is sensibly spherical; and conceive that at A, B, C, &c. there are local irregularities, perhaps large in extent as compared with the depth of L, but very small as compared with AB in fig. 1. Trace the inner surface DEF by making $\mathrm{AD}=\mathrm{L}a$, $\mathrm{BE}=\mathrm{L}b$, $\mathrm{CF}=\mathrm{L}c$, &c. These lines, however, are to be made geometrically equal only when the density of the matter is the same as that above L; if the density about A is less than that above L, take the geometrical length AD greater than La in the same proportion. Then the attraction of the shell on the point L will be strictly equal to 0. Moreover, its attraction on the point U will be sensibly the same as if its form were free from irregularities. For, the attractions on U are all *added* together; the irregularities are local and numerous, and are partly additive and partly subtractive; and by hypothesis we have excluded all the irregularities near L or U, which, individually, can be important. And it may be accepted as a universal principle, that when a result is produced by the *addition* of a great number of small components which are liable individually to

small irregularities + or − affecting the ratio or multiplier of each, the sum of all the components will be sensibly free from the effects of these irregularities.

53. In like manner, if we divide the nucleus by planes parallel to the tangent at L or U, as shown by the dotted lines in fig. 2 (or indeed in any other way), the attractions of the slices thus formed are additive, both in their effect on L and in their effect on U. Therefore, for the nucleus generally, by the same reasoning as that above, the effect of irregularities in the outline (and therefore the effect of the irregularities in the outline of the earth, on which these depend) will be, as I conceive, sensibly evanescent. But this does not apply to irregularities in the geological constitution of the earth at a small distance below L; because these irregularities, or rather that one irregularity, may be sensible in proportion to the whole change of attraction between U and L. This is a source of uncertainty from which no experiments made on the earth itself can be perfectly free. We must trust in a great measure to the general regularity of stratification, &c. of the district, for supporting us in the confidence that there is no great disturbance in the law of attractions of the nucleus upon the points U and L.

54. To illustrate in some degree the difference in the attractions and changes of attraction depending on different slices of a sphere, I have supposed a homogeneous sphere divided into twenty slices by equidistant planes parallel to the tangent at L, and have computed (by formulæ easily investigated) the attraction of each slice upon the point L at the surface, and upon a point raised above the surface by $\frac{1}{10}$th part of the radius. Omitting the factor π, the results are as follows:—

No. of slice.	Attraction on point at the surface.	Attraction on point elevated $\frac{1}{10}$ radius.	Decrease by the elevation of the point.
1	·17019	·10401	·06618
2	·14548	·11404	·03144
3	·12941	·10644	·02297
4	·11640	·09811	·01829
5	·10519*	·09002	·01517
6	·09515	·08231	·01284
7	·08600	·07501	·01099
8	·07756	·06808	·00948
9	·06963	·06144	·00819
10	·06217	·05509	·00708
11	·05512	·04901	·00611
12	·04833	·04315	·00518
13	·04191	·03745	·00446
14	·03568	·03207	·00361
15	·02973	·02666	·00307
16	·02391	·02161	·00230
17	·01836	·01655	·00181
18	·01295	·01165	·00130
19	·00766	·00693	·00073
20	·00249	·00232	·00017

* I was not aware till I made this calculation that the plane which bisects the radius drawn to L divides the sphere into two segments whose attractions on L are equal.

These different slices, it may be remarked, correspond to equal surfaces on the sphere; and upon these it is not improbable that the irregularities may mainly depend.

55. In much of the preceding reasoning, it will be remarked, I have tacitly assumed that large continental elevations or large marine depressions, as we find them on the earth, do not interfere materially with the general law of attraction based on the spherical distribution of matter. For the reasons which seem to sustain this assumption, I would refer to a paper by me (printed in the Philosophical Transactions, 1855) on the Attractions of Mountain Masses. It will also be remarked that I have not introduced the consideration of the earth's rotation. I conceive its effects to be extremely insignificant; but the formulæ applying to it are so unmanageable, that I have not pursued it to details.

Considering now that it is sufficiently shown that, on the supposition that the surface in the neighbourhood of U is truly spherical, we may use the method of article 2, with no other uncertainty than that explained in article 53: I shall proceed with the corrections for the inequalities of the surface near U.

56. First, I shall investigate the attraction of the matter included between two horizontal planes, figure 3, upon points U and L in these planes, whose distance or the separation of the planes is equal to the distance UL in figure 2.

Divide the whole of the matter into cylindrical rings, of which UL is the axis: let the internal and external radii of one of these rings be ϱ and $\varrho+\delta\varrho$. Call the azimuth of any part of the ring θ; the end-surface of the prism included between θ and $\theta+\delta\theta$ is $\varrho.\delta\varrho.\delta\theta$. Let z be the vertical ordinate measured upwards from the lower plane; the solid content of the part of the prism included between z and δz is $\varrho\delta\varrho.\delta\theta.\delta z$: its attraction on the point L, supposing its density to be d, is $\frac{d.\rho\delta\rho.\delta\theta.\delta z}{\rho^2+z^2}$; and the resolved part of this, in the vertical direction, is $\frac{d.\rho\delta\rho.\delta\theta.z\delta z}{(\rho^2+z^2)^{\frac{3}{2}}}$. Integrating with respect to z between the limits $z=0$ and $z=c=\text{UL}$, we have $d.\varrho\delta\varrho.\delta\theta.\left(\frac{1}{\rho}-\frac{1}{(\rho^2+c^2)^{\frac{1}{2}}}\right)$. Integrating with respect to θ for the whole circumference, we have $2\pi.d.\left(\delta\varrho-\frac{\rho\delta\rho}{(\rho^2+c^2)^{\frac{1}{2}}}\right)$. Integrating with respect to ϱ, we have $2\pi.d.\{\varrho+c-(\varrho^2+c^2)^{\frac{1}{2}}\}$. This is the attraction upwards on the point L. The attraction downwards on the point U will be the same; and thus the difference of attractions on U and L, estimated in the downwards direction, will be $4\pi.d.\{\varrho+c-(\varrho^2+c^2)^{\frac{1}{2}}\}$.

If the planes be continued without limit, or ϱ be infinite, this expression becomes $4\pi c.d.$ Now the attraction of a shell whose thickness is c, computed as in article 2, is 0 for the point at the inner surface of the shell, and $4\pi c.d$ for the point at the outer surface, and therefore the difference is $4\pi cd$. Hence it is indifferent whether we consider the difference of attractions at the upper and lower stations (independent of the change in the attraction of the nucleus caused by the change of distance from it),

as produced by a shell of matter, or as produced by the matter between two parallel planes of unlimited extent.

If the extent of the planes be limited, their form being circular, let $\varrho=nc$; then the difference of attractions is $4\pi c.d.\left\{n+1-\left(n+\frac{1}{2n}-\text{\&c.}\right)\right\}=4\pi c.d.\left\{1-\frac{1}{2n}\right.$ nearly. If (as in the Harton experiment) $r=\frac{1}{4}$ mile, $\varrho=3$ miles $=12c$ nearly, this $=4\pi c.d\left\{1-\frac{1}{24}\right\}$ nearly. Thus it appears that $\frac{23}{24}$ of the effect is produced by the matter within three miles of the pendulum-stations. It is evident therefore that it is not necessary to give attention to the small inequalities of the ground at any great distance from the pendulum-stations.

57. The inequalities which we have to consider are entirely at the surface, and do not in any case exceed in vertical measure one-tenth of the depth of the mine. They may therefore be considered as being actually at the surface; and, if their horizontal extent is not very great, each may be considered as collected at its centre of gravity. Its effect on the upper station will be 0; its effect on the lower station will be $\frac{c}{(\varrho^2+c^2)^{\frac{3}{2}}}\times d\times$ volume; where ϱ is the horizontal distance of the centre of gravity. An eminence will increase the attraction upwards, and a depression will diminish it. But as our only object is to find the difference of attractions on the two stations, we may estimate the whole, with changed sign, as an effect on the upper station; then an eminence will increase the attraction downwards, and a depression will diminish it.

There is however one depression which it is desirable to consider in a different way, namely that of the sea. The depth of the sea itself is less important than the depression below the table-land, which continues with little change of level to the edge of the cliffs; and a sufficiently accurate estimate may be formed of the measure of depression (including the effect of the attraction of the water) considered as uniform. The line of cliff may be considered as straight. Let a be the distance from the pendulum-station to the straight line of cliff, measured perpendicularly to that line; b the depth of the depression; let x be parallel to a, and y parallel to the line of cliff. The matter $d\times\delta x\times\delta y\times b$ is at the distance $(x^2+y^2)^{\frac{1}{2}}$ from the upper station, and therefore its vertical attraction on the lower station is $\frac{d.b.c.\delta x.\delta y}{(x^2+y^2+c^2)^{\frac{3}{2}}}$, or $\frac{d.b.c.\delta x.\delta y}{(x^2+y^2)^{\frac{3}{2}}}$ nearly. Integrating first with respect to y, from $-\infty$ to $+\infty$, we have $\frac{2d.b.c.\delta x}{x^2}$. Integrating then with respect to x, from a to ∞, we have $\frac{2d.b.c}{a}$.

These formulæ will suffice for our purpose. It is only necessary further to remark, that, as the unit of measure is absolutely arbitrary, and as the numerical and graphical operations are a little facilitated by using for unit the "depth of the mine," I have, in all the subsequent calculations, adopted that quantity (1256 feet) as unit

of measure. In all the preceding formulæ, c must now be made $=1$. The earth's radius corresponding to Harton is then 16621·7, its attraction at the lower station is $\frac{4\pi}{3}\times 16621{\cdot}7\times \mathrm{D}$, or $69625\times\mathrm{D}$, and at the upper station $\left\{69625-\frac{8\pi}{3}\right\}\times\mathrm{D}$. The attraction of the shell upon the upper station, or the difference of attractions of the indefinite horizontal stratum upon the upper and lower stations, is $4\pi\times d$. The formula applicable to the depression of the sea will be $\frac{2b}{a}\times d$; and that applicable to any other elevation or depression will be $\frac{\text{volume}}{(\rho^2+1)^{\frac{3}{2}}}\times d$; where all linear measures are to be referred to the mine-depth as unit.

58. On my making known that information on the inequalities of the ground would be required for the final calculations, the Mayor and Corporation of South Shields directed that the necessary surveys should be made, and the execution of this work was entrusted to Christopher Thompson, Esq., Surveyor for the Corporation. This gentleman entered fully into my views, and, after the proper surveys for elevation, furnished me with a map extending about three miles in all directions round Harton, with the elevations above high water in feet marked at numerous points. I found that a line might be drawn, nearly ten depths distant from the upper station, touching the cliffs of Tynemouth and the cliffs south-east of Harton, and ranging for some distance along the coast of Durham. I therefore drew a line parallel to this at ten depths distance from the upper station, and divided the whole country into squares (with sides of one depth each) whose sides were parallel and perpendicular to this. These squares I grouped as appeared most convenient, as will be seen in the Map, Plate XII. fig. 4 (the principal object being to secure a proper representation of Jarrow Slake and the Valley of the Tyne), and adopted for each group the elevation in feet above high water which Mr. Thompson's elevations of special points suggested. The elevation of the upper pendulum-station was 74 feet. Consequently, the vertical measure which was to be used for computing the "volume" in the formula above was $\frac{\text{Elevation}-74}{1256}$. Of this formula, a small table was prepared. The quantity ρ was measured graphically from the map, and $\sqrt{\rho^2+1}$ was formed graphically from it.

59. Adopting for the sea, so far as its effects are principally sensible (and conceiving the water replaced by an equivalent quantity of ground with surface lower than the sea), the elevation -15 feet, to which corresponds

$$b=\frac{-15-74}{1256}=\frac{-89}{1256}=-{\cdot}07087,$$

the factor of d applicable to the depression of the sea beyond the straight line at distance 10 is

$$\frac{-2\times{\cdot}07087}{10}=-{\cdot}014174.$$

The elements for computing the effect of the other inequalities, and the factors of d, are as follows; that for 19* being inserted by estimation:—

No.	Elevation in feet above high water.	Surface.	ρ.	Attraction. +	Attraction. −	No.	Elevation in feet above high water.	Surface.	ρ.	Attraction. +	Attraction. −
1	− 15	......	......	......	·014174	23	15	3	2·6	......	·006421
2	0	9	10·7	......	·000427	24	60	4	1·7	......	·005920
3	0	8	9·5	......	·000541	25	55	4	1·7	......	·008021
4	5	7	10·0	......	·000379	26	80	4	1·7	·002537	
5	5	6	9·8	......	·000346	27	74	4	1·7	·000000	
6	30	6	8·3	......	·000361	28	90	8	3·0	·003292	
7	70	12	7·0	......	·000108	29	90	12	5·5	·000866	
8	80	7	6·9	·000099	......	30	90	12	8·5	·000244	
9	130	12	7·8	·001102	......	31	0	6	16·4	......	·000080
10	125	10	9·7	·000438	......	32	0	4	15·3	......	·000065
11	0	4	7·5	......	·000543	33	0	12	11·0	......	·000524
12	15	4	5·8	......	·000925	34	− 5	5	7·0	......	·000890
13	5	2	5·2	......	·000743	35	30	4	9·7	......	·000151
14	50	6	3·9	......	·001739	36	40	20	6·8	......	·001669
15	80	6	3·7	·000514	......	37	20	2	4·0	......	·001229
16	100	16	4·5	·003381	......	38	74	25	4·4	·000000	
17	120	12	7·0	·001244	......	39	105	16	6·5	·001387	
18	120	24	10·5	·000748	......	40	90	21	8·5	·000426	
19	− 15	50	18·0	......	·000605	41	140	16	9·3	·001029	
19*				......	000600	42	100	24	10·4	·000435	
20	15	2	7·9	......	·000186	43	125	16	12·0	·000372	
21	0	2	6·6	......	·000395	44	120	10	11·0	·000272	
22	− 10	9	4·5	......	·006143						

The sum of all the factors for the effects of inequality is —·044799.

As the attraction of the shell is $12·566368\times d$, and as the effects of inequality amount to $-·044799\times d$, the complete effect of the ground above the lower station is $12·521569\times d$.

60. We are now in a position to obtain a result requiring only for its complete numerical exhibition a knowledge of the specific gravities of the mine-rocks. The formula for gravity at the lower station has been found to be

$$69625\times\mathrm{D},$$

and that at the upper station

$$(69625-8·3776)\times\mathrm{D}+12·5216\times d.$$

Hence $$\frac{\text{Gravity below}}{\text{Gravity above}}=1+0·00012032-0·00017984\times\frac{d}{\mathrm{D}}.$$

But the experiments gave (see article 45)

$$\frac{\text{Gravity below}}{\text{Gravity above}}=1·00005185\pm·00000019.$$

Hence $$0·00006847\pm·00000019=0·00017984\times\frac{d}{\mathrm{D}};$$

or $$\frac{\mathrm{D}}{d}=2·6266\pm·0073,$$

where (as has been explained in article 46) the last term ought probably to be doubled, to give the uncertainty depending on the pendulum-experiments only.

SECTION VIII.—*Estimation of the Mean Specific Gravity of the Surface Rocks; and final conclusion on the Mean Density of the Earth, as determined from the Pendulum Experiments.*

61. By the kindness of Mr. ARKLEY, I was furnished with a complete account of the strata passed through in sinking the Harton shaft, and with specimens of a great number of the rocks. These were submitted to Professor W. H. MILLER, who with great labour determined their specific gravities. I place below the entire catalogue and measure of the strata, and the results at which Professor MILLER arrived. Professor MILLER remarks that there is inevitable uncertainty in the estimation of the weight of those specimens which were drawn from beds described as containing water.

No.	Description of Rock.	Thickness. ft.	Thickness. in.	Specific Gravity.	Temperature of water, Centigrade. °
1	Soil	1	0		
2	Yellow Clay	6	4½		
3	Blue laminated Clay	7	0		
4	Gravelly Clay	69	2		
5	Red Clay (resting on the Stone Head)	10	0		
6	Soft blue Shale, and a little Water	7	0		
7	Coal		3		
8	Dark Grey Shale	7	6	2·5698	15·9
9	Coal		3		
10	Dark Grey Shale	12	3	2·5273	16·4
11	Coal and a little Water		4		
12	Strong dark Grey Shale mixed with Sandstone	9	2		
13	Strong Grey Sandstone and Water	7	10	2·5341	8·8
14	Grey Shale	18	0	2·6170	9·2
15	Bituminous Shale		5		
16	Coal	1	0		
17	Thill or Grey Shale	4	2		
18	Sandstone Girdle		10		
19	Grey Shale, Sandstone Girdles, and Water	14	2		
20	White Sandstone Girdle and Water	1	2		
21	Grey Shale	1	6		
22	White Sandstone Girdle and Water	3	5		
23	Bituminous Shale	2	9		
24	Coal		4		
25	Thill or Grey Shale	10	0	2·5325	9·7
26	Grey Sandstone	3	2	2·6325	9·5
27	Grey Shale	2	6		
28	Grey Sandstone with partings of Bituminous Shale	2	8		
29	Blue Shale	5	8		
30	Coal	1	2		
31	Dark Grey Shale	4	6		
32	Grey Shale	10	0		
33	Coal	1	2		
34	Band of Grey Shale		4		
35	Coal		6		
36	Sandstone with very hard Girdles	7	0	2·8072	17·25

TABLE (continued).

No.	Description of Rock.	Thickness. ft.	in.	Specific Gravity.	Temperature of water, Centigrade. °
37	Grey Shale and Water	3	4		
38	Coal		6		
39	Bituminous Shale		3		
40	Coal		3		
41	Grey Shale with Sandstone Girdles	8	4		
42	Coal		3		
43	Grey Shale	5	2		
44	Coal		2		
45	Grey Shale with thin Sandstone Girdles and Water	23	7	2·5525	15·8
46	Coal, Brassey Band, and Coal	2	10	1·3117	16·1
47	Grey Shale	4	0	2·5716	10·0
48	Grey Shale, Sandstone Girdles, and Water	15	1	2·5687	10·3
49	Sandstone and Water	45	0	2·6695	8·8
50	Dark Grey Shale	12	0	2·5962	9·9
51	Coal	1	8	1·3474	7·7
52	Grey Shale or Thill	4	0		
53	Grey Sandstone and Water	5	2		
54	White Sandstone with Shale partings and Water	116	8	2·8016	16·0
55	Dark Grey Shale	16	3	2·6132	16·4
56	Coal		8		
57	Dark Grey Shale	16	3	2·6233	4·8
58	Coal		5		
59	Brown Shale		6		
60	Grey Sandstone with Shale partings and salt water, emitting Carburetted Hydrogen Gas	7	3½	2·5410	16·9
61	Red and White Sandstone, porous and wet	53	8½	2·8163	8·4
62	Coal mixed with hard Bituminous Shale		3	1·7010	8·4
63	Soft Grey Shale	7	0	2·5184	9·5
64	Strong Bituminous Shale mixed with Nodules of Clay Ironstone	1	5	2·2587	9·5
65	Coal		2½		
66	Strong light-coloured Grey Sandstone with Shale partings and very hard Girdles	10	0	2·6595	9·1
67	Dark Grey Shale	10	7	2·5734	17·5
68	Coal } Bituminous Band } Coal } Bituminous Band } Coal }	1	8	1·4839	17·5
69	Hard Grey Shale with very hard Sandstone Girdles	27	4	2·6223	7·7
70	Dark Grey Shale with Girdles of Clay Ironstone	15	10	2·5416	8·9
71	Foul Coal		5		
72	Dark Grey Shale	2	6	2·7073	18·5
73	Hard dark Grey Shale with Sandstone Girdles	12	0	2·2660	9·5
74	Coal		5		
75	Grey Shale	1	8		
76	White Sandstone with very hard Girdles and Water	24	11	2·5334	9·0
77	Coal 0 ft. 4 in., Slaty Coal 1 ft. 1 in.=1 ft. 5 in.	1	5		
78	Grey Shale	2	7		
79	Coal		6		
80	Dark Grey Shale with Sandstone Girdles and Water	14	1	2·6434	9·3
81	White Sandstone with Shale partings	13	11½	2·5613	7·8
82	Grey Shale	1	4		
83	Coal, strong and good	2	6		
84	Splint Coal		6		
85	Grey Shale	1	6		
86	Strong White Sandstone, very hard in places, and water	43	8½	2·4969	15·5
87	Bituminous Shale		3½		

Table (continued).

No.	Description of Rock.	Thickness. ft.	in.	Specific Gravity.	Temperature of water, Centigrade.
		ft.	in.		c
88	Black Slaty Coal	1	5½		
89	Grey Shale with Sandstone Girdles	15	8		
90	Bituminous Shale with Clay Ironstone Girdles	3	0		
91	Coal		1½		
92	Dark Grey Shale with Sandstone Girdles	16	1		
93	Hard White Sandstone with frequently hard Shale and small Ironstone Nodules very coarse, and a little Water	64	7	2·5483	8·5
94	Dark Blue Shale with Nodules and Ironstone Girdles	6	11	2·5178	9·5
95	Grey Shale with Sandstone Girdles	6	0		
96	Bituminous Shale	7	5	2·5209	17·1
97	Coal, strong and good	·	6	1·2956	18·1
98	Grey Shale with Sandstone Girdles	21	0		
99	White Sandstone with a little Water	19	0	2·3250	
100	Grey Shale with Ironstone Girdles	6	0		
101	Coal	1	2		
102	Bituminous Shale		1	2·3942	4·8
103	Coal		2½		
104	Bituminous Shale	3	0	2·6000	9·1
105	Bituminous Shale mixed with Coal pipes		6		
106	Grey Shale	5	6		
107	Hard Shale	21	0	2·6066	9·6
108	Grey Shale with Sandstone Girdles	27	0	2·6641	7·3
109	Strong Grey Sandstone with Shale partings, and very hard Sandstone Girdles with a little Water	40	5	2·6146	9·6
110	Blue Shale Girdle	2	0½		
111	White Sandstone Girdle with Shale partings	2	3		
112	Coal	1	10		
113	Grey Shale with Ironstone Nodules	4	0		
114	Strong Grey Shale with Sandstone Girdles	10	0		
115	Strong dark Grey Shale with Sandstone Girdles	14	0		
116	Splint Coal		5		
117	Grey Shale with Sandstone Girdles	10	0		
118	Strong White Sandstone with very hard Girdles	12	0	2·6481	15·2
119	Very hard Sandstone	8	0	2·6763	7·7
120	Hard Grey Shale with Sandstone and Ironstone Girdles	33	1	2·6021	7·4
121	Black Slaty Bituminous Shale	1	0		
122	Coal		9		
123	Bituminous Shale		3		
124	Shale	4	0		
125	Hard Grey Shale with very hard Sandstone Girdles	7	9		
126	Coal		3		
127	Hard Grey Shale with mild and also very hard Sandstone Girdles	25	3	2·8178	5·3
128	Coal		8½		
129	Bituminous Shale	3	2		
130	Coal		3½		
131	Shale	4	2	2·5591	16·8
132	Bituminous Shale		5½		
133	Splinty Coal		2½		
134	Grey Shale	1	2		
135	Dark Grey Shale with Ironstone Girdles	6	2		
136	Grey Shale with Ironstone Girdles	11	3		
137	Grey Shale with Shale partings and very hard Sandstone Girdles	15	0		
138	Dark Grey Shale with Ironstone Girdles	8	0		
139	White Sandstone with very hard Girdles	5	6		
140	Grey Sandstone with Shale partings and very hard Girdles	21	1½	2·6146	8·3
141	Bituminous Shale		4		
142	Top Coal, Splint, Bottom Coal } Bensham Coal Seam.	6	0	1·2869	8·7

62. Without pretending to extreme accuracy, we may consider the thickness and specific gravity of the beds as represented, with sufficient accuracy for the present purpose, by the following numbers:—

1211 feet of rocky and shaly beds . .	specific gravity 2·56
30 feet of coaly beds	specific gravity 1·43
15 feet completely worked out . .	specific gravity 0·00

From these the mean specific gravity is found to be 2·50.

63. Substituting this for d in the expression at the end of the last Section, we get for the mean density of the Earth, as determined from these experiments,

$$6{\cdot}566 \pm {\cdot}0182,$$

where the last term (expressing the probable error) ought to be at least doubled.

64. The value thus obtained is much larger than that obtained from the Schehallien experiment, and considerably larger than the mean found by BAILY from the torsion-rod experiments. It is extremely difficult to assign with precision the causes or the measures of the errors of any of these determinations; and I shall content myself with expressing my opinion, that the value now presented is entitled to compete with the others, on, at least, equal terms.

XV. *Supplement to the "Account of Pendulum Experiments undertaken in the Harton Colliery;" being an Account of Experiments undertaken to determine the Correction for the Temperature of the Pendulum.* *By* G. B. AIRY, *Esq., Astronomer Royal.*

Received February 13,—Read March 6, 1856.

SECTION IX.—*Introductory and Historical.*

65. IN order to remove, as far as possible, the slight uncertainty in the result of the Harton Experiment depending on the mean difference of temperature of the Pendulums, I again borrowed the two Pendulums and the Clock SHELTON from the Royal Society, and prepared them for use in the following manner. Two rooms were selected at the Royal Observatory, which appeared well adapted for a series of pendulum experiments. One is the Laundry of the Dwelling House; a room on the basement story, almost entirely sunk below the level of the Lawn and Front Court, but bordered on two sides by a dry area; with stone floor laid immediately upon the solid ground. The chimney-opening and other crevices were carefully stopped up, and a hot-air-stove with funnel leading into the chimney was planted in the room; by means of this stove the temperature of the room could be maintained with reasonable steadiness nearly to 90° FAHR. The Pendulum 8 with Clock EARNSHAW were mounted in the usual way in this room. A screen was placed between the stove and the pendulum, which effectually prevented the pendulum and its thermometers from receiving any sensible heat by radiation. The other room is the Lower Record Room, a room with stone floor laid (except at its centre) upon the solid ground, warmed by a heating apparatus below the centre of the floor. The Pendulum 1821 with Clock SHELTON were mounted in one side of this room. As a hot-air-pipe rises in the centre of the room, this pipe also was screened to prevent unfair radiation on the pendulum, &c.; although here the necessity for such caution was far less pressing than in the Laundry.

66. The plan proposed for operations was; to compare the clocks, by carrying a Solar Chronometer from one to the other, and observing their readings at the time of coincidence of beats (a method which admits of extreme accuracy when the clocks are so near together that the chronometer can be carried without risk of disturbance); to divide the operations into Four Series, numbered in sequence Fifth, Sixth, Seventh, Eighth; to make no alteration whatever in the pendulums, nor even to dismount them, between one series and another; but to make Pendulum 1821 hot and 8 cool in the Fifth and Seventh Series, and Pendulum 8 hot and 1821 cool in the Sixth and Eighth Series; and through the extent of each Series to keep up the Swings in

incessant sequence, as in the observations at Harton. The uncontrolled conduct of the experiments was entrusted to Messrs. DUNKIN and ELLIS; and I am confident it will be found that they are inferior to none that have ever been made.

In all cases, the fires for warming the rooms were lighted nearly twenty-four hours before the observations began.

67. When the pendulum-boxes were opened at Greenwich, two of the four thermometers which had been used at Harton were missing, and all attempts to trace them have hitherto failed. Two thermometers belonging to the Royal Observatory (one by THOMAS JONES and one by TROUGHTON and SIMMS) were substituted for them. It would have been desirable to use the very same thermometers which were used in the Harton experiment; but it is probable that, with the considerable range of temperature used in these experiments, the accidental differences between two thermometers made by makers of repute will not produce any sensible effect on the result.

68. The following is the journal of operations:—

Fifth Series. Pendulum 1821 in heat.

1856. Jan. 1.—Observations commenced a little before noon. Mr. DUNKIN observed Pendulum 1821 from the beginning of Swing 83 to the beginning of Swing 85. Mr. ELLIS observed Pendulum 8 from the beginning of Swing 83, and 1821 from the end of Swing 85, to the beginning of Swing 87.

Jan. 2.—Mr. DUNKIN observed 8 from the end of Swing 87 to the beginning of Swing 89; Mr. ELLIS observed 8 from the end of Swing 89 to the beginning of Swing 91; and Mr. ELLIS observed it from the end of Swing 91 to the end of Swing 92. Mr. DUNKIN observed 1821 from the end of Swing 87 to the end of Swing 92, nearly two hours after midnight. This closed the Fifth Series.

Sixth Series. Pendulum 8 in heat.

Jan. 4.—Commencing about $1\frac{1}{2}$ hour before noon. Mr. DUNKIN observed 1821 from the beginning of Swing 93 to the beginning of Swing 97. Mr. ELLIS observed 8 from the beginning of 93 to the beginning of 95; Mr. DUNKIN from the end of 95 to the beginning of 97.

Jan. 5.—Mr. ELLIS observed 1821 from the end of 97 to the beginning of 99; Mr. DUNKIN from the end of 99 to the beginning of 102; Mr. ELLIS at the end of 102. Mr. ELLIS observed 8 from the end of 97 to the end of 102, about $1\frac{1}{2}$ hour after midnight. This closed the Sixth Series.

Seventh Series. Pendulum 1821 in heat.

1856. Jan. 8.—Commencing about 2 hours before noon. Mr. DUNKIN observed 1821 from the beginning of 103 to the beginning of 105; and Mr. ELLIS from the end of 105 to the beginning of 107. Mr. ELLIS observed 8 from the beginning of 103 to the beginning of 107.

Jan. 9.—Mr. DUNKIN observed 1821 from the end of 107 to the end of 112. Mr. DUNKIN observed 8 from the end of 107 to the beginning of 109; Mr. ELLIS from the end of 109 to the beginning of 111; and Mr. DUNKIN from the end of 111 to the end of 112, about 1½ hour after midnight. This closed the Seventh Series.

Eighth Series. Pendulum 8 in heat.

Jan. 11.—Mr. DUNKIN observed 1821 from the beginning of 113 to the beginning of 117. Mr. ELLIS observed 8 from the beginning of 113 to the beginning of 115; Mr. DUNKIN from the end of 115 to the beginning of 117.

Jan. 12.—Mr. ELLIS observed 1821 from the end of 117 to the end of 118; Mr. DUNKIN from the beginning of 119 to the beginning of 121; and Mr. ELLIS from the end of 121 to the end of 122. Mr. ELLIS observed 8 from the end of 117 to the end of 122, about 2 hours after midnight. This terminated the Eighth Series, and the whole operation.

SECTION X.—*Comparisons of Clocks.*

69. The clocks were compared by means of a Solar Chronometer, sometimes three times and sometimes four times at each interruption of Swings. When there are three comparisons, all are made by one observer; when there are four, they are usually (but not in every case) made by two observers. The discordance of the different comparisons in each group does not exceed $0^s{\cdot}01$ or $0^s{\cdot}02$.

The following Table contains the Mean of each group of comparisons, and the computation of the Log. Rate of SHELTON upon EARNSHAW.

No. of Swing.	Approximate Time of Comparison.		Number of Comparisons.	Mean of Times by SHELTON.	Mean of Times by EARNSHAW.	Interval by SHELTON.	Interval by EARNSHAW.	Rate $\frac{\text{SHELTON}}{\text{EARNSHAW}}$.	Log. Rate $\frac{\text{SHELTON}}{\text{EARNSHAW}}$.
	1856. Jan.	h		h m s	h m s	h m s	h m s		
83	0.	23	4	6 5 14·00	21 46 41·87	3 11 48·25	3 11 29·47	1·0016345	0·00070928
84	1.	2	4	9 17 2·25	0 58 11·34	4 7 39·50	4 7 15·52	1·0016164	0·00070142
85	1.	6	4	13 24 41·75	5 5 26·86	4 30 40·92	4 30 14·58	1·0016245	0·00070495
86	1.	10	3	17 55 22·67	9 35 41·44	4 24 11·66	4 23 46·18	1·0016100	0·00069865
87	1.	15	3	22 19 34·33	13 59 27·62	3 32 26·34	3 32 5·72	1·0016203	0·00070312
88	1.	18	3	1 52 0·67	17 31 33·32	3 58 44·00	3 58 20·91	1·0016146	0·00070065
89	1.	22	3	5 50 44·67	21 29 54·23	3 42 9·58	3 41 48·11	1·0016133	0·00070008
90	2.	2	4	9 32 54·25	1 11 42·34	3 58 2·00	3 57 39·00	1·0016130	0·00069995
91	2.	6	4	13 30 56·25	5 9 21·34	4 22 3·42	4 21 38·06	1·0016155	0·00070104
92	2.	10	3	17 52 59·67	9 30 59·40	3 36 56·83	3 36 35·90	1·0016105	0·00069887
	2.	14	4	21 29 56·50	13 7 35·30				
93	3.	22	4	6 5 12·00	21 39 43·29	3 21 39·00	3 21 19·75	1·0015936	0·00069154
94	4.	2	4	9 26 51·00	1 1 3·04	4 13 24·75	4 13 0·56	1·0015935	0·00069150
95	4.	6	4	13 40 15·75	5 14 3·60	4 0 22·92	3 59 59·83	1·0016035	0·00069584
96	4.	10	3	17 40 38·67	9 14 3·43	3 53 35·00	3 53 12·42	1·0016138	0·00070030
97	4.	14	3	21 34 13·67	13 7 15·85	4 40 26·33	4 39 59·23	1·0016131	0·00069999
98	4.	18	3	2 14 40·00	17 47 15·08	3 45 54·67	3 45 32·73	1·0016212	0·00070351
99	4.	22	3	6 0 34·67	21 32 47·81	3 37 2·83	3 36 41·77	1·0016198	0·00070291
100	5.	2	4	9 37 37·50	1 9 29·58	4 4 8·50	4 3 44·81	1·0016191	0·00070260
101	5.	6	4	13 41 46·00	5 13 14·39	4 9 8·00	4 8 43·81	1·0016209	0·00070338
102	5.	10	4	17 50 54·00	9 21 58·20	4 12 17·75	4 11 53·26	1·0016205	0·00070321
	5.	14	4	22 3 11·75	13 33 51·46				
103	7.	22	4	5 48 34·50	21 18 12·38	3 37 41·00	3 37 19·62	1·0016396	0·00071149
104	8.	2	4	9 26 15·50	0 55 32·00	4 19 3·25	4 18 37·71	1·0016458	0·00071418
105	8.	6	4	13 45 18·75	5 14 9·71	4 44 9·25	4 43 41·31	1·0016415	0·00071231
106	8.	10	3	18 29 28·00	9 57 51·02	3 39 8·33	3 38 46·78	1·0016417	0·00071239
107	8.	14	3	22 8 36·33	13 36 37·80	4 12 2·34	4 11 37·53	1·0016433	0·00071309
108	8.	18	3	2 20 38·67	17 48 15·33	3 43 52·66	3 43 30·66	1·0016405	0·00071188
109	8.	22	3	6 4 31·33	21 31 45·99	3 38 56·17	3 38 34·69	1·0016378	0·00071071
110	9.	2	4	9 43 27·50	1 10 20·68	4 9 18·00	4 8 53·52	1·0016393	0·00071136
111	9.	6	4	13 52 45·50	5 19 14·20	4 1 12·17	4 0 48·49	1·0016390	0·00071123
112	9.	10	3	17 53 57·67	9 20 2·69	4 7 8·08	4 6 43·89	1·0016341	0·00070910
	9.	14	4	22 1 5·75	13 26 46·58				
113	10.	22	4	5 56 22·50	21 18 57·71	3 58 47·00	3 58 23·91	1·0016143	0·00070052
114	11.	2	4	9 55 9·50	1 17 21·62	4 6 8·00	4 5 44·14	1·0016183	0·00070225
115	11.	6	4	14 1 17·50	5 23 5·76	4 4 32·17	4 4 8·39	1·0016234	0·00070446
116	11.	10	3	18 5 49·67	9 27 14·15	3 40 25·00	3 40 3·57	1·0016230	0·00070429
117	11.	14	3	21 46 14·67	13 7 17·72	5 1 35·00	5 1 5·65	1·0016247	0·00070503
118	11.	18	3	2 47 49·67	18 8 23·37	3 52 38·00	3 52 15·35	1·0016254	0·00070533
119	11.	22	3	6 40 27·67	22 0 38·72	3 15 40·08	3 15 21·02	1·0016262	0·00070568
120	12.	2	4	9 56 7·75	1 15 59·74	4 14 49·25	4 14 24·41	1·0016273	0·00070616
121	12.	6	4	14 10 57·00	5 30 24·15	4 55 24·00	4 54 55·10	1·0016322	0·00070828
122	12.	11	3	19 6 21·00	10 25 19·25	3 46 3·00	3 45 40·97	1·0016270	0·00070603
	12.	15	4	22 52 24·00	14 11 0·22				

SECTION XI.—*Auxiliary Tables for the reduction of the observations of Coincidences.*

70. The range of the Intervals of Coincidences in these experiments is not included in the tables given in Section III. The table of log $\frac{n-2}{n}$ which is placed below includes all that are required here. The method of using it is the same as in the preceding Series.

The tables used before for the Correction for Arc were sufficient for the present experiments.

The range of temperature in the experiments now made greatly exceeded that for the experiments in the mine. In extending a table, so as to include an increased range of physical conditions, there will always be some doubt as to the propriety of selection of that mathematical function which is assumed to increase proportionally with the indications of the physical measurer. I have here assumed that the changes of the logarithms of the temperature-correction are simply proportional to the changes of thermometer-reading. The table below, therefore, is formed from the table in Section III. by simple addition and subtraction of a constant number.

The table used for barometrical correction is the same which was used before.

n.	Log. $\frac{n-2}{n}$.	*n*.	Log. $\frac{n-2}{n}$.	*n*.	Log. $\frac{n-2}{n}$.	*n*.	Log. $\frac{n-2}{n}$.	*n*.	Log. $\frac{n-2}{n}$.
269·5	9·99676502	271·5	9·99678894	273·5	9·99681250	277·8	9·99686202	279·8	9·99688454
·6	6622	·6	9012			278·9	6315	280·9	8566
·7	6742	·7	9130	276·0	9·99684148	·0	6429	·0	8677
·8	6863	·8	9249	·1	4263	·1	6542	·1	8788
·9	6983	·9	9367	·2	4377	·2	6655	·2	8899
270·0	7103	272·0	9486	·3	4492	·3	6768	·3	9011
·1	7222	·1	9604	·4	4606	·4	6881	·4	9122
·2	7342	·2	9722	·5	4721	·5	6994	·5	9233
·3	7462	·3	9840	·6	4835	·6	7107	·6	9344
·4	7582	·4	9·99679958	·7	4949	·7	7219	·7	9455
·5	7702	·5	9·99680076	·8	5064	·8	7332	·8	9566
·6	7821	·6	0194	·9	5178	·9	7444	·9	9677
·7	7940	·7	0311	277·0	5293	279·0	7557	281·0	9788
·8	8060	·8	0429	·1	5407	·1	7669	·1	9·99689899
·9	8179	·9	0546	·2	5521	·2	7781	·2	9·99690009
271·0	8299	273·0	0664	·3	5635	·3	7893	·3	0119
·1	8418	·1	0781	·4	5748	·4	8006	·4	0230
·2	8537	·2	0898	·5	5862	·5	8118	·5	0341
·3	8656	·3	1016	·6	5975	·6	8230	·6	0451
·4	8775	·4	1133	·7	6088	·7	8342	·7	0562

Thermometer.	Correction to Log Rate.	Thermometer.	Correction to Log. Rate.	Thermometer.	Correction to Log. Rate.
40°·0	9·99997937	50°·5	0·00000104	88°·0	0·00007840
·5	8040	51·0	0207	·5	7943
41·0	8144	·5	0310	89·0	8046
·5	8247	52·0	0413	·5	8149
42·0	8350	·5	0516	90·0	8252
·5	8453	53·0	0619	·5	8355
43·0	8556	·5	0722	91·0	8458
·5	8659	54·0	0825	·5	8561
44·0	8762	·5	0928	92·0	8664
·5	8865	55·0	1032	·5	8767
45·0	8968			93·0	8870
·5	9071	83·0	0·00006808	·5	8973
46·0	9175	·5	6912	94·0	9076
·5	9278	84·0	7015	·5	9179
47·0	9381	·5	7118	95·0	9282
·5	9484	85·0	7221	·5	9385
48·0	9587	·5	7324	96·0	9488
·5	9690	86·0	7428	·5	9591
49·0	9793	·5	7531	97·0	9694
·5	9·99999897	87·0	7634	·5	9797
50·0	0·00000000	·5	7737	98·0	9900

By the application of the numbers from these two tables, together with those for Arc and Barometer, in the same manner as in the Harton Experiments, there is obtained the "Corrected Log. Rate of Pendulum upon Clock" referred to Barometer 0·000 and Thermometer 50°; but affected by the full amount of the errors of the table for corrections depending on the Thermometer.

SECTION XII.—*Abstract of the Computations of the Rate of each detached Pendulum upon its Clock.*

71. The whole of the reductions, for both stations, are included in the following tables.

Pendulum 1821. Fifth Series.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Interval.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 1821 upon SHELTON.
				s			°	in.	
83	3	3	31	272·39	1·67	0·71	84·36	29·775	9·99692466
84	3	3	46	272·81	1·92	0·54	85·03	29·750	3098
85	3	3	48	273·05	2·08	0·54	83·95	29·705	3178
86	3	3	45	273·10	2·09	0·59	84·49	29·645	3351
87	3	3	32	273·01	1·85	0·77	85·26	29·605	3401
88	3	3	38	273·24	2·12	0·75	85·78	29·585	3818
89	3	3	34	273·23	1·97	0·77	84·71	29·525	3553
90	3	3	44	273·44	2·06	0·61	83·31	29·460	3474
91	3	3	48	273·10	1·87	0·51	84·98	29·435	3356
92	3	3	33	273·16	1·75	0·71	85·15	29·430	3487

Pendulum 1821. Sixth Series.

93	3	3	35	279·14	2·01	0·74	53·36	29·570	9·99693872
94	3	3	47	278·89	2·02	0·54	54·41	29·545	3754
95	3	3	43	279·04	1·93	0·57	53·38	29·535	3701
96	3	3	37	279·00	2·22	0·78	53·26	29·515	3739
97	3	3	46	279·09	2·05	0·57	53·25	29·465	3738
98	3	3	31	278·96	2·16	0·89	52·91	29·430	3635
99	3	3	32	278·98	2·04	0·82	52·71	29·400	3562
100	3	3	45	279·02	1·94	0·54	52·85	29·370	3533
101	3	3	44	279·21	1·95	0·58	52·35	29·355	3653
102	3	3	43	279·34	2·11	0·63	51·91	29·310	3744

Pendulum 1821. Seventh Series.

103	3	3	41	272·22	2·08	0·71	89·48	28·970	9·99693256
104	3	3	50	271·15	2·22	0·59	93·21	28·985	2754
105	3	3	54	270·83	2·02	0·46	95·60	29·010	2796
106	3	3	35	271·54	1·93	0·74	92·00	29·025	2959
107	3	3	42	271·96	1·93	0·61	91·31	29·050	3286
108	3	3	38	271·62	2·00	0·72	93·70	29·070	3420
109	3	3	36	271·30	1·98	0·74	94·09	29·085	3124
110	3	3	48	271·31	2·08	0·58	94·11	29·070	3114
111	3	3	46	271·35	1·99	0·58	94·08	29·080	3142
112	3	3	38	271·05	1·90	0·68	96·49	29·110	3382

Pendulum 1821. Eighth Series.

No. of Swing.	Number of Coincidences.		Number of Intervals.	Mean Interval.	Arc of Vibration.		Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 1821 upon SHELTON.
	Beginning.	End.			Beginning.	End.			
				s			°	in.	
113	3	3	44	281·12	1·85	0·57	43·05	29·745	9·99693913
114	3	3	46	280·83	1·76	0·53	43·80	29·800	3731
115	3	3	45	280·93	1·95	0·55	43·80	29·880	3891
116	3	3	36	281·16	1·75	0·60	42·26	29·975	3824
117	3	3	50	281·15	1·84	0·48	41·98	30·075	3759
118	3	4	36	281·09	2·08	0·80	41·68	30·190	3780
119	3	3	31	281·31	1·83	0·76	41·46	30·265	3928
120	3	3	46	281·36	2·20	0·60	40·88	30·330	3902
121	3	3	54	281·46	2·14	0·55	40·44	30·380	3906
122	3	3	40	281·65	1·85	0·67	40·54	30·420	4123

Pendulum 8. Fifth Series.

No. of Swing.	Number of Coincidences.		Number of Intervals.	Mean Interval.	Arc of Vibration.		Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 8 upon EARNSHAW.
	Beginning.	End.			Beginning.	End.			
				s			°	in.	
83	3	3	29	277·91	1·70	0·75	47·80	29·775	9·99691320
84	3	3	46	277·88	1·96	0·56	47·81	29·750	1281
85	3	3	43	277·91	1·94	0·59	47·80	29·705	1310
86	3	3	44	277·91	2·01	0·60	47·68	29·645	1289
87	3	3	31	277·83	2·11	0·90	47·41	29·605	1246
88	3	3	37	277·91	1·96	0·72	47·28	29·585	1216
89	3	3	40	277·91	1·86	0·60	48·08	29·525	1319
90	3	3	44	277·86	2·04	0·62	48·18	29·460	1314
91	3	3	43	277·88	2·14	0·67	48·05	29·435	1337
92	3	3	33	277·97	1·79	0·72	47·96	29·430	1368

Pendulum 8. Sixth Series.

93	3	3	37	269·56	2·21	0·71	89·69	29·570	9·99690259
94	3	3	48	270·15	1·97	0·58	88·80	29·545	0699
95	3	3	41	270·38	2·21	0·74	84·45	29·535	0168
96	3	3	38	270·39	1·99	0·72	86·71	29·515	0591
97	3	3	47	269·77	2·06	0·60	91·09	29·465	0722
98	3	3	32	270·43	2·17	0·89	90·00	29·430	1394
99	3	3	36	270·82	1·90	0·71	86·78	29·400	1079
100	3	3	47	270·39	2·05	0·61	90·14	29·370	1255
101	3	3	45	270·08	2·14	0·68	89·19	29·355	0721
102	3	3	39	271·21	2·03	0·76	85·26	29·310	1253

Pendulum 8. Seventh Series.

103	3	3	26	278·20	1·94	0·68	52·24	28·970	9·99692447
104	3	3	47	277·73	1·99	0·58	52·45	28·985	1945
105	3	3	48	277·71	2·18	0·67	51·91	29·010	1876
106	3	3	33	277·76	2·15	0·89	51·03	29·025	1818
107	3	3	41	277·94	1·93	0·69	50·79	29·050	1870
108	3	3	37	277·91	2·23	0·86	50·06	29·070	1803
109	3	3	40	278·25	1·93	0·73	48·83	29·085	1832
110	3	3	48	278·32	2·32	0·69	48·61	29·070	1930
111	4	3	40	278·36	2·08	0·76	48·71	29·080	1967
112	3	3	36	278·53	1·94	0·80	48·23	29·110	2052

Pendulum 8. Eighth Series.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Interval.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 8 upon EARNSHAW.
				s			°	in.	
113	3	3	45	270·14	2·20	0·68	94·28	29·745	9·99691921
114	3	3	47	270·57	2·02	0·61	92·44	29·800	2012
115	3	3	42	270·21	2·08	0·72	93·11	29·880	1775
116	3	3	37	270·51	1·92	0·74	94·44	29·975	2399
117	3	3	53	271·08	1·98	0·50	91·29	30·075	2397
118	3	3	36	270·93	2·10	0·83	93·39	30·190	2787
119	3	3	35	269·97	2·10	0·82	95·80	30·265	2146
120	3	3	49	269·78	2·11	0·58	98·00	30·330	2313
121	3	3	52	270·78	2·07	0·55	95·78	30·380	3047
122	3	3	42	270·90	1·95	0·66	94·39	30·420	2917

SECTION XIII.—*Computation of Log. Rate of Pendulum* 1821 *on Pendulum* 8; *combination of individual results; and conclusion on the amount of temperature correction, and on the result of the Harton Experiment.*

72. The quantity $\text{Log.}\,\frac{\text{Rate of Pendulum 1821}}{\text{Rate of Pendulum 8}}$ is formed, as a similar quantity is formed in Section VI., by combining together

$$\text{Log.}\,\frac{\text{Rate of Pendulum 1821}}{\text{Rate of S}\textsc{helton}} + \text{Log.}\,\frac{\text{Rate of S}\textsc{helton}}{\text{Rate of E}\textsc{arnshaw}} - \text{Log.}\,\frac{\text{Rate of Pendulum 8}}{\text{Rate of E}\textsc{arnshaw}}.$$

Results for the Log. Rate of Pendulum 1821 upon Pendulum 8.

Fifth Series. No. of Swing.	Log. Rate Pendulum 1821 / Rate Pendulum 8.	Sixth Series. No. of Swing.	Log. Rate Pendulum 1821 / Rate Pendulum 8.	Seventh Series. No. of Swing.	Log. Rate Pendulum 1821 / Rate Pendulum 8.	Eighth Series. No. of Swing.	Log. Rate Pendulum 1821 / Rate Pendulum 8.
83	0·00072074	93	0·00072767	103	0·00071958	113	0·00072044
84	71959	94	72205	104	72227	114	71944
85	72363	95	73117	105	72151	115	72562
86	71927	96	73178	106	72380	116	71854
87	72467	97	73015	107	72725	117	71865
88	72667	98	72592	108	72805	118	71526
89	72242	99	72774	109	72363	119	72350
90	72155	100	72538	110	72320	120	72205
91	72123	101	73270	111	72298	121	71687
92	72006	102	72812	112	72240	122	71809

73. We have now to consider how the individual results of each Series are to be combined. And here it is to be remarked, that the principal cause of error is not, in these experiments, the uncertainty of comparisons of clocks (which was almost the only source of error in the experiments at the mine). It is probable that the amount of uncertainty from this cause is here nearly unappreciable. The principal sources of error will lie in the circumstances of the pendulum-observations; chiefly, perhaps,

in the insecurity of the indications of the temperature of the pendulums, as inferred from the readings of the neighbouring thermometers. After consideration of these points, I have thought it best to give equal weights to the results. And for comparison with the simple mean of these, I have taken the simple mean of the thermometer-readings. Thus I obtain,—

	$\text{Log.} \frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}$.	Temperature of 1821.	Temperature of 8.
Fifth Series	0·00072198	84°·71	47°·80
Sixth Series	0·00072827	53·04	88·21
Seventh Series	0·00072347	93·41	50·29
Eighth Series	0·00071985	41·99	94·29

The interpretation of these numbers is not without difficulty. From the care taken by the observers, it was supposed that the material state of the pendulums would be sensibly the same during the whole continuance of the observations, and it was expected therefore that the results of the Fifth and the Seventh Series would be in close accordance, and that the results of the Sixth and the Eighth Series would be in close accordance. Now the fact is, that the results of the Fifth and Seventh Series agree pretty well, when regard is had to the circumstances of forced temperature (the discordance 0·00000149 corresponds to $0^{s}\cdot3$ per day nearly, or to $0^{\circ}\cdot72$ of temperature); but the discordance between the Sixth and Eighth is 0·00000842 (which corresponds to $1^{s}\cdot68$ per day, or to $4^{\circ}\cdot09$ of temperature). It would therefore seem, at first sight, that there is strong reason to suppose that some change took place in one of the pendulums after the Seventh Series.

74. As there is no reason whatever for supposing a change before the end of the Seventh Series, let us take the mean of the Fifth and Seventh Series, and compare it (A.) with the Sixth Series alone, (B.) with the mean of the Sixth and Eighth Series; and from these comparisons let us in each case obtain a relation between the two pendulums nearly independent of temperature. Thus we shall have,—

Comparison A.

	$\text{Log.} \frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}$.	Temperature of 1821.	Temperature of 8.
Mean of Fifth and Seventh Series	0·00072273	89°·06	49°·04
Sixth Series	0·00072827	53·04	88·21
Mean of the two lines	0·00072550	71·05	68·62

Comparison B.

Mean of Fifth and Seventh Series	0·00072273	89·06	49·04
Mean of Sixth and Eighth Series	0·00072406	47·52	91·25
Mean of the two lines	0·00072340	68·29	70·15

When it is considered that corrections for temperature have already been applied, founded on the best materials to which at present we have access, it will be per-

ceived that the two "Means of the two lines" are almost strictly independent of temperature. Now we have, in the Harton Experiment, other means of determining the relation between the pendulums. In article 45,

$$\text{Log.}\frac{\text{Rate Pendulum 8 below}}{\text{Rate Pendulum 1821 above}}=9\cdot99928558$$

$$\text{Log.}\frac{\text{Rate Pendulum 1821 below}}{\text{Rate Pendulum 8 above}}=0\cdot00073694.$$

Subtracting the first from the second, and dividing by 2, the effect of the mine is eliminated, and that of temperature is sensibly eliminated; and we obtain

$$\text{Log.}\frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}=0\cdot00072568.$$

This agrees so nearly with the result of Comparison A, as greatly to increase the presumption that some change took place in one pendulum after the Seventh Series.

75. Let then z be the increase (in units of the 8th decimal of the logarithm) which ought to be made for every degree of temperature. Taking the difference between the two first lines of Comparison A,

$$554=z\times75\cdot19,$$

whence $$z=7\cdot37 \text{ nearly}$$

$$=\frac{1}{28}\text{ of the correction employed in our tables.}$$

As the mean excess of temperature at the lower station in the Harton Experiment was $7^{\circ}\cdot13$, the correction to be added to the rate below is 53, or the correction to the gravity is 106. Therefore (see article 45),

$$\text{Log.}\frac{\text{Gravity below}}{\text{Gravity above}}=0\cdot00002358, \text{ or } \frac{\text{Gravity below}}{\text{Gravity above}}=1\cdot00005429.$$

Using this number in the equation of article 60,

$$0\cdot00006603=0\cdot00017984\times\frac{d}{\text{D}},$$

whence $\frac{\text{D}}{d}=2\cdot7236.$

And the Earth's mean density (article 62) will result 6·809.

76. If, however, we used Comparison B in the same way, we should have

$$133=z\times83\cdot75,$$

whence $$z=1\cdot59 \text{ nearly,}$$

or the alteration to be made in the Earth's mean density is less than one-fourth of that resulting from Comparison A. This gives for the Earth's mean density 6·623 nearly.

Addendum.

On communicating with Professor Stokes, in reference to the effect of the Earth's rotation and ellipticity in modifying the numerical results of the Harton Experiment, I was favoured by that gentleman with an investigation, which, with his permission, I subjoin as a valuable addition to my own paper.

"I shall suppose the surface of the Earth to be an ellipsoid of revolution, and will employ the notation made use of in my paper on Clairaut's Theorem, published in the fourth volume of the Cambridge and Dublin Mathematical Journal. In this,

V is the potential of the Earth's mass.

r, θ are the polar coordinates of any point in or exterior to the Earth's surface; r being measured from the centre, and θ from the axis of rotation.

a is the equatorial radius.

ε the ellipticity.

ω the angular velocity.

m the ratio of the centrifugal force to gravity at the equator.

E the mass of the Earth.

ν the angle between the normal and radius vector at any point of the surface.

In the following investigation, small quantities of the second order are neglected, ε and m being regarded as small quantities of the first order.

If
$$U=V+\frac{\omega^2}{2}r^2\sin^2\theta,$$
the differential coefficients
$$\frac{dU}{dr},\quad \frac{1}{r}\cdot\frac{dU}{d\theta}$$
will give the components of the force along and perpendicular to the radius vector; and, g being the force of gravity,
$$g=-\cos\nu\cdot\frac{dU}{dr}+\sin\nu\cdot\frac{1}{r}\cdot\frac{dU}{d\theta};$$
which becomes, since ν and $\frac{dU}{d\theta}$ are small quantities of the first order,
$$g=-\frac{dU}{dr}.$$
Let v be measured along the vertical; then
$$\frac{dg}{dv}=\cos\nu\cdot\frac{dg}{dr}-\sin\nu\cdot\frac{1}{r}\cdot\frac{dg}{d\theta},$$
or, to the first order,
$$\frac{dg}{dv}=\frac{dg}{dr}=-\frac{d^2U}{dr^2}.$$

Let c be the depth of the mine; then if $\left(\frac{c}{a}\right)^2$ be neglected, we shall have for the

value of the fraction $\frac{\text{gravity below}}{\text{gravity above}}$ (which I will call F), calculated on the supposition that all the attracting mass is internal to both stations,

$$F=1-\frac{c}{g}.\frac{dg}{dv},$$

where, after differentiation, r is to be put equal to the radius vector of the surface, namely $a(1-\varepsilon\cos^2\theta)$. Now the value of V (Article 5 of the paper referred to) is

$$V=\frac{E}{r}-\left(\frac{E\varepsilon}{a}-\frac{1}{2}\omega^2a^2\right)\frac{a^3}{r^3}\left(\cos^2\theta-\frac{1}{3}\right),$$

which is true, independently of any particular hypothesis respecting the distribution of matter in the interior of the Earth; so that

$$U=\frac{E}{r}-\left(\frac{E\varepsilon}{a}-\frac{1}{2}\omega^2a^2\right)\frac{a^3}{r^3}\left(\cos^2\theta-\frac{1}{3}\right)+\frac{\omega^2}{2}r^2\sin^2\theta$$

and

$$g=-\frac{dU}{dr}$$

$$=\frac{E}{r^2}-3\left(\frac{E\varepsilon}{a}-\frac{1}{2}\omega^2a^2\right)\frac{a^3}{r^4}\left(\cos^2\theta-\frac{1}{3}\right)-\omega^2r\sin^2\theta,$$

whence

$$-\frac{dg}{dv}=-\frac{dg}{dr}$$

$$=\frac{2E}{r^3}-12\left(\frac{E\varepsilon}{a}-\frac{1}{2}\omega^2a^2\right)\frac{a^3}{r^5}\left(\cos^2\theta-\frac{1}{3}\right)+\omega^2\sin^2\theta.$$

Putting now $r=a(1-\varepsilon\cos^2\theta)$, $\omega^2=m\frac{E}{a^3}$, we find

$$g=\frac{E}{a^2}(1+2\varepsilon\cos^2\theta)-\frac{3E}{a^2}\left(\varepsilon-\frac{m}{2}\right)\left(\cos^2\theta-\frac{1}{3}\right)-m\frac{E}{a^2}(1-\cos^2\theta)$$

$$=\frac{E}{a^2}\left\{1+\left(\frac{5m}{2}-\varepsilon\right)\cos^2\theta+\varepsilon-\frac{3m}{2}\right\}$$

$$-\frac{dg}{dv}=\frac{2E}{a^3}(1+3\varepsilon\cos^2\theta)-\frac{12E}{a^3}\left(\varepsilon-\frac{m}{2}\right)\left(\cos^2\theta-\frac{1}{3}\right)+\frac{mE}{a^3}(1-\cos^2\theta)$$

$$=\frac{2E}{a^3}\left\{1+\left(\frac{5m}{2}-3\varepsilon\right)\cos^2\theta+2\varepsilon-\frac{m}{2}\right\}.$$

Whence

$$-\frac{1}{g}.\frac{dg}{dv}=\frac{2}{a}\left\{\begin{matrix}1+\left(\frac{5m}{2}-3\varepsilon\right)\cos^2\theta+2\varepsilon-\frac{m}{2}\\ -\left(\frac{5m}{2}-\varepsilon\right)\cos^2\theta-\varepsilon+\frac{3m}{2}\end{matrix}\right\}$$

$$=\frac{2}{a}\{1-2\varepsilon\cos^2\theta+\varepsilon+m\};$$

and therefore

$$F=1+\frac{2c}{a}\{1-2\varepsilon\cos^2\theta+\varepsilon+m\}.$$

Now the method adopted in the 'Account of Experiments,' &c., article 57, gives

$$-\frac{1}{g}\cdot\frac{dg}{dr}=\frac{2c}{r}=\frac{2c}{a}(1+\varepsilon\cos^2\theta),$$

whence

$$F=1+\frac{2c}{a}(1+\varepsilon\cos^2\theta).$$

Therefore, if R be the ratio of the value of F−1 given above, to F−1 as calculated by the method of the 'Account of Experiments,'

$$R=\frac{1-2\varepsilon\cos^2\theta+\varepsilon+m}{1+\varepsilon\cos^2\theta}=1-3\varepsilon\cos^2\theta+\varepsilon+m.$$

If l be the geocentric latitude of the place, we may in the small term replace θ by $90°-l$; and since $\cos^2\theta=\sin^2 l=\frac{1}{2}(1-\cos 2l)$, we find

$$R=1+m-\frac{\varepsilon}{2}+\frac{3\varepsilon}{2}\cos 2l.$$

Now

$$m=\frac{1}{289}=0\cdot00346$$

$$\varepsilon=\frac{1}{300\cdot8}=0\cdot00333$$

$$l, \text{ for Harton, } =54°\ 48';$$

$$R=1+0\cdot00346-0\cdot00334=1\cdot00012.$$

That R should have been so very nearly equal to unity, depends upon an accidental numerical relation between the values of m, ε, and l. At the equator, R−1 would have been as great as 0·00679.

In article 60 of the 'Account,' F−1 was found =·00012032; whence R.(F−1)=·00012033; which only alters the final value of the mean density in the ratio of 6836 to 6835, giving for result

6·565.

At the equator, the correction to the deduced value 6·566 would have been −·077."

XV. *On Periodical Laws discoverable in the Mean Effects of the larger Magnetic Disturbances.*—No. III.

By Colonel EDWARD SABINE, *R.A., D.C.L., Treas. and V.P.R.S.*

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HAVING at length completed the analysis of the larger disturbances of the horizontal and vertical magnetic forces at Toronto during five years of hourly observation, with a view to the development of the periodical laws which regulate the occurrence of the occasional disturbances of those elements, and of their theoretical equivalents, the Inclination and Total Force, I now propose to lay before the Royal Society a condensed view of the mode in which the investigation has been made, and of its results.

The hourly observations of the Bifilar and Vertical Force Magnetometers during the five years terminating June 30, 1848, were received at Woolwich, from Toronto, precisely in the state in which they are printed in the second and third volumes of the 'Observations at the Toronto Observatory'; namely, the readings, uncorrected for temperature, at every hour of Göttingen time, arranged in Monthly tables, accompanied by corresponding tables of the temperature of the magnets, shown by thermometers of which the balls were enclosed in the same case with the magnets, and which were read contemporaneously with the Bifilar and Vertical Force scales. The Monthly tables of the scale-readings and of the temperatures were summed before their transmission to Woolwich, both in vertical and horizontal columns, and *means* were taken of all the days in the month at the different hours, and of all the hours of the day on the different days, forming "hourly means" and "daily means." In this state the observations were received at Woolwich and subsequently printed; they were, in fact, printed from the original manuscripts.

The first step taken at the office at Woolwich was to rewrite the whole of the observations of the five years in scale-divisions, corresponding to the respective readings, but reduced to a uniform temperature of 55°, which was taken as a convenient approximate mean temperature: for this purpose each of the observations had to receive a correction, proportioned to the difference between the recorded contemporaneous reading of the thermometer and the standard temperature of 55°, and computed by a coefficient representing the change in the scale-reading produced by an alteration of 1° of the thermometer. The coefficient was obtained directly from the observations themselves, by ascertaining the factor which would best satisfy the differ-

ences of the scale-readings in different natural temperatures, when the magnetometers were mounted and in use.

The formation of the Monthly tables of the "scale-readings reduced to a uniform temperature of 55°," from the tables "uncorrected for temperature," was performed by two Non-commissioned Officers of the Royal Artillery, working independently of each other; the correctness of their work was proved by the accordance of the two independent computers; the daily and hourly means were then taken in the same manner by two independent computers, and were additionally checked by their comparisons with the daily and hourly uncorrected means calculated at Toronto; these means being also reduced, for the purpose of this comparison, to the standard temperature of 55° (excepting in a very few instances, in which the observations on days of *excessive* disturbance had been omitted in the sums and means of the uncorrected readings computed at Toronto, but were restored in the sums and means of the corrected tables). The new tables thus formed, of the scale-readings reduced to 55°, then passed into my hands, when I satisfied myself, by a careful examination, that a difference of fourteen scale-divisions for the Bifilar magnetometer, and of four scale-divisions for the Vertical Force magnetometer, above or below what might be taken as a *normal* value,—viz. the mean value at the same hour during the same month, or for several preceding and several succeeding days,—would constitute a convenient minimum limit for the disturbances of largest amount; since, on the one hand, it would be a greater departure from the normal value than could reasonably be ascribed to any other cause than that of a disturbance in the earth's magnetism, whilst, on the other hand, the number of disturbances that would be thereby separated, would form a sufficient body to permit their periodical laws (if such existed) to be investigated. Having determined on this limit, I proceeded to mark provisionally, with a pencil, every observation which differed in the Bifilar fourteen scale-divisions or more, and in the Vertical Force magnetometer four scale-divisions or more, from their respective normals. I then recomputed the normals, omitting the observations provisionally marked as disturbed, and compared afresh all the observations, including the provisionally-marked ones, with the new normals, altering the markings where required; and I continued this process until the normal in every case included every observation which differed in the Bifilar less than fourteen, and in the Vertical Force magnetometer less than four scale-divisions from itself, and excluded every observation which differed in the Bifilar fourteen, and in the Vertical Force magnetometer four scale-divisions or more from itself. The excluded observations were then marked, finally, with a surrounding ring in ink. In this state the Tables were returned to the Office, and the correctness of the markings, and of the normals excluding the larger disturbances, was examined by a separate computer.

Two computers, working separately, and having their work compared, then formed a table of the marked disturbances of each element, during the five years, arranged

chronologically, showing the day, the hour, and the amount of disturbance, (*i. e.* the difference from the normal,) in scale-divisions; and on the receipt of this table from the Office, I proceeded to distribute the disturbances according to the years, months, and hours of their occurrence, separating them into disturbances increasing, and disturbances decreasing, the respective forces, and forming annual, monthly, and hourly tables; the correctness of the distribution and of the calculations in each of the tables being in every case examined by a second person.

In course of the process of marking the disturbances, it became evident that there were times, occasional, but by no means frequent, when the change in the mean monthly scale-reading, *i. e.* the mean of all the hours and all the days in the month, from one month to the next, was so considerable as to cause the regular hourly normals of the month to be inapplicable to its earlier or later portions. In such cases, the difficulty was met, and more suitable normals obtained for the earlier or later portions of the month, by taking the hourly means of the last fortnight of the one month and the first fortnight of the next; or by a mean of the normals of the two months combined; or, in a very few instances in which the departure from an uniformly progressive change was greatest, by normals derived from periods of less duration than a month.

The Tables showing the normal values finally adopted, the periods for which they were employed, and the periods from which they were derived, together with the annual, monthly, and hourly tables of the aggregate values of the larger disturbances of the horizontal and vertical forces in the several years, months and hours, will be found in full detail in the third volume of the Toronto Observations, which is now in the press.

The Disturbances of the *Inclination*, which equalled or exceeded $1'{\cdot}0$, and of the *Total Force* which equalled or exceeded ·0004 of the whole force, (both measured from the respective normals at the same hour and in the same month,) were obtained from the observed disturbances of the Horizontal and Vertical Forces in the following manner:—Tables were formed, in the first column of which were placed, in chronological order, the larger disturbances of the Vertical Force, separated as already described, and in the second column those of the Horizontal Force, each expressed in terms of the respective forces, by the conversion of the scale-divisions in which the disturbances were observed into parts of the respective forces by means of the scale-coefficient. At a large proportion of the hours of contemporaneous observation, when one of the two components of the force exhibited a disturbance which by its amount was brought into the category of the larger disturbances, the other component was also disturbed. In such cases, there were contemporaneous entries in both columns; but when one of the components only was so affected, the entry in the corresponding column of the other component was blank. These blanks were now all filled up, by inserting for the component which did not exhibit a disturbance of sufficiently large amount to have been classed as a large disturbance, and separated

accordingly, the difference, whatever that might be, between the observation at that hour and its proper normal. These two columns then exhibited all the larger disturbances of both the Horizontal and Vertical components whenever either component was disturbed, with the contemporaneous difference of the other component from its mean or normal value in the cases when one only of the two components exhibited a large disturbance. The entries in the two columns had each their proper signs prefixed, + if the disturbance or difference from the normal were in augmentation of the force, and − if in diminution of the force. These two columns then expressed the values of $\frac{\Delta Y}{Y}$ for the Vertical Force, and $\frac{\Delta X}{X}$ for the Horizontal Force, for every hour at which either $\frac{\Delta Y}{Y}$ equalled or exceeded four scale-divisions, or ·00026 parts of the Vertical Force, or $\frac{\Delta X}{X}$ equalled or exceeded fourteen scale-divisions, or ·0012 parts of the Horizontal Force. A third and fourth column were then filled in, the third expressing the values of $\Delta\theta$, or the disturbances of the Inclination, and the fourth the values $\frac{\Delta\varphi}{\varphi}$, or the disturbances of the Total Force (in parts of φ the Total Force at Toronto) corresponding to the entries in the first and second columns, and computed by the formulæ—

$$\Delta\theta=\sin\theta\cos\theta\left(\frac{\Delta Y}{Y}-\frac{\Delta X}{X}\right)$$

$$\frac{\Delta\varphi}{\varphi}=\cos^2\theta\,\frac{\Delta X}{X}+\sin^2\theta\,\frac{\Delta Y}{Y}.$$

From the third and fourth columns all the disturbances of the Inclination ($\Delta\theta$) which equalled or exceeded $1'\cdot0$ in amount, and all the disturbances of the Total Force $\left(\frac{\Delta\varphi}{\varphi}\right)$ equalling or exceeding ·0004 in amount were taken, as forming respectively a sufficient body of the larger disturbances of each element to permit their periodical laws to be investigated and shown. These disturbances were then dealt with in regard to classification and tabular arrangement, in the same manner as that which has been already explained in treating of disturbances of the horizontal and vertical components of the force.

In deriving the disturbances of the Inclination and Total Force from those of the Horizontal and Vertical Forces, all the calculations and arrangements in tables were prepared under the superintendence of Mr. MAGRATH, Chief Clerk, by the Non-commissioned-officers of the Royal Artillery employed in the Woolwich Office, every part of the process having had the advantage of two independent computers.

To complete the view of the periodical laws of the magnetic disturbances at Toronto, a revision has been made of the analysis of the larger Disturbances of the Declination, the results of which were contained in a former paper presented to the Royal Society in 1852. These disturbances have now been computed from normals on

which the same labour has been bestowed as on those of the Horizontal and Vertical Forces, so as to cause them to include all the observations which differ less, and exclude all those which differ more than a certain fixed value from themselves. In this revision also the fixed or standard measure of a large disturbance has been increased from five scale-divisions (3'·6 of arc) to seven scale-divisions (or 5'·0 of arc); the experience gained in the first or experimental examination having led to the belief that the higher standard is on the whole to be preferred.

General Conclusions.

Decennial Period.—In a communication made to the Royal Society on the 18th of March, 1852, it was shown that the larger disturbances of the Declination, both at Toronto and at Hobarton, indicated by the variation in their numbers and aggregate values in different years, the existence of a *periodical inequality*, of which the extreme and opposite phases were five years distant from each other, and the years 1843 and 1848 were respectively the epochs of minimum and maximum. The examination which led to this conclusion comprehended the years from 1841 to 1848 inclusive, and was definite in respect to 1843 as the year of minimum, inasmuch as 1841, 1842, and 1843 showed a progressive *decrease* in the number and aggregate value of the larger disturbances in each year, whilst from 1843 to 1848 there was a progressive *increase* of both in successive years. It was noticed in the same communication, that the regular *diurnal* variation of each of the three magnetic observation-elements at Toronto and Hobarton, the Declination, the Horizontal Force and the Vertical Force, and of each of the two derived elements, the Inclination and the Total Force, exhibited evidences of a corresponding periodical inequality in the amplitude or extent of the diurnal variation, the years 1843 and 1848 being also epochs respectively of minimum and maximum. The observations of one of the elements (the Declination), extending uninterruptedly at both stations over eleven entire years, or from 1841 to 1851 inclusive, distinctly pointed out 1848 as the year of maximum as well as 1843 as the year of minimum.

This discovery of the existence of a periodical inequality, common to the magnetic disturbances of larger amount, and to those more regular diurnal magnetic variations of which the sun has been long recognized as the primary cause (inasmuch as they conform systematically to laws depending on solar time), was regarded as affording presumptive evidence of the subsistence of a causal connexion common to those two classes of phenomena, which presumption was corroborated by facts adduced in the same communication, proving that the disturbances are themselves subject, on the average, to a well-marked diurnal period, which is also regulated by solar time.

The periodical inequality thus manifested, having its opposite phases of maximum and minimum separated by an interval of five years, and of which the cycle might therefore be conceived to include about ten of our solar years, did not appear to connect itself with any of those divisions of time with which we are conversant as

depending upon the relative circumstances of the sun, and the earth and her satellite. The cycle might or might not be one of regular and unfailing recurrence. The observational evidence to which we are indebted for a knowledge of its existence, though sufficiently decisive as far as the period of observation extended, could only be viewed, in reference to a permanently cyclical character of the phenomenon, as fragmentary, and as the commencement of an investigation which would require to be pursued in one or more of the permanent magnetic observatories established in our own and other countries. Had no other circumstance presented itself to give an additional interest to an investigation which thus held out a fair promise at least of making known laws of definite order and sequence in phenomena which have excited so much attention of late years, but of which so little is even yet known;—had for example the decennial period, which appeared to prevail with precisely corresponding features in two distinct classes of the magnetic variations, connected itself with no other periodical variation either of a terrestrial or cosmical nature with which we are acquainted,—there might have been indeed little reason to apprehend that the investigation would have been suffered to drop: but the interest and importance of the inquiry have doubtless been greatly enhanced by the remarkable coincidence, which it was the object of the paper communicated to the Royal Society in March 1852 to announce, between the above-described periodical inequality by which the magnetic variations referable to solar influence are affected, and the periodical inequality which has been discovered by M. SCHWABE to exist in the frequency and magnitude of the solar spots. The coincidence, as far as we are yet able to discover, is absolute; the duration of the period is the same, and the epochs of maximum and minimum fall in both cases in the same years. The regularity with which the alternations of increase and decrease have been traced by M. SCHWABE in his observations of the solar spots, which have been now continued for about thirty years, must be regarded as conferring a very high degree of probability on the systematic character of causes, which are as yet known to us only by the visible appearances which they produce on the sun's disk, and by the disturbances which they occasion in the magnetic direction and force at the surface of our globe. As a discovery which promises to raise terrestrial magnetism to the dignity of a cosmical science, we may feel confident, that, although the Colonial Observatories have been brought to a close, the investigations which they have thus successfully commenced will be pursued to their proper accomplishment, in those national establishments which have a permanency suitable for such undertakings.

The conclusions which have been drawn, both in the Philosophical Transactions, and in the introductory discussions in the Toronto and Hobarton volumes, regarding the periodical laws of the disturbances at those stations, have been hitherto confined exclusively to the disturbances of a single element, the Declination. It was fully recognized that each of the other two observational elements, viz. the Horizontal and he Vertical Forces, might be expected to furnish concurrent but strictly independent

evidence of the periodical affections to which the magnetic disturbances were subject; but the work to be accomplished for the elaboration of that evidence was considerably greater than in the case of the Declination, from the necessity of eliminating the influence of changes of temperature on the magnets employed in measuring the variations of the horizontal and vertical forces, before those disturbances could be separated for analysis. The labour required in the different processes of reduction has now been gone through; and it remains to bring together in one view the evidence which the three observational and the two derived elements furnish of the periodical laws, decennial, annual and diurnal, which regulate the occurrence of the larger disturbances.

In respect to the decennial period, it must be regarded as a fortunate circumstance, that the five years of hourly observation, which were commenced before the existence of any inequality of longer duration than a year was suspected, began with 1843, the year of minimum, and closed with 1848, the year of maximum disturbance, so that the variation has been followed through a complete phase. This has been strictly the case in the Declination and Horizontal Forces, and with a single exception in the Vertical Force also, the exception being caused by the interruption of the observations of that element, for purposes explained elsewhere, during the months of October, November and December 1843, and January and February 1844. These months have been supplied in the year ending June 1844, from hourly observations made with the same apparatus in the preceding year, viz. in the months of October, November and December 1842, and January and February 1843; thus rendering the five years of the Vertical Force complete for the investigation of the *annual* and *diurnal* variations, but of course in regard to the *decennial* period the months taken from a different year, even though it be the adjacent one, are not a *perfect* substitute. The effect of this substitution has been in fact to swell the aggregate value of the disturbances of the Vertical Force in the year nominally ending June 30, 1844, but really comprising five months taken from a preceding year, so as to make them slightly exceed the aggregate value in the year ending June 30, 1845. A similar slight excess in the aggregate value of the disturbances of the Horizontal Force in the year ending June 1844 over the aggregate value in the year ending June 1845, is found when the same five months of the preceding year are substituted for its own months; but when in the case of the Horizontal Force (the observations of which were not suspended as were those of the Vertical Force) the actual observations throughout the year ending June 1844 are taken, the true progression is restored, and the apparent anomaly disappears.

The variations of the three magnetic elements being measured by instruments wholly distinct and unconnected with each other, each element affords an independent evidence of the progressive increase in the aggregate values of the larger disturbances during the period under examination. The sum of the aggregate values of each

element in the five years, divided by 5, gives the mean annual value of that element, which we may take in each case, for the purpose of comparison with the actual aggregate values in the different years, as equal to 1·00; we have then the ratios of the disturbances of the different elements in the different years as follows:—

TABLE I.

	Declin.	Hor. Force.	Vert. Force.	Mean.
Year ending June 30, 1844	0·52	0·35	0·65*	0·44
Year ending June 30, 1845	0·64	0·47	0·58	0·57
Year ending June 30, 1846	0·82	0·55	0·73	0·70
Year ending June 30, 1847	1·39	1·14	1·23	1·25
Year ending June 30, 1848	1·63	2·49	1·80	1·97

The final column has been added to show the *mean* ratio of disturbance in each year as derived from the three elements, measured by the aggregate value in each year and in each element of all the disturbances which exceeded a certain definite magnitude, that magnitude being taken the same throughout the five years.

It is seen by this Table, that in the year ending June 1847 the ratio of disturbance is above twice as great, and in the year ending June 1848 nearly four times as great as in either of the years ending June 1844 or June 1845. In the year ending June 1848, which is the year of maximum, the proportion is nearly five times as great as in the year ending June 1844, which is the year of minimum. The evidence of the existence of a decennial period borne by the disturbances of the Declination, and announced to the Royal Society in March 1852, receives therefore the fullest confirmation, from the variations in different years of the disturbances of the Horizontal and Vertical forces.

Fig. 1, Plate X. has been drawn in illustration of the progressive increase of disturbance in each of the three elements between the year ending June 1844 and the year ending June 1848. The broken horizontal line represents the mean or *average* annual disturbance in each element, and is the zero-line, or the unit, with which the *actual* aggregate values of the disturbance of each element in each year are compared; the Declination is represented by a dark continuous line, the Horizontal Force by a light continuous line, and the Vertical Force by a dotted line. The rate of increase of disturbance is seen to be much slower in the first half than in the second half of the five years.

Annual Period.—The sum of the aggregate values of the disturbances of each element in the five years, divided by 12, gives the *average monthly* disturbance-value

* In the deduction of this number, five months of the preceding year have been substituted for five months of the year ending June 1844; it has not been included therefore in the final column showing the mean ratios in each year.

for that element, which being taken=1·00 and compared with the *actual* monthly disturbance-values, gives the ratios in the following Table:—

TABLE II.

Months.	Declination.	Horizontal Force.	Vertical Force.	Mean.
July	0·94	0·61	0·71	0·75
August	1·16	0·75	1·08	0·99
September	1·62	1·71	1·61	1·64
October	1·31	1·48	1·29	1·36
November	0·78	0·98	0·75	0·84
December	0·76	0·58	0·61	0·65
January	0·57	0·56	0·57	0·57
February	0·84	0·94	0·74	0·84
March	1·11	0·94	1·08	1·04
April	1·42	1·50	1·49	1·47
May	0·98	0·90	1·12	1·00
June	0·53	0·36	0·50	0·46

The evidence afforded by each of the three observational elements, in regard to annual variation, is to one and the same effect, viz. January and June are the months of minimum disturbance, September and April the months of maximum disturbance. The aggregate value of the disturbances in the equinoctial months is about three times as great as in the solstitial months. Of the two equinoctial months the value is somewhat higher in each element in September than in April; and of the two solstitial months December is higher than June, also in each of the three elements.

Fig. 2, Plate X. has been drawn in illustration of the annual variation which has been thus described. The broken horizontal line is the *mean* monthly disturbance of each element (*i. e.* the sum of the disturbances in the 12 months divided by 12). The dark continuous line for the Declination, the light continuous line for the Horizontal Force, and the dotted line for the Vertical Force, show in each case the variation in the proportions which the *actual* disturbances in each month bear to the *mean* monthly disturbance in the same element. The correspondence of the three elements could scarcely be more perfect.

The annual variation which has been thus deduced has reference exclusively to the variable amount in the different months of the *aggregate values* of the disturbances of each element, without distinguishing apart or separating the disturbances which cause easterly deflections and those which cause westerly deflections; or those which increase and those which decrease the horizontal and vertical forces. When this separation is made, we continue to find that each of the two portions into which the disturbances of each element are divided exhibits distinctly and notably the same general features which have been derived from their conjoint consideration. The equinoxes are in all cases the epochs of maxima and the solstices of minima.

But when we study more carefully the relative prevalence of *disturbances of particular character* at different periods of the year,—which we may do by forming tables

of the relative proportion which the aggregate values in the different months of the easterly disturbances bear to the aggregate values in the same months of the westerly disturbances, and the aggregate values of the disturbances which decrease the force, bear to the aggregate values of those which increase it,—we find that indications present themselves of an annual variation of a different kind from that which has been hitherto discussed, namely an annual variation in the *character* of the disturbances of two at least of the elements which have been observed; and although a greater length of time and a greater amount or continuance of observation may be required for the satisfactory establishment of such a periodical variation, its present indication ought not to be overlooked, since the range of the variation is of considerable magnitude, and its systematic character as distinctly marked as could well be expected in an annual variation derived from not more than five years. The elements in which these phenomena are most distinctly noticeable, are the Declination and the Vertical Force, and the correspondence between the indications of these two elements is in many respects very remarkable. In both elements, when the relative proportions are taken,—in the Declination of the aggregate values in the different months of easterly and westerly disturbances, and in the Vertical Force of disturbances which decrease and disturbances which increase the force,—we find that in both cases the proportions vary from a minimum at the southern solstice to a maximum at the northern solstice, the equinoxes being intermediate. At the northern solstice easterly disturbances are in considerable excess, as are disturbances which decrease the Vertical force; at the southern solstice, the excess of both is on the other side; westerly disturbances then predominate, as do the disturbances which increase the Vertical force. The relative proportion of the aggregate values of easterly to westerly disturbances of the Declination, and of disturbances which decrease the Vertical Force to those which increase it, varies from the one solstice to the other roughly as about 3 to 1; and in both elements nearly alike.

In the Horizontal Force, the disproportion between the values of the disturbances which increase the force and those which decrease it is so great (decreasing disturbances greatly preponderating at all periods of the year), that a variation corresponding to that of the two other elements is not so simply arrived at; but it may be stated generally that the proportion of decreasing disturbances is greater at the epoch of the southern solstice than at that of the northern solstice.

Diurnal Variation.—Before we proceed to examine the diurnal variation of the Declination, Inclination and Total force which it is the average effect of the larger disturbances to produce, it may be desirable to show the proportions in which the disturbances of the three observed elements occur at the different hours. This is expressed in the following Table by the proportion which the *actual* aggregate values in the five years of the disturbances at each particular hour bear to the *mean* or *average* disturbance at all the hours taken as unity.

TABLE III.

Toronto Astronomical Hours.	Declination.	Horizontal Force.	Vertical Force.	Toronto Civil Hours.
18	1·05	1·00	1·21	6 A.M.
19	1·17	1·40	1·15	7 A.M.
20	1·27	1·20	0·80	8 A.M.
21	1·11	1·00	0·54	9 A.M.
22	0·87	1·00	0·36	10 A.M.
23	0·66	0·90	0·34	11 A.M.
0	0·49	0·87	0·46	Noon.
1	0·30	0·76	0·63	1 P.M.
2	0·40	0·66	0·77	2 P.M.
3	0·40	0·66	0·87	3 P.M.
4	0·53	0·61	1·04	4 P.M.
5	0·56	0·66	1·07	5 P.M.
6	0·84	0·59	1·01	6 P.M.
7	0·98	0·76	1·05	7 P.M.
8	1·22	0·75	0·89	8 P.M.
9	1·82	0·90	0·74	9 P.M.
10	1·55	1·03	0·85	10 P.M.
11	1·25	1·14	0·93	11 P.M.
12	1·35	1·22	1·39	Midnight.
13	1·52	1·58	1·58	1 A.M.
14	1·21	1·60	1·61	2 A.M.
15	1·13	1·37	1·73	3 A.M.
16	1·34	1·14	1·51	4 A.M.
17	1·05	1·02	1·41	5 A.M.

From the systematic increase and decrease of the ratios at the different hours, it is obvious that the disturbances of each element, when viewed on the average of a sufficient body of observations, are regulated by laws which have a diurnal period. The diurnal variation thus presented is far however from being alike in each of the three elements; the maximum disturbance takes place indeed in all the elements during the hours of the night, and the minimum disturbance during the hours of the day; but the particular hours of maximum and minimum are very different in the three cases. The hour of maximum in the Declination, for example, is 9 P.M. when the disturbances of the horizontal and vertical forces are both even *less than the hourly average*; and the horizontal and vertical forces do not reach their hours of maximum until, respectively, 2 and 3 A.M., when the disturbances of the Declination have notably declined. So in respect to the hour of minimum: that of the Declination, 1 P.M., is nearly midway between that of the vertical force at 11 A.M., and that of the horizontal force at 4 P.M.; the disturbance of the horizontal force being still high when that of the vertical force is at a minimum, and the disturbance of the vertical force being still high when that of the horizontal force is a minimum. Speaking generally, the disturbances of the three elements are above the average in the hours of the night and early morning, and below the average during the hours of the day; to the latter, however, there is an exception in the vertical force, which is above the average from 4 to 7 P.M. In the Declination the aggregate value of the disturbances at the hour of maximum is about six times as great as at the hour of minimum; in

the horizontal force about 2·7 as great, and in the vertical force about five times as great.

But in the ratios of the Declination-disturbances at the different hours shown in the preceding Table, we have the joint effects of two classes of disturbances, those which produce easterly and those which produce westerly deflections; and in the ratios of the disturbances of the horizontal and vertical forces at the different hours, we have the further complication, that the variations of the horizontal and vertical forces do not bear a simple relation to those of their theoretical equivalents to which they are due, viz. the Inclination and Total Force, but involve quantities dependent on the resolution of forces, which, when the Inclination is great, as it is at Toronto, have a tendency to mask the simplicity of the variations of the Inclination and of the Total Force, as they would appear if they were the subjects of direct observation. In the following Table therefore are placed the proportions at the different hours in which the six classes of phenomena respectively vary, viz. the disturbances which produce easterly and those which produce westerly deflections, those which increase and those which decrease the Inclination, and those which increase and those which decrease the Total Force.

TABLE IV.

Toronto Astronomical Hours.	Disturbances of the Declination producing		of the Inclination producing		of the Total Force producing		Toronto Civil Hours.
	Easterly Deflection.	Westerly Deflection.	Increase of Inclination.	Decrease of Inclination.	Increase of Force.	Decrease of Force.	
18	0·45	1·82	0·82	0·83	0·27	1·91	6 A.M.
19	0·35	2·23	1·29	0·51	0·26	1·91	7 A.M.
20	0·26	2·58	1·14	0·62	0·37	1·12	8 A.M.
21	0·21	2·25	1·05	1·37	0·22	0·65	9 A.M.
22	0·28	1·62	0·96	1·47	0·25	0·28	10 A.M.
23	0·39	1·01	0·89	1·80	0·39	0·07	11 A.M.
0	0·24	0·80	0·93	1·75	0·67	0·04	Noon.
1	0·21	0·41	0·87	1·70	1·25	0·10	1 P.M.
2	0·20	0·65	0·65	1·92	1·79	0·08	2 P.M.
3	0·22	0·62	0·71	1·36	2·21	0·06	3 P.M.
4	0·32	0·80	0·61	1·46	2·77	0·07	4 P.M.
5	0·44	0·71	0·79	1·35	2·96	0·07	5 P.M.
6	1·05	0·57	1·04	0·72	2·39	0·07	6 P.M.
7	1·44	0·39	1·14	1·02	2·56	0·09	7 P.M.
8	1·95	0·28	1·12	0·55	1·99	0·09	8 P.M.
9	3·09	0·22	1·17	0·58	1·23	0·31	9 P.M.
10	2·41	0·45	1·18	1·39	0·81	0·83	10 P.M.
11	2·02	0·27	1·19	0·84	0·53	1·19	11 P.M.
12	1·76	0·82	1·10	0·85	0·46	2·14	Midnight.
13	1·79	1·19	1·32	0·70	0·26	2·57	1 A.M.
14	1·37	1·00	1·33	0·37	0·22	2·70	2 A.M.
15	1·28	0·94	1·05	0·48	0·28	2·81	3 A.M.
16	1·48	1·21	0·90	0·48	0·18	2·48	4 A.M.
17	0·91	1·23	0·76	0·55	0·28	2·24	5 A.M.

We learn from this Table, that the laws which regulate the occurrence of easterly and westerly disturbances are not, on the one hand, similar, nor, on the other hand,

are they always complementary to each other. Thus, from 1 P.M. to 5 P.M. both classes of the Declination-disturbances are considerably below the average, and from 1 A.M. to 5 A.M. both classes, with a slight exception, are above the average; whilst from 6 P.M. to 11 P.M. easterly disturbances greatly exceed, and westerly fall greatly short of the average; and from 6 A.M. to 11 A.M. westerly exceed, and easterly fall short of the average. In the Inclination and Total Force the complementary character of the opposite affections of each element is more extensively manifested; thus the disturbances which increase the Inclination are below the average from about noon and the early hours after noon, when those which decrease it are above the average; and are below the average from about midnight and the early hours after midnight, when those which increase the same element are above the average. In the Total Force, from 1 A.M. to 8 A.M. the disturbances which increase the force are greatly above, as those which decrease the force are greatly below the average; a contrast which is reversed from 1 A.M. to 8 A.M., the disturbances which decrease the force being then greatly above, whilst those which increase it are greatly below the average. In neither of the two elements however does the complementary character exclusively prevail. It may be remarked, that in all the instances which have been thus brought into view, touching successively the diurnal variations of the disturbances of each of the three elements, the parallel cases which have been cited, whether of identity or of contrast, fall without exception on homonymous hours; a circumstance which affords additional evidence of the systematic character of the affections of which we are treating.

There does not appear to be any uniform contemporaneous connexion between the prevalence of either easterly or westerly Declination-disturbances and of those which either increase or decrease the Inclination or the Total Force. Thus, for example, the hours at which the disturbances which increase the Total Force are most notably above the average are from 1 P.M. to 9 P.M.; whilst we find that for half that period, or from 1 P.M. to 5 P.M., the Declination-disturbances are characterized by a very low proportion of easterly disturbances, and for the other half of the period, or from 6 P.M. to 9 P.M., by a very high proportion of easterly disturbances; and without multiplying instances of dissimilarity, it may be remarked generally, that the more the six classes of disturbances are examined and compared with each other, the less reason does there appear to conclude that there is any uniform interaccompaniment of the variations of different elements.

As the instrument by which the variations of the Declination are observed is more simple in construction than those required for the variations of the Inclination and Total Force, and the disturbances of the Declination are therefore more easily observed and more generally known, a somewhat disproportionate consideration has been frequently given to them in the discussion of these phenomena, which it may be desirable briefly to remark upon. Thus the knowledge of the magnetic disturbances having been chiefly drawn from those of the Declination, it has been very generally

and very naturally imagined that the early hours of the night, or from 8 P.M. to 11 P.M., are those at which magnetic disturbances principally take place; that about 11 P.M., or a little after, they begin to subside, disappearing almost wholly in the day-time, and reappearing again possibly the following evening at the same hour as on the preceding evening, in supposed analogy with certain atmospheric disturbances which manifest a tendency to recur at the same hours on successive days. It is in this supposed analogy that the term of magnetic *storms* appears to have originated. An examination of the observations of the three elements at but a single station (as Toronto for example), teaches us that this view requires to be considerably modified. The disturbances of the Declination which reach a maximum at 9 P.M. have indeed already subsided considerably at 11 P.M.; but those of the Inclination show no abatement until about 2 A.M.; whilst those of the Total Force, which are much below their average value at 9 P.M., increase progressively to their maximum, which is only reached at 3 A.M., or nearly six hours after the maximum of the Declination-disturbances has taken place. In like manner, the hours of the afternoon in which the Declination is but little disturbed, and which have been supposed in consequence to be hours in which an intermission of disturbance takes place, are seen by the Table to be precisely those hours at which the disturbances which increase the Total Force have their principal development, being then in the proportion of nearly ten to one when compared with the homonymous hours after midnight. When these remarkable phenomena are more fully studied, the aspect they present is that of a disturbance continuing frequently through several successive days, changing from one element to another, and affecting each at different hours and in different modes, in conformity with laws, the average operation of which it has been the object of this investigation to ascertain; and wearing the appearance consequently, when only a single element is regarded, of a limitation to those hours when that element in particular is affected, but which appearance ceases when the phenomena are more generally apprehended. It was the supposed analogy between magnetical and atmospherical disturbances, which led, in the commencement of the British Colonial Observatories, to the *simultaneous* observations and record of these two great, and, as we have now reason to believe, distinct branches of natural phenomena; and as the inquiry advances, we are continually becoming acquainted with additional circumstances to strengthen the persuasion, that the causes of these occasional and previously supposed "irregular" manifestations of disturbing magnetical influence must be sought in a more distant source than in variations of the meteorological phenomena.

There is another misapprehension in regard to the nature of the occasional disturbances which has followed very naturally from the limitation of the view to the disturbances of a single element: an inference has sometimes been drawn in favour of a *local origin* of a particular disturbance (in contradistinction to the general fact of their simultaneous occurrence at extremely distant parts of the globe) from

the circumstance, that though the disturbance was manifested by the Declination at one station, no indication of it was shown by the contemporaneous observations of the Declination at another and a distant station. Now, simultaneity at stations separated by considerable intervals of longitude implies a difference in the solar hours; and the observations at Toronto show that a difference in the solar hour may determine the question, whether a disturbance, which may nevertheless be common to both stations, may or may not be traceable at both by simultaneous observations of a single element only. Towards the attainment of a just conclusion, therefore, in regard to a possible local origin, it is indispensable that a more extensive generalization should be made, and that contemporaneous affections of the *three* elements should be brought into the comparison. Nor can this condition of the inquiry be dispensed with even in comparing the phenomena at stations under the same meridian, but separated by large intervals of latitude, unless it be first shown that the same law of solar hours prevails at both stations in regard to the occurrence of the disturbances of each particular element. It need scarcely be said that the general simultaneity of the disturbances has a very important bearing upon their theory, inasmuch as it militates decidedly against the supposition of their originating in atmospherical peculiarities, and tends to assign them with far greater probability to a cosmical source. That some disturbances may have a local origin, is undoubtedly possible, but no such case has yet, I believe, been established on adequate evidence.

For the purpose of viewing in its simplest form and expressed in numerical value the influence which, on a daily average, the larger disturbances exercise on the Declination, Inclination and Total Force, we must revert to the aggregate values in the five years which supplied the ratios of disturbance at the different hours in each of the six classes of phenomena contained in Table IV. From these values we obtain readily and immediately for each hour the excess in the aggregate amount of easterly over westerly, or of westerly over easterly deflection, and of disturbances which increase or decrease the Inclination or the Total Force over those which respectively decrease or increase those elements. Hence we easily form a table containing, for each of the elements at every hour, the numerical excess in the aggregate values of whichever kind of disturbance predominates at that hour; and by dividing the excess by 1550, which is the number of days of observation in the five years, we have the mean daily effect corresponding to the values of the larger disturbances of each of the elements at the different hours; or the average diurnal variation of each element due to the larger disturbances. This is shown in Table V., in which the diurnal variation of the Declination and Inclination is expressed in decimals of a minute of arc, and that of the Total Force in parts of the Total Force at Toronto, which in absolute value, and employing British units, may be taken with sufficient approximation at 13·9.

TABLE V.

Toronto Astronomical Time.	Mean diurnal variation occasioned by the larger disturbances.			Toronto Civil Time.
	Declination.	Inclination.	Total Force.	
h	′	′	Parts of the Total Force.	
18	0·29 W.	+0·10	−·000092	6 A.M.
19	0·41 W.	+0·18	−·000092	7 A.M.
20	0·52 W.	+0·16	−·000047	8 A.M.
21	0·46 W.	+0·13	−·000032	9 A.M.
22	0·30 W.	+0·10	−·000008	10 A.M.
23	0·11 W.	+0·09	+·000007	11 A.M.
0	0·11 W.	+0·09	+·000015	Noon.
1	0·03 W.	+0·09	+·000027	1 P.M.
2	0·09 W.	+0·05	+·000042	2 P.M.
3	0·08 W.	+0·07	+·000054	3 P.M.
4	0·09 W.	+0·05	+·000068	4 P.M.
5	0·04 W.	+0·08	+·000073	5 P.M.
6	0·18 E.	+0·14	+·000058	6 P.M.
7	0·34 E.	+0·14	+·000062	7 P.M.
8	0·52 E.	+0·16	+·000047	8 P.M.
9	0·87 E.	+0·16	+·000016	9 P.M.
10	0·61 E.	+0·15	−·000022	10 P.M.
11	0·53 E.	+0·16	−·000047	11 P.M.
12	0·33 E.	+0·14	−·000098	Midnight.
13	0·26 E.	+0·18	−·000125	1 A.M.
14	0·17 E.	+0·19	−·000132	2 A.M.
15	0·16 E.	+0·14	−·000138	3 A.M.
16	0·15 E.	+0·12	−·000123	4 A.M.
17	0·02 W.	+0·10	−·000109	5 A.M.

From this Table we find that the range of the diurnal variation of the different elements is as follows: viz. of the Declination 1′·39, being from 0′·52 W. at 8 A.M. to 0′·87 E. at 9 P.M.;—of the Inclination 0′·14, being from a minimum increase of 0′·05 at 2 P.M. to a maximum increase of 0′·19 at 2 P.M.;—and of the Total Force ·000211 parts of the total force at Toronto, being from a maximum decrease of ·000138 at 3 A.M. to a maximum increase of ·000073 at 5 P.M. As the larger disturbances of each element, which have been separated by the processes and subjected to the analysis described in this communication, can by no means be supposed to include the whole of the disturbances of the class to which they belong, we can only regard the extent of the diurnal variation, as stated above, to be in each case a *minimum limit*, which would be certainly somewhat exceeded, if by any mode of proceeding we could succeed in separating the minor effects of the same causes; but we have no reason to suppose that the epochs of maxima and minima, or the laws of intermediate progression, would sustain any material alteration thereby. And as the aggregate values of the disturbances are taken from the five years which include a complete quinquennial or semi-decennial period, the *mean* diurnal variation deduced from them must be considered as subject to a small quinquennial variation, analogous to that which has been found to exist in the ordinary solar diurnal variation. And as the sum of the variation values at the different hours taken with their proper

signs in no case equals zero, but has a sensible magnitude in each element, the absolute values of these elements must also be affected with a very small cyclical variation due to the disturbances, of which the period will also be quinquennial.

In concluding this paper, I may venture to congratulate the Royal Society on the success which has attended the attempt, commenced at its recommendation and carried out under its auspices, to investigate the laws of these remarkable and mysterious phenomena, the Magnetic Disturbances, by the philosophical method of extensive and systematic observation; and on the proof which this paper contains of the sufficiency for their purposes of the instrumental means devised by the Committee of Physics for observing the variations of the Horizontal and Vertical Forces; and of which, especially in regard to the Vertical Force magnetometer, doubts have been sometimes expressed, arising apparently in some cases from the want of sufficient skill in the artist who attempted to construct the instrument, and in others from the absence of any personal experience in those who expressed opinions as to its probable performance. By means of these two instruments, a full confirmation has now been given to the existence of periodical laws of systematic order and regularity in phenomena previously regarded as irregular, the periodical character of the decennial, annual and diurnal variations being as clearly shown by the disturbances of the Inclination and Total Force as by those of the Declination; thus accomplishing the first important step towards a physical theory of the Disturbances by the direct connexion which they are now shown to have with the Sun.

The conclusions which can be drawn from the observations at a single station are necessarily limited to the theory of the phenomena as they present themselves at a single point on the earth's surface. May it not be hoped, that the fruits which have recompensed the labour bestowed on the Toronto observations, may encourage some amongst the numerous physicists in Europe and America, who signified their desire to cooperate with the Royal Society in this inquiry, and to adopt the methods and processes of observation which have been followed out at Toronto, to apply themselves to the deduction of the laws of the occasional disturbances which, from the example of Toronto, they may expect to be able to disentangle from the great mass of observations on which their labour has been already bestowed; provided that those observations have been made with the care and perseverance which have distinguished those made by the Officers and Non-commissioned Officers of the Royal Artillery at the Toronto Observatory? Few may be willing to face a heavy labour of reduction before experience has shown that results will follow from the labour; but some may be expected to do so when an example is before them that this additional labour bestowed on their observations will not be without its recompense: a very few stations at which the investigation should be as full and as satisfactory as at Toronto, might, if widely distant from each other on the earth's surface, suffice to form a general theory of the phenomena of the magnetic disturbances.

The observations at the St. Helena and Hobarton observatories are undergoing a

similar reduction and analysis, for the purpose of showing the periodical laws which regulate the occurrence of the larger disturbances of the three magnetic elements at an equatorial station, and at one nearly antipodal to Toronto. Should health be continued to me, and should no unforeseen circumstance occur to interrupt the progress of the investigation, I shall hope to avail myself of a future opportunity of submitting the results to the Royal Society.

Woolwich, February 1, 1856.

XVI. *On the ultimate arrangement of the Biliary Ducts, and on some other points in the Anatomy of the Liver of Vertebrate Animals.* By LIONEL S. BEALE, *M.B., Professor of Physiology in King's College, London; Physician to King's College Hospital. Communicated by* F. KIERNAN, *Esq., F.R.S.*

Received June 14,—Read June 21, 1855.

IN his valuable paper published in the Philosophical Transactions for 1833, Mr. KIERNAN describes and figures the anastomoses between branches of the duct in the left triangular ligament of the human liver. He also refers to communications existing between the ducts in other situations, as in the membranous bridge stretching over the fissure for the umbilical vein, and upon the inferior surface of the diaphragm. In the same paper, this author gives a diagram of the manner in which he supposed the ducts to terminate in the lobules of the liver, and subjoins the following remarks:—"No such view of the ducts as that represented in this figure can be obtained in the liver. The interlobular ducts are in the figure seen anastomosing with each other. I have never seen these anastomoses, but I have seen the anastomoses of the ducts in the left lateral ligament, and from the results of experiments related in this paper, I believe the interlobular ducts anastomose; I have never injected the lobular biliary plexus to the extent represented in this figure."

Since the appearance of this important communication, the subject has been much investigated both in this country and on the continent; but as far as I can ascertain, no observer has yet succeeded in *demonstrating* the manner in which the ducts terminate, or has been able to show conclusively the precise relation which the hepatic cells bear to the biliary ducts. Various hypothetical views have been offered.

MÜLLER considered that the ducts terminated in blind extremities; E. H. WEBER, under the name of "Vasa aberrantia," described ducts which formed a network in the transverse fissure of the liver, uniting the right and left hepatic ducts. In 1850 the same observer described ducts terminating in blind extremities upon the surface of the external lobules of the cat's liver.

KRUKENBERG, SCHRÖDER VAN DER KOLK, RETZIUS, THEILE, BACKER, LEIDY and others, have adopted the view that the hepatic cells lie within a basement membrane; but with reference to the arrangement of the cells within the tube, there is much difference of opinion among them.

LEREBOULLET, in his memoir on the "Foie gras," published in 1853, advocates a similar view; but his representations are very diagrammatic, and for the most part taken from preparations examined by low powers.

HENLE, GERLACH, HYRTL and NATALIS GUILLOT look upon the finest gall-ducts as

communicating with spaces between the hepatic cells into which the bile escapes, and is received by the most minute ducts.

Handfield Jones and Kölliker describe the hepatic cells as forming a solid network composed of columns of cells, not bounded by any basement membrane, but lying between the meshes of the capillary network.

Handfield Jones thinks the ducts terminate by blind extremities, which lie amongst the cells at the peripheral parts of the lobule. The small cells lining these ducts are considered by him as the chief agents concerned in the secretion of bile, and he looks upon the function of the hepatic cells as totally distinct from this. Busk and Huxley and Dr. Carpenter concur in this view, which places the liver in the same category as the suprarenal capsules, follicles of Peyer, spleen, &c. Kölliker offers the supposition that the finest ducts impinge upon the columns of the network of hepatic cells, and makes the following remarks with reference to this point:—"Often as I have sought for a direct communication of the finest canals with the hepatic networks, I have not directly observed it; which is indeed by no means surprising, if we consider the softness of the parts with which we have to do; but unfortunately the result is a *hiatus* in the minute anatomy of the parts, which can hardly be made good by hypotheses*."

The conflicting opinions of observers appear to have been based upon inference and hypothesis rather than upon direct observation, and are embodied in diagrammatic figures; some authors, agreeing with Kiernan, regard the liver as arranged upon the type of true glands, while the latest authorities have endeavoured to establish the view that this important organ is more nearly related to the ductless glands.

My own observations have been made upon the livers of several different animals, and I have tried very numerous methods of preparation, some with considerable success. The results of the examination of injected specimens precisely accord with the observations made some months before upon uninjected preparations.

The points which I hope to establish are as follows:—

1. That the hepatic cells lie within an exceedingly delicate tubular network of basement membrane.

2. That the smallest biliary ducts† are directly continuous with this network.

3. That in favourable specimens, injection forced in from the duct, will pass into every part of the tubular network, even quite to the centre of the lobule, and that the capillary network can be injected in the same preparation.

In carrying out my investigations upon the anatomy of the ducts, I have been led

* An excellent abstract of the views upon the structure of the liver, up to the year 1852, will be found in Professor Kölliker's 'Mikroskopische Anatomie.'

† In the following pages the word "duct" is used to denote the tubes which carry off the secretion, in contradistinction to the secretory tubes or "*cell-containing network*" in which the secretion is formed.

naturally to examine other points in the minute anatomy of the liver. Some of these observations accord with the notions previously entertained, while others are at variance with the opinions usually held. In this communication I wish to restrict myself, as far as possible, to the consideration of the anatomical arrangement of the biliary ducts and the relation which these bear to the secreting cells of the liver, and shall therefore only allude to some of the more general points incidentally. It may, however, be desirable to offer a few remarks upon the nature of the lobule as it occurs in the livers of different animals.

Lobules.—If by a lobule is understood a perfectly circumscribed portion of hepatic substance containing within itself all the structural elements necessary for the formation of bile, which can be separated from adjacent portions, the only livers in which such lobules are to be demonstrated, so far as is yet known, are those of the pig, and Polar bear (according to Müller). In the pig each lobule possesses a distinct capsule of its own, and can be readily separated from its neighbours. The capsule is perforated by branches of the vessels and duct which are distributed in its interior. This arrangement of separate lobules would permit a considerable amount of movement upon each other, whilst the capsule would have the effect of preventing undue engorgement and distension of each individual lobule, a provision which seems to be especially needed in the pig.

There is not this division into distinct and separate lobules in the livers of other animals which I have examined. As numerous authors have described, there is a sort of mapping out of small spaces produced by the arrangement of the smallest vessels and ducts, to be seen more or less distinctly in the livers of all vertebrate animals; but it is impossible to separate these apparently isolated portions from each other, without tearing the hepatic tissue of which they are composed, thereby leaving a rough and jagged surface. In fact, while the lobules of the pig's liver are provided each with its separate capsule, and in many instances separated from its neighbours by a certain quantity of areolar tissue, and the branches of the vessels and duct for their supply, the so-called lobules of the liver of man and other animals are only separated from each other at certain points by the fissures in which the portal vessels and duct lie. There is no distinct capsule, nor areolar, nor fibrous tissue between them, and the capillaries and the cell-containing network of one lobule are continuous at various points with those of its neighbours.

Most writers seem to have regarded the pig's liver as the type of structure to which all others should be referred; but it seems more natural to look upon this beautiful liver as exceptional, and bearing the same relation in structural peculiarity to the livers of other animals, as the much-divided kidney of the porpoise bears to the corresponding solid organ of man and most other mammalia.

It may be said that in a *physiological* point of view the livers of all vertebrate animals are composed of lobules, but in a strictly *anatomical* sense this term must be restricted to the livers of the pig and Polar bear.

So also it is only in the liver of the pig that a prolongation of the areolar tissue can be demonstrated between the ultimate portions of secreting structure.

In the human liver areolar tissue is abundant in the large portal canals, but it gradually diminishes in quantity as the branches of the vessels become smaller, and their most minute ramifications are entirely free from it, as Bowman, Henle, and Vogel have previously observed. In the liver of the seal it is very sparing in quantity in the portal canals, but in some of the rodent animals it seems to be reduced to a minimum (rabbit, rat, mouse). In the rabbit a very little areolar tissue is found even in the largest portal canals.

In subjecting sections of uninjected specimens to examination, the smallest branches of the vessels are inevitably stretched and torn by manipulation, and thus a striated appearance, closely resembling that of fibrous tissue, is produced.

As is well known, the vessels of the kidney and of other glands at their point of entrance, are invested with a covering of areolar tissue, which is gradually lost as they divide into smaller branches in the glands. A similar arrangement occurs in the case of the liver, but from the large size of the gland this investment is prolonged for a considerable distance into its interior.

Method of preparing Specimens.

In consequence of the softness of the liver, I have been unable to demonstrate the arrangement of the minute ducts and their connexion with the cell-containing network without previous hardening. This hardening may be effected in several different ways. By soaking small pieces of liver in strong syrup for several weeks, they become so firm, that very thin sections can be readily obtained. The liver may also be hardened in alcohol, and afterwards rendered transparent by soda. I have also succeeded in obtaining good sections from pieces which had been soaked in the mixture of alcohol and acetic acid, recommended by Mr. L. Clarke in investigating the structure of the cord. The fluid, however, which has afforded the most satisfactory results, consists of alcohol, to which a few drops of solution of soda have been added. In this mixture the liver becomes perfectly firm and transparent at the same time*.

In my investigations I have employed several other solutions, but the above are most worthy of notice. Some of the livers were hardened in the state usually obtained from dead animals, others were partially injected from the duct in order to force the bile into the smaller ducts before they were hardened, and in some instances the bile duct was tied some hours before the death of the animal.

* The advantage of this solution seems to depend upon the opposite action of the two fluids. The alcohol precipitates albuminous compounds, and renders them hard and opake. The soda, on the other hand, softens and dissolves them, rendering them transparent. In conjunction, these operate in rendering the tissue quite *hard* and *transparent* at the same time. I am still prosecuting experiments with this fluid. Large preparations have been preserved in the alkaline fluid with advantage. I have succeeded in making some most beautiful preparations, showing the ossification of the bones in the embryo at different periods in this manner.

Method of injecting the Ducts of the Liver and the Cell-containing Network.

Although I felt certain of the correctness of my observations upon various uninjected specimens examined two years since (in 1853), it appeared to me that it must be possible to inject both the minute ducts and the hepatic cell-containing network, if they were really continuous, and for a long time past I have been endeavouring to effect this object. It would be useless here to describe all the different plans I have resorted to, and I shall therefore only refer to the method which succeeded. After failing many times to drive the injection into the smallest ducts, it occurred to me to try to empty the ducts of their bile before injecting them. This object was at last readily obtained by forcing water into the portal vein, and causing the vessels to become distended. In consequence of this distension, the ducts were compressed, and the bile was forced from the small into the larger branches. After continuing the injection of the water for a short time, bile trickled from the duct and the gall-bladder became filled—soon bile ceased to escape, and almost pure water flowed. After the liver had been thoroughly drained, by being placed in soft cloths for some hours, the duct was injected with Prussian blue*. The injection passes readily with slight force, and can be seen entering the lobules upon the external surface of the liver. It does not appear gradually round the circumference of the lobule as when the portal vein is injected, but it forms small roundish points, almost like extravasations, here and there at the outer part of the lobule, and it spreads from thence towards the centre.

After the duct has been injected in this manner, it becomes necessary to distend the capillary vessels with injection, otherwise a thin section has a confused appearance. For this purpose plain clear size is the best. The portal vein is injected with it, and the injection continued until the whole organ is fully distended. The vessels

* Before resorting to Prussian blue I had tried numerous different substances. Opake injections always caused the preparation to look confused under high powers, and it was impossible completely to remove the adhering particles of injection from the section by washing. In consequence the specimen never looked clear. Transparent injections seemed likely to afford more favourable results, and several were tried. Most of them passed through the walls of the ducts, colouring all the adjacent tissues, and afforded no better success than opake injections. Carefully prepared Prussian blue combines the advantages of an opake injection with those of a coloured solution. The particles will not pass through basement membrane, and at the same time they are so minute, that when suspended in fluid, the appearance of a solution is produced. Injections of Prussian blue may be examined by reflected or by transmitted light.

The Prussian blue is prepared in the usual manner, by adding a solution of ferrocyanide of potassium to a dilute solution of perchloride of iron. The mixture should have the appearance of a dark blue solution, and should contain no flocculi. It should not form a deposit until it has stood for some time. About a third of its bulk of spirit should be slowly added and the mixture well stirred. The advantage of the spirit is, that it hardens the delicate walls of the finer ducts as it comes in contact with them, and thus tends to prevent their rupture. The mixture must be strained through two or three layers of fine muslin before use. The Prussian blue may be injected by means of a small half-ounce syringe, or by the pressure of a column of fluid about four feet in height placed in a long tube.

are then stopped and the liver allowed to become cool. When the size has set, very thin sections can be obtained without difficulty. The section must be carefully washed before examination, in order to remove loose cells, &c. from the surface which would interfere with its definition. It may be placed in thin syrup, glycerine or dilute alcohol, and will bear examination with a quarter, or with an eighth of an inch object-glass (400 diameters).

By injecting the ducts, the lymphatics are also injected. If the duct does not burst, colourless fluid returns by the lymphatics, which may thus be injected with plain size, while the colouring matter remains in the duct. If a small duct rupture, the injection often enters the abundant plexus of lymphatics in the portal canals, and may even reach the thoracic duct, as occurred to KIERNAN, and once to myself, in the case of a rabbit.

A human liver which has afforded many excellent specimens, has been injected with four colours, two transparent, and two opake; as follows, artery with vermilion; portal vein with flake-white; hepatic vein with lake, and the duct with Prussian blue.

Evidence of the existence of a Tubular Basement Membrane in which the Liver-cells are contained.

It is not uncommon to find cells with shreds of delicate membrane attached to them in specimens which have been slightly hardened in dilute alcohol. A drawing which has not been published shows a cell from the rabbit's liver enclosed in a membrane, which can be traced beyond it for some distance as a very narrow contracted tube.

This delicate basement membrane is also well displayed in certain specimens in which a curious chemical change has taken place in the contents of the tube. In a section of dog's liver, which had been soaking for some time in a weak solution of soda, most of the cells appeared to have been dissolved at their outer part, and in consequence, a fusion of the matter of which they were composed had taken place, causing the formation of a highly refracting mass within the basement membrane, the outline of which was rendered very distinct. This preparation is represented in Plate XV. fig. 18, and at *a* one of the tubes, separated and drawn out with its contents contracted within it, is shown.

I have a drawing of a somewhat similar change in the liver of a flounder which had been treated with soda, and afterwards by acetic acid, causing the precipitation of some of the constituents of the bile which had been previously dissolved by the soda. By pressure some of these highly refractive masses were broken, and by examination with a very dull light, the continuity of the delicate basement membrane could be traced between them.

When sections of liver have been soaked for some time in strong syrup or glycerine, the cells in the interior of the tubes shrink from exosmosis, and the delicate

tubular membrane contracts upon its contents. The tubes therefore become much narrower, and, except in the highly refracting nature of the contents, the altered cell-containing network closely resembles that of the capillaries.

In a properly prepared liver, it is often possible to show the cell-containing network in one section and the capillary network in another. When the vessels are distended with clear size, the meshes of the *cell-containing network* are seen; but if a section be well washed and placed in glycerine, the sharp well-defined outlines of the *capillary vessels* are brought into view. In such a section from which the cells have been removed, I have seen in some places, just at the thin edge of the specimen, stretched across the space between two capillary vessels, the exceedingly delicate basement membrane of the tube in which the cells lie, recognizable rather from the small quantity of débris and granular matter which adheres to it, than from any positive characters distinguishable in the membrane itself. This appearance can only be seen under the influence of a very dull light.

The network is capable of being distended to a considerable extent in every part of the lobule without rupture. Often the injection accumulates to such an extent as completely to obscure the cells, and in consequence of the pressure thus caused, the vessels are rendered invisible. Even in such specimens the appearance cannot be mistaken for extravasation or vascular injection. In tubes which are only partially injected, the injection often accumulates a little on each side, gradually shading off as it were towards the centre, while towards the adjacent capillary vessel it forms a distinct, well-defined and dark line.

The cells which escape into the surrounding fluid from an injected specimen have portions of injection adhering to them, or are deeply stained with it.

A portion of the cell-containing network of the human subject injected is shown in Plate XV. fig. 19. The cells represented in the drawing are much altered from the mode of preparing the specimen. Their position, however, is shown. In the pig, portions of the cell-containing network, in different states, are represented in Plate XV. figs. 23, 24, 26, 28. In the seal, in Plate XIV. figs. 15 & 16. In the rabbit, in fig. 11 *c*. In the dog, in Plate XV. fig. 18. In the human fœtus, in Plate XV. fig. 17, and in Plate XIV. fig. 12.

If a section be made through the hepatic tissue exactly at right angles to a small branch of the intralobular vein, the cells are seen to form rows which radiate from the centre towards the circumference of the lobule, as has been noticed by MÜLLER, VALENTIN, BOWMAN, THEILE, GERLACH and others. In sections made in many other directions this radiated arrangement is not to be shown. The drawing in Plate I. fig. 1 (not published) is an exact copy of a section of horse's liver, and is the most beautiful specimen of this arrangement which I have seen. The rows increase in number as they pass from the centre. They are connected at irregular intervals by oblique or transverse branches, often much narrower than a cell, and containing only nuclei and granular matter. These communicating branches are best seen in injected specimens.

The cells of which these rows are composed, for the most part form single lines, although here and there, where the tubes are wider than usual, two cells may be seen lying transversely across the tube. In a specimen in which these radiating tubes had been injected, not only was the existence of a basement membrane proved, but one distinct from, and not adhering to the walls of the capillary vessels. Although the meshes of the capillary network are slightly elongated towards the centre of the lobule, such an arrangement could not produce the appearance observed. If the cells lay promiscuously in the meshes of the capillaries without being circumscribed by any basement membrane, the radiating lines would necessarily be connected transversely by very numerous cells.

Some of these tubes from the human liver are represented in a drawing not published, which was copied from an injected specimen*.

The contents of the cell-containing network are liable to considerable alterations in volume, a change which can be readily effected artificially. In the same specimen the diameter of the network varies to a limited extent, according to the size and number of the cells within the basement membrane. It is usually about $\frac{1}{1000}$th of an inch in diameter in most mammalian animals, and is considerably wider than the narrowest part of the small ducts with which it is immediately continuous.

The width of the spaces between the tubes of the network, or in other words, the diameter of the vessels, varies much in different parts of the lobule of the human liver, being much wider at the portal surface, where the small branches of the vein enter, than at a greater depth where capillaries are alone found.

This point is shown in sections from different parts of the lobule of the liver of the fœtus in Plate XV. fig. 17.

In some situations, then, it is demonstrable that the basement membrane of the cell-containing network is distinct from the walls of the capillaries; but in the greater part of the lobule, where the two membranes come into close contact, they are incorporated, so that really the majority of the liver-cells are nearly surrounded with blood, from which they are only separated by one thin layer of delicate structureless membrane.

In the fœtus there is a distinct interval between the wall of the tube in which the secreting cells lie, and that of the capillary vessels; so that when a good section is obtained, two distinct lines are seen between the liver-cells and the cavity of the capillaries. These two lines are separated by a transparent, apparently structureless substance, in which no trace of fibres can be detected, as represented in fig. 17.

These circumstances impress me with the notion that the liver is originally composed of two distinct networks which intimately interdigitate with, or fit into, each other,—one containing the secreting cells, the other the blood. The walls of these

* I have not been able to demonstrate the presence of cæcal tubes connected with the branches of the network in the lobules of the liver, but from the arrangement of those of the "vasa aberrantia," presently to be described, and from other appearances which I have observed, I think it likely that such exist in small number.

two sets of tubes gradually become incorporated, except in those situations where the capillary network is less dense, or where the meshes of the cell-containing network are more widely separated from each other; in which cases a distinct limitary membrane to the tubes containing the liver-cells can be demonstrated even in the adult. The cell-containing network and the vascular network can be alternately distended. The membrane is very permeable to water in both directions, as I have proved by first forcing fluid from the vessels into the duct, and afterwards in the opposite direction. Subsequent injection proved that no rupture had taken place. The greatest force which can be applied will not cause the bile to pass through this delicate membrane into the capillary vessels. It is permeable to bile only in one direction.

Of the contents of the Tubular Network of Basement Membrane, and of the Arrangement of the Liver-cells within it.

Within the tubular network lie the hepatic cells, with a certain quantity of granular matter and cell débris, and in some instances, free granules of yellow colouring matter and free oil-globules. The liver-cells have been described by many writers. They are not arranged in any definite or regular manner within the tubes of basement membrane. Their form appears to be determined to a great extent by mutual adaptation and pressure. In adult animals they usually contain one nucleus, and within this is a bright point (nucleolus) and faint granules: rarely cells containing two nuclei are found. In the embryo, however, the cells contain two nuclei very commonly, and often cells are met with which contain several (from two to six). Besides the nucleus, the liver-cell contains, as is well known, coloured granules and oil-globules, apparently suspended in a viscid albuminous material. The coloured granules are not constantly present. Often they are numerous in cells near the centre of the lobule, and entirely absent in those near the margin, while the reverse is usually the case with reference to the oil-globules. On some occasions, cells which contained no coloured granules when first examined, were found to contain several after having been allowed to soak for some time in dilute acetic acid. The granules of colouring matter are, however, not unfrequently present in the cells near the centre of the lobule, as well as in those containing much oil at the circumference. In some instances I have observed that the cells near the centre of the lobule were smaller and were surrounded with a greater quantity of granular matter than those nearer the margin.

Many observers have endeavoured to show that the cells of the liver are arranged in a definite manner. Professor Lereboullet, one of the latest writers on this point (1853), describes the cells as arranged in double rows, between which injection can be forced so as to separate them, and he gives two diagrams to illustrate his views on this point. I have never seen anything like this arrangement, and, as I have before remarked, injection passes very readily at the outer part of the tubes. In mammalia,

for the most part, the cells form a single row within the basement membrane (human subject, pig, dog, cat, rabbit, horse, seal, Guinea-pig and others). In some situations the tube contains two cells lying transversely across it, but this appearance is often produced by two tubes being placed in close juxtaposition. Sometimes, when the ducts have been injected, two cells may be forced across the tube by the pressure of the injection, Plate XV. fig. 24. The cells do not completely fill the tubes, neither are they always placed quite close together, but a certain quantity of granular matter is seen to intervene. The action of certain reagents in causing the fusion of the contents of the cells has been alluded to, but it is doubtful if these so-called cells in all instances have a distinct membranous wall. Injection runs sometimes on one side and sometimes upon the other side of the tube. Often it entirely surrounds the cells, and not unfrequently presses them to one side and separates them a little from each other, Plate XV. fig. 23.

From these circumstances it would appear that the cells are not firmly adherent to the walls of the tube; indeed many of them often appear to lie free within its cavity. but exhibit a tendency to cohere to each other when set free.

In the only embryos of mammalia which I have examined, there is often more than one row of cells within the tubes, frequently two or three (human subject, calf), Plate XV. fig. 17. The cells are smaller, but in many situations the diameter of the tubes is greater than in the adult.

In birds, the tubes in some parts are wide enough to contain two or three rows of cells, but in others there is only room for one row (duck, linnet, turkey, starling, fowl). In the chick several rows of cells lie within the tubes of the network, Plate XV. figs. 29 & 30.

In those reptiles which I have examined, for the most part several cells are seen lying across the tube (frog, adder, field-snake, newt).

In fishes also the tubes are wide and contain several in the transverse direction (flounder, frog-fish, sturgeon, herring, cod), Plate XV. figs. 36 & 37.

It may be remarked generally that the cell-containing network is widest among fishes, and most narrow in mammalia. The similarity of the arrangement of the cells in the tubular network in the embryos of the higher animals with that existing in the adult, as well as in the embryonic condition of the lower, is a point of interest, of which there are many analogous examples in the case of other organs.

The changes in volume which the hepatic cells so readily undergo, renders it very difficult to ascertain their dimensions with accuracy.

OF THE DUCTS OF THE LIVER.

The duct, like the artery, lies close to the portal vein in the portal canals, and usually on the same side. In the large portal canals the external coats of the duct and artery are incorporated with that of the vein, a point well seen in the horse when the duct is very large. The position of the vessels remains nearly the same in the

smaller canals, but they often change the side upon which they lie. Usually, one branch of the duct and artery accompanies the portal vein, but not unfrequently there are two or more branches of these vessels around the vein.

Anastomosis of the ducts near the trunk from which they come off.—The anastomosis between the larger ducts and of the larger branches of the interlobular ducts with each other in the human liver is pretty numerous, but they are confined to the trunks near their origin. In some animals the branches resulting from the division of the small ducts communicate with each other at a distance from the parent trunk at the margin of the lobule. These communications may be very few in number, or so numerous as to form a network upon the surface of the lobule.

The anastomoses of different interlobular ducts with each other round the lobules, according to my observations, are very rare; and out of numerous injections, I have been able to satisfy myself of its actual occurrence only in one instance.

Kiernan observed, that if the left hepatic duct were injected with size or mercury, the injection returned by the right duct. It seems not improbable that this may have resulted from the numerous communications between these ducts in the transverse fissure of the liver, rather than from the union of interlobular branches. Not only do the right and left hepatic ducts anastomose by the intervention of small tortuous branches in the transverse fissure of the human liver, as E. H. Weber many years ago demonstrated, but the branches which come off from these trunks are connected by smaller ones which arise not far from the point at which they are given off. Many of my preparations show a most complete and intimate network at a short distance from the parent trunks in the transverse fissure, and to a less extent in the portal canals. This curious and beautiful network is shown in Plate XIII. figs. 2. 3 & 5. The smallest of these branches seem to have no fibrous coat, and in the larger ones this is much thinner than in the ordinary ducts. In the portal canals these anastomosing branches connected with the ducts are much less numerous, and some of the small communicating branches lie in the thick coats of the ducts, Plate XIII. fig. 6 *a*.

In the fœtus the communications are less numerous and the branches less tortuous, but they are to be shown more readily than in the adult, in consequence of their being surrounded by a much less quantity of areolar tissue. In the dog and in the calf I have seen similar communications (Plate XIV. fig. 13 *a*), but they are less numerous than in the human subject.

In the instances above referred to, an irregular network of ducts, more or less extensive, is to be demonstrated near the trunks, but the arrangement is not constant in all animals, nor, so far as I can ascertain, are these communications numerous enough to entitle them to the name of interlobular plexus or network. I have not been able to demonstrate such a network, although I am not prepared to say that it is absent, in the pig, seal, rabbit, horse, cat and monkey. If these communications exist in these animals, they can only be present in a limited extent.

Of the Sacculi of the Ducts.—The so-called "glands" of the ducts have been recognized by KIERNAN and subsequent observers, but the attention of most anatomists has been directed to those which surround the large ducts, rather than to those which are found in the walls of ducts of smaller size.

The so-called "glands" are small cavities of a rounded or oval form, or more or less branched, communicating with the cavity of the duct by a very constricted neck. A very simple sacculus is represented in Plate XIII. figs. 7 & 8, from a small duct of the pig's liver. These glands are for the most part situated in the coats of the duct, and when injected scarcely project beyond its external surface. The larger branched glands, however, usually extend for some distance from the duct. The glands are usually situated all round the tube (pig, dog, monkey, cat, seal). In the smallest ducts in which they occur in the human subject, they are generally confined to two lines on opposite sides of the tube, but not unfrequently a few may be found between these lines of orifices. These orifices, which were described by KIERNAN, and which lie tolerably close together, forming a straight row of openings on each side of the duct, are best seen when the small duct is laid open. They are, however, almost all of them the openings of small branches of ducts, many of which anastomose in the fibrous coat of the branch to which they belong, and probably very few only should be regarded as sacculi or glands, which are scarce in the small ducts of the human liver. The little dilatations connected with the ducts shown in Plate XIII. fig. 6 *b*, are examples of these so-called "glands" upon some of the smallest ducts in which they occur in the human liver. In the largest ducts in the human subject they are found all round the tube, but in unopened ducts many may escape notice in consequence of the pressure of the glass cover, by which the glands on the upper and lower surfaces are rendered almost invisible, while an undue prominence is given to those at the sides of the tube.

The largest sacculi in the pig are branched, and are situated at the point where a smaller duct leaves the large trunk.

In the human subject, at the point where the smaller ducts open into the large one, the lining membrane is so arranged as to form a valve which would tend to prevent the passage of the bile from the large into the small ducts, more especially as these latter run obliquely for some distance in the coats of the larger duct.

In the pig, glands are numerous upon ducts from the $\frac{1}{125}$th to the $\frac{1}{100}$th of an inch and more. They are situated all round the tube, and are rarely seen upon ducts below the $\frac{1}{200}$th of an inch in diameter. They are not present upon the small interlobular ducts, the walls of which are very thin. In the human subject, in the seal, and in the monkey, they are rare upon ducts below the $\frac{1}{100}$th of an inch in diameter. In the fœtus they are common upon ducts $\frac{3}{500}$ths of an inch in diameter. They are rarely met with upon ducts in the cat's liver less than $\frac{3}{200}$ths of an inch, and in those of the hedgehog below $\frac{9}{1000}$ths of an inch.

Vasa aberrantia.—The most curious appendages to the ducts are the "vasa aber-

rantia," originally described by WEBER in the transverse fissure of the liver. They have been previously alluded to when the anastomoses of the ducts were described. THEILE regarded these ducts, so well named by WEBER, as irregular and branched mucous glands. Some observers, and lately LEREBOULLET, have altogether failed to discover these curious branches. The cæcal appendages connected with these ducts are very numerous and of uniform size. Their coats are much thinner than those of the ordinary gall-ducts. They are lined with epithelium, principally of the sub-columnar form.

These curious ducts are most numerous in the transverse fissure, but are also found in the upper part of the umbilical fissure. I have seen these ducts in all the large portal canals, and in those not more than a quarter of an inch in diameter, they are found occasionally. Many of the smaller ducts at their origin from a larger trunk in a portal canal have cæcal appendages like a branch of the vasa aberrantia, and on the other hand straight branches may be traced from these aberrant ducts in some instances in the transverse fissure, to the hepatic tissue, Plate XIII. fig. 4 *a*.

The vasa aberrantia are always closely surrounded by areolar tissue, in which lymphatics are very numerous, and adipose tissue not unfrequently abundant.

The arrangement of the vessels is peculiar. The arteries and veins form a network, in the meshes of which the vasa aberrantia lie. Each small branch of the artery is accompanied by two branches of the vein, lying one on each side of it, which communicate by numerous transverse branches, some of which pass over the artery, and others under it. It is interesting to observe that this peculiar arrangement of the arteries and veins exists in the coats of the *gall-bladder*, in the *transverse fissure* of the liver, and in the *portal canals*. It is only to be seen in good double injections; but when the vessels are thoroughly injected with different colours, a most beautiful appearance is produced. The veins pour their blood into large branches of the portal vein. This peculiar arrangement of the veins doubtless has the effect of ensuring free circulation through them under different circumstances, and admits of their being compressed or stretched to a great degree without obstruction to the passage of blood through them. Some of these vessels are represented in Plate XIII. fig. 1 *a*.

In the transverse fissure of the adult human liver the vasa aberrantia lie nearer to the trunk of the portal vein than to the hepatic tissue, and they can be removed without cutting into it. In the liver of the fœtus, on the other hand, the quantity of areolar tissue is much less, and the vasa aberrantia lie so close to the hepatic tissue, that it is impossible to remove them without a thin layer of the latter, into which they enter at numerous points. In the fœtus these ducts are less numerous than in the adult, and their course is less tortuous. They occur in small patches in which the branches are seen to be very numerous, and the anastomoses very frequent. The epithelium is more abundant, and the cells large and of a dark colour, and the injection does not run so readily as in the adult.

These facts appear to me to militate strongly against the notion of these ducts being modified and anastomosing mucous glands, as THEILE supposes. From their arrangement it seems not improbable that they are really altered secreting tubes, and at one time formed a part of the secreting structure of the liver. As the portal vein increases in size at the termination of intra-uterine life, it is not unreasonable to suppose that some of the hepatic tissue close to it would be removed to some extent, to make room for the enlarged vein; in such a case, many of the vessels would degenerate into fibrous tissue, and the branched and anastomosing vasa aberrantia would represent all that remains of the hepatic tubular network.

In the very thin edge of a horse's liver which was composed principally of fibrous tissue, I have been able to trace the gradual alteration of the ducts and the ultimate complete disappearance of secreting cells.

Spread out, as it were, upon the surface of the portal vein in the rabbit, there is a very thin layer of hepatic tissue, and I have been enabled to trace the different stages between the vasa aberrantia and the cell-containing network, most distinctly, in a specimen in which the ducts were injected, and the vein distended with plain size.

Function of the glands and vasa aberrantia.—To the sacculi of the ducts, the office of secreting the mucus in the bile has been assigned, but cavities opening into a tube by a narrow neck, not the $\frac{1}{5000}$th of an inch in diameter, seem ill-adapted for pouring out a viscid tenacious mucus; neither is it easy to suppose how this mucus, secreted at certain points, would become thoroughly mixed with the mass of the bile as it flowed through the ducts. If these cavities were filled with mucus, one would hardly expect that injection would enter them so readily as it does. The complicated and highly tortuous vasa aberrantia possess no characters which would lead to the inference of their being mucous glands.

Again, the bile of the rabbit, in which animal these glands are very few in number, and only found upon the largest ducts, affords as bulky a precipitate of mucus upon the addition of alcohol as that of the pig, in which these glands are so numerous and so distinct.

It seems highly probable that these little cavities, as they are found in the adult liver, are to be looked upon as reservoirs for containing bile, whilst it becomes inspissated and probably undergoes other changes. The numerous vessels and lymphatics which surround them, and the similarity of arrangement of those around the vasa aberrantia, and of the vessels of the gall-bladder, still further strengthen this view of their office. I have not yet minutely investigated the arrangement of the glands of the ducts in many different animals, nor have I been able to ascertain if ducts, corresponding to the vasa aberrantia, are usually present. I have preparations showing them in the pig, in the rabbit, and in the monkey.

Of the finest branches of the Ducts, and of their connexion with the cell-containing Network.

Mammalia.—In well-injected preparations the smallest branches of the duct can be readily traced up to the secreting cells, Plate XIV. fig. 10. The walls of these ducts are composed of basement membrane only, lined with delicate epithelium. In the human liver, and in that of most mammalia, except the pig, a few of the finest branches of the duct can be followed for some distance beneath the surface of the lobule, and are seen apparently lying amongst the secreting cells.

In many animals, particularly in the rabbit, and to a much more limited extent in the dog and in the human subject, these smallest branches of the duct are connected together so as to form a network of ducts, distinguished by their diameter being much less than that of the network in which the liver-cells lie, and by the character of their epithelium. These branches appear to lie in the most superficial part of the cell containing network, without being connected with the cells which surround them; but they join, or are continuous with, a portion of the cell-containing network at a deeper part of the lobule. The most superficial portion of the cell-containing network is connected with small ducts which do not penetrate amongst its meshes. This arrangement is shown in a very simple form in Plate XIV. fig. 16, which represents a section from a seal's liver, in which the ducts and cell-containing network were both well injected with transparent blue injection: at *c*, some of the superficial branches of the duct, and at *d*, branches which penetrate beneath the surface of the lobule, are represented. Some of these branches, which appear to lie amongst the secreting cells at the peripheral part of the lobule, have been figured by GERLACH, and are, as he justly observes, much narrower than the cells amongst which they lie. He makes them terminate by open mouths, into which the bile is poured from the cells. The narrowness of these tubes has been advanced as an argument against the possibility of the cells lying within tubes, but from the description just given it will be seen that this objection does not apply.

In the pig, numerous fine branches of the duct are, as it were, applied to the surface of the capsule of the lobule (Plate XIII. fig. 9), which is perforated at frequent intervals by many very narrow short branches, which are immediately connected with a network which partly lies amongst the fibres of the capsule. This network may be looked upon as the most superficial portion of the cell-containing network, and the narrow tubes of which it is composed are found to contain oil-globules, granular matter, and a few very minute cells, Plate XV. fig. 25. When the liver is very fatty, however, this most superficial portion contains numerous secreting cells filled with oil-globules, Plate XV. fig. 28. I have not been able to trace any ducts passing amongst the meshes of the most superficial part of the cell-containing network, as I have described in animals in which the distinct lobules do not exist. In the last figure, the very narrow ductal portion of the tube is seen to dilate considerably at the

point where the secreting cells commence. This drawing is an exact copy of a beautiful preparation of a very fatty pig's liver which had been injected, and the parts which were injected in the preparation are represented in the drawing by the dotted shading. The continuity of the narrow duct with the wide tubes in which the secreting cells lie is well shown, in consequence of this part being accidentally separated from the adjoining portion.

The extremely delicate nature and narrowness of the ducts will readily explain the difficulty of demonstrating their continuity with the cell-containing network, particularly in the pig's liver, for they almost invariably break at this constricted portion, especially as this partly lies in the substance of the capsule of the lobule. I have seen the same anatomical point in many specimens, but in none more distinctly than in the preparation referred to, which still preserves its essential characters (June 9th, 1856).

In Plate XV. fig. 19, the connexion between the small ducts and the cell-containing network is shown in an injected specimen of human liver. The duct is greatly distended with injection, and the hepatic cells have been much altered in preparing the specimen. The communications between the finest branches of the duct are few in the human liver; a branch can often be followed for some distance amongst the meshes of the peripheral part of the cell-containing network, until it is seen to become continuous with it at some distance from the surface. These branches can only be seen in injected specimens.

In Plate XV. figs. 20 & 21, the connexion between the ducts and secreting network is shown in an uninjected liver, which had been treated with soda, and kept for some time in strong syrup. The liver-cells have been destroyed by the mode of preparation. The narrow duct is well seen at *a*, fig. 21. The other specimen has been subjected to considerable pressure.

In the human fœtus, the connexion between the duct and cell-containing network is shown under the influence of a low power in Plate XIV. fig. 14 *a*, and also in figs. 12 & 13.

In a seal's liver which I injected, the hepatic cells were small, and the injection very readily passed into the cell-containing network. The small ducts are comparatively few in number, but their course was very easily traced. Often they could be readily followed upon the thin wall of a small portal vein, as shown in Plate XIV. figs. 15 & 16. In the latter figure, small branches of the duct are seen passing amongst the superficial branches of the cell-containing network to join some of those at a deeper part. At *c*, two or three branches are seen, which were connected with that portion of the network nearest to the large duct.

Fig. 11 is a drawing of a section of a rabbit's liver, taken from a very thin and emaciated animal, in which the cells were much smaller and more granular than in health. The tubes of the network were shriveled, and in many situations contained only granular matter. In a thin section of this liver, narrow granular lines,

communicating at intervals with each other (cell-containing network), were seen separated by wide clear spaces (capillaries) intervening between them. In this liver the very fine ducts were injected to a greater extent than can be effected usually, and a network of anastomosing branches round the portal vein is shown in the drawing. Even in this specimen the injection has not entered many of the small branches of the duct. The injection ran very readily, and in some places distended the tubes of the cell-containing network to a very great extent, in which case of course the clear intervening space (capillary) was obliterated.

Birds.—In the fowl and in the turkey, the ducts form a network continuous with the network in which the liver-cells lie; but the tubes are of much less diameter than those of the latter. I have been able to trace the continuity in the livers of the two birds above referred to, by injection. The abundance of epithelium in the ducts of most birds forms a great obstacle to the passage of the injection, and the excessive thinness of the capillary vessels prevents the preliminary injection of much water without rupture.

In the chick of the fifteenth and also in that of the twenty-first day, the continuity between the ducts and cell-containing netwoik is shown in Plate XV. figs. 29 & 30.

Reptiles.—In the newt (*Triton cristatus*), the course of the ducts, in an uninjected specimen, is shown in fig. 31. The tubes in which the hepatic cells lie were much stretched and pressed in these specimens. I have not yet been able to inject the ducts of the newt or frog.

I have succeeded in making a good injection of the ducts of the adder, and was enabled to trace their continuity with the wide tubes containing the liver-cells. The large branch of the portal vein is surrounded with a plexus of ducts.

Fishes.—The greatest difficulties presented themselves in the investigation of the anatomy of the ducts in this class. The ducts become exceedingly narrow before they join the very wide tubes of the network containing the cells. I have not been able to harden the liver sufficiently to obtain very thin sections, and in tearing out the finer ducts they always break off in the narrow part.

Numerous attempts at injecting the liver failed altogether in consequence of the vessels rupturing when the water was thrown in. Another source of great inconvenience is the frequent occurrence of entozoa and their ova. In consequence of the tubes being almost always full of cells containing oil, and free oil-globules, it is difficult to obtain a specimen for examination which is not rendered obscure by the escape of numerous cells into the fluid in which the preparation is placed.

In the common flounder, the connexion between the ducts and cell-containing network has been demonstrated in an uninjected specimen. In this specimen the secreting tubes were much shrunken and of less diameter than usual, a circumstance to which the distinctness of their outline is due.

I have injected the ducts and a part of the cell-containing network of the sturgeon

and Lophius. In Plate XV. figs. 32, 33, 34 & 35, some of the ducts and portions of the network are represented. The mode of branching of the ducts, and an anastomosis, is shown in fig. 33. Even in the very fatty liver of the cod, I once traced the continuity of the ducts and the wide tubular network distended with cells containing oil and free oil-globules.

In injecting the livers of fish it is necessary to dilute the injecting fluid with weak spirit, or it will not penetrate to the finest branches. Often the particles of the injection accumulate in some of the finer ducts, forming what appears to be a rounded and slightly dilated extremity, for the further continuity of the tube cannot in all cases be seen. Indeed so perfect is the resemblance, that it is only by examining numerous different specimens that one becomes convinced of the fallacy.

In all four classes of vertebrata, I have seen both in injected, and also in uninjected specimens, the communications between the finest ducts and the cell-containing network. Of the nature of this continuity there can, I think, be no doubt. I cannot conceive any other explanation of the facts I have observed, or of the appearances I have copied from my preparations. The observations upon uninjected specimens, shown in Plate XV. figs. 18, 20 & 21, were made in 1854, many months before I had succeeded in injecting the ducts.

In all vertebrate animals which I have examined, the duct becomes much narrowed just before it becomes continuous with the tubes of the network in which the secreting cells lie. The arrangement of the most minute ducts varies somewhat in different animals, as has been described. In some they form a network of very narrow tubes, continuous with those in which the liver-cells are contained; in others these communications are excessively few in number; while in some, I do not think they exist at all. But I would not express myself positively upon this point, for I feel persuaded that in the most perfect injection which I have yet made, the minute ducts have not all been injected, and from a cursory examination of these preparations alone, only a very imperfect idea of the number of the ducts or of the beauty of their arrangement and relation to the other structural elements of the liver in the natural state of the organ, can be formed.

Diameter of the finest ducts.—The diameter of the finest ducts can only be obtained approximatively; for when not injected, they can only be demonstrated distinctly in fortunate specimens, and are probably somewhat narrower than during life. When injected, on the other hand, they are usually distended, and sometimes to a very considerable extent.

In the pig, the smallest branches containing a little injection are not more than the $\frac{1}{3000}$th of an inch in diameter; in the human subject, about $\frac{1}{2500}$th; in the seal, $\frac{1}{3000}$th; in some fishes, not more than $\frac{1}{5000}$th.

The diameter of the cavity of the tube and the total diameter of ducts of different sizes are shown in the following Table:—

	External diameter.	Internal diameter.
Pig	·008	·004
	·006	·003
	·001	·001
Human subject	·01	·0045
	·002	·002
Human fœtus	·01	·0045
Cat	·015	·0075
Monkey	·02	·01
Seal	·001	·001
	·0003	·0003

The diameter of the ducts from which my drawings were taken can be readily determined by reference to the scales appended to the Plates.

The narrowing of the excretory duct, just before it becomes connected with the secreting tube, is seen in other glands; thus the straight and ductal portion of the renal tube is narrower than the convoluted and secreting portion. It must, however, be borne in mind, that although the excretory portion of the tube is so small, its cavity, through which the secretion passes, is wider than in that part in which the secretion is separated. In the liver, where the secretion is highly elaborated and slowly removed in a more concentrated form, we should naturally expect to find the contrast between the ductal and secreting portions of the gland still greater than in most other secreting organs.

Epithelium of the small ducts.—As is well known, the larger ducts have a thick lining of columnar epithelium, the cells of which become shorter and altogether smaller in the finer ducts. In the narrowest tubes the cells of epithelium are somewhat flattened, and usually of a circular form. Sometimes they are oval or angular, depending probably upon being stretched. They have a faintly granular appearance, and rarely a nucleus can be seen within. The quantity of this epithelium varies very much; sometimes it completely lines the tube. In some instances it is so abundant as apparently to leave no distinct cavity in the duct, while it is not uncommon to find some of the finest ducts containing only a very few cells scattered at irregular intervals over the basement membrane, of which the coats of these small ducts are alone composed. In a perfectly normal condition, when the minute ducts are undisturbed by manipulation, and are examined in a proper medium, they are generally seen to be lined by epithelium; but from the extreme minuteness of these ducts and tender character of their walls, it is scarcely surprising that one should fail in making out distinctly the epithelium in every instance. The epithelium of the small ducts presents very similar characters in all the animals in which I have been able to demonstrate it. In the human subject it is shown in Plate XV. figs. 20 & 21, and in the seal in Plate XIV. fig. 16, at *b*.

This ductal epithelium does not pass by insensible gradations into the secreting

hepatic cells, but usually ceases at the point where the narrow duct expands into the wide secreting tubes of the network. In a few instances I have seen tubes containing liver-cells lined with this delicate epithelium, observations which confirm those of Mr. WHARTON JONES, who has seen hepatic cells in the smaller ducts. In these cases the cells may have accidentally entered the ductal portion of the tube.

The arrangement above described is very similar to that which occurs in the gastric gland. It is only in the lower part (stomach tube) that the cells of spheroidal epithelium (which alone there is every reason to believe take part in the secretion of the gastric juice) are found. The upper portion or duct (stomach-cell) is lined with columnar or subcolumnar epithelium*. The secreting cells are not arranged with any order or regularity round the basement membrane of the tube, as in the kidney, but appear to fill its cavity; so that the secretion having escaped from the cells with or without their liquefaction, must pass off by the slight interstices between them. As I have before remarked, the same irregularity occurs in the arrangement of the secreting cells in the tubular network of the liver, and as may be observed in a less remarkable degree, in the secreting portion of many other glands, as the pancreas, lacteal glands, sebaceous and sweat glands.

The conclusions to which I have been led from my observations, may be summed up as follows:—

1. That the liver of vertebrate animals essentially consists of two "solid" tubular networks mutually adapted to each other. One of these networks contains the liver-cells, the other the blood.

2. The cell-containing network is continuous with the ducts. The small delicate epithelial cells lining the latter channels contrast remarkably with the large secreting cells, which are not arranged in any definite manner within the tubes of the network.

3. The duct is many times narrower than the tubular cell-containing network at the point where it becomes continuous with it.

4. Injection passes sometimes on one, and sometimes on the other side of the tube, or between the cells, where two or more lie across the tube. As injection can thus be made to pass readily *from* the ducts into the network and around the cells, it follows that there can be no obstacle to the passage of the bile along the same channels in the opposite—its natural direction.

5. In some animals, the most minute ducts are directly connected with the tubes of the cell-containing network. Of these branches, some pass amongst the most superficial meshes to join the network at a deeper part. In other animals the finest ducts form a network which is continuous with that in which the liver-cells lie.

6. The interlobular ducts do not anastomose, but the branches coming off from the trunk are often connected with each other, as well as with the parent trunk, near their origin from it, by intervening branches.

* TODD and BOWMAN's Physiology, vol. ii.

7. The walls of the smallest ducts are composed of basement membrane only. The thick complex coat of the larger ducts contains within it small cavities (the so-called glands of the ducts), by which the bile in these thick-walled tubes would be brought into close proximity with the arteries, veins and lymphatics, which are very abundant wherever the ducts ramify.

8. The office of the vasa aberrantia, which are so numerous in the transverse fissure of the human liver and in the larger portal canals, appears to be similar to that of the cavities in the walls of the ducts. It is worthy of remark, that the network of vessels ramifying so abundantly in the coats of the gall-bladder, in the transverse fissure, and in the large portal canals, is arranged in a similar manner, each branch of artery being accompanied by two branches of the vein.

9. The liver is therefore a true gland, consisting of a formative portion and a system of excretory ducts directly continuous with it. The secreting cells lie within a delicate tubular network of basement membrane, through the thin wall of which they draw from the blood the materials of their secretion.

Explanation of the Plates.

The dotted shading in the Plates corresponds to the Prussian blue injection represented in the drawing. By tracing this shading, the extent to which the injection had penetrated in the preparation is shown.

To ascertain the diameter of any object represented, it is only necessary to compare it with one of the scales at the foot of the Plates, the divisions of which are *magnified in the same degree as the drawing.*

PLATE XIII.

Fig. 1. Vasa aberrantia, from the transverse fissure of the human liver, injected with Prussian blue. *a.* Branches of portal vein injected with white lead. *b.* Branches of hepatic artery injected with vermilion. *c.* Branches of duct or vasa aberrantia, in some places only partially injected.

Figs. 2 & 3. Ducts with vasa aberrantia, from the transverse fissure of the human liver, injected with Prussian blue.

Fig. 4. Dilated portion of duct from the transverse fissure of the human liver, giving off long straight branches to secreting structure.

Fig. 5. Vasa aberrantia and parietal appendages of ducts from the transverse fissure. This drawing represents the portion of fig. 3 marked *a*, magnified 8 diameters.

Fig. 6. Interlobular duct with lateral appendages and one or two irregular branched ducts or vasa aberrantia. The greater number of the lateral channels are seen not to project beyond the outer surface of the fibrous coat of the duct, *a.*

Fig. 7. *b*. Interlobular duct with parietal sacculi of the pig, injected with Prussian blue. *c*. Branch of artery.

Fig. 8. One of the sacculi marked *a*, fig. 10, magnified 215 diameters. *a*. Fibrous coat of duct. *b*. Cavity of sacculus filled with injection. *c*. Epithelium.

Fig. 9. A very small lobule of the pig's liver, showing the ducts dividing into branches upon the surface of the capsule.

PLATE XIV.

Fig. 10. Transverse section of human liver, showing the general arrangement of the ducts in the interlobular fissures. *a*. Branches of portal vein injected with white lead. *b*. Small branches of intralobular vein, injected with lake. *c*. Capillaries of lobule, represented only in a few situations. *d*. Branches of duct injected with Prussian blue passing towards the lobules, and giving off few branches in their course. The imperfect manner in which the lobules are mapped out in the human subject and in most other mammalian livers, is shown in this figure.

Fig. 11. Interlobular duct of the rabbit, injected with Prussian blue. *a*. Trunk of duct. *b*. Small branches passing off to cell-containing network, and anastomosing with each other. *c*. Commencement of cell-containing network at the margin of the lobule, showing its continuity with the finest ducts. In this specimen the liver-cells were much disintegrated from disease. The tubes of the cell-containing network were occupied with granular matter in which very few liver-cells could be detected. The injection consequently penetrated very readily for a considerable distance, as shown by the tinting.

Fig. 12. Interlobular duct, parietal sacculi, and branches of communication (*a*) from the human fœtus. At *b* small branches of the ducts are seen passing off to, and becoming continuous with, the cell-containing network of the lobule. Injected with Prussian blue.

Fig. 13. A corresponding preparation from the fœtal calf near the full time. The sacculi are large and distinct. In this figure the outline of the portal vein, which was injected with plain size, is shown. *a*. Branches of duct passing to cell-containing network of lobule.

Fig. 14. Interlobular ducts and smaller branches from the human fœtus, showing their anastomoses, and, at *a*, their continuity with the cell-containing network at the margin of the lobule. *b*. Section of a small branch of the portal vein with ducts around it, which have been cut across in making the section. In this preparation the ducts were injected with carmine.

Fig. 15. A small portal canal from the liver of the seal. The portal vein injected with plain size, and the duct with Prussian blue. *a*. Artery. *b*. Duct.

Small branches of the duct upon the transparent portal vein are seen to be continuous with the network of tubes in which the liver-cells lie, and into which network a little injection has passed from the duct. The part of the cell-containing network represented in this figure is situated immediately round the portal canals, which contain but little areolar tissue in the seal.

Fig. 16. Small ducts from the seal injected with Prussian blue, showing their continuity with the cell-containing network of the lobule. At *a*, the outlines of a few capillaries of the portal vein injected with colourless size are seen. *b*. A small branch of the portal vein. The narrowest portions of the duct in this preparation have been distended considerably by the injection.

PLATE XV.

Fig. 17. Portion of cell-containing network of human fœtus at the point of entrance of a small branch of the portal vein which is about to divide into capillaries. The meshes of the cell-containing network at this point are much wider than at a greater distance from the circumference of the lobule, in consequence of the vessels being larger. The outlines of the tubes of the network and of the walls of the capillaries are seen to be separated by an intervening, perfectly transparent material.

Fig. 18. Part of the cell-containing network of the dog's liver where its branches are parallel, altered by being soaked for some time in dilute caustic soda. A few cells are seen, but the greater number are fused together, and the tubes are chiefly occupied by a yellow, transparent, highly refracting substance, in which numerous oil-globules are contained. *a*. A tube stretched and separated from its neighbours, showing its basement membrane and contents.

Fig. 19. A small duct from the human liver at the point where it becomes continuous with the cell-containing network. The cells are much altered in character from the mode of preparing the specimen. The duct is very much distended with injection.

Fig. 20. Section of the liver of a man, aged 43, which had been hardened. Uninjected. This preparation shows the continuity of the most minute ducts with the cell-containing network. The epithelium of the smallest ducts is seen in this specimen. The hepatic cells are destroyed by the action of soda. This preparation has been subjected to great pressure between the glasses.

Fig. 21. A similar preparation, not so much flattened by pressure. *a*. Finest ducts. *b*. A venous capillary cut across. *c*. A small artery.

Fig. 22. An interlobular duct dividing into smaller branches upon the surface of a

lobule of pig's liver: injected with Prussian blue. The injection has entered the superficial portion of the cell-containing network, producing the mottled appearance represented.

Fig. 23. A portion of cell-containing network near the surface of a lobule from the pig's liver, injected and isolated. The injection has pressed the cells towards one side of the tube, and partially separated them from each other, and has thus prevented the thin wall of the tube from being in close contact with them.

Fig. 24. Part of cell-containing network (pig), showing liver-cells, granular matter and oil-globules in the interior of the tube. In one part the tube is somewhat dilated.

Fig. 25. Most superficial portion of the cell-containing network (pig), lying partly in the fibrous capsule of the lobule, and partly within the capsule. The tubes contain a few small cells, free oil-globules and granular matter. Partially injected.

Fig. 26. Portion of cell-containing network within the capsule of the lobule, injected with Prussian blue. Pig.

Fig. 27. A small portion of fig. 25 distended with injection.

Fig. 28. Connexion of duct with cell-containing network from a fatty liver of the pig: injected. The small trunk *a* gives off several smaller branches, *b*, not more than $\frac{1}{3000}$th of an inch in diameter, although distended by the injection which has reached the cell-containing network. The tubes of the latter are four or five times wider than the smallest ducts.

Fig. 29. Part of cell-containing network and finest ducts of chick on the 15th day of incubation, from a preparation which had been hardened in alcohol and soda.

Fig. 30. A similar preparation on the 21st day. The tubes contain much oil.

Fig. 31. Branches of duct of newt (*Triton cristatus*). At *a* the commencement of the tubes of the cell-containing network.

Fig. 32. Ducts and their connexion with the cell-containing network. Sturgeon. Injected.

Fig. 33. Ducts of *Lophius piscatorius*; injected. Natural size.

Figs. 34, 35. Finest ducts of Lophius and their connexion with cell-containing network; injected.

Fig. 36. Part of cell-containing network. Lophius.

Fig. 37. A similar preparation with branches of capillary vessels, *a*.

Fig. 38. Portion of a large duct (Lophius), with a branch, showing epithelium in the interior, an internal coat composed of circular muscular fibres, and an external thicker coat consisting of longitudinal fibres.

XVII. *On the Enumeration of* x-*edra having Triedral Summits, and an* (x—1)-*gonal Base. By the Rev.* THOMAS P. KIRKMAN, *M.A. Communicated by* A. CAYLEY, *F.R.S.*

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IT is easily proved that no x-edron has a face of more than $x-1$ angles, and that, if it has an $(x-1)$-gonal face, it has at least two triangular faces. The object of this paper is to determine the number of x-edra which have an $(x-1)$-gonal face, and all their summits triedral, or, which is the same thing, the number of x-acra which have an $(x-1)$-edral summit, and all their faces triangular.

We may call the $(x-1)$-gonal face the base of the x-edron. All the faces will be collateral with that base, and k of them will be triangular faces. If we suppose those k triangles to become infinitely small, in any x-edron A, we have as the result an $(x-k)$-edron B, having only triedral summits, none of whose triangular faces was a triangle of A. And it is evident that there is only one $(x-k)$-edron B from which A can be cut by sections that shall remove no edge entirely, and shall leave untouched no triangle of B. It is plain also that B cannot have more, but may have fewer, triangles than A; for if the vanishing of a triangle of A gave rise to two triangles in B, B, having two contiguous triangles, and all its summits triedral, would be a tetraedron.

If now we suppose the triangular faces of B to vanish, of which there must at least be two, there will arise a polyedron C, having only triedral summits, and fewer faces than B. In like manner C is reduced by the vanishing of its triangles to a solid of still fewer faces, and, by this continual evanescence of triangular faces, we shall finally arrive, either at a tetraedron, or a pentaedron with two triangular faces.

Hence it appears, that every x-edron having its base $(x-1)$-gonal, and all its summits triedral, can be cut from one of these two simple solids, by reversing the above process, *i. e.* by cutting away k summits of the base of a polyedron B, having $k-h$ triangular faces, so as to leave none of those $k-h$ triangles untouched. And by this process no polyedron A can be twice generated. It is to be remembered that we are all through handling no polyedra but those whose summits are all triedral.

The pentaedron on a 4-lateral base, has around that base the faces 3434. This I call a *doubly reversible* polyedron, as it exhibits in the faces about the base the series 34 repeated, and reads backwards and forwards the same.

If we cut every base summit of the tetraedron, we obtain a heptaedron having around its hexagonal base the faces 353535. This I call a *trebly reversible* heptaedron.

If we cut a summit of either triangle of the pentaedron, we obtain the system 35344 about the pentagonal base. This is a *reversible* hexaedron.

By cutting the first triangle of this with the pentagon and the second with the adjoining quadrilateral, we obtain the system 4364354, which I call an *irreversible* octaedron; it does not read backwards and forwards the same.

The double reversible 3434, by one section of its triangles, becomes 435435, a *double irreversible*, and by another, 436344, a *reversible* heptaedron.

And the system 353535 gives, by cutting the triangles in one way, the *irreversible* 437346345, and, in another way, the *trebly irreversible* 436436436.

These six varieties comprise all the polyedra that have only triedral summits. They are all *irreversible*, *reversible*, *doubly irreversible*, *doubly reversible*, *trebly irreversible*, or *trebly reversible.*

For if any polyedron exhibited in the faces about the base the fourfold repetition of any period of m faces containing k triangles, the vanishing of the $4k$ triangles would give rise to a fourfold repetition of a period of $m-k$ faces, which would contain k' triangles, k' being not more than k; and this system around the base would reduce, by the vanishing of the $4k'$ triangles in it, to a fourfold repetition of a period of $m-k-k'$ faces; and we should obtain at last a base of $4(m-\mathrm{K})$ sides, admitting no further reduction by the vanishing of triangles, *i. e.* we should obtain a pyramid having a tetraedral summit; which is impossible.

Problem.—An x-edron P being given on an $x-1$-gonal base, and having k triangular faces, it is required to determine how many $(x+k+l)$-edra can be cut from it by the removal of $k+l$ base summits, so that none of the k triangles shall remain untouched, and so that no $(x+k+l)$-edron shall be the reflected image of any other.

First, let P have about its base a series of x faces which read differently both backwards and forwards from every face, *i. e.* let it be an *irreversible* x-edron.

As no two contiguous faces about the base can be triangles, if $x>4$ (for then there would be a tetraedral summit at least, at one extremity of their common side), $k \not> \frac{1}{2}(x-1)$, and $k+l \not> x-1$.

We are bound to cut each of the k triangles once, which can be done in 2^k ways, giving 2^k different irreversible arrangements of $x+k-1$ faces. Next we have to cut l of the remaining $x-1-k$ angles about the base of P. These may be any l out of $x-1-k$, and this gives us $2^k.\frac{(x-1-k)^{l|-1}}{\overline{l+1}}$ arrangements of our $k+l$ sections. But these will not be all different arrangements. Any one of them will contain e cases of twice cut triangles of P, thus made into e pentagons, and consequently $l-e$ triangles which are not cut from triangles.

Let us suppose $e=2$, the case in which two pentagons are newly made, standing thus,

$$\ldots c353de353f\ldots$$

Of these four triangles, two were introduced in the distribution of our k sections,

and two others in that of the l sections, which may happen in 2^2 different ways. This arrangement, $c353de353f$, in which $cdef$ are supposed not to be triangles, will then be found 2^2 times with every disposition of the remaining $k-2$ triangles first cut, and with every $(l-2)$ sections of angles not in triangles, of which angles the number is $x-1-2k$. We have therefore counted the completed arrangement $\ldots c353de353f\ldots$ in the number $2^k.\frac{(x-1-k)^{l|-1}}{\lceil l+1}$,

$$2^2.2^{k-2}.\frac{(x-1-2k)^{l-2|-1}}{\lceil l-2+1} \text{ times,}$$

instead of

$$2^{k-2}.\frac{(x-1-2k)^{l-2|-1}}{\lceil l-2+1} \text{ times.}$$

The same error has been made with every value of $e \ngtr k$, and $e \ngtr l$, or $e \ngtr$ the least of k and l, and this with every set of e twice cut triangles that can be selected out of k. Hence there is an error made in excess, in $2^k.\frac{(x-1-k)^{l|-1}}{\lceil l+1}$, of

$$(2^e-1).2^{k-e}.\frac{(x-1-2k)^{l-e|-1}}{\lceil l-e+1}.\frac{k^{e|-1}}{\lceil e+1},$$

for every value of $e \ngtr$ the least of k and l; for, in supposing e twice cut triangles, we assume that $e \ngtr k$, and that $k+l-k \nless e$. The number required in the problem is thus proved to be

$$2^k.\frac{(x-k-1)^{l|-1}}{\lceil l+1}-\Sigma_e.(2^e-1)2^{k-e}.\frac{(x-1-2k)^{l-e|-1}}{\lceil l+1-e}.\frac{k^{e|-1}}{\lceil e+1},$$

for all positive values of e not greater than either k or l: which was to be found.

This function I shall denote by the symbol $ii.(x, k, l)$. It expresses the number of $(x+k+l)$-edra that can be made from any x-edron having an $(x-1)$-gonal base and k triangular faces, of which no two are contiguous, by removing $k+l$ of the summits about the base, so that no edge shall be entirely removed, and that no one of the k triangles shall remain untouched. Of course $k+l \ngtr x-1$. Its values are

$$ii(x, k, 0)=2^k; \quad ii(x, k, 1)=2^{k-1}(2x-3k-2);$$

$$ii(x, 1, l)=2.\frac{(x-2)^{l|-1}}{\lceil l+1}-\frac{(x-3)^{l-1|-1}}{\lceil l};$$

$$ii(x, k, 2)=2^{k-1}.[(x-3)(x-4)-k.(x-1-2k)]-3.2^{k-3}.k.(k-1);$$

$$ii(x, 2, l)=4.\frac{(x-3)^{l|-1}}{\lceil 1+l}-4.\frac{(x-5)^{l-1|-1}}{\lceil l}-3.\frac{(x-5)^{l-2|-1}}{\lceil l-1};$$

&c. &c.

Let $\mathrm{I}(x, k)$ be the total number of irreversible x-edra on an $\overline{x-1}$-gonal base that have k triangular faces. Then

$$\mathrm{I}(x, k)\{ii.(x, k, l)\}, \text{ part of } \mathrm{I}(x+k+l, k+l),$$

is the whole number of $(x+k+l)$-edra that can be cut, from irreversible x-edra having k triangles, so as to have $k+l$ triangles. Others can be cut to be also part of

$I(x+k+l,\ k+l)$, from other x-edra having k' triangles, by removing $k'+l'$ summits about the base, none of the k' triangles being untouched, if $k'+l'=k+l$.

Next let our subject of operation be an irreversible containing more than one period of faces about the base. If it be a doubly irreversible, the base will be $2x$-gonal, as every face will be opposite to a similar one. We have then two periods each of x faces, and if we operate on one of these so as to remove angles, leaving no triangle in the period untouched, and then repeat the operation exactly in order in the other period, we shall obtain a doubly irreversible for our result.

Let $I^2(2x+1,\ 2k)$ be the whole number of doubly irreversible $(2x+1)$-edra on a $2x$-gonal base, having $2k$ triangles. Then $ii.\left(x+1,\ k,\ \frac{l}{2}\right)$ is the number of ways in which we can remove $k+\frac{l}{2}$ summits from one period, and therefore

$$I^2(2x+1,\ 2k)ii\left(x+1,\ k,\ \frac{l}{2}\right),\ \text{part of } I^2(2x+1+2k+l,\ 2k+l),$$

is the whole number of doubly irreversibles having $2k+l$ triangles, and a $(2x+2k+l)$-gonal base, that can be cut from all the $I^2(2x+1,\ 2k)$ polyedra before us.

If l is not an even number, $ii\left(x+1,\ k,\ \frac{l}{2}\right)$ is to be considered nothing; for it is impossible to remove a fractional number of summits.

We can cut also from these $I^2.(2x+1, 2k)$ doubly irreversibles a number of singly irreversibles. If each of these doubly irreversibles were single, it would give rise to $ii(2x+1, 2k, l)$ singly irreversibles; but the double character of the subject of operation causes every method of removing $2k+l$ angles, which is not alike in both periods of the subject, to appear twice in the number just written. That is, we are to subtract from this number all the doubly irreversibles that can be made, and take half the remainder, which is, after division,

$$\tfrac{1}{2}\left\{ii(2x+1,\ 2k,\ l)-ii\left(x+1,\ k,\ \frac{l}{2}\right)\right\}.$$

The second term of this is zero when l is odd. We obtain thus for the number of singly irreversibles that can be cut to have $2k+l$ triangles from all the $I^2(2x+1,\ 2k)$ under consideration,

$$I^2.(2x+1,\ 2k).\tfrac{1}{2}\left\{ii(2x+1,\ 2k,\ l)-ii\left(x+1,\ k,\ \frac{l}{2}\right)\right\},$$

a part of $I(2x+1+2k+l,\ 2k+l)$.

Next let us consider the operations that can be effected on triply irreversible $(3x+1)$-edra having $3k$ triangles. Let their number be $I^3(3x+1,\ 3k)$. It is easily proved by a repetition of the preceding argument, that $ii\left(x+1,\ k,\ \frac{l}{3}\right)$ is the number of triply irreversibles that can be cut from each of them, and that

$$\tfrac{1}{3}\left\{ii(3x+1,\ 3k,\ l)-ii\left(x+1,\ k,\ \frac{l}{3}\right)\right\}$$

is that of the singly irreversibles, where the appearance of a fraction in the function ii, reduces it, as it always must, to zero. That is, we obtain

$$\mathrm{I}^3(3x+1,\ 3k).ii\left(x+1,\ k,\ \frac{l}{3}\right),\quad \text{as } \mathrm{I}^3.(3x+3k+l+1,\ 3k+l),$$

and

$$\mathrm{I}^3(3x+1,3k)\tfrac{1}{3}\left\{ii(3x+1,\ 3k,\ l)-ii\left(x+1,\ k,\ \frac{l}{3}\right)\right\}$$

as a portion of

$$\mathrm{I}(3x+3k+l+1,\ 3k+l).$$

From an irreversible no reversible can be cut by this removing of summits; for as the summits of triangles of a reversible correspond in pairs, the arrangement of faces about the base will still be reversible, if all the triangles are supposed to vanish.

Let us now operate on a reversible polyedron, whose faces about the base read backwards and forwards alike.

There will be a certain period *abc....klm* reversed, in one of the three ways,

abc....klmlk....cb,

abc....klmmlk....cb,

or *abc....klmmlk....cba*.

There is in any of these what may be called an axis of reversion, which in the first passes through the faces a and m, in the second through a and between two m's, in the third between two a's and between two m's.

It is evident that the number of triangles about the base of a reversible cannot be odd, unless the axis of reversion passes through a triangle; as all faces recur in order reversed, through which that axis does not pass; and the base must be $(2x+1)$-gonal, if the axis passes through one face only. The third case, of an axis of reversion passing through no face, does not occur when all the summits are triedral.

First let the base be even, and let the number of triangles be even also; we have to consider the operations practicable upon a $(2x+1)$-edron R reversible, with $2k$ triangles. Some $(2x+1+2k+l)$-edra can be cut from it reversible, and some irreversible, by the removing of $2k+l$ summits of the base, leaving none of the $2k$ triangles untouched.

A reversible so cut will have on either side of its axis of reversion half the $2k+l$ added triangles, unless it passes through an added triangle, in which case it will have on either side $\frac{1}{2}(2k+l-1)$ of them.

Let $l=2l'$; then the number of possible operations on one side of the axis of reversion, which exhibits x summits, is $ii(x,\ k,\ l')$; each of which gives by repeating it backwards one of our reversible $(2x+1+2k+2l')$-edra. As the axis does not pass through a summit, l cannot be odd, for the added triangles are all in pairs.

If then $\mathrm{R}(2x+1,\ 2k)$ be the total number of reversible $(2x+1)$-edra having $2k$ triangles, we obtain

$$\mathrm{R}(2x+1,2k).ii\left(x,\ k,\ \frac{l}{2}\right),\ \text{part of } \mathrm{R}(2x+1+2k+l,\ 2k+l).$$

Now take the reversible $(2x+1)$-edron R′ having $2k+1$ triangles; the axis of reversion passes through one of them. As this triangle cannot be untouched, it must be twice touched, so that we have only $(2k+1+l-2)$ other sections to make, one half of these on the $x-1$ summits on one side of the axis, neglecting the summit of the central triangle. The possible operations are

$$ii\left(x, k, \frac{l-2}{2}\right);$$

or, if $R(2x+1, 2k+1)$ be the total number of reversible $(2x+1)$-edra having $2k+1$ triangles, we obtain

$$R(2x+1, 2k+1)ii\left(x, k, \frac{l-2}{2}\right), \text{ part of } R(2x+2+2k+l, 2k+1+l).$$

We take now a reversible R″ having a $(2x-1)$-gonal base, and $2k$ triangles.

The axis of reversion passes through a summit and through a face, which we shall suppose to be not a triangle. It is not difficult to prove that it can be no triangle if $2x-1>3$, all the summits being triedral.

We have on either side of the axis $\frac{1}{2}(2x-2)$ summits, that in the axis being neglected, and half the $2k$ triangles. We have to distribute $k+\frac{l}{2}$ sections on these $x-1$ summits. The number of ways to do it is

$$ii\left(x, k, \frac{l}{2}\right),$$

which requires l to be even; or if l be odd, we may cut the summit in the axis, and distribute $k+\frac{l-1}{2}$ sections on the $x-1$ summits, giving $ii\left(x, k, \frac{l-1}{2}\right)$. These operations reversed on the other side of the axis give us all the possible results.

If then $R(2x, 2k)$ be the total number of reversible $2x$-edra having $2k$ triangles, we can cut from all these

$$R(2x, 2k)\left\{ii\left(x, k, \frac{l}{2}\right)+ii\left(x, k, \frac{l-1}{2}\right)\right\}$$

$(2x+2k+l)$-edra reversible, having $2k+l$ triangles. One of the terms in the second factor is always zero. The polyedra so cut are a portion of $R(2x+2k+l, 2k+l)$.

When $k+l=x-1$, all the angles about the base of the reversible with k triangles are cut, and the result is of necessity reversible.

But if $k+l<x-1$, some of the results of $k+l$ sections will be irreversible. The whole number of results, if we treated the n-edron R or R′ or R″ as irreversible, would be $ii(n, k, l)$, but these are not all different.

They will all, except the reversible ones, have a different order on the two sides of the axis of reversion; and each irreversible will occur twice, the second time reverted by exchanging the arrangements of the two sides of the axis, so as to make a polyedron and its reflected image. As we are not to count these reflexions, we have to

subtract from $ii(n, k, l)$ all the possible reversible results, and divide the remainder by two. That is, by what precedes, we obtain from R, R′ and R″

$$\tfrac{1}{2}\left\{ii(2x+1, 2k, l)-ii\left(x, k, \frac{l}{2}\right)\right\},$$

$$\tfrac{1}{2}\left\{ii(2x+1, 2k+1, l)-ii\left(x, k, \frac{l-2}{2}\right)\right\},$$

$$\tfrac{1}{2}\left\{ii(2x, 2k, l)-ii\left(x, k, \frac{l}{2}\right)-ii\left(x, k, \frac{l-1}{2}\right)\right\}.$$

It is to be understood in these formulæ that

$2k+l<2x$ in the first,

$2k+1+l<2x$ in the second,

$2k+l<2x-1$ in the third.

If we multiply the first by $R(2x+1, 2k)$, the second by $R(2x+1, 2k+1)$, and the third by $R(2x, 2k)$, we obtain the corresponding portions of $I(2x+1+2k+l, 2k+l)$, $I(2x+2+2k+l, 2k+1+l)$, and $I(2x+2k+l, 2k+l)$.

Let us next operate on a doubly reversible $(4x+1)$-edron, with $4k$ triangles. All these are cut from the pentaedron 3434, and by the addition of an even number of triangles in each period. The operations by which doubly reversibles are cut from a doubly reversible are simply those whereby reversibles are cut from the reversible period of $2x$ summits, containing $2k$ triangles, being in number $ii\left(x+1, k, \frac{l}{4}\right)$ by what has preceded. Or, if $R^2(4x+1, 4k)$ be the number of $(4x+1)$-edra doubly reversible, with $4k$ triangles,

$$R^2(4x+1, 4k).ii.\left(x+1, k, \frac{l}{4}\right)=R^2(4(x+k)+l+1, 4k+l),$$

there being no more; for one of these latter can be cut from nothing but a doubly reversible with $4k+l$ faces fewer.

From the same $(4x+1)$-edra can be cut doubly irreversibles, namely so many as the irreversibles producible from sections of one period, or from a reversible $(2x+1)$-edron with $2k$ triangles. This number is, as just proved,

$$\tfrac{1}{2}\left\{ii\left(2x+1, 2k, \frac{l}{2}\right)-ii\left(x+1, k, \frac{l}{4}\right)\right\},$$

which multiplied by $R^2.(4x+1, 4k)$, constitutes a portion of $I^2.(4x+4k+l+1, 4k+l)$.

As a doubly reversible is also a reversible, reversibles can be cut from it. It is to be observed that, as a reversible, it has two axes of reversion, as indeed every $2m$-ly reversible has. Thus the enneaedron 35363536 has an axis through the two pentagons and another through the hexagons. If we operate on one side of the axis for irreversible results, and revert our operations on the other side, we obtain reversibles. The number of such results in either position of the axis of reversion is that of the irreversibles producible by $\left(2k+\frac{l}{2}\right)$ sections of a reversible $(2x+1)$-gon with $2k$

triangles, or

$$\tfrac{1}{2}\left\{ii\left(2x+1,\ 2k,\ \frac{l}{2}\right)-ii\left(x+1,\ k,\ \frac{l}{4}\right)\right\};$$

whence we obtain, from both positions of the axis,

$$R^2.(4x+1,\ 4k).\left\{ii\left(2x+1,\ 2k,\ \frac{l}{2}\right)-ii\left(x+1,\ k,\ \frac{l}{4}\right)\right\}$$

as part of $\quad R(4x+1+4k+l,\ 4k+l)$.

If we forget for a moment the character of one of these $R^2(4m+1,\ 4k)$, and treat it as an irreversible, we obtain by $4k+l$ sections $ii(4m+1,\ 4k,\ l)$ results. Of these the doubly reversibles can occur only once, every doubly irreversible will occur twice, in one result as the reflexion of the other; every reversible will occur twice, the operations in the first period in the second result being those of the second period in the first; and every irreversible will occur four times, twice by the exchange of the operations on the first period for those on the second, and twice again by the reversion of all the operations, producing reflected images of two preceding results.

That is, if we subtract from $ii(4x+1,\ 4k,\ l)$ all the doubly reversibles, twice the reversibles, and twice the doubly reversibles that can be cut from a doubly reversible $(4x+1)$-gon having $4k$ triangles, by $4k+l$ sections, there remains four times the number of irreversibles that can be cut from it, by $4k+l$ sections.

This remainder, divided by 4, is

$$\begin{aligned}&\tfrac{1}{4}\Big[ii(4x+1,\ 4k,\ l)-ii\left(x+1,\ k,\ \frac{l}{4}\right)\\&\qquad-2\left\{ii\left(2x+1,\ 2k,\ \frac{l}{2}\right)-ii\left(x+1,\ k,\ \frac{l}{4}\right)\right\}\\&\qquad-\left\{ii\left(2x+1,\ 2k,\ \frac{l}{2}\right)-ii\left(x+1,\ k,\ \frac{l}{4}\right)\right\}\Big]\\&=\tfrac{1}{4}\left[ii(4x+1,\ 4k,\ l)+2ii\left(x+1,\ k,\ \frac{l}{4}\right)-3ii\left(2x+1,\ 2k,\ \frac{l}{2}\right)\right],\end{aligned}$$

which, multiplied by $R^2(4x+1,\ 4k)$, is to be added to $I(4x+1+4k+l,\ 4k+l)$.

It remains that we handle now trebly reversible $(6x+1)$-edra, having $6k$ triangles.

If $x>1$, the number of triangles in a triply reversible cannot be less than $6k$, as they are all cut from the heptaedron 535353, by an even number of sections in every period. By operating on one reversible period of $2x$ summits and $2k$ triangles for reversibles by $\frac{1}{3}(6k+3l)$ sections, we obtain all the triply reversibles. The number of these so found is $ii.\left(x+1,\ k,\ \frac{l}{2}\right)$, giving so many $(6x+1+6k+3l)$-edra triply reversible with $6k+3l$ triangles, for each subject; and in all

$$R^3(6x+1,\ 6k)\left(ii\left(x+1,\ k,\ \frac{l}{2}\right)\right)=R^3(6k+1+6k+3l,\ 6k+3l).$$

The number of triply irreversibles is equal to that of the irreversibles producible

from a reversible period of $2x$ summits and $2k$ triangles by $\frac{1}{3}(6k+3l)$ sections, and is, by what precedes,

$$\tfrac{1}{2}\left\{ii(2x+1,\ 2k,\ l)-ii\left(x+1,\ k,\ \frac{l}{2}\right)\right\},$$

from each subject of operation. This, multiplied by $R^3(6x+1, 6k)$, is to be added to $I^3(6x+1+6k+3l,\ 6k+3l)$.

The reversibles obtainable from a triply irreversible are found only about one axis of reversion. Thus 735373537353 has only one axis through a heptagon and a pentagon. We are to cut by $\frac{1}{2}(6k+2l)$ sections on one side of this axis all possible irreversibles from a reversible system of $3x$ summits and $3k$ triangles. These results reverted on the other side of the axis, will give all possible reversibles. Among these will be all triply reversible $(6x+1+6k+2l)$-edra, with $6k+2l$ triangles, for these are all reversible; and none of these can occur more than once. We have these to subtract from our results, leaving

$$ii(3x+1,\ 3k,\ l)-ii\left(x+1,\ k,\ \frac{l}{3}\right)$$

reversibles from every subject; which number, multiplied by $R^3(6x+1,\ 6k)$, forms part of $R(6x+1+6k+2l,\ 6k+2l)$.

If we treated a triply reversible as an irreversible by $6k+l$ sections, we should obtain

$$ii(6x+1,\ 6k,\ l) \text{ results.}$$

Among these every triply reversible is found once; every triply irreversible twice, one place showing the reflected image of the other; every reversible three times, each time the same operations commencing at a different period; and every irreversible six times, being begun both backwards and forwards in three different periods. If then we subtract from $ii(6x+1,\ 6k,\ l)$ every triply reversible, twice the triply irrevisibles, and thrice the reversibles that can be made by $6k+l$ sections of a triply reversible $(6x+1)$-edron having $6k$ triangles, there will remain six times the number of irreversibles that can be so cut from the same. This remainder, after division by 6, is

$$\tfrac{1}{6}\Big\{ii(6x+1,\ 6k,\ l)-ii\left(x+1,\ k,\ \frac{l}{6}\right)-ii\left(2x+1,\ 2k,\ \frac{l}{3}\right)+ii\left(x+1,\ k,\ \frac{l}{6}\right)$$
$$-3ii\left(3x+1,\ 3k,\ \frac{l}{2}\right)+3ii\left(x+1,\ k,\ \frac{l}{6}\right)\Big\}$$
$$=\tfrac{1}{6}\left\{ii(6x+1,\ 6k,\ l)+3ii\left(x+1,\ k,\ \frac{l}{6}\right)-ii\left(2x+1,\ 2k,\ \frac{l}{3}\right)-3ii\left(3x+1,\ 3k,\ \frac{l}{2}\right)\right\},$$

which, multiplied by $R^3(6x+1,\ 6k)$, forms part of $I(6x+1+6k+l,\ 6k+l)$.

It is most convenient to treat the case of the triply reversible having only three triangles by itself. From this heptaedron can be cut one triply reversible by cutting every summit of the base. One triply irreversible only can be made, a decaedron, by cutting each triangle once. Two reversibles can be cut, by four sections,

differing in the manner of cutting the triangle through which the axis of reversion does not pass, giving two reversible 11-edra. Three irreversibles can be cut from it, one by three, another by four, a third by five sections, as is evident on a moment's consideration, giving a 10-edron, 11-edron, and 12-edron.

We can now collect into one group all the formulæ above deduced, which contain the complete solution of our problem; to find the number of these x-edra on an $(x-1)$-gonal base. There is no ambiguity in the case of two $(x-1)$-gonal faces, for the figure is always identical with itself whichever be considered the base, and can have only two triangles.

Let $\mathrm{II}(x, k, l)$ or $\mathrm{IR}(x, k, l)$ be the number of irreversible $(x+k+l)$-edra that can be cut to have $(k+l)$ triangles from an irreversible or reversible x-edron having k triangles, the capital on the right denoting the subject of operation: $(k+l)<x$.

$$\mathrm{II}(x, k, l) = ii(x, k, l),$$

$$\mathrm{II}^2(2x+1, 2k, l) = \tfrac{1}{2}\left\{ii(2x+1, 2k, l)-ii\left(x+1, k, \frac{l}{2}\right)\right\},$$

$$\mathrm{II}^3(3x+1, 3k, l) = \tfrac{1}{3}\left\{ii(3x+1, 3k, l)-ii\left(x+1, k, \frac{l}{3}\right)\right\},$$

$$\mathrm{I}^2\mathrm{I}^2(2x+1, 2k, l) = ii\left(x+1, k, \frac{l}{2}\right),$$

$$\mathrm{RR}(2x+1, 2k, l) = ii\left(x+1, k, \frac{l}{2}\right),$$

$$\mathrm{RR}(2x+1, 2k+1, l) = ii\left(x, k, \frac{l-2}{2}\right),$$

$$\mathrm{RR}(2x, 2k, l) = ii\left(x, k, \frac{l}{2}\right)+ii\left(x, k, \frac{l-1}{2}\right),$$

$$\mathrm{IR}(2x+1, 2k, l) = \tfrac{1}{2}\left\{ii(2x+1, 2k, l)-ii\left(x+1, k, \frac{l}{2}\right)\right\},$$

$$\mathrm{IR}(2x+1, 2k+1, l) = \tfrac{1}{2}\left\{ii(2x+1, 2k+1, l)-ii\left(x, k, \frac{l-2}{2}\right)\right\},$$

$$\mathrm{IR}(2x, 2k, l) = \tfrac{1}{2}\left\{ii(2x, 2k, l)-ii\left(x, k, \frac{l}{2}\right)-ii\left(x, k, \frac{l-1}{2}\right)\right\};$$

$$\mathrm{R}^2\mathrm{R}^2(4x+1, 4k, l) = ii(x+1, k, \tfrac{1}{4}l),$$

$$\mathrm{I}^2\mathrm{R}^2(4x+1, 4k, l) = \tfrac{1}{2}\{ii(2x+1, 2k, \tfrac{1}{2}l)-ii(x+1, k, \tfrac{1}{4}l)\},$$

$$\mathrm{RR}^2(4x+1, 4k, l) = ii(2x+1, 2k, \tfrac{1}{2}l)-ii(x+1, k, \tfrac{1}{4}l),$$

$$\mathrm{IR}^2(4x+1, 4k, l) = \tfrac{1}{4}\left[ii(4x+1, 4k, l)+2ii\left(x+1, k, \frac{l}{4}\right)-3ii(2x+1, 2k, \tfrac{1}{2}l)\right];$$

$$\mathrm{R}^3\mathrm{R}^3(6x+1, 6k, l) = ii(x+1, k, \tfrac{1}{6}l), \quad \mathrm{R}^3\mathrm{R}^3(7, 3, 3)=1,$$

$$\mathrm{I}^3\mathrm{R}^3(6x+1, 6k, l) = \tfrac{1}{2}\{ii(2x+1, 2k, \tfrac{1}{3}l)-ii(x+1, k, \tfrac{1}{6}l)\},$$

$$\begin{aligned}
\mathrm{RR}^3(6x+1,\, 6k,\, l) &= ii(3x+1,\, 3k,\, \tfrac{1}{2}l) - ii(x+1,\, k,\, \tfrac{1}{6}l), \quad \mathrm{RR}^3(7,\, 3,\, 1)=2,\\
\mathrm{IR}^3(6x+1,\, 6k,\, l) &= \tfrac{1}{6}\{ii(6x+1,\, 6k,\, l)+3ii(x+1,\, k,\, \tfrac{1}{6}l)\\
&\qquad -ii(2x+1,\, 2k,\, \tfrac{1}{3}l)-3ii(3x+1,\, 3k,\, \tfrac{1}{2}l)\},\\
\mathrm{IR}^3(7,\, 3,\, 2) &= \mathrm{IR}^3(7,\, 3,\, 1)=\mathrm{IR}^3(7,\, 3,\, 0)=1:\\
\mathrm{I}^n\mathrm{R}^m(x+1,\, k,\, x-k) &= 0,
\end{aligned}$$

for if all the summits of a reversible are cut, the result is reversible.

In $\mathrm{I}^n\mathrm{I}^m(x, k, l)$, $\mathrm{I}^n\mathrm{R}^m(x, k, l)$, $\mathrm{R}^n\mathrm{R}^m(x, k, l)$, the second capital marks the character and multiplicity of the x-edra having k triangles, from which are cut the $(x+k+l)$-edra having $k+l$ triangles, of which the character and multiplicity are denoted by the first.

To show the use of these equations, we can easily by trial verify the following :—

$$\begin{array}{lll}
\mathrm{R}^3(4,\, 3)=1, & \mathrm{R}^2(5,\, 2)=1, & \mathrm{R}(6,\, 2)=1,\\
\mathrm{R}(7,\, 2)=1, & \mathrm{R}^3(7,\, 3)=1, & \mathrm{I}^2(7,\, 2)=1,\\
\mathrm{R}(8,\, 2)=2, & \mathrm{I}(8,\, 2)=1, & \mathrm{I}(8,\, 3)=1,\\
\mathrm{R}(9,\, 3)=2, & \mathrm{R}(9,\, 2)=2, & \mathrm{R}^2(9,\, 4)=1,\\
\mathrm{I}(9,\, 3)=3, & \mathrm{I}(9,\, 2)=2, & \mathrm{I}^2(9,\, 2)=2\,;
\end{array}$$

then to find the decaedra on a 9-gonal base, with only triedral summits, we first write down the classes,

$$\begin{aligned}
&\mathrm{R}(10,\, 2)=\mathrm{R}(8,\, 2).\mathrm{RR}(8,\, 2,\, 0)\,;\\
&\mathrm{I}(10,\, 2)=\mathrm{R}(8,\, 2).\mathrm{IR}(8,\, 2,\, 0)+\mathrm{I}(8,\, 2).\mathrm{II}(8,\, 2,\, 0)\,;\\
&\mathrm{I}(10,\, 3)=\mathrm{R}^3(7,\, 3).\mathrm{IR}^3(7,\, 3,\, 0)+\mathrm{R}(7,\, 2).\mathrm{IR}(7,\, 2,\, 1)+\mathrm{I}^2(7,\, 2).\mathrm{II}^2(7,\, 2,\, 1)\,;\\
&\mathrm{I}^3(10,\, 3)=\mathrm{R}^3(7,\, 3).\mathrm{I}^3\mathrm{R}^3(7,\, 3,\, 0)\,;\\
&\mathrm{R}(10,\, 4)=\mathrm{R}(6,\, 2).\mathrm{RR}(6,\, 2,\, 2)\,;\\
&\mathrm{I}(10,\, 4)=\mathrm{R}(6,\, 2).\mathrm{IR}(6,\, 2,\, 2).
\end{aligned}$$

That is, by what precedes,

$$\begin{aligned}
\mathrm{R}(10,\, 2)&=2.ii(4,\, 1,\, 0)=2.2=4\,;\\
\mathrm{I}(10,\, 2)&=2.\tfrac{1}{2}\{ii(8,\, 2,\, 0)-ii(4,\, 1,\, 0)\}\\
&=2.\tfrac{1}{2}\{4-2\}=2\,;\\
\mathrm{I}(10,\, 3)&=1.\tfrac{1}{6}\{ii(7,\, 3,\, 0)-ii(3,\, 1,\, 0)\}+1.\tfrac{1}{2}ii(7,\, 2,\, 1)+1.\tfrac{1}{2}ii(7,\, 2,\, 1)\\
&=\tfrac{1}{6}\{8-2\}+\tfrac{12}{2}+\tfrac{12}{2}=13\,;\\
\mathrm{I}^3(10,\, 3)&=1.1=1\,;\\
\mathrm{R}(10,\, 4)&=1.ii(3,\, 1,\, 1)=1.(6-5)=1\,;\\
\mathrm{I}(10,\, 4)&=1.\tfrac{1}{2}\{ii(6,\, 2,\, 2)-ii(3,\, 1,\, 1)\}=1.\tfrac{1}{2}\{72-108+41-1\}=2.
\end{aligned}$$

Next to find the hendecaedra, we write down

$I(11, 5) = R(6, 2).IR(6, 2, 3) = 0;$

$R(11, 5) = R(6, 2).RR(6, 2, 3);$

$R(11, 4) = R(7, 2).RR(7, 2, 2) + R^3(7, 3).RR^3(7, 3, 1);$

$I(11, 4) = R(7, 2).IR(7, 2, 2) + I^2(7, 2).II^2(7, 2, 2) + R^3(7, 3).IR^3(7, 3, 1);$

$I^2(11, 4) = I^2(7, 2).I^2I^2(7, 2, 2);$

$R(11, 3) = R(8, 2).RR(8, 2, 1);$

$I(11, 3) = R(8, 2).IR(8, 2, 1) + I(8, 2).II(8, 2, 1) + I(8, 3).II(8, 3, 0);$

$R(11, 2) = R(9, 2).RR(9, 2, 0);$

$I(11, 2) = R(9, 2).IR(9, 2, 0) + I(9, 2).II(9, 2, 0) + I^2(9, 2).II^2(9, 2, 0);$

$I^2(11, 2) = I^2(9, 2).I^2I^2(9, 2, 0).$

That is—

$$
\begin{array}{lr}
R(11, 5) = 1.ii(3, 1, 1) & = 1; \\
R(11, 4) = 1.ii(4, 1, 1) + 1.2 & = 5; \\
I(11, 4) = 1.\tfrac{1}{2}\{ii(7, 2, 2) - ii(4, 1, 1)\} & \\
\qquad + 1.\tfrac{1}{2}\{ii(7, 2, 2) - ii(4, 1, 1)\} + 1.1 & = 11; \\
I^2(11, 4) = 1.ii(4, 1, 1) & = 3; \\
R(11, 3) = 2.ii(4, 1, 0) & = 4; \\
I(11, 3) = 2.\tfrac{1}{2}\{ii(8, 2, 1) - ii(4, 1, 0)\} & \\
\qquad + 1.ii(8, 2, 1) + 1.ii(8, 3, 0) & \\
\qquad = 2.7 + 1.16 + 1.8 & = 38; \\
R(11, 2) = 2.ii(5, 1, 0) = 2.2 & = 4; \\
I(11, 2) = 2.\tfrac{1}{2}\{ii(9, 2, 0) - ii(5, 1, 0)\} + 2.ii(9, 2, 0) & \\
\qquad + 2.\tfrac{1}{2}\{ii(9, 2, 0) - ii(5, 1, 0)\} = 2.1 + 2.4 + 2.1 & = 12; \\
I^2(11, 2) = 2.ii(5, 1, 0) & = 4.
\end{array}
$$

As a verification, it may be worth while to write down these eighty-two 11-edra, to show the faces in order about the 10-gonal base.

$R(11, 5)$ is 3537353636;

$R(11, 4)$ are 5383535453, 4383453635, 3464363636, 4373537345, 3463536437;

$I(11, 4)$ are 4438353635, 6346436363, 5437436363, 3543835354, 3637374354, 6353653463, 6353644373, 6346354373, 5353736345, 6353644373, 4437363536;

$I^2(11, 4)$ are 5353653536, 6346363463, 5437354373;

R(11, 3) are 4439344535, 5347435535, 4355534636, 3445443737;

I(11, 3) are 5444439353, 5734446353, 6634455353, 7534545353, 8435445353, 5463447353, 5553456353, 5643546353, 5436444383, 8436354443, 5436634463, 6436356344, 5436453473, 7346354534, 5436543563, 6436355435, 5439344453, 5438435453, 5437443653, 5437535453, 5436444383, 5436383444, 5436354435, 5436374354, 5443744373, 5534653463, 6443743643, 5443743734, 5443834634, 6443834543, 6534643643, 5534734634, 5534643734, 6534734543, 5443753463, 5443843553, 5534653463, 5534644373;

R(11, 2) are 4534843544, 44431034444, 5435653454, 6344644364;

I(11, 2) are 4534934444, 6344653454, 6734445443, 6643545443, 5734445534, 5643545534, 5834444543, 4834444634, 5743544543, 4743544634, 6444374534, 6553455443;

$I^2(11, 2)$ are 5553455534, 7444374443, 6544365443, 6453464534.

The dodecaedra are found by rather less calculation than the hendecaedra, forming only eight classes, as follows:—

$I(12,5) = R^2(7,3).IR^3(7,3,2)+I^2(7,2).II^2(7,2,3)+R(7,2).IR(7,2,3)$,

$R(12,5)=R(7,2)RR(7,2,3)+R^3(7,3).RR^3(7,3,2)$,

$I(12,4) =I(8,3)II(8,3,1)+I(8,2).II(8,2,2)+R(8,2).IR(8,2,2)$,

$R(12,4)=R(8,2).RR(8,2,2)$,

$I(12,3) =I(9,3).II(9,3,0)+I(9,2)II(9,2,1)+R(9,3).IR(9,3,0)+R(9,2).IR(9,2,1)$,

$R(12,3)=R(9,3).RR(9,3,0)+R(9,2)RR(9,2,1)$,

$I(12,2) =I(10,2)II(10,2,0)+R(10,2).IR(10,2,0)$,

$R(12,2)=R(10,2).RR(10,2,0)$.

These are, by what precedes, all given numbers; and, by continuing the process, we can finally obtain all the x-edra on an $(x-1)$-gonal base, numbered in their proper classes, which have only triedral summits.

I have generalized the expressions $I^nI^m(x, k, l)$, $R^nR^m(x, k, l)$, $I^nR^m(x, k, l)$, which the theory requires for enumerating all the x-edra having an $(x-1)$-gonal base, and any summits whatever; but the formulæ are not worth producing. The number of distinctions to be made is too great to be of any ready use. If the x-edra having an $(x-1)$-gonal base were classed and enumerated according to their summits, it would be possible to count all the $(x+h)$-edra on the same $(x-1)$-gonal base, by removing summits not in the base, thus producing *crown-faces*, and by the vanishing of edges about the crowns and base, thus producing faces contiguous, but not collateral. That is, it would be possible to enumerate and classify the N-edra.

XVIII. *On the Representation of Polyedra. By the Rev.* THOMAS P. KIRKMAN, *M.A. Communicated by* A. CAYLEY, *F.R.S.*

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To every p-edral q-acron, or solid having p faces and q summits, corresponds a q-edral p-acron, whose q faces have the same order (of succession) and rank (as to number of edges) with the q summits, and whose p summits correspond in the same way to the p faces, of the q-acron.

When $p=q$, the corresponding pair will sometimes be identical figures, as to the number, rank and arrangement of their faces and summits; whilst at other times, as will always be the case if p is not $=q$, the two figures will differ. When they differ they may be called a *sympolar pair* of *heteropolars*, or simply a *sympolar pair*; when they are the same figure, it may be called *an autopolar polyedron.*

An elegant method of representing an immense number of sympolar pairs and autopolars, may be deduced from the property enunciated in the theorems following, A. and B.

Def.—Any most-angled face of a polyedron being taken as its base, the angles of the base may be called *base-summits,* the remaining angles of the faces either collateral or synacral with the base may be termed *wall-summits,* and all summits lying only in faces neither collateral nor synacral with the base, we may name *crown-summits.*

If a polyedron has base-summits $a_1a_2a_3\ldots$, wall-summits $b_1b_2b_3\ldots$, and crown-summits besides, the latter may consist of a system $c_1c_2c_3\ldots$ lying in faces that contain also some of $b_1b_2b_3\ldots$, and an interior system $d_1d_2d_3\ldots$ not lying in faces containing any of $b_1b_2b_3\ldots$. The summits $d_1d_2d_3\ldots$ may be looked at as wall-summits referred to $c_1c_2c_3\ldots$ as base-summits, and as crown-summits referred to $b_1b_2b_3\ldots$ as base-summits. And there may be any number of systems of crown-summits interior to $d_1d_2d_3\ldots$, as $e_1e_2e_3\ldots$ leading up to $f_1f_2f_3\ldots$ &c.

A. If in any q-acron there are either no crown-summits, or if from each of the wall-summits $b_1b_2b_3\ldots$ there passes an edge to one of the crown-summits $c_1c_2c_3\ldots$, and from each of $c_1c_2c_3\ldots$ an edge to one of $d_1d_2d_3\ldots$, and from each of $d_1d_2d_3\ldots$ an edge to one of $e_1e_2e_3\ldots$, and so on, the q summits of the q-acron are the angles of a closed q-gon, whose q sides are all edges of the q-acron.

B. If in any p-edron there are either no faces $\gamma_1\gamma_2\gamma_3\ldots$, of which none has any summit collateral or synacral with the most-angled summit S of the p-edron, or if each of the faces $\beta_1\beta_2\beta_3\ldots$ about S is collateral with some one of $\gamma_1\gamma_2\gamma_3\ldots$, a closed

polygon of p sides can be drawn on the faces of the figure, so as to have a side on every face, and to pass through no summit.

There is no use in perplexing further the enunciation of B, as its truth follows from, and its extent is parallel with, that of A. Nor is it worth while to trouble the reader with demonstrations of these properties, since they are not properties of all polyedra, a negative which may however be worth the proving. It may suffice, and may perhaps be useful, to show the connexion between these closed polygons and a pleasing mode of representation.

Let us suppose that in any p-edral q-acron we have traced the closed p-gon through the faces, and the closed q-gon through the summits, and let the edges of the p-gon be numbered in order $1\ 2 \ldots p$, and the angles of the q-gon $1\ 2 \ldots q$. Thus are all the faces and summits of the p-edral q-acron numbered, in the circles $1\ 2\ 3 \ldots p 12..$, and $1\ 2\ 3 \ldots q 12..$

Any edge of the figure may be read *abcd*, *a* and *c* being its left and right summits, and *b* and *d* its upper and lower faces, or *cdab*, which is the same thing, turning the figure about. In the same face *d* we read, passing towards the right from *c* to the summit *e*, the edge *cfed* or *edcf*, in the two faces *d* and *f*. Thus it appears that, in the reading of the edges, any consecutive and external duad, as *cd* in *abcd*, will occur reversed and internal, as *dc* in *edcf*, and *vice versâ*. We can thus represent the $p+q+2$ edges of the p-edral q-acron by as many quadruplets, so formed, that every contiguous internal or external duad shall occur again reversed as an external or internal duad; the quadruplets being all read from left to right.

Let xyx_1y_1, $x_2y_2x_3y_3$ be two of these quadruplets. The former is an edge at the xth summit and in the yth face, and also at the x_1th summit and in the y_1th face. The like is conceived of the latter. At the points xy and x_1y_1 referred to right axes, write *a*, at x_1y_1 and x_2y_2 write *b*, and so on with all the $p+q+2$ edges *abcd*....

The result is a paradigm of the figure and its sympolar. The horizontal multiplets will be the faces, denoted by their edges, the vertical ones the summits so denoted, or *vice versâ*. The edges in summit or face will stand in their true order. For the closed q-gon through the summits, if it leaves any face before it has completed the circuit thereof, must return to it to complete that in the same direction; otherwise it would cross its own path and pass more than once through one or more summits, which is impossible, as it has only q sides.

Every pyramid is autopolar. If the base be $(2n+1)$-gonal, the system of edges is denoted by $4n+2$ quadruplets in pairs of the form *abcd*, *dcba*; or as well by as many in pairs of the form *abcd*, *dabc*. Either edge in any pair lies between the poles of the faces through the other. I call these two edges (*a*A) *a gamic pair*, and either is the *gamic* of the other. Thus the pentagonal pyramid is represented by either of the systems,

a 1356	*b* 2416	*c* 3526	*d* 4136	*e* 5246
A 6531,	B 6142,	C 6253,	D 6314,	E 6425,

a 1126 b 2536 c 3446 d 4356 e 5226
A 6112, B 6253, C 6344, D 6435, E 6522,

giving the paradigms

.	.	A	d	.	B
.	.	.	B	e	C
a	.	.	.	C	D
D	b	.	.	.	E
.	E	c	.	.	A
b	c	d	e	a	.

a	E	.	.	.	A
A	.	.	.	e	B
.	.	.	d	B	C
.	.	c	C	.	D
.	b	D	.	.	E
e	a	b	c	d	.

The first arrangement may be so folded that A shall fall upon a, B upon b, &c. The second cannot. Every e-gonal face is polar to an e-edral summit, the face and summit showing letters on their edges of like names and succession. And every contiguous duad, as aD, in a horizontal line, is contiguous also in a vertical line, if we observe that the extremes of any multiplet are a contiguous duad. This shows that any angle aD in a face is also an angle in a summit, a property which the paradigm of course always has, whether of autopolar or heteropolar figure. It is observable, that in the first arrangement no edge a meets its gamic A in a point; whilst in the second we see the angles aA and cC, which may be denominated *nodal angles*, in the *nodal face* aEA, at the *nodal summit* aAe.

The $2m$-gonal pyramid can only be represented by pairs of quadruplets of the second form, $abcd$, $dabc$. Thus for $m=3$, the system

a 1127 b 2637 c 3547 d 4457 e 5367 f 6217
A 7112, B 7263, C 7354, D 7445, E 7536, F 7621

gives this paradigm, showing two nodal gamic pairs aA and dD,

a	F	.	.	.	.	A
A	.	.	.	.	f	B
.	.	.	.	e	B	C
.	.	.	d	C	.	D
.	.	c	D	.	.	E
.	b	E	.	.	.	F
f	a	b	c	d	e	.

The reason why the $(2m+1)$-gonal pyramid has the first arrangement as well as the second, is, that every base-summit may be taken for the pole of the wall-face opposite it. In the $2m$-gonal no summit is opposite to a face, nor can the interval between a base summit and its polar-wall face be constant. This is best seen by inspection of the schemes

ABCDEAB, ABCDEAB, ABCDEAB.
$d\ e\ a\ b\ c\ d$ $c\ b\ a\ e\ d\ c$ $b\ a\ e\ d\ c\ b$

ABCDEFAB, ABCDEFAB.
$e\ f\ a\ b\ c\ d\ e$ $c\ b\ a\ f\ e\ d\ c$

The triangles made by three adjoining letters in all the three upper examples, as AB*d*, D*ab*, correspond; but this is the case only in the second of the lower ones.

I add the following examples:—

a 1357 *b* 2617 *c* 3527 *d* 4137 *e* 5847
A 7531, B 7162, C 7253, D 7314, E 7485,

which forms

.	.	A	*d*	.	*g*	B	.
.	.	.	.	*f*	B	C	.
a	.	.	.	C	.	D	.
D	.	.	.	.	.	E	G
.	F	*c*	.	.	.	A	E
G	*b*	.	.	.	.	.	F
b	*c*	*d*	*e*	*a*	.	.	.
.	.	.	*g*	*e*	*f*	.	.,

an autopolar octaedron, having a pentagonal, two quadrilateral, and five triangular faces. This can be folded to lay *a* upon A, &c.

a 1228 *b* 1382 *c* 1864 *d* 1473 *e* 4657 *f* 5668 *g* 3758
A 8122, B 2138, C 4186, D 3147, E 7465, F 8566, G 8375,

and

a 1138 *b* 1473 *c* 1824 *d* 2574 *e* 2765 *f* 5667 *g* 3728
A 8113, B 3147, C 4182, D 4257, E 5276, F 7566, G 8372,

give

.	B	D	C	.	.	.	A
a	A	.	.	.	.	.	*b*
b	.	.	.	.	.	*d*	G
d	.	.	.	.	*c*	E	.
.	.	.	.	.	E	G	F
.	.	.	*e*	*f*	F	.	G
.	.	*g*	D	*e*	.	.	.
c	*a*	B	.	*g*	*f*	.	.

a	.	B	C	.	.	.	A
.	.	.	D	E	.	G	C
A	.	.	.	.	.	*b*	G
b	*c*	.	.	.	.	*d*	.
.	*d*	.	.	.	*e*	F	.
.	.	.	.	*f*	F	E	.
.	*e*	*g*	B	D	*f*	.	.
c	*g*	*a*	.	.	.	.	.

These are both autopolar octaedra having, like the preceding, a pentagonal, two quadrilateral, and five triangular faces. They are all three different figures. In the first, the pentagon *bcdea* has every other face either collateral or synacral with it, for *f*BC and DEG, the only ones not collateral with it, have the summits *f*C*ae*, and *a*DG*b* common with it, as is evident from the duads *f*C and DG. And its two quadrilaterals A*dg*B and AEF*c* have the common side A.

The second octaedron has also its pentagon either collateral or synacral with every other face, but its two quadrilaterals have not a common side.

The third has a crown triangle A*b*G, neither collateral nor synacral with the pentagon *feg*BD, for none of the angles A*b*, *b*G, GA are in the summits of that pentagon.

The following gives a sympolar pair of octaedra, having each a pentagon, two quadrilaterals, and five triangles :—

$$
\begin{array}{lllllll}
a\,1228, & b\,2338, & c\,3344, & d\,4554, & e\,5566, & f\,7667, & g\,8118, \\
h\,1142, & i\,2243, & j\,3456, & k\,3678, & l\,8541, & m\,6587, & n\,7788;
\end{array}
$$

$$
\begin{array}{cccccccc}
h & . & . & l & . & . & . & g \\
a & i & . & h & . & . & . & . \\
. & b & c & i & . & . & . & . \\
 & & & & & & & \\
. & . & . & d & c & m & . & l \\
. & . & k & . & j & e & f & . \\
. & . & . & . & . & f & n & m \\
g & a & b & . & . & . & k & n.
\end{array}
$$

Here are two distinct heteropolars; one has two quadrilaterals, *deml* and *kjef*, having a common side *e*; the other has two quadrilaterals, *cjkl* and *glmn*, that have no common side. The reader will find no difficulty in drawing these octaedra, by joining the angles of a pentagon to three included wall-summits.

It is to be observed, that in all these sets of quadruplets representing any p-edral q-acron, if we collect those which contain any given numeral in the same place, we shall find that in the two adjoining places they exhibit circles of the same numbers differing by one cyclical step. Thus, collecting from the above the quadruplets containing 8 in the fourth place,

8118, 1228, 2338, 3678, 7788,

show the circle 81237 in the first and third places. And those containing 1 in the second place, 8118, 1142, 4185, show the circle 814 in the first and third.

From this property of the closed p-gon and q-gon of props. A. and B, it is possible that some light may be thrown, when the matter is better handled, on the classification of polyedra, such as may lead to the solution of the problem of their enumeration.

It is easy to prove that there are polyedra on which the closed polygons cannot be drawn.

For suppose the q-gon of prop. A. drawn on a q-acron. In making the circuit of any face G which we enter across an edge FG, which is not an edge of the q-gon, we add to the number of summits counted in F and other faces, all the summits of G, except two, these two having been enumerated in the circuit of F from which we enter G. That is, counting first all the summits of the base, we add to these for every m-gon whose circuit we proceed to make, $m-2$ summits more. The number of faces, connected with each other and with the base by edges, not part of the closed q-gon, whose circuits the closed q-gon makes and includes, will be $\alpha_3+\alpha_4+\alpha_5+\ldots+\alpha_k$; where α_m is the number of m-gons among them, and α_k that

of the k-gons, the base being one of these. The whole number of summits will therefore be

$$\alpha_3+2\alpha_4+3\alpha_5+\ldots.+(k-2)\alpha_k+2=q;$$

for we have counted *all the summits* of one k-gon, viz. of the base.

In order, then, that such a q-gon should be possible, it is necessary that among the p faces of our p-edral q-acron, there should be α_3 triangles, α_4 quadrilaterals, α_5 pentagons, &c., of which the above equation can be affirmed. Now if q should be odd, and all the p faces even-angled, this equation becomes

$$2\alpha_4+4\alpha_6+6\alpha_8+\&c.=2r+1,$$

which is impossible. Hence it appears, that if the number of summits of a q-acron be odd, while the faces are all even-angled, the closed q-gon cannot be drawn through its summits. I find exceedingly few polyedra on which the closed p-gon and q-gon cannot be drawn. In fact, it is far from being necessary to their existence, that all the conditions of the theorems A and B should be fulfilled.

If we cut in two the cell of the bee by a section of its six parallel edges, we have a 13-acron, whose faces are one hexagon and nine quadrilaterals. The closed 13-gon cannot be drawn. But if a line be drawn from the triedral vertex to the opposite angle of one of the quadrilaterals about that vertex, and this quadrilateral supposed broken into two triangles having that line for their common edge, we shall then have a 13-acron whose faces are one hexagon, eight quadrilaterals, and two triangles; and whose summits are nine *triaces* and two *tessaraces*. Of this figure the paradigm can be constructed. Here I would fain beg the reader's permission to call a 5-edral summit a *pentace*, a 6- or 7-edral summit a *hexace* or a *heptace*. The words are at least convenient in speaking of the summits of polyacra.

As authorities and analogy are alike divided about the spelling of the word polyedron, I have pleased myself herein. Why *polyhedron* of necessity, and yet not *perihodic*?

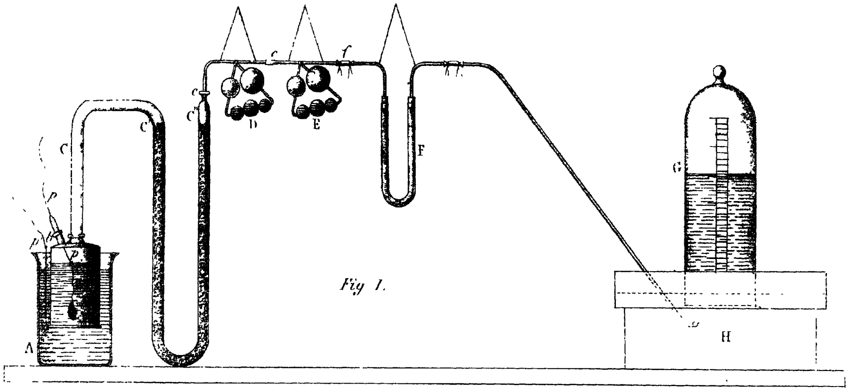

Fig 1.

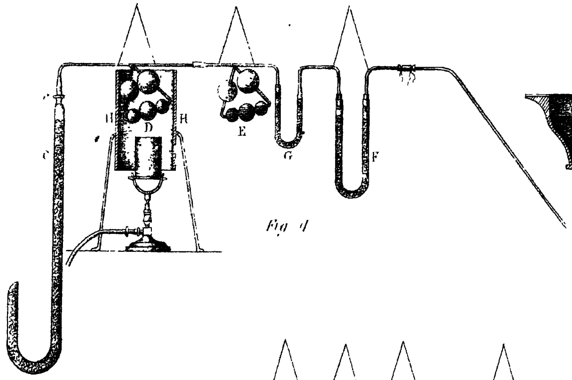

Fig 4

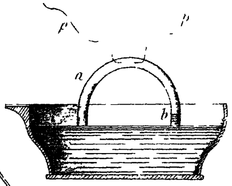

Fig 5


Fig. 3

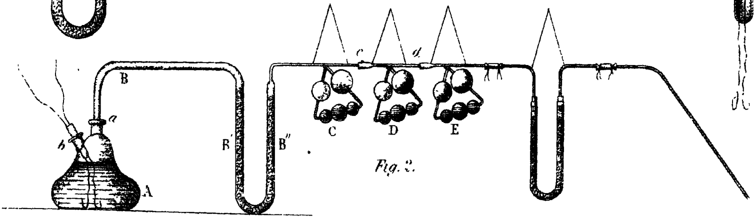

Fig. 2.

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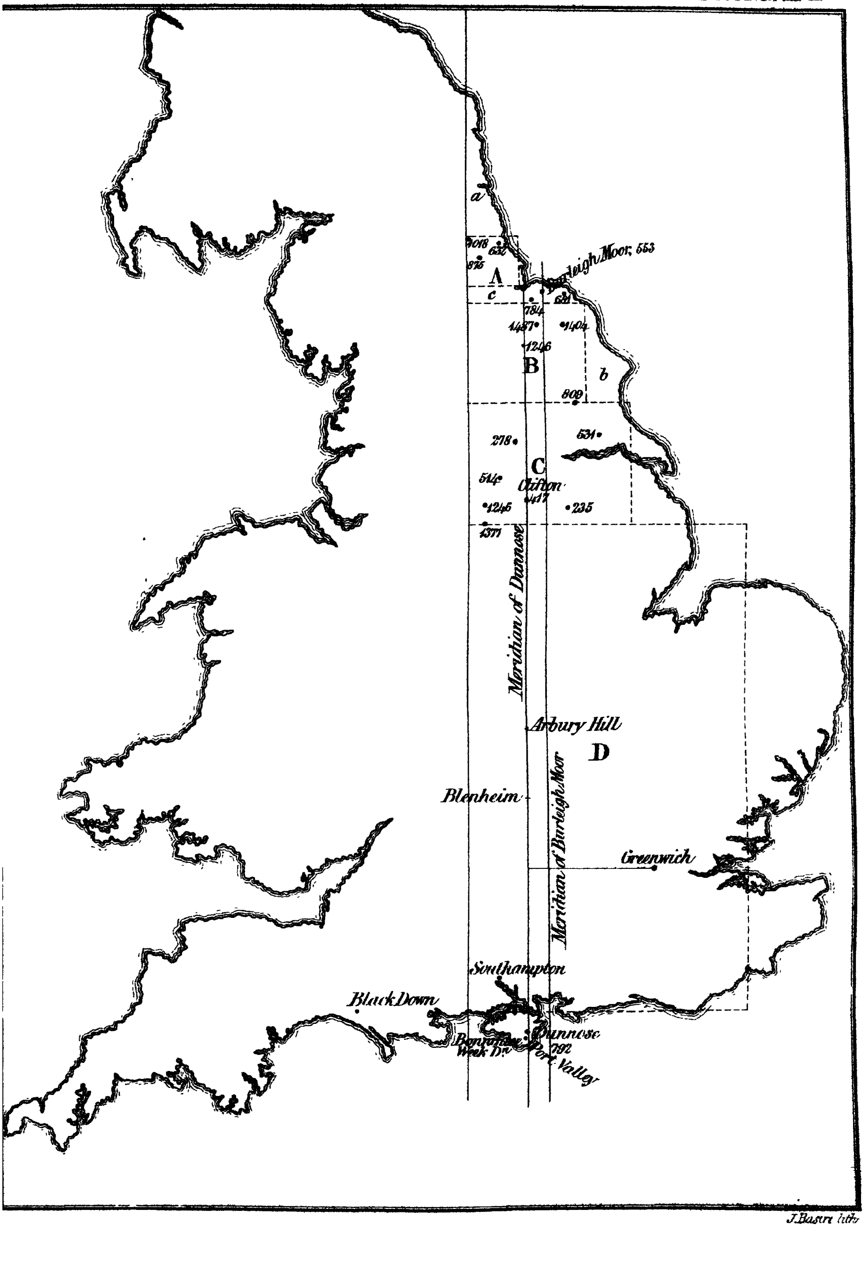

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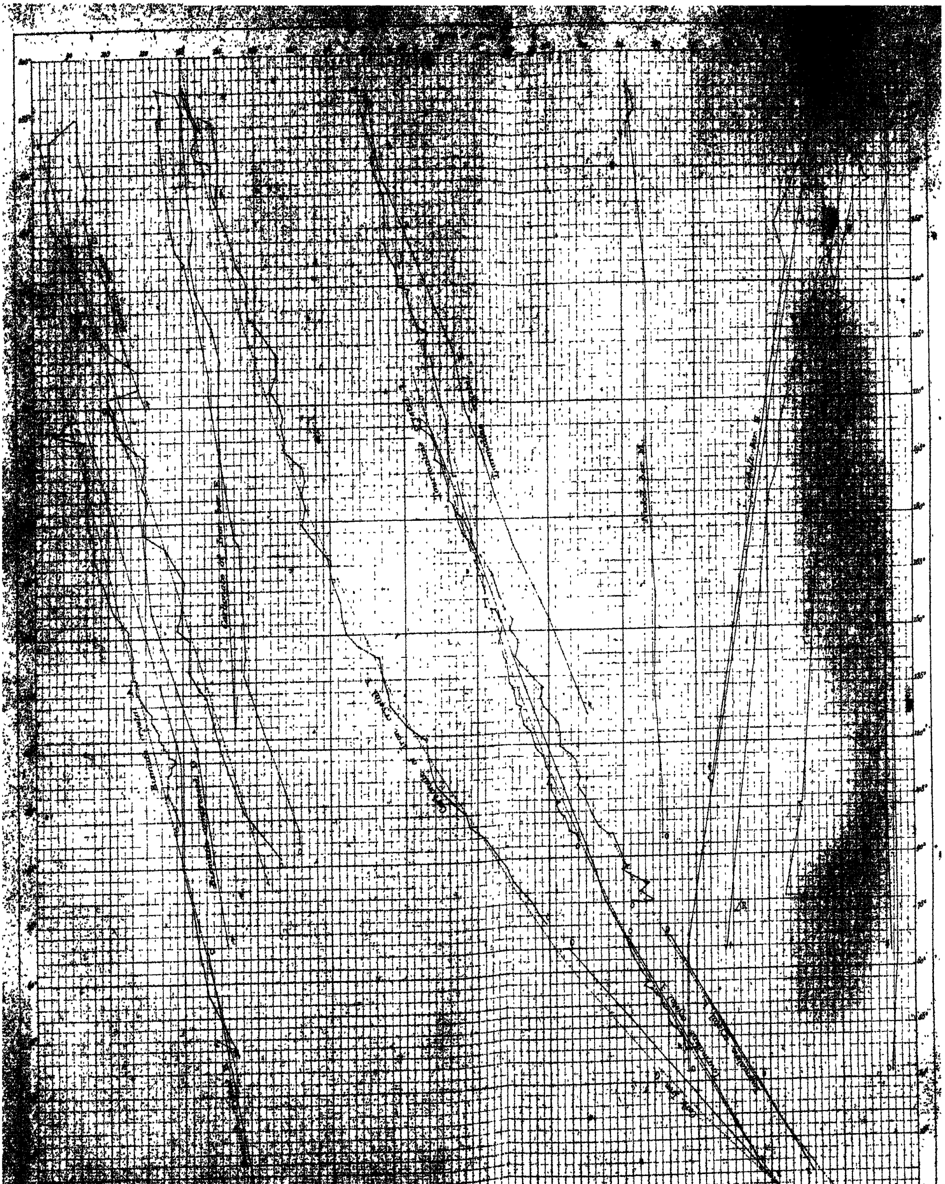
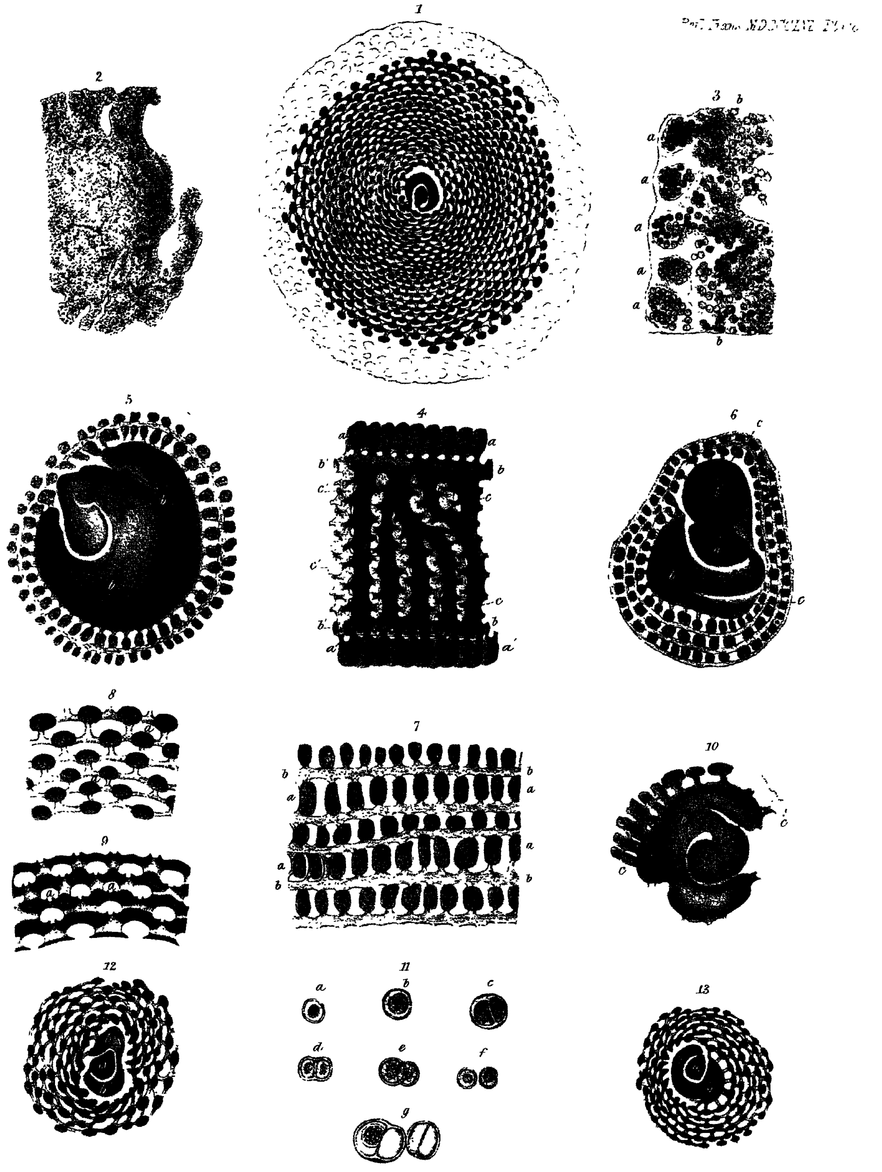
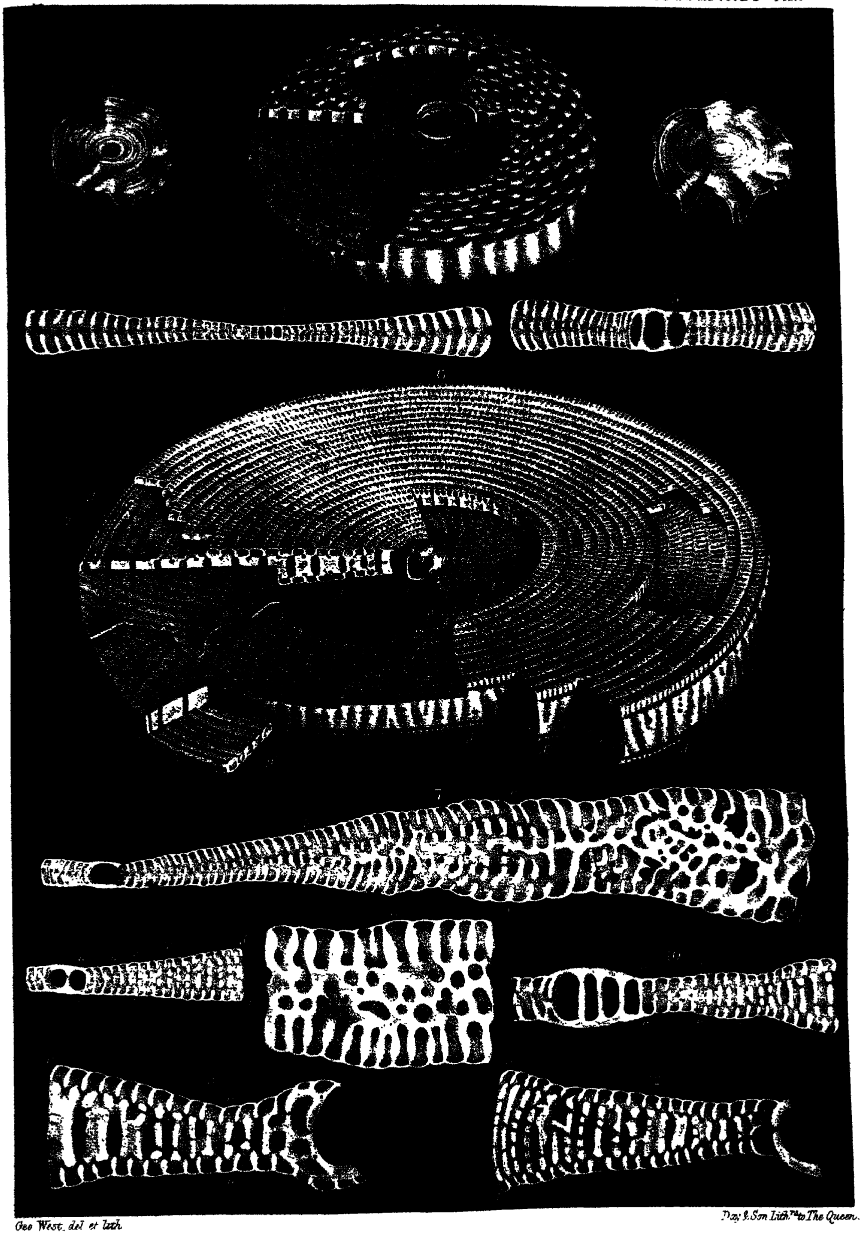


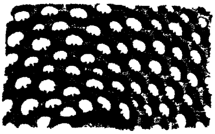


[illegible] Lens Aldous lith Day & Son, Lith[rs] to The Queen



Geo West. del et lith.

Day & Son Lith.rs to The Queen.



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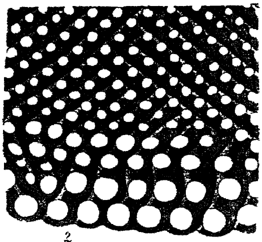

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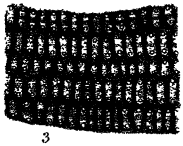

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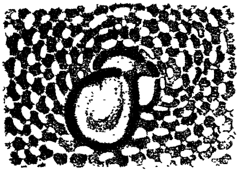

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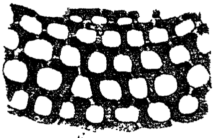

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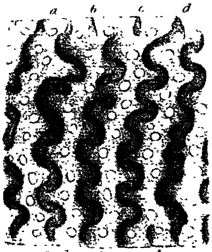

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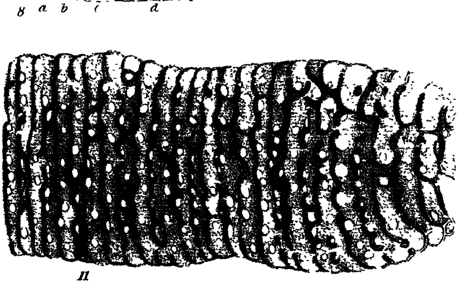

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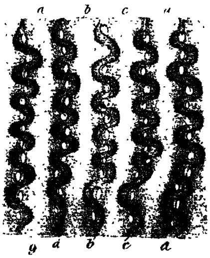
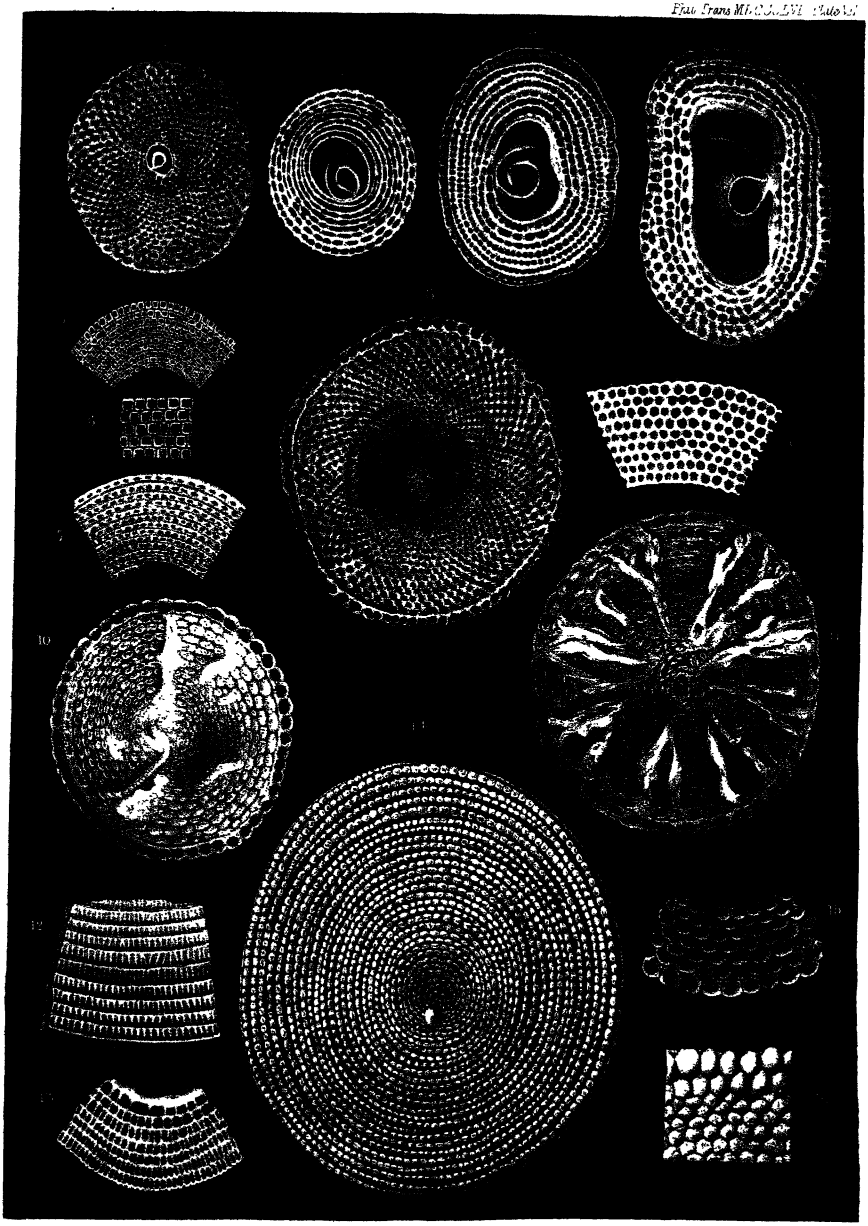
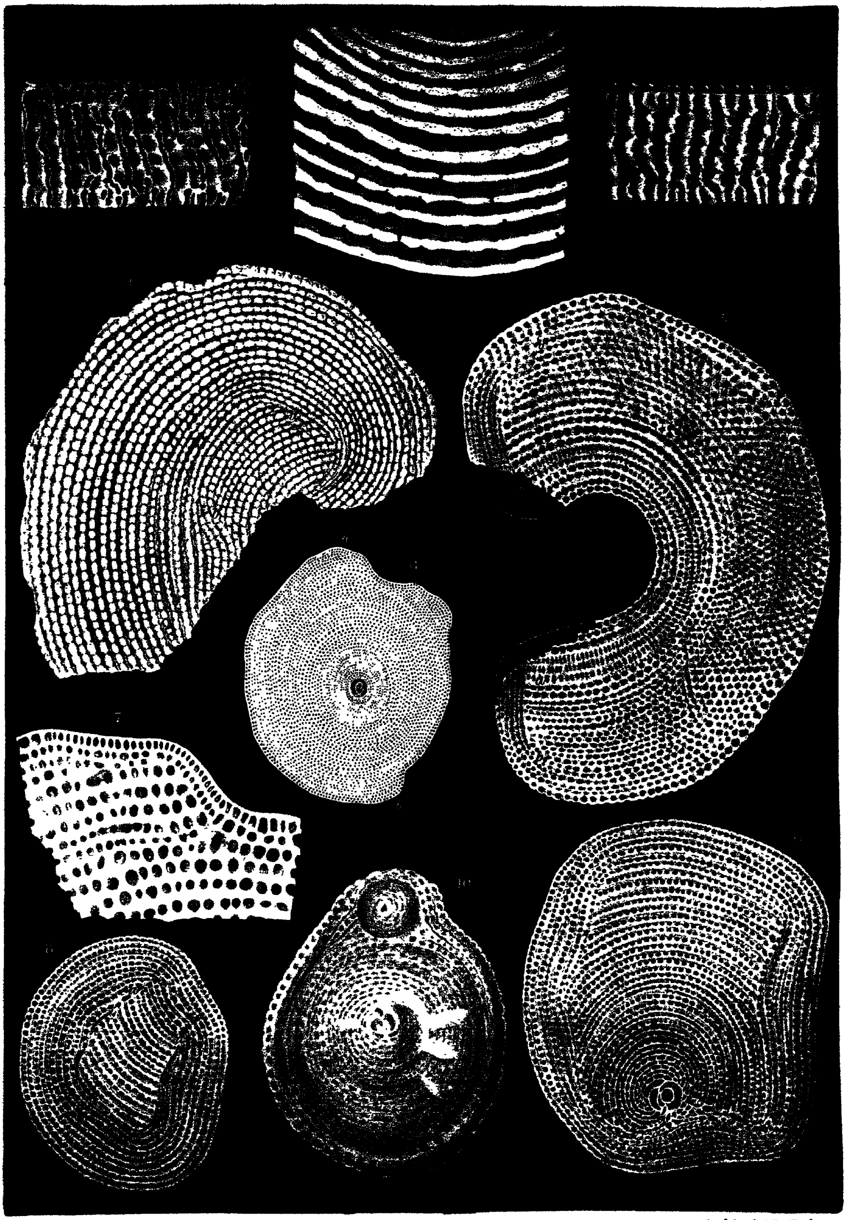


9 a b c d

George West, del. et lith.

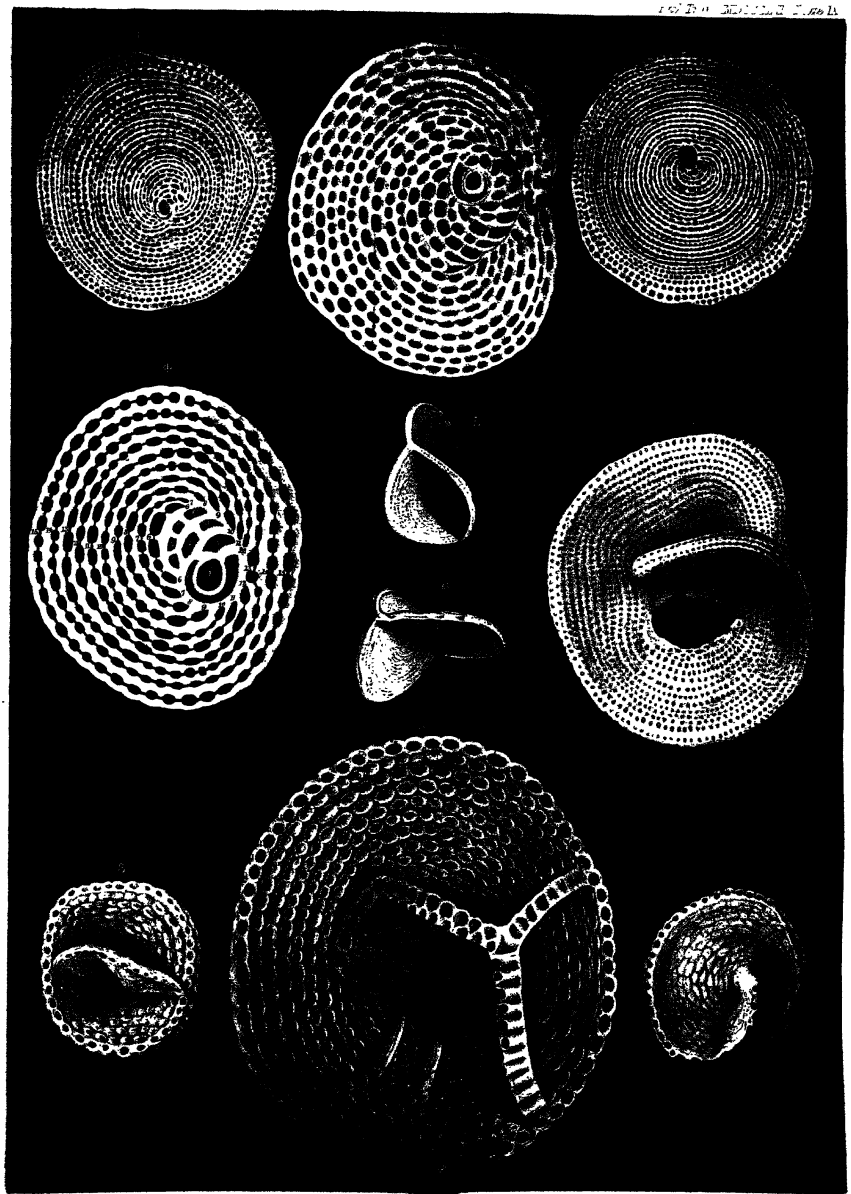
Bays & Son Lith.rs to The Queen





Lens Aldous. lith

Day & Son, Lith^rs to The Queen



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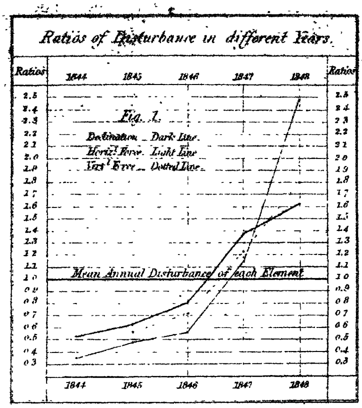
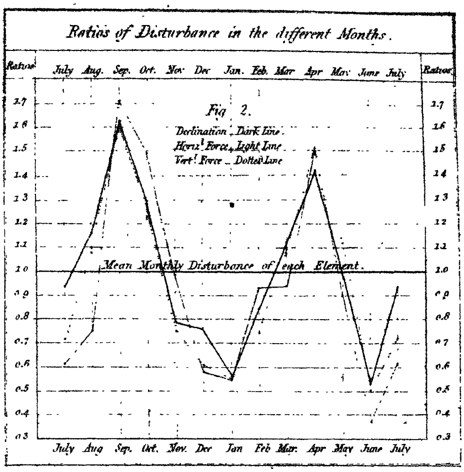
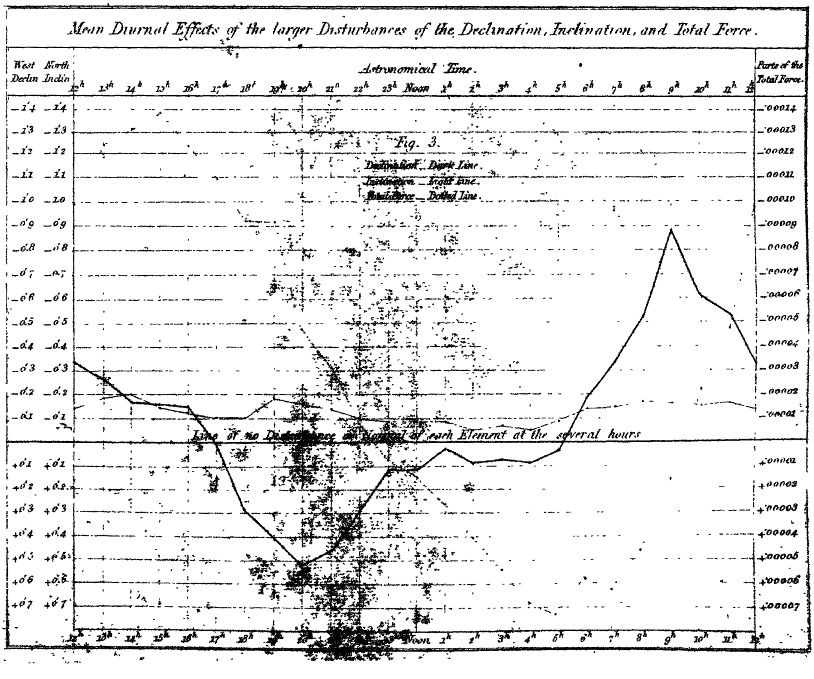
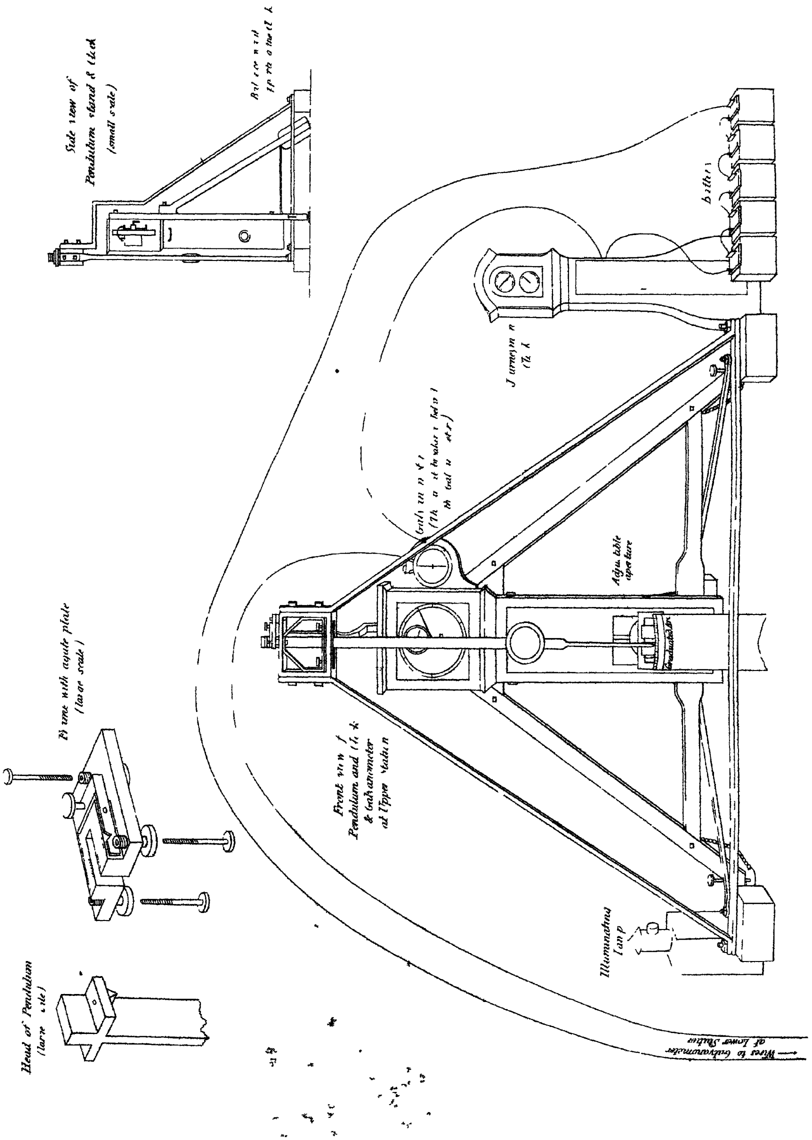

Ratios of Disturbance in different Years.
Fig. 1.
Declination _ Dark Line.
Horiz.l Force _ Light Line
Vert.l Force _ Dotted Line.
Mean Annual Disturbance of each Element
Ratios
1844
1845
1846
1847
1848


Ratios of Disturbance in the different Months.
Fig. 2.
Declination _ Dark Line.
Horiz.l Force _ Light Line.
Vert.l Force _ Dotted Line
Mean Monthly Disturbance of each Element.
Ratios
July Aug. Sep. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July


Mean Diurnal Effects of the larger Disturbances of the Declination, Inclination, and Total Force.
Fig. 3.
Declination _ Dark Line.
Inclination _ Light Line.
Total Force _ Dotted Line.
West Declin
North Inclin
Astronomical Time.
Parts of the Total Force.
Line of no Disturbance or Normal of each Element at the several hours

J. Basire sc.


Side view of
Pendulum stand & clock
(small scale)
Head of Pendulum
Pendulum and
& Galvanometer
Adjustable aperture
Illuminating
Wires to Galvanometer
of Lower Station

J. Basire sc

Fig 1.

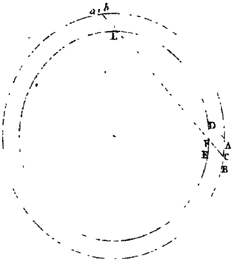


Fig 2.

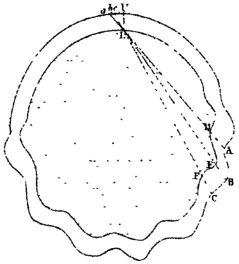


Fig 3.

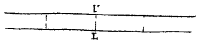
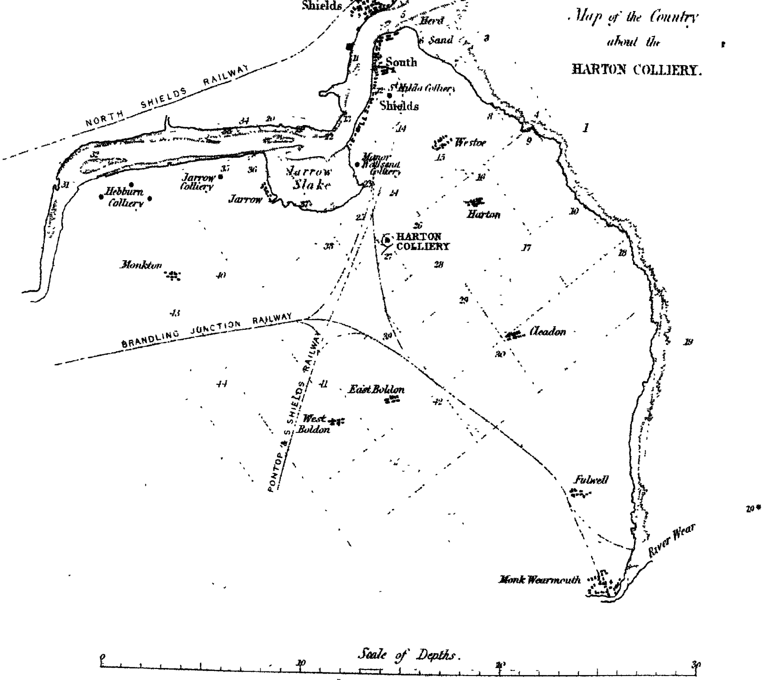


Fig 4.
Map of the Country about the HARTON COLLIERY.



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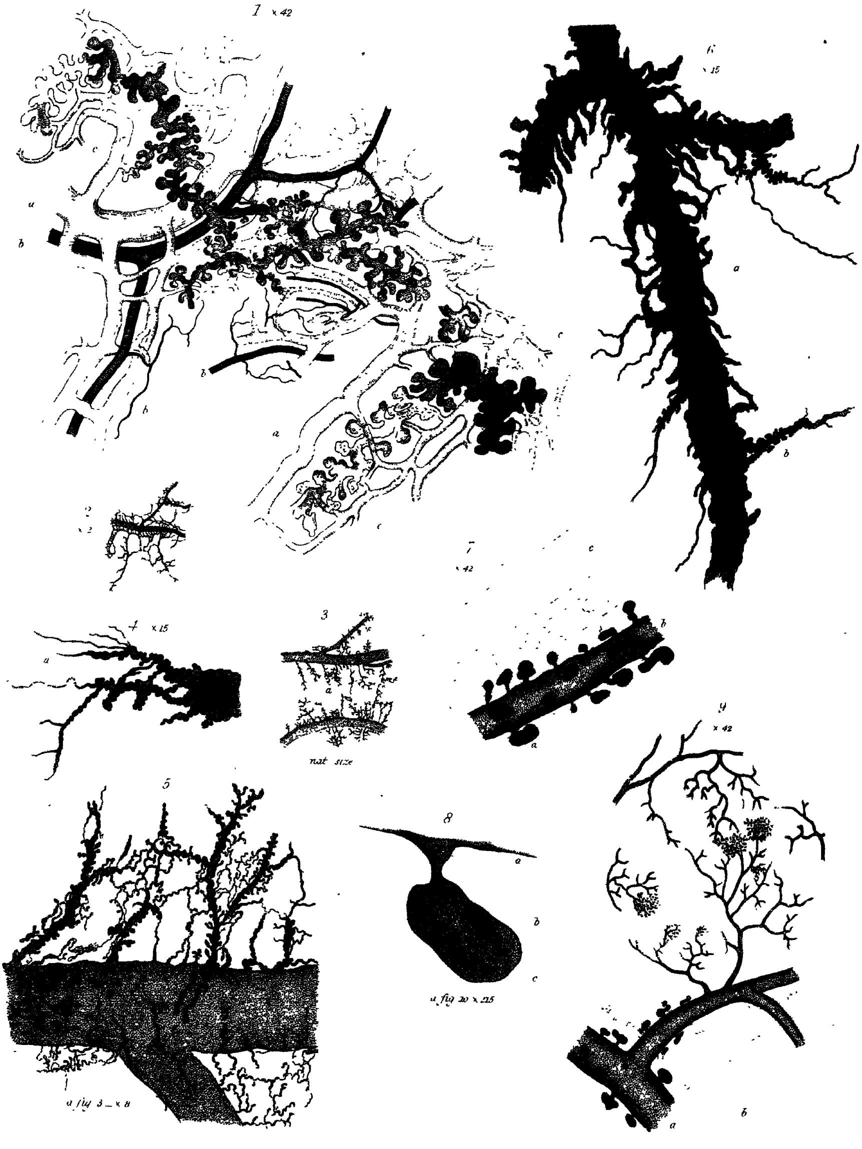

Scale
Parts of an English Inch. 100ths × 15 · 100ths × 42 · 1000ths × 215

Lionel Beale ad nat del

J Basire, sc

Scale
Parts of an
English Inch

T. Bastre sc.

Scale
Parts of an inch ...
Lineal 1/4 Inch ...

PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCLVI.

VOL. 146.—PART II.

LONDON:

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MDCCCLVI.

CONTENTS.

ERRATUM.

Page 618, line 19, *for* Makerstown *read* Brisbane.

ADJUDICATION of the MEDALS of the ROYAL SOCIETY for the year 1856 by the PRESIDENT and COUNCIL.

The COPLEY MEDAL to M. HENRI MILNE-EDWARDS, for his Researches in Comparative Anatomy and Zoology.

The RUMFORD MEDAL to M. LOUIS PASTEUR, for his Discovery of the Nature of Racemic Acid and its relations to Polarized Light, and for the Researches to which he was led by that discovery.

A ROYAL MEDAL to Sir JOHN RICHARDSON, for his Contributions to Natural History and Physical Geography.

A ROYAL MEDAL to Professor WILLIAM THOMSON, for his various Physical Researches, relating to Electricity, to the Motive Power of Heat, and to other subjects.

The BAKERIAN LECTURE was delivered by Professor WILLIAM THOMSON, F.R.S., and entitled "On the Electro-dynamic Qualities of Metals."

XIX. *On the Structure, Functions, and Homologies of the Manducatory Organs in the Class* ROTIFERA. *By* PHILIP HENRY GOSSE, *Esq. Communicated by* THOMAS BELL, *V.P.R.S., Pres. L.S.*

Received January 5,—Read February 22 and March 1, 1855.

1. AN examination of the whole alimentary system in the class ROTIFERA would include—The buccal funnel; the manducatory apparatus, with its muscular bulb and the muscular bands by which it is retained *in situ*; the salivary glands; the œsophagus; the pancreatic glands; the stomach; the hepatic follicles; the cæca; the intestine; the rectum; and the cloaca. Of these, however, the manducatory organs will occupy my principal attention; and I shall endeavour to trace them throughout the class; and to show, that the various forms which they assume can all be reduced to a common type. I also propose to inquire, what are the real homologues of these organs in the other classes of animals; and what light we can gather, from their structure, on the question of the zoological rank of the ROTIFERA.

2. Considering the attractive appearance which the Wheel-animals present, and the facilities which are afforded for their investigation by their abundance—since they are found in great variety in almost every river, lake, pond, and ditch,—it is rather remarkable that they have been so little studied.

3. The foundation of our acquaintance with the organization of the ROTIFERA was broadly laid by Professor EHRENBERG, in the 'Transactions of the Berlin Academy,' as early as 1830; and in 1838 he published his 'Infusionsthierchen,' a work which, in spite of its mistakes and deficiencies, must be considered a magnificent monument of industry and scientific acumen. It is true, LEEUWENHOEK, BAKER, JOBLOT, and other early microscopists had figured a few species; and RÖSEL, SCHAFFER, and especially MÜLLER, had attempted to resolve the internal anatomy of some; but all that had been accomplished amounted to little more than vague guessing, when EHREN-

BERG took them up. He penetrated into the minute organization of these animals; resolved and figured, with a precision and minuteness up to that time unattempted, the nutritive, circulatory, nervous, muscular and generative systems; and, though the details of some of the organs which he has described existed only in his imagination, and the functions of others which he clearly saw were quite misunderstood, it would be absurd to deny that his elaborate plates and descriptions are generally faithful representations of these minute creatures.

4. M. DUJARDIN, like his great predecessor, included the ROTIFERA, by the name of *Systolides*, among the INFUSORIA, in his work on the subject published in 1841. In many particulars his observations, which were largely polemical, have advanced our knowledge of the subject; but in some respects they are a retrogression. His personal acquaintance with species was greatly inferior to that of the Prussian zoologist, and insufficient for satisfactory generalisation.

5. Since then, I am not aware that any naturalist has attempted, *from personal observation*, a revision of the class, or even of any considerable number of species; and the science has advanced, chiefly, by isolated papers on individual species, and by critical examinations of the facts already accumulated. These memoirs I shall enumerate in chronological order.

6. In 1843, Dr. KÖLLIKER published a memoir * on *Megalotrocha*, confining himself to the segmentation of the egg, and the (so-called) seminal threads. Dr. OSKAR SCHMIDT, in 1846, gave a résumé of what was then known of the organization of the ROTIFERA generally †. In the same year, the late Dr. MANTELL, in his 'Thoughts on Animalcules,' though chiefly founded on EHRENBERG, added some information of interest and value on the development of the young in *Stephanoceros* and *Melicerta*. In the 'Annals of Natural History' for 1848, Dr. DOBIE described with minuteness two new species of *Floscularia*; and Mr. BRIGHTWELL recorded his important discovery of the diœcious character of a ROTIFERON which I have since named *Asplanchna*. The same species formed the subject of a valuable memoir by Mr. DALRYMPLE ‡, read before the Royal Society in February 1849. Meanwhile, however, some additional observations had appeared by Dr. LEYDIG §, on the egg-segmentation of *Notommata*, *Euchlanis*, and *Megalotrocha*; and in the same year (1848), Dr. FREY had published a work on the class generally ||, which I have not been able to see.

In 1850 I published several memoirs on ROTIFERA in the 'Transactions of the Microscopical Society' and the 'Annals of Natural History,' viz. "On the Habits of *Melicerta ringens* ¶" (January); "On the Anatomy of *Notommata aurita* ¶" (May); "On *Asplanchna priodonta* **" (July); and "On *Notommata parasita* ¶" (December).

* FRORIEP's Neue Not., No. 28. p. 17.

† WIEGMANN's Archiv, 1846, p. 68.

‡ Philosophical Transactions, 1849.

§ Isis, 1848, p. 170.

|| Ueber die Bedeckungen der wirbellosen Thiere. Göttingen, 1848. This work I know only by a citation in SIEBOLD and STANNIUS' 'Comparative Anatomy.'

¶ Trans. Micr. Soc. i. pp. 58, 93, 143.

** Ann. Nat. Hist. July 1850.

In 1851 I published a catalogue of 108 species that I had observed, including descriptions of five new genera and thirty-two new species*. In this latter year M. d'Udekem published in the 'Memoirs of the Brussels Academy' two papers†, the one on the circulatory system in *Lacinularia socialis*, the other on *Floscularia cornuta*; the latter as if a new species, but which had been described and figured by Dr. Dobie four years before.

The former of these animals, *Lacinularia socialis*, became the subject of two very valuable papers in the year 1852; the one by Mr. Huxley‡, and the other by Dr. Leydig §. In the following year appeared a "Memoir on the Anatomy of *Melicerta ringens*," by Professor Williamson ||, closely followed by a second ¶ on the same species, by myself.

7. In this enumeration I believe I have included all that has been published on the subject of the Rotifera, *from original observation*: should anything, however, have been omitted, it is because it has eluded my most careful scrutiny.

8. What changes may have been produced on the state of our knowledge of the nervous, vascular, glandular, muscular and reproductive systems of the Rotifera, by these researches, it is not my province here to inquire. My present business is with the digestive system, and particularly with the organs of manducation; and of these, I think it is no more than the truth to say, our knowledge is almost exactly where Ehrenberg placed it five-and-twenty years ago.

9. This assertion seems strange after so many memoirs, of more or less value, by different observers, on various species; but the wonder is much lessened by a re-examination of their subjects. Of the thirteen monographs enumerated, two are on *Floscularia*, three on *Melicerta*, three on *Lacinularia*, three on *Asplanchna*, and two on *Notommata*. Now in *Melicerta* and *Lacinularia* the manducatory apparatus is of the same type, *and that an abnormal one*; in *Floscularia* and *Asplanchna* the types of structure differ widely from the former, and as greatly from each other. These three types are in fact unintelligible in themselves, and can be explained only by tracing the organs from their normal development, through their modifications and degenerations. Nothing then, in fact, has been attempted towards an explanation of the jaws, in the great families of *Hydatinæa*, *Euchlanidota*, and *Brachionæa*—the true types of Rotifera—except my own two memoirs on *Notommata* in 1850. And I freely confess, that, in spite of many and persevering efforts, I had not, at that time, been able to attain any satisfaction on the true structure of these organs.

10. Nothing is easier than to see the forms of the various parts, *in outline*, in some one aspect; to obtain, for example, a dorsal or ventral view of the jaws in *Brachio-*

* Ann. Nat. Hist. Sept. 1851. † Bull. de l'Acad. Belg. xviii. pp. 39, 43. ‡ Journ. Micr. Soc. i. (Trans.) p. 1.
§ Sieb. et Köll. Zeitschr., Feb. 1852. || Journ. Micr. Soc. i. (Comm.) p. 1.
¶ Ibid. p. 71. Note.—Since the presentation of this paper to the Royal Society, there has appeared in the 'Journal of Microscopical Science' an abstract of another memoir by Dr. Leydig, "On the Structure and systematic Position of the Rotifera." The original I have not yet had an opportunity of examining.—P. H. G.

nus, or a lateral one in *Furcularia*: but when we have done this, we have gone but a little way towards such an understanding of the whole form, and the relation of the complex parts to each other, as will enable us to project them, or to form an ideal model of the whole. A little more light is shed on the structure by crushing the animals between plates of glass, or under the graduated pressure of a compressorium: but the distortion of the parts is very great; and some of them, especially those which form the intermediate piece (*incus*), on which the greatest obscurity rests, are almost invariably broken into a multitude of separate fragments in the process of flattening; and thus the result merely tantalizes the expectation.

11. Little more seems to be yet known of the structure of the manducatory organs (I speak of the normal forms) than can be obtained by these modes, and the published descriptions of them are vague and unsatisfactory.

12. Thus Ehrenberg describes the general structure as follows:—"The normal intestinal canal in the Rotifera consists first of a globose muscular throat-bulb (*schlundkopf*), in which are fixed two toothed jaws, and the anterior opening of which is placed in the midst of the wheel-organ, somewhat towards the belly [side]*."—"In all the forms there is a moving manducatory organ (*kauorgan*), furnished, in forty-eight genera, with indubitable teeth, and evidently ministering to the alimentary canal †."

Again, in proposing a division of the class, according to the teeth, he groups the Rotifera as, i. Toothless (*Agomphia*); ii. Free-toothed (*Gymnogomphia*), where the teeth resemble the fingers of a hand fastened to the jaw-frame behind, but free in front; iii. Bound-toothed (*Desmogomphia*), where the teeth are fastened across upon the jaw, like an arrow upon a bow ‡.

The most laboured description of the normal structure the same zoologist has given under the genus *Hydatina*, which he professedly makes the vehicle for a *résumé* of his acquaintance with the typical forms of the class. "The nutritive system," he observes, "consists of a great buccal cavity, chiefly enclosed and formed by the rotatory organ, as an upper lip; in the bottom of which, near the ventral side, lies the spherical four-muscled throat-bulb, with two many-toothed jaws. In each jaw are held five conical teeth, resembling a hand, somewhat converging towards the bottom, diminishing in size inwardly. Sometimes a minute sixth tooth appears to be developed. The five teeth of each jaw, which are perceived, on crushing the body between glass plates, to be the only hard and solid parts, are jointed into a cartilaginous frame, which serves for the attachment of the muscles, and has the form of a shoulder-blade. This is the proper jaw, which is composed of several parts. Inwardly, both jaws are in connexion with a frame of cartilaginous throat-arches, which is very complicated, and seems more fitted for the support and attachment of the masticatory muscles than for proper activity §."

13. I shall presently show that these descriptions are both very imperfect and very

* Trans. Berl. Acad. 1830, p. 29. † Infusionsth. p. 385. ‡ Ibid. p. 386. § Ibid. p. 414.

incorrect; and the figures, with which they are illustrated, are but rude approximations to verisimilitude.

14. M. Dujardin sums up his observations on the same organs in the following words. After alluding to the Tardigrada, which he includes in his Systolides, the genus *Enteroplea*, which is said to be toothless, and the genera *Floscularia* and *Lindia*, which he describes (incorrectly) as lacking a ciliated mouth, he observes,—

"All the other Systolides, having jaws enclosed in a muscular pharyngeal bulb, which is moveable and protractile, at the bottom of a ciliated vestibule, may be distinguished according to the form of these jaws. Thus the *Rotifers* have their jaws in the form of a stirrup, opposed by the base, and bearing two or more small teeth, laid parallel, like arrows on a bow. The outer border, which is semicircular, furnishes a point of attachment for the muscles of the pharyngeal bulb; and, drawn by them, it is strongly elevated and depressed, to produce, during manducation, the movement of the jaws. Their inner border is composed of two transverse bars, a little arched outwardly. . . . . All the others . . . have jaws more resembling those of articulated animals, and composed of an assemblage of articulated pieces, which we may, up to a certain point, compare to the two pairs of mandibles and jaws, to the lips, to the tongue, and to the labial palpi of insects. In fact, in many of the Systolides we observe a central odd piece, on which are articulated two simple branches, which bear upon each other, or meet as by a hinge, in the midst of moveable and articulated pieces, supporting the jaws properly so called, and transmitting to them, thus, all the effort of the median muscular mass, to make them bite upon the prey, by furnishing to them a *point d'appui*, when the lateral muscles draw back the external branches which carry the jaws *."

15. The eminent French naturalist, in this description of the jaws, which appears to be drawn from the type found in such genera as *Diglena* and *Albertia*, has touched a key which might have unlocked the structure, not in a few genera only, but in the whole class of Rotifera. As it is, however, I cannot agree with Dr. Oskar Schmidt, that the arrangement and function of the teeth, and of the surrounding bulb, are so manifest as to need no further observation †.

16. It may not be out of place to describe the manner in which the following observations were made. The desideratum was to obtain views of *various aspects* of the same animal; particularly of the dorsal and ventral aspects, of the lateral, and of the vertical or frontal. But the minuteness of the objects, ranging from $\frac{1}{300}$ to $\frac{1}{50}$th of an inch, precluded the possibility of affixing them to a needle, or other machinery, by which they could be made to revolve while in the field of vision; and the more, since, being aquatic animals, they must be viewed immersed in water; a momentary removal causing the death of most species. The incessant activity of these little animals is also a great bar to accuracy of observation, if they be allowed freedom; and if we confine them by means of the compressorium, the form of their

* Infusoires, p. 583.

† Wiegmann's Archiv, 1846, p. 69.

bodies causes them to present almost invariably the same aspect to the observer, viz. that in which the greatest width is at right angles to the line of vision; so that we may examine specimen after specimen, till patience is exhausted, and acquire no new information, because we look each time at the same aspect.

17. In the course of experiments with various chemical reagents on these animals, I found that a solution of potash had the effect of instantly dissolving the flesh, and most of the viscera; leaving the general integument, the walls of the pharyngeal bulb, and all the solid parts of the manducatory apparatus uninjured*. In most cases, also, the last-named organs are expelled from the visceral cavity by the contraction of the integuments; so that they float at large, in brilliant clearness, undimmed by intervening tissues, and as patent to observation as when crushed between plates of glass; with the advantage of all the parts being unbroken, and retaining their relative positions. Now, by turning the screw of the compressorium, flattening or deepening the drop of water, waves were communicated to it, by means of which the floating bulb, being nearly globular, was made to revolve irregularly, and thus to present, in succession, various aspects to the eye.

18. The observations which I am about to record were made with one of POWELL and LEALAND's microscopes, with a power of 560 diameters; except those on *Stephanoceros* and *Diglena*, on which the powers employed were 220 and 300 diameters.

19. For the sake of precision in description, it may be well here to mention a few terms that I shall employ, and to define the sense in which they will be used. The symmetry of the ROTIFERA is truly bilateral, the genera *Stephanoceros* and *Floscularia* alone retaining a lingering remnant of radiism, in the arrangement of the frontal lobes; even in these, however, the whole of the anatomy besides, both external and internal, is bilateral. In most cases this arrangement is obvious; the motions of the animal, like those of the footed larvæ of insects, being performed on the belly, with the head foremost. Where this is not the case, as with those genera which, either with or without an enveloping tube, adhere to foreign substances by the tip of the foot, and elevate the body in an erect position, the *dorsal* aspect is always determinable by the eye or eyes being towards that surface, by the stomach and intestine passing down it, and by the cloaca being on that side of the foot. The *ventral* aspect has the manducatory apparatus, and the ovary. The *anterior* extremity carries the vibratile cilia; the *posterior* is terminated by the foot. In such species as are clothed with a lorica, I shall call the anterior termination of the dorsum, the *occipital* edge; and the anterior termination of the venter, the *mental* edge. Other terms will be defined in the process of description.

20. As the manducatory bulb, with its complex contents, is the principal subject of these observations, I shall commence with it; treating the other parts as accessory to it. In most ROTIFERA this organ forms a prominent object, conspicuous from its

* Dr. LEYDIG, I find, has used the same agent in examining the teeth in *Lacinularia*; but he does not appear to have employed it in the investigation of other ROTIFERA (SIEBOLD and KÖLL. Zeitschr. 1852).

form and motions. By old observers the vigorous workings of its internal parts caused it to be mistaken for a heart; but it has long been recognized as an organ of mastication. Dr. Ehrenberg sometimes calls it *Kauorgan,* but more generally *Schlundkopf,* and its contents *Kau-apparat.* M. Dujardin speaks of it as *le bulbe pharyngien.* Von Siebold calls it a *pharynx*; and English zoologists have generally used the term *gizzard.* I hope to prove that it is neither a gizzard nor a pharynx, but a true *mouth*: in the mean time, however, whatever its homological value, it is doubtless a form of apparatus which has no parallel in other classes of animals, and therefore deserves a proper appellation. I propose then to appropriate to the sub-globose muscular bulb, which contains the manducatory organs in most Rotifera, the term *mastax.*

21. The *mastax,* with its contents, is found in the highest degree of development in the genera *Brachionus, Euchlanis,* and some of the *Hydatinæa.* It is usually more or less globose in form, composed of three lobes, which are confluent anteriorly, with a common rounded outline; but separated posteriorly, one lobe diverging towards the ventral side, the others laterally, and a little dorsally; so that a posterior aspect would assume the outline of a trefoil. The general form is sub-hemispheric in *Brachionus* and its allies; that of an oblate spheroid in *Euchlanis*; a prolate spheroid in *Notommata aurita*; cordate in *Notommata petromyzon*; sub-triquetrous in *Plagiognatha*; triglobular in *Notommata clavulata*; purse-like in *Mastigocerca*; irregularly oblique in *Synchæta* and *Polyarthra*; and wanting in *Floscularia.*

22. In substance it varies from a state in which its walls are thick and solid, composed of dense muscular fibre, with little cavity, as in *Brachionus,* to one in which it forms a capacious sac, with thin, apparently membranous, parietes, as in *Furcularia.*

23. The anterior side of the *mastax* is perforate, its walls here merging into the tube of the *buccal funnel.* It is perforate also on its dorsal aspect; whence the *œsophagus* issues to join the stomach.

24. Let us now examine in detail the *mastax,* in the modifications which it assumes in various species. In *Brachionus urceolaris* (Plate XVI. figs. 1 to 10), it is a dense, colourless, highly-refractive mass of muscles, sub-hemispherical, distinctly trilobate posteriorly, and cleft deeply on the ventral side of its anterior surface (fig. 1). Within it are placed two geniculate organs (*b*), which, from their resemblance in form and action to hammers working on an anvil, I have elsewhere named *mallei*; and a third (*f*), still more complex, which I call the *incus.* These three pieces are not arranged in the same plane; for the *mallei* approach each other a little dorsally, while the *incus* is placed on the ventral side of the centre, its stem pointing considerably towards the same side. Thus each of the three organs corresponds to, and occupies the centre of, one of the lobes of the *mastax.* This obliquity of the parts with respect to each other, and to the planes of the body, is one cause of the great difficulty which attends an endeavour to reconcile the various aspects of this organ in any intelligible manner.

25. Each *malleus* consists of two principal portions, articulated with each other by

a powerful joint, which seems to be ginglymate in its character, admitting of motion in one plane only. The two opposing surfaces appear to be united by elastic ligaments, while their irregularities lock into each other.

26. The inferior portion of the *malleus*, which I shall call the *manubrium* (*c*), is an irregularly-curved piece, shaped somewhat like the scapula of a mammal, knobbed at its head, and flattened at its lower or free end, where also it is twisted to one side. Ridges run down it, on both the interior and exterior surfaces. The head is obliquely truncate; and it is this oblique surface (*d*), that is jointed to the superior portion (*e*), which, from its prevalent form in other genera, rather than in this, I call the *uncus*.

27. The *uncus*, when at rest, is placed nearly at right angles to the *manubrium*, but is capable of considerable change of relative position by means of the joint. It consists of five or six *finger-like processes* (figs. 2, 3), set parallel to each other, and separated by narrow interspaces, which appear to be occupied by a thin membrane. These are not joints, moveable *inter se*, but resemble the teeth of a comb in their mode of origination. They are slender, rough, and enlarged at their tips; and the ultimate one of the series, next the dorsal side (fig. 3), is an offshoot from the penultimate. Their extremities are bent abruptly downward, approaching each other; and they are so arranged, that the whole *uncus* forms a segment, about one-third, of a cylinder, or a drum with incurving sides, with a broad truncate end (fig. 4).

28. The *incus* (*f*) also consists of distinct articulated portions. The principal are two stout *rami* (*g*), together vase-like in ventral aspect (fig. 1), resting on what appears to be a slender pedicel (*fulcrum*, *h*). But viewed laterally, the *fulcrum* is seen to be a thin plate (fig. 5), to one edge of which the *rami* are jointed, in such a manner, that they can open and close, like a pair of shears. Each *ramus* is a thick, somewhat trigonal piece, with the outer side rounded, the upper side hollowed, and the inner side flat, and in contact with the corresponding face of its fellow, in a state of repose. The *uncus* of each *malleus*, respectively, falls into the concavity of each *ramus*; and is fastened to it by a stout triangular muscle (*i*), which is seen passing from the hollow of the *ramus* to the under surface of the *uncus*.

29. Such are the firm parts; which, whatever their material, have great strength, solidity, and density. Their density, however, is not uniform; for, in some parts, they appear to merge insensibly into membrane, or into the muscular bands. They are perfectly transparent and colourless, and have a high refractive power. Their substance is not affected by a solution of potash, but is instantly dissolved, without effervescence, by hydrochloric, nitric, and acetic acid. Sulphuric acid also dissolves it, without any ebullition that appears to originate from the contact.

30. The special muscles which move these organs are numerous, and several of them massive. The walls of the *mastax* themselves are, as has been already said, muscular, and so thick as to leave the internal cavity but small. There are indications of muscles, which I have not been able satisfactorily to define; but the following are well made out. A thick muscle (*j*) embraces the upper and outer angle of the

articulation of the *malleus*, and is inserted in the wall of the *mastax* immediately over against it. A semi-crescentic band (*k*) is inserted, by its broad end, into the inferior and basal part of the *uncus*, and by its slender end into the middle of the inner side of the *manubrium*. The former of these (*j*) may be considered as an *extensor*, the latter (*k*) as a *flexor*. A broad and powerful band (*l*) is inserted along the whole inner side of the *manubrium*, and also apparently into the basal part of the *uncus*, and passes inwards towards the posterior or lobate wall of the *mastax*; into which it probably merges, as I have not been able to trace its inferior insertion.

31. The movements of these organs are very complex. The most conspicuous is an alternate approach and recession of the two *unci*, by a perpendicular motion on the hinge-joint. The opposing faces come into successive contact, and bruise down the particles of food in the manner of mullers. But a moment's observation shows that there are other movements besides this. The *manubria* move also at the same time; their free extremities are made to approach each other, as the *unci* mutually recede; and that with a peculiar twist, which greatly alters the apparent figure of these organs. [See figs. 6 to 8; of which 6 represents the right *manubrium* when the *unci* have receded, 8 the same when the *unci* approach, and 7 an intermediate condition.] The *incus* also has considerable motion. Sometimes the *fulcrum* is elevated and the *rami* depressed, so that the former is invisible: the *rami* open and shut with the working of the *mallei*, being fastened to them by the strong triangular muscle above-mentioned; but it is also evident that they have a motion of separating and closing independent of the *mallei*, though this is comparatively limited in extent, and not very often exercised. Again, when substances are brought into contact with the jaws, which, for any reason, are not acceptable, they are thrown up through the *buccal funnel*, by a peculiar scoop-like action of the *unci*, which is very curious to witness.

32. I have seen this action many times; but, in particular, on one occasion, in which much intelligence seemed to be displayed. I was watching a *Brachionus pala* in water, in which a number of that beautiful, mulberry-like animalcule, *Syncrypta volvox*, were revolving. One or two of these had been devoured, and were very visible in the intestinal canal of the *Brachionus*, which appeared excited by the enjoyment to unusual efforts. The mode in which it directed its ciliated flaps towards the spot where a *Syncrypta* was whirling, or suddenly stretched forward to the extent of the long foot, as if it would seize the prey, seemed to indicate a perception of its presence; as did, still more, the manner in which it depressed the lip-like lobe of the rotatory organ on one side, when the prey was in the vortex on that side, and the eager haste with which it shrank down into the lorica, the instant the animalcula dropped at length into the *buccal funnel*. Now, however, arose a difficulty; the black, millstone-like *unci* opened and stretched forward to grasp the little victim; they touched the globular investing case, but could not embrace it. The *Brachionus* redoubled his efforts; the jaws gaped vigorously, but could only scrape the sides of the little globe, which at every touch slipped away, the expanse of the *unci* being not

quite sufficient to grasp it. At length the animal appeared indignant; the jaws no more endeavoured to grasp, but, with a very distinct and sudden upward jerk, threw out the prey, which until then had been retained and pressed downwards by the contraction of the sides of the *buccal funnel*. Several times I saw this scene occur, with the same violent efforts, and the same ill success. A smaller *Syncrypta*, however, was bruised down by the jaws, while under my eye, and passed quickly into the stomach.

33. This incident I have described with some minuteness, not only as illustrating the particular movement in question, but also because of its bearing upon the general functions of the organs under review—the prehension and manducation of food. And it may be well, before I notice the modifications of the *mastax* in other forms, to say a few words on the structure and functions of the *buccal funnel*, and of some other contiguous organs.

34. The broad vibratile disk of *Brachionus* is deeply cleft at its mental edge, the incision reaching down to the summit of the *mastax*. The sides of the cleft can be brought into contact; and hence the structure is visible only at certain times, as when food is taken in. But the interior is a wide infundibuliform cavity, narrowing to a slender tube at its lower extremity, where its sides merge into the parietes of the *mastax*. This is the cavity which I have called the *buccal funnel*.

35. In *Brachionus amphiceros* the strong setiform cilia of the disk-lobes overarch the incision; and its upper edges, for a short distance downward, are irregularly jagged or crenated. In other species, as *B. Bakeri* and *B. dorcas*, the margins are smooth. In every case, however, the interior surface of the *funnel* is set with fine cilia, and currents, or ciliary waves, may constantly be seen pouring down the tube.

36. The sides of the *funnel*, in all cases, are formed of irregular bulbous masses of transparent flesh, which may be presumed to be muscular. Those masses which encircle the rim are usually large, and more regular. The parts are very flexible and mobile. The tube can be quite closed by the contact of its walls, even while the upper part of the *funnel* remains expanded: in *B. Bakeri* I have seen a globose mass occasionally pushed up from behind the tube to a considerable distance up the *funnel*, and presently retracted. It may be that some function analogous to taste is exercised by this organ.

37. Attached to the tube of the *funnel*, resting on the summit of the *mastax*, are seated a pair of large, clear, vesicular organs (see fig. 9, *n*), which, from their appearance and their situation, may be assumed to be *salivary glands*. In *Brachionus* they are of great size, and are generally two- or three-lobed. In *Asplanchna* (fig. 56), also, they are very large, and kidney-shaped. In the genera *Euchlanis* (fig. 12) and *Anuræa*, similar, and probably homologous vesicles are seated on the *œsophagus*, just below its exit from the *mastax*.

38. The tube of the *buccal funnel* (*m*) invariably opens upon the spot where the *unci* meet. The particles of food, or the minute animalcules, which form the prey of

the ROTIFERA, are drawn by the discal vortices into the *funnel*, and lodged at this point, within the *mastax*. The *unci*, and the *rami* of the *incus*, conjointly work on them; and they are speedily dismissed to the tips of the *rami*, immediately below which, on the dorsal side of the *mastax*, the *œsophagus* opens (*p*); a membranous tube, capable of great expansion and contraction, but varying much in length and diameter in different genera. A current of water appears to be almost constantly setting through the *funnel*, along the *rami*, in a direction towards their extremities, and thence through the *œsophagus* into the stomach.

39. In general, the ciliary vortices are sufficient to bring the prey within the *funnel*; but in several genera of the family *Euchlanidota*, as *Metopidia*, *Colurus*, *Monura*, and *Stephanops*, there is a curious accessory organ, which aids in the capture of prey; at least, I am sure it is so employed in several species of *Metopidia*.

40. Thus, in *M. acuminata* (fig. 11), the frontal region is formed by an arched fleshy process occipitally, which is approached by a small one at the mental side; and between these is the wide entrance of the *buccal funnel*. The occipital process is protected by a horny crystalline plate, forming a segment of a sphere, and when viewed laterally taking the appearance of a curved horn. It can be partially protruded and retracted, and also bent down to meet the mental lobe.

41. This apparatus, when the animal is taking food, is kept in vigorous action. A strong vortex is produced by the ciliary wheels; and as the floating atoms whirl by, the moveable plate is thrown forward with a grasping motion, the fleshy head being, at the same time, protruded; and, when the lobes are in contact, retracted. This is repeated almost every instant, with manifest eagerness and discrimination, the manducatory apparatus working vigorously all the while.

42. The same curious organ is frequently employed in another way. It is bent considerably downward; and, as the animal crawls deliberately up and down the stems of aquatic plants, it is used to rake and grub among the floccose deposits, the minute *Diatomaceæ*, &c., that adhere to them. (See fig. 11.)

43. Taking the structure described in *Brachionus* as the standard, I now proceed to examine how it is modified in other genera. In *Euchlanis deflexa* (figs. 12 to 15), the *fulcrum* of the *incus* (*h*) is thinned off ventrally to a blade-like edge, which is minutely jagged, and dilated laterally at the foot. The *rami* are very large (*g*), expanding at the sides in a triangular, pointed form; and arching across the *mastax* towards the *dorsum*, where each terminates in an elongated, curved, descending spine. The two points approach each other in a forcipate manner.

44. The *rami* are crossed at right angles by the four-fingered *unci* of the *mallei*, which by their motions evidently open the *rami*; though these latter do not appear, as usual, to be separable to the *fulcrum*, but to be united into one piece, with an ovate excavation between them (fig. 14), that does not reach to the *fulcrum*.

45. The *manubria* are more developed than in *Brachionus*. Instead of the thickened knob, to which the *uncus* is articulated, the upper portion of the *manubrium*

forms a broad laminar dilatation, down which run several carinæ, or ridges, of solid material; the interspaces, apparently, being filled with membrane, or softer and more fleshy substance (*c*). The ridges enclose three *areas*, of which the central one extends through the whole length of the *manubrium*, and the two external ones are smaller, and compose the dilatation. This structure is worthy of notice, since it is highly characteristic; and, as we shall presently find, will help us to identify this organ under very modified forms.

46. The parietes of the *mastax* are much thinner than in *Brachionus*. A stout muscle (*j*) embraces the articulation of the *malleus*, including a portion of the *manubrium*, and of the *uncus*. In *E. hipposideros*, a fan-shaped muscular band spreads along the interior side of both these parts, filling the angle, and stretching from one to the other: it is evidently a powerful *flexor*.

47. In this genus we see a structure of the *incus*, which prevails extensively in the class: each *ramus* is produced into an angular projection on either side of the *fulcrum*. To this projection, which I shall call *alula* (*o*), a muscular band is fastened, which passes down, and is inserted into the fundus of the *mastax*. Another more slender band or ligament connects the projection with the foot of the *fulcrum*.

48. The tube of the *buccal funnel* (*m*) is very wide. The *œsophagus* (*p*) is also wide, and short. On the latter duct are seated two globose clear *salivary glands* (*n*), each of which encloses a spherical nucleated nucleus. These must not be confounded with the *pancreatic glands*, which are much larger, and seated on the stomach, below the entrance of the *œsophagus*.

49. In *Notommata aurita* (figs. 16 to 21), while there is much resemblance to *Euchlanis*, the structure is in some respects peculiar. The form of the *mastax* is prolate, the longitudinal diameter exceeding the transverse,—a figure which is dependent on the fact, that the *fulcrum* of the *incus*, and the *manubria* of the *mallei*, are greatly produced in length, and are all extended nearly in the same direction; viz. that of the longitudinal axis of the body (see figs. 16 & 17).

50. The *manubrium*, as in *Euchlanis*, is three-looped, and dilated at the summit, unsymmetrically (see fig. 20). The *uncus* is broad, composed of five fingers, which are somewhat divergent, and is arched transversely as well as longitudinally. It is furnished, on that side which is next to the *dorsum*, with a peculiar, semicrescentic process (*q*), strengthened by a carina: the points of these two processes are opposed to each other at their tips, beyond the tips of the *rami* of the *incus*.

51. The *fulcrum* of the *incus*, when viewed ventrally, might be mistaken for a straight slender rod, with a round, dilated foot (fig. 16). It really consists of two slender curved rods, united by a thin lamina: the exterior of these, which is dilated laterally at the foot, is more curved than the interior, whose extremity it receives in the hollow of its own (fig. 17), as the chord of a bow joins the extremity of the arc. The summit of the *fulcrum* is obliquely truncate, and to this oblique surface are articulated, by a hinge-joint, the two *rami*.

52. The *rami* here take the form of moveable blades or jaws (figs. 16, 21), which arch across the vault of the *mastax* towards the *dorsum*, and receive, on their convex surfaces, the *unci*, which are tied to them near their extremities. The *rami* are capable of being widely opened (fig. 21), when several jagged teeth are seen on their opposing edges, which lock into each other when closed. In the act of expansion, an obtusely pointed *lamina* is seen below their arch (fig. 21, *r*), which is capable of being slightly protruded or contracted, independently of the motion of the *rami* or *unci*. The *rami* themselves, though opened and closed with the *mallei*, are not dependent on the action of the latter, for they evidently possess a powerful spontaneous motion in opening and closing. This movement is doubtless produced by the muscular bands which (as we saw in *Euchlanis*) connect the lateral processes (*alulæ*) of the *rami*, which are greatly developed, with the fundus of the *mastax*, and with the foot of the *fulcrum* (*t* and *u*).

53. The muscles of the *malleus* differ from those which we have seen in *Brachionus* and *Euchlanis*. A long band passes from the summit of the *manubrium* to the fundus of the *mastax*; another ties its lower extremity to the paries, immediately below it; while a third passes upward from the inner face of the same piece, probably to the inferior surface of the *uncus* (fig. 16).

54. The entrance of the *buccal funnel* into the *mastax* is, in this species, protected by a vault of many complex pieces, which appear solid (fig. 16, *v*), though of such tenuity and fragility that I have not been able to resolve them satisfactorily.

55. *Notommata clavulata* (figs. 22 to 26), a species of large size and of peculiar beauty, is remarkable for the great development of the *mallei*. The *buccal funnel* is here shallow, but richly ciliated; its short tube merges insensibly into the *mastax* (fig. 23), which consists of three lobes, more than usually marked, and nearly spherical. Each *manubrium* is, as usual, dilated above, where the lateral loops are trigonal, and attenuated below. A broad trapezoidal *uncus* (figs. 25, 26) is articulated to it, of eight fingers, of which the second and third are branches of the first. The fingers are arched both transversely and longitudinally, and their extremities are connected by a web (fig. 26). A transverse process crosses their inferior surface, doubtless a point of attachment to the ligament, which fastens them to the *ramus* of the *incus*, and which corresponds to the triangular ligament (*i*) described in *Brachionus* (§ 28). The elasticity of this ligament is well shown in the working of the jaws; for the *uncus* is elevated to a considerable distance above the level of the *incus* (see fig. 24), when the appearance and action of the pair are exactly those of curved dentate mandibles, opening and snapping across the tube of the *buccal funnel*. The *incus* is placed nearly horizontally, or at right angles with the plane of the *manubria*; but, during the vigorous working of these organs, they are alternately depressed and elevated to so great a degree, that the *fulcrum* appears now above, now below their level.

56. The *fulcrum* is rather short, much compressed, and thickened at its free extremity; its direction scarcely deviates from that of the *rami*,—in this respect contrast-

ing widely with *Not. aurita*. The *rami*, when closed, assume a pyriform or lozenge-like outline, but are cleft to their base; and as the *unci* are attached to the terminal half of their length, they open widely (fig. 24).

57. The whole manducatory apparatus in *Anuræa acuminata* bears a resemblance, singularly exact, to this of *Not. clavulata*, notwithstanding the external diversity of the two animals. The similarity extends even to the manner in which the *unci*, which are seven-fingered, are protruded into the *funnel*, and fiercely snapped.

58. *Notommata petromyzon* (Plate XVII. figs. 27 to 31) is remarkable for the extreme simplicity of the parts under review; a circumstance which makes it peculiarly valuable as a study. It is one of those species in which the ciliated facial disk is very oblique, being nearly on the plane of the *venter*. Hence the *buccal funnel* is short at all times, and can be quite obliterated, the entrance of the *mastax* opening on the facial surface. This organ is a delicate sac with membranous walls, of obcordate form (fig. 31), deeply trilobate at the fundus.

59. The *fulcrum* of the *incus* (fig. 31) is thin and blade-like; straight, except that its free extremity is slightly incurved, very deep, and truncate at its articulate extremity. The *rami* (fig. 29), when united, form an isosceles triangle, cleft to the *fulcrum* and arching downwards.

60. The *mallei* are equally simple. The *manubrium* (figs. 28, 31) is a slender rod, with a projecting process near its articulation. The *uncus* (fig. 28) consists of two fingers, membranous in texture, and at times evanescent, which work on the *ramus* near its extremity; and two muscular threads are seen connecting the former with the latter.

61. Notwithstanding the simplicity of the organs in this instance, a comparison with *Not. aurita* will show that the structure is essentially the same in the two species.

62. In *Notommata lacinulata* (figs. 32 to 34) the *mastax* is very large, subtrihedral, with the orifice at the surface of the disk or protruding from it; so that there is no *buccal funnel*. The *incus*, though somewhat simple, is very large; and the *rami*, when closed, form a hemispherical *dome* of thin texture; so as to resemble, when viewed obliquely from above (fig. 33), a globe of glass standing on a pedestal (the *fulcrum*). The similitude is enhanced by lines passing in different directions over the vault, like the astronomical circles. The *mallei* (fig. 34) are slender rods, hooked at the bottom; and soldered at the upper part across the dome, where they become very much attenuated, without any distinct division into *manubrium* and *uncus*.

63. The *rami* of the *incus* are divided to their base; in use, they are protruded considerably, and are distinctly organs of prehension; their edges being employed vigorously, in nibbling at the floccose matter that accumulates on aquatic plants, as the little animal crawls, by means of its two-toed foot, up and down the stalks.

64. M. Dujardin has constituted a genus (*Plagiognatha*) for this species and *Not. felis* of Ehrenberg; mainly, however, because they have "mâchoires à branches parallèles tournées du même côté, et recourbées vers le bord cilié, avec une tige cen-

trale (*fulcrum*) droite, très-longue, élargie à sa base *." This description is so vague, that it might embrace a multitude of widely remote species and genera; while it does not at all indicate the true peculiarity of the organs it professes to define.

65. *Furcularia gibba* (figs. 35 to 37) comes, in the form of its jaws, very near to *Not. lacinulata*; but most of the species of this genus, as described by Professor EHRENBERG (*not by M. DUJARDIN*, who includes in it a number of dissimilar species, already well defined and separated by his Prussian predecessor), are distinguished by the *manubria* being dilated laterally at their free extremity, so as to resemble the foot of a towel-horse. These expansions are doubtless for the attachment of muscles (which, however, I have not been able to define); and as the simple *uncus* is affixed to the edge of the *ramus* only by its point (fig. 36), a greater play is probably afforded it by such muscles, in the combined action of the jaws upon the prey.

66. The *rami* are broad, glassy, vaulted, cleft throughout, capable of widely opening (fig. 36), and produced into long decurved points (fig. 37). Their lower edges are thickened, so as to form a marginal band.

67. *Furcularia marina* has its manducatory organs formed on another type, approaching that of *Diglena*; as I shall presently notice.

68. *Notommata gibba* (figs. 38 to 40) prepares us for the remarkable modification of these organs which we find in *Synchæta* and *Polyarthra*. It is a minute species, having much of the appearance and habits of *Not. lacinulata*; but remarkable for the length and lozenge-form outline of the *mastax*, owing to the great posterior development of the ventral lobe, which is itself dependent on the great elongation of the *fulcrum*. The *manubria* (fig. 39) are long and incurved at their free extremities; the *unci* are single-toothed, and soldered to the *rami*. The latter are curved, glassy blades; along the middle of each runs a line, which is difficult to understand: after much study, I think it to be the angle of depression of the surface, as represented ideally at fig. 40.

69. In *Synchæta* and *Polyarthra*, the *mastax* and its included organs attain their maximum of development as regards dimensions, though not as regards complexity. In some species of the former genus fully one-third of the entire bulk is occupied by these organs. Owing, however, to the extreme delicacy of the parts, particularly of the *mallei*, and the unequal refraction produced by the prismatic form of the animals, the structure is unusually difficult to resolve. EHRENBERG evidently did not understand it: the points of the *unci* he appears to have seen, in *S. pectinata*, but no more; and the frontal styles he mistook for accessory jaws †. In *Polyarthra*, he merely says that "there are two single-toothed jaws ‡." His figures give little or no light on the true structure in either genus.

70. DUJARDIN knows nothing of *Synchæta*, no species of which he seems to have seen; but concludes, most groundlessly, that it is not distinct from *Hydatina*. Of *Polyarthra* he merely says, "Mâchoires unidentées §;" and though he figures a spe-

* Infusoires, p. 651. † Infusionsthier. p. 437. ‡ Ibid. p. 440. § Infus. p. 641.

cies*, he gives no details of the internal structure, but represents two projecting articulated setæ, which he calls "appendices de la bouche." Of the latter, however, I must confess, after the examination of dozens of specimens, I can discern no trace, with a power of 560 diameters.

71. In 1850 I had discovered, in a species of *Synchæta*, what seemed to me a most anomalous condition of the manducatory organs. As the species appeared new, I described it†, in 1851, by the name of *S. mordax*. The peculiarity consisted in two pairs of hooked jaws, exactly resembling the mandibles and maxillæ of a beetle, now and then projected from the front, and opened with a sudden snapping motion, and instantly withdrawn. Each pair moved independently of the other, but in evident connexion with two pairs of bulbous muscles, seated deep in the breast. They could scarcely be discerned, when withdrawn beneath a sort of membranous lip that formed the frontal outline; and no trace of them could be recognized after the animal had been subjected to the compressorium.

72. Until recently, this structure completely baffled my endeavours to solve it. It was totally unlike anything that I was acquainted with in the whole class; and yet I was quite sure of the exactitude of the observation, having witnessed the phenomena on many occasions. Very lately, however, I have succeeded, by means of the solution of potash, in defining the whole structure of the solid parts, and can demonstrate them at pleasure. I found, with surprise, that there is in them no deviation from the normal type, while the function and homology of the organs are greatly illustrated by their action.

73. The *mastax*, as before stated, is of large size, ventricose, globose, or subcubical in figure, with both the *incus* and the *mallei* so much bent, as to form, when viewed laterally (fig. 43), each two sides of a quadrangle. The *fulcrum* is a slender, compressed rod, slightly arcuate. The *rami* are thin, elastic blades, nearly straight when viewed laterally (fig. 43), but arched in a forcipate manner; the internal edges not in contact, but approaching at the points, which are somewhat twisted. The *mallei* are slender; the *manubrium* much bent; the *uncus* a single, pointed finger, connected with the *ramus* by a delicate membrane, cut into teeth,—at least in *S. tremula* (fig. 42). The points of the *unci* form the anterior, as the points of the *rami* form the posterior, pair of snapping teeth.

74. The vigorous action of these jaws would lead us to expect powerful muscles, and we find them peculiarly thick and bulbose. The great convergent pair with clubbed summits, that form a conspicuous V-shaped object in the midst of the animal, are muscles, which envelope the *mallei* (fig. 41), and are seen, during the momentary protrusion of their tips, like thick gums around the bases of the teeth. A thick, clavate muscle also proceeds from the articulation of the *uncus*, and lines the globosity of each lateral lobe of the *mastax* (fig. 41). In *S. mordax* a pair of muscular bulbs are placed, one on each side of the foot of the *fulcrum*: these are probably

* Infus. pl. 21. fig. 6, A. B.

† Ann. Nat. Hist., Sept. 1851.

the extremities of fasciæ, that extend to the *rami*; as the motions of the latter are synchronous with twitchings in these bulbs.

75. There is a subtile clear membrane stretched over the whole apparatus, but not in contact with it; for it has a slight power of independent contraction and protrusion. It is of considerable breadth, with a blunt point in the centre, which is simple in *S. mordax*, bifid in *S. tremula* (fig. 41). As there is no *buccal funnel*, this must be the occipital (or rather frontal) margin of the *mastax*, the teeth being evidently extruded from beneath it; there must also be a mental, or inferior, margin. but I have not defined it.

76. *Polyarthra* differs little from *Synchœta* in the form of its jaws. The *mastax* is more oblique (fig. 47), its central lobe being much produced, and pointing towards the venter. The muscles of the upper portion are so dense (fig. 44) as to shut out all sight of the interior, until they are dissolved away by potash (fig. 46).

77. The *rami* are very broad, somewhat square at the base, flat, but much arched longitudinally (fig. 47). They open and shut vigorously, with a snapping action, but are not protruded from the front: their whole interior edges come into contact.

78. The *mallei* are simple, slender, bent rods, apparently without distinct articulation (fig. 48). During life they are thick, and irregular in outline (fig. 46), owing. doubtless, to their being invested with muscle, as in *Synchœta*.

79. There is a clear vaulted membrane, spreading like a dome over the jaws, when viewed vertically (*i. e.* from the front), which is certainly the paries of the *mastax*: it has an advancing and receding margin (fig. 49), which is placed considerably towards the mental aspect, where the *mastax* opens at the bottom of a shallow *funnel*.

80. From these forms, especially that of *Synchœta*, the transition is easy to *Diglena, Eosphora, Albertia*, and *Furcularia marina*. Thus, in *Diglena forcipata* (figs. 50, 51). the *incus* is a true forceps; the *rami* projecting in nearly the same plane as the *fulcrum*, and the hooked points coming into contact. The edges have a peculiar structure, being delicately ridged transversely; while from the terminal points of the ridges spring slender setæ in a double row, which project so as to oppose those of the other *ramus*. This curious structure I find also in *Diglena grandis*. The *fulcrum* is compressed.

81. The *mallei* have nearly straight *manubria*, slightly enlarged at the free extremity, and clubbed towards the joint, where they are invested with thick muscle. The *uncus* is a simple curved piece, so far as its point of attachment to the *ramus*; but at this point there is articulated to it a stout curved acute *spine*, which, when the *rami* are closed, crosses its fellow of the opposite *uncus*, and plays over the setæ of the *ramus* (fig. 50). In *Diglena grandis* there are *two* spines on each *uncus*, the one much shorter than the other; as there are also in *Eosphora aurita*. In the latter species these tooth-like spines are projectile, as a formidable pair of jaws; and I doubt not that such is their function in *Diglena* also, though I have not seen them so used; especially as, in both genera, the ciliated disk is prolonged on the prone surface

nearly in the longitudinal plane of the venter; so that the aperture of the *mastax* is brought to the surface, without the intervention of any appreciable *funnel.*

82. Scarcely differing from this structure is that of the curious little *Albertia vermiculus,* first found by M. DUJARDIN, living entozoically in *Lumbricus* and *Limax,* and which I have found in the intestine of *Nais proboscidea* (figs. 52, 53). Whether the *uncus* has an accessory tooth, or not, I cannot certainly say, owing to the minuteness of the parts; the animal being only $\frac{1}{100}$th of an inch in length, and the dental apparatus being unusually minute in proportion. It is, however, frequently protruded, to the extent of fully half of the apparatus, from the front, and vigorously snapped.

83. *Furcularia marina* (figs. 54, 55) is another species, in which these organs are greatly protrusile; the whole of the long *mallei* being sometimes exterior to the frontal disk (fig. 54). The *incus* (fig. 55) seems to me a simple, much-arcuated forceps; and the *mallei* to be without *unci,* the *manubria* (simple incurved rods) being articulated to the bases of the *rami.* M. DUJARDIN describes and figures them differently*, but I believe he is mistaken.

84. We are now prepared to understand the form of dental apparatus, which has hitherto appeared perfectly anomalous in this class,—that of *Asplanchna,* as described and figured by Mr. BRIGHTWELL and Mr. DALRYMPLE, in the case of *A. Brightwellii,* and by myself in that of *A. priodonta.* To these must be added *Notommata myrmeleo,* and *N. syrinx,* of EHRENBERG; referrible doubtless to the same genus. A moment's comparison of the jaws of *A. priodonta* (Plate XVIII. figs. 56 to 59) with those of *Synchæta tremula,* or of *Diglena forcipata* (Plate XVII. figs. 41, 50) either opened or closed, will show that the former constitute an *incus,* with the *fulcrum* much diminished; the very row of teeth that runs, like the edge of a saw, along the interior of the *ramus* (figs. 58, 59), having its counterpart in the ridges and setæ of *Diglena*; as also in the teeth of these organs in *Notommata aurita.*

85. The *mallei,* at first sight, seem wholly wanting in *Asplanchna*; but they are really present, though in a state of extreme attenuation and degeneration. Mr. DALRYMPLE thought he saw an occasional glimpse of an accessory curved point, outside the tip of each jaw, in *A. Brightwellii*; and I have invariably observed the same in both that species and *A. priodonta* (figs. 58, 59). It is a curved pointed rod, which, at its lower end, that ordinarily would be free, is attached to a process, which is itself attached, I believe, by a transverse ligament, to the *ramus,* near its articulation. It is most evanescent; yet it is not dissolved by treatment with potash, and I have no doubt that it represents the *malleus.*

86. The *fulcrum,* though short and thin, is very deep; the depth in fact exceeding the length (fig. 59); and it affords a clear insight into the nature of the articulation of the *rami*—a ginglymus of the simplest kind; the articulating parts having straight parallel edges, united by an interposed membrane.

87. Another peculiarity in this genus is the absence of the *mastax*; at least in its

* Infus. p. 649; and pl. 22. figs. 4, C, D, E.

ordinary form of a chamber inclosing the trophi. The *fulcrum* is attached to a reniform muscular *cushion* (fig. 56); with the *rami* projecting freely into the *buccal funnel*: and, under graduated pressure, protruded from the front, when they snap vigorously. But when we consider that the normal form of the *mastax* is that of three lobes, of which one belongs to the *incus*, and two to the *mallei*, it is natural to expect that the evanescence of the *mallei* would be accompanied by the evanescence of their muscular lobes. And this I conclude is the true state of the case: the *cushion* is the lobe of the *incus*, and therefore the sole representative of the *mastax*; the lateral lobes having become obsolete.

88. I was unwilling to interrupt the regular gradation, through which we have traced the degeneration of the *mallei* to evanescence; but I now retrace my course a little, to notice how the same organs degenerate in other modes. In a beautiful and common species, well known to most microscopists, *Mastigocerca carinata* (figs. 60 to 62), the dental apparatus occurs under an unusual form. The *mastax* is a somewhat slender sac, much produced in length, and with the component lobes greatly and irregularly developed. The *incus* has a *fulcrum* of great length and slenderness, a straight rod with a dilated foot (fig. 62, *h*). The *rami* are small, forcipate, and resembling those I have lately described; but with the *alulæ* greatly produced (*o*). The *mallei* have long, slender, incurved *manubria*, and simple *unci*.

89. But the remarkable circumstance is the non-symmetrical character of the apparatus. The left side is much more developed than the right. The left *alula* of the *incus* (*o*) descends to a greater distance than the right (*o'*); and its extremity is dilated into an expansion, with several irregular angles, to which muscular threads are attached. The *ramus* also of the same side is larger than its fellow. So with the *mallei*. The *manubrium* of the right (*c'*) is comparatively short, very slender, and of uniform thickness; with a long, slender, rod-like *uncus* (*e'*), doubly bent in the middle. The left is much longer, irregularly swollen, clubbed at the articulation, and bearing a thick, curved, knobbed *uncus*, which terminates at a point not precisely opposite the tip of its fellow (*e*)*.

90. In *Monocerca* (fig. 63),—from which *Mastigocerca* can scarcely be said to differ generically, though Professor Ehrenberg places it in another family,—the right *malleus* entirely disappears, not a vestige of it remaining; though the left (*b*) is long, and well developed. The *incus* is a straight rod, with a high *carina* (*f*), with the *rami* almost obliterated: the *alulæ*, however, are rather large, but unequal.

91. This want of symmetry is a remarkable character of the genus, and is displayed in other particulars. For example, in *Monocerca bicornis*, the little projecting tubular organ, which Ehrenberg has called the respiratory tube, but which I consider a rudimentary *antenna*, is double; but the two are unequal. In the same species, and also in *M. porcellus*, the lorica terminates frontally in two *spines*, of

* In *Notommata parasita* there is a similar want of symmetry, the right *manubrium* being much shorter than the left (see my memoir in Trans. Micr. Soc. iii. p. 143).

which, again, the left is considerably larger than the right. The lorica, in *Mon. rattus*, and much more prominently in *Mast. carinata*, is furnished with a *dorsal carina*, running along the median line; the elevation of this ridge is not perpendicular, but leans considerably over to the right side. Even the single *foot-spine*, which is so characteristic of the genus, must be considered either as representing the ordinary pair soldered together, or as single through the obsolescence of the other. That the latter is the true solution is the more probable, since, in *Mastigocerca*, there is, at the base of the foot-spine, another spine, very minute, but distinctly jointed to the foot-bulb. This unsymmetrical development is not without parallel in higher animals; of which it will be sufficient to allude to a single example, in the case of the projecting tooth of the Narwhal.

92. The figure of the *incus-fulcrum* in *Monocerca porcellus*, and the obsolescence of its *rami*, make an approach to the structure which we find in a very curious form, *Scaridium longicaudum* (Plate XVII. figs. 64, 65). The whole organization of the manducatory apparatus is here so abnormal, that I shall describe it in detail; especially as Dujardin has not noticed the genus at all, and Ehrenberg confines himself to the vague remark, that "the *Schlundkopf* (Pharynx) is oblique, with unequal, pincer-toothed (one-toothed) jaws."

93. The *mastax* is somewhat obconic, or shaped like the heart of a mammal; a muscular sac, on the parietes of which transverse *rugæ* appear (β). The eye (x), a large flattened capsule, with the crimson pigment not quite filling it, is attached to its occipital surface; apparently not connected, as usual, with the large occipital ganglionic sac (y); which, however, presses upon it from above and behind. The summit of the *mastax* projects into a point, which, though within the level of the ciliated ridges of the front (fig. 64), seems to be in contact with the surrounding water, without the intervention of a *buccal funnel*. This point is cleft deeply; the incision being transverse, from right to left, but obliquely upwards (fig. 65). The occipital division separates widely from the mental one, with a gaping or snapping action, very frequently performed; but with so great a rapidity and suddenness, that it needed long-continued observation to enable me to understand the parts. The structure is as follows.

94. Behind the occipital division, and from its point, spring several arched setæ; and an apparatus of hooked teeth is visible within its concavity. The mental division (δ) is slightly bifid, with rounded points; between which, at the moment of gaping, several hooked setæ are projected, and instantly retracted (ϵ). These setæ, or teeth, are connected, by prolongations of their bases, which are doubly geniculate and appear jointed, with the summit of the *fulcrum* (g); and must therefore be the representatives of the *rami*. The *fulcrum* itself is a straight rod, with a strongly developed *carina* (h), arcuated somewhat in the same way as that of *Notommata aurita* (see fig. 17).

95. The *mallei* consist each of a thin *uncus* (e), working on the *ramus*, and of a

manubrium (*c*), singularly looped, but of such extreme tenuity, as to require the most delicate adjustment of focus to resolve it. Indeed, I have never been able to see more than one in a lateral view; but infer the existence of the other, from the symmetrical appearance of the apparatus, in a dorsal aspect. Singular as the form of this organ is, I think we can recognise in it the three loops which constitute the solid framework of the *manubrium* in *Euchlanis*, &c. (§ 45).

96. We have now traced the same common organs of manducation, through various phases, from what I ventured to call their normal development in *Brachionus* and *Euchlanis*. Viewed generally, these modifications may be considered as successive degenerations of the *mallei*, and augmentations of the *incus*. I shall now return to the same starting-point, and trace a chain of modifications in another direction; a chain of fewer links, it is true, but all tending to the same point, the degeneration and final obsolescence of the *incus*, and (in the final stage) of the *mallei* also.

97. Though the types of structure, in the manducatory organs of the species which we have now to consider, are few, being not more than three or four, the species and genera are numerous; and they may all be distinguished, by a remarkable peculiarity, from those with which we have been hitherto engaged. They no longer assume a prone position when at rest, with the venter towards the support, but take an erect posture, the body elevated in the same line as the foot, the tip of which is the point of attachment. Many of the members of one of the great groups, and all of another, inhabit cylindrical cases, made of gelatinous matter, thrown off from their own bodies, absurdly called "loricæ" by Ehrenberg, for the purpose of giving a semblance of unity to his artificial arrangement, but really having not the slightest analogy with the stiff integument of *Brachionus*, *Euchlanis*, &c., which is an organic part of the animal.

98. The genera *Pterodina* and *Triarthra* may seem exceptions to this generalization; for the former has a distinct lorica, as has also the allied genus which I have named *Pompholyx*; and *Triarthra* has a posterior stylet, which, with the anterior pair, has been compared with the pinnæ of *Polyarthra*; while yet all these display modifications of the manducatory apparatus, belonging to the type which I am about to describe.

99. I am not in this place occupied with the principles of a new arrangement of the Rotifera, and shall therefore merely say, that the above exceptions are apparent rather than actual; though they may be considered as osculant groups.

100. Dismissing these, I come to examine the manducatory organs as they appear in the genera *Triarthra*, *Pompholyx*, *Pterodina*, *Œcistes*, *Limnias*, *Melicerta*, *Conochilus*, *Megalotrocha*, *Lacinularia*, and *Tubicolaria*. So far as my examinations reach (which include eight of these ten genera), there is no appreciable variation in the structure of these organs in them all; and in one of the two, which I have never been so fortunate as to meet with, the deficiency has been well supplied; since it is the species (*Lacinularia socialis*) which forms the subject of the admirable memoirs of Mr. Huxley and Dr. Leydig, already referred to (§ 6).

101. Professor EHRENBERG had simply described the apparatus as consisting of teeth, which, like the arrow on a bow, are fastened across the jaw*; and M. DUJARDIN does little more than repeat the description. He says, "all these animals present a pair of jaws, almost in form of a stirrup, composed of an arch traversed by a bar, on which lean, by the free extremity, three parallel teeth springing from the bow of the stirrup, which is engaged in the fleshy bulb†."

102. Dr. LEYDIG, who complains of the inaccuracy of EHRENBERG's figures, describes the apparatus as consisting of "two bent quadrangular plates, across which several lines are stretched; the three foremost, which are stouter than the rest, jutting out as three teeth." He says that the two plates have a shears-like figure; and notices at their union "an apophysis (the *fulcrum*), which seems to enter into the circular mass of the gizzard‡."

103. Mr. HUXLEY, on the other hand, saw the analogy of this type with that which I have already considered; but not with the stirrup-like form which is found in *Philodina*, &c. He thus describes it: "The armature of the pharyngeal bulb is composed of four separate pieces. Two of these (which form the *incus* of Mr. GOSSE) are elongated triangular prisms, applied together by their flat inner faces§: the upper faces are rather concave, while the outer faces are convex; and upon these the two other pieces (the *mallei* of Mr. GOSSE) are articulated. The last are elongated —concave internally, convex externally—and present two clear spaces in their interior; from their inner surface a thin curved plate projects inwards. At its anterior extremity this plate is brownish, and divided into five or six hard teeth, with slightly enlarged extremities. Posteriorly the divisions become less and less distinct. and the plate takes quite the appearance of the rest of the piece.

"This is essentially the same structure as that of the teeth of *Notommata* [=*Asplanchna*], described by Mr. DALRYMPLE, and by Mr. GOSSE (on the anatomy of *Notommata aurita*); and *very different from the true 'stirrup-shaped' armature*."

104. Professor WILLIAMSON describes the apparatus with elaborate care as he finds it in *Melicerta*. His remarks are too long to quote, but they agree mainly with what was already known. He notices "two broad elongated plates," which he calls "*crushers*," from which "proceed laterally numerous parallel bars, somewhat thickened at their inner extremities, where they are attached to the plates; whilst at their opposite ends they are united with others of the same side by a curved connecting bar, from the outer sides of which are given off various loops and processes. . . . From the upper extremities of the two crushers there project, upwards and backwards, two slender prolongations, united by a kind of double hinge-joint near their apex, where they not only play upon each other, but also on a third small central fixed point,

* Infusionsth. p. 386. † Ibid. p. 615. ‡ SIEB. and KÖLL. Zeitsch. 1852, p. 463.

§ Mr. HUXLEY says, these are "not described by LEYDIG;" but they are his "bent quadrangular plates," *ut supra*.

lodged in a little conglobate cellular mass *." Professor WILLIAMSON denies that the transverse teeth move on the plates, as affirmed by EHRENBERG, since they are firmly united with them. He further states, that "the conglobate organ in which the apparatus is imbedded [*i. e.* the *mastax*] is composed of numerous large cells, each of which contains a beautiful nucleus with its nucleolus;" and supposes that muscular threads penetrate it to reach the dental apparatus †. The statement of the cellular character of the *mastax*, and the presumption of penetrating muscles, are alike negatived by my observations, not only of this species, but of the whole range of the ROTIFERA. The able and learned Professor has probably been misled, in the former conclusion, by some overlying tissues, perhaps similar to the salivary glands in *Euchlanis* (see § 48).

105. My own observations on the same species, published in the same journal, did not succeed in dispelling the obscurity which still rested on the structure; and I shall therefore here merely allude to them, as a part of the bibliography of the subject. It appears, however, that though more attention has been bestowed on this type than on any other in the class, it still needs solution. I shall therefore attempt to give it in detail, as it appears in *Limnias ceratophylli* (Plate XVIII. figs. 66 to 71).

106. The *mastax* consists (fig. 66, *a*) of three subglobose lobes, as usual (not *four*, as stated by LEYDIG, and by EHRENBERG before him); one on each side appropriated to each *malleus* (*b*), and the third descending towards the ventral aspect, which envelopes the *incus* (*f*). The *mallei* are more intimately united to the *rami* of the *incus* than in the former type; each *uncus* forming, with its *ramus*, a well-defined mass of muscle, enclosing the solid parts, and in form approaching the quadrature of a globe (ζ); two flat faces opposing and working on each other (fig. 66). Across the upper surface of the mass the *uncus* is stretched (fig. 71, *e*), as three long parallel fingers, arched in their common direction, and imbedded in the muscular substance; their points just reaching the opposing face of the *ramus*, and meeting the points of the opposite *uncus*, when closed. The *manubrium* (*c*) is much disguised, by being greatly dilated transversely, forming three bow-like loops of little solidity, to the chord of which the fingers are soldered, *not articulated*. The surface of the dense muscular mass displays *striæ* parallel to the fingers, and, as it were, continuing their number towards the dorsal extremity, becoming fainter till they are imperceptible. These *striæ* do not disappear when the muscular parts are dissolved by potash; and hence I infer the existence of a delicate investiture of solid substance, similar to that of the teeth, &c., enclosing the muscular mass.

107. The *incus*, which cannot be separated from the *mallei*, thus consists of two portions (*g*), corresponding to the *rami* in *Brachionus*, &c., each of which forms the

* These two prolongations are, as I conceive, but the lateral outlines of the *fulcrum*; and the joint is not at the point where they unite, as Professor WILLIAMSON supposes, but at the point of their widest separation, whence the *rami* diverge.

† Journ. Micr. Sci. i. (Comm.) p. 4.

lower part of the quadrantic mass above described. At the ventral extremity they are articulated to a slender *fulcrum* (*h*), which is a little bent downward. The solid framework of each *ramus* sends off, from its inferior surface, a slender curved process (*o*), which is connected with the extremity of the *fulcrum*, and is probably the analogue of the *alula*.

108. The action of this apparatus is as follows:—The ciliary vortices, produced by the waves of the coronal disk, pass together through the upper sinus, and are hurled in one stream along the centre of the face, nearly to the projecting chin. Here is placed the orifice of the *buccal funnel*, a perpendicularly descending tube of considerable width (fig. 66), slightly funnel-shaped at the top, the interior surface of which is strongly ciliated. It descends straight upon the *mustax*, over the part where the *unci* unite. But just above this point there are two valves projecting from the walls of the tube, also well ciliated. These can be brought into contact, or separated in various degrees, at will; and being very sensitive, they regulate the force of the inflowing current, and doubtless exclude hurtful or useless substances. The current now flows along the two *rami* of the *incus*, as already described; and, passing between their separated points, descends into the *œsophagus*, a slender duct opening beneath them (*p*), and leading into the digesting stomach.

109. As this current passes, the manducatory apparatus acts upon the particles of food which it brings in its course. The quadrantic masses approach each other and recede, with a rapid rolling movement, in the direction of the curvature of the *mallei*; while, at the same time, the *rami* of the *incus* open and close their points, rise and sink, and occasionally perform a kind of shoveling action. The points of the fingers of the *unci*, meeting each other, doubtless pierce and tear the *Infusoria* swallowed, and the striated faces of the quadrantic masses bruise, squeeze, and grind them down.

110. When the muscular investiture is dissolved away by potash, the essential identity of the whole structure with that of the type already described becomes abundantly evident. A comparison with *Notommata clavulata*, for example (compare fig. 71 with figs. 25 and 26), will show this. Even the *mallei*, which in some aspects present difficulty (figs. 67 and 69), when viewed vertically are but little changed: the fingers are parallel, instead of divergent, and the ansate character of the *manubrium* is lost; but three areas, enclosed by loops or carinæ of solid substance (fig. 71), reveal their true nature.

111. Another well-marked and easily recognized family, in this group, is that which Ehrenberg has named *Philodinæa*, the first forms of Wheel-animals which attracted the notice of microscopists. The *buccal funnel* is here rather long and slender, and always permanent; the manducatory apparatus having no power of materially altering its relative position in the body, and never being brought into contact with the exterior.

112. The *mastax* in *Rotifer macrurus* (figs. 72 to 78), which may be considered as

a typical species, consists of the usual three globose lobes, of which the two lateral are somewhat produced above, to embrace the termination of the *funnel.* The dental apparatus differs very little from the type which we have seen in *Limnias* and its fellows. The *mallei* and the *incus* are soldered together, into two sub-quadranti-globular masses (ζ), which appear to be muscular, but invested with a solid integument. These contain the *rami* of the *incus*, and are crossed by two strong teeth, which rise from below the exterior edge of the mass, and descend upon its interior face.

113. The *manubria* are more obsolescent than in *Limnias*; but they may still be recognized, in a vertical aspect (fig. 74), as three loops, of which the central one is chiefly developed (*c*); and in a dorsal aspect (fig. 72), as a translucent, reniform lobe, descending on the exterior of each quadrant. Numerous delicate striæ cross the quadrantic masses, on *each* side of the principal teeth, and these are permanent (fig. 74) after potash-treatment. A very minute and rudimentary *fulcrum* (fig. 75, *h*) is seen in the lateral aspect.

114. The structure and action of an apparatus of this type may be made more clear by a homely illustration. Suppose an apple to be divided longitudinally, leaving the stalk attached to one-half. Let this now be again split longitudinally, so far as the stalk, but not actually separating either portion from it. Draw the two portions slightly apart, and lay them down on their rounded surfaces (fig. 76). They now represent the quadrantic masses in repose, the stalk being the *fulcrum*, and the upper surfaces being crossed by the teeth. By the contraction of the muscles of which they are composed, the two segments are made to turn on their long axes, until the points of the teeth are brought into contact, and the toothed surfaces rise and approach each other (fig. 77). The lower edges, however, do not separate, as the upper edges approach, but the form of the masses alters, becoming more lenticular; so that when the toothed surfaces are brought into their closest approximation (fig. 78), the outline has a subcircular figure. It is on account of this change of form, that I presume the masses themselves to be partially composed of muscle.

115. In *Philodina* (e. g. *P. roseola* and *P. megalotrocha*) the quadrants are connected by an elastic ligament, which crosses from the interior face of one to the other, just below the points of the teeth. I find no trace of this in *Rotifer*. The teeth are generally but two; in *P. aculeata*, however, I find, as EHRENBERG does, *three*, and he indicates the same number in *P. macrostyla* and *Monolabis conica*, species which I have not met with.

116. It has been usual to call the divisions of the dental apparatus, in this family, "stirrup-shaped;" but this comparison is grounded on a misapprehension of their true form, which I have proved, by numberless examinations, to be that of the quadrature of a sphere, as above explained.

117. It is no less evident that Mr. HUXLEY's remark, above cited, that the jaws in *Lacinularia* (=*Limnias*) are "very different from the true stirrup-shaped armature" (so-called), is founded on error; the two forms bearing the very closest analogy, as

will appear by a comparison of the figures 66 and 72, while I have already proved the essential identity of the former with the structure in *Brachionus*, &c.

118. We are now arrived at the most aberrant forms of the ROTIFERA, the genera *Floscularia* and *Stephanoceros*. In the former (figs. 79 to 82), the position of the dental apparatus is even more abnormal than its structure. The teeth appear to be enclosed in no *mastax*, and are placed far down in the abdominal cavity; nearer to the cloaca, in fact, than to the flower-like disk. I will endeavour to explain this.

119. The whole of the upper part of this elegant animal's body is lined with a very sensitive, contractile, partially-opake membrane, which, a little below the disk, recedes from the walls of the body, and forms a *diaphragm* with a highly contractile, and versatile central orifice. At some distance lower down another *diaphragm* occurs, and the ample chamber thus enclosed forms a kind of *crop*, or receptacle for the captured prey. Below the second *diaphragm* is another capacious chamber, which we must consider as a stomach, since digestion evidently commences in it, and it opens into the intestine.

120. The *mastax*, as I have above stated, is wholly wanting; the dental apparatus, which is very small, evidently springing from the common paries of the stomach, just below the second *diaphragm*. That this absence of the *mastax* is real, and not illusive, is proved by the facts, that the *Infusoria* swallowed pass into the stomach, where they accumulate in its wide cavity; that the jaws are seen to act on one and another, according as they come within reach; and that, after such action, they pass off again into the same cavity, to undergo another mastication, when chance again brings them within reach of the teeth.

121. From the ventral side of the ample *crop* that precedes the stomach, there springs, in *F. ornata*, a perpendicular membrane, or *veil* (fig. 79), extending partly across the cavity*. This is free, except at the vertical edge, by which it is attached to the side of the chamber; and being ample, and of great delicacy, it continually floats and waves from side to side. At the bottom of this *veil*, but on the dorsal side, are placed the jaws, consisting of a pair of curved, unjointed, but free *mallei*, with a membranous process beneath each.

122. Each *malleus* (fig. 80) is an *uncus* of two slender, arched, divergent fingers, united by a subtile web†: the back of each curves downwards, where, expanding and becoming membranous, it is connected with some delicate but definite processes (figs. 80, 81) with rounded outlines, which I should have supposed to be muscular bulbs, but that they remain after treatment with potash. They are probably analogous to the loops in *Limnias* and *Rotifer*, representing the *manubrium*.

* Dr. DOBIE considers this waving *veil*, in his *Floscularia cornuta*, to be a slit in the *diaphragm*, fringed with vibratile cilia, the motion of which, as he thinks, gives rise to the peculiar serpentine movement always observed at this point. (Ann. Nat. Hist. 1848.)

† M. D'UDEKEM describes the jaw as "a simple plate, armed with one sole tooth." (Bull. de l'Acad. Roy. de Belg. xviii. 43.)

123. Across the *uncus*, about midway between the bifurcation and the tip, on the inferior surface, a membrane originates, which projects transversely and perpendicularly downwards to some distance, curving at the bottom towards its fellow of the opposite *malleus* (fig. 80). I incline to think this the vanishing representative of the quadrantic mass in *Limnias* and *Rotifer* = the *ramus* of the *incus*.

124. The whole apparatus is very minute, and in some specimens can, either not at all, or with much difficulty, be detected; so that we may consider the dental apparatus of the ROTIFERA degraded to its lowest point in this genus.

125. *Stephanoceros*, the most elegant of ROTIFERA, affords peculiar facilities for observation; since it is by far the largest animal in the Class, reaching to one-fifteenth of an inch in height*. There are two capacious *crops* (each bounded by a *diaphragm*), of which the lower seems to answer to that of *Floscularia*. The jaws are placed in the latter, not in the stomach, which is distinctly separated from it. They are evidently imbedded in its dorsal paries, working freely in the cavity, without an enclosing *mastax* †.

126. EHRENBERG's figures of the apparatus seem more than usually incorrect; it is but fair to say, however, that he admits his observation to be susceptible of doubt. Each *malleus* is an *uncus*, of three curved divergent fingers (figs. 84, 85), whose extremities are united by an indented membrane, like the foot of a water-fowl. There is no distinct *manubrium*, but the posterior part of the *uncus* forms a knob, which is enclosed in a large muscular bulb.

127. The *incus* consists of two very mobile and widely separable *rami*, somewhat quadranti-globular, but much flattened, and each furnished with a lengthened process, which unites with its fellow to form the hinge, without a *fulcrum* (fig. 83). The *uncus* is connected with the *ramus* by an elastic ligament, by which means the latter is stretched open vigorously, while the teeth of the *malleus* act on the prey imprisoned in the *crop*.

128. Thus I have shown that the masticatory apparatus in the whole class of ROTIFERA is modeled on one common plan. The organs, indeed, are considerably modified, and sometimes so much disguised as to be unrecognizable, on cursory examination: but a careful scrutiny proves that every modification (for I have omitted none, that I am acquainted with, in which there is any variation of importance) is, without any violence, referrible to the common type.

129. This form of the trophi, alone, isolates the ROTIFERA from other animals, and proves, in concurrence with other points of organization, that this class is a very natural and well-marked group; since we find one type of structure running through the whole, which is yet widely different from that of any other class of animals.

130. As no group of animals, however, nor any set of organs, is so isolated as to

* EHRENBERG can only have seen small specimens, since he mentions one-third of a line as the dimensions of the species. I have, however, seen several of the size mentioned in the text.

† EHRENBERG figures one, indeed, of the usual three-lobed form, but I fear it is imaginary.

have no relations of affinity with others, it is a subject of interest to inquire, What is the homological value of the complex apparatus we have been considering? or, What organ or set of organs do they represent in other classes?

131. It has been usual to compare the jaws of the ROTIFERA with the gastric teeth in the higher CRUSTACEA, but I do not think there is in them any homology with these appendages. It is true that the great subcartilaginous crop in *Asplanchna* (fig. 56), a structure unparalleled by any other genus in the whole class, presents a curious similarity, in form and appearance, to the stomach of some of the Decapods, particularly that of *Cancer pagurus*; but the resemblance diminishes on examination; and the bruising teeth of the Crab have no more analogy with the expanding jaws of the *Asplanchna*, than with the complex apparatus of the more normal ROTIFERA. The resemblance lies in the circumstance that, in each case, the viscus consists of membranous walls, stretched, as it were, over a subcartilaginous framework, of a somewhat cubical shape. But the positions of the afferent and efferent orifices differ importantly, in the two viscera, in relation to the angles of the framework, *in situ* as well as *inter se*; the pyloric orifice in the stomach of the Crab being situate at the extremity most remote from the cardiac, while, in the crop of the *Asplanchna*, the efferent orifice is placed close to the afferent, at the ventro-anterior angle.

132. But again; this viscus in the CRUSTACEA is a true digestive stomach; whereas the crop of *Asplanchna* is merely a temporary receptacle, separated from the stomach by a long *œsophagus*: as there is thus no homology between the viscera themselves, there can be none between their respective appendages.

133. If the manducatory organs of the ROTIFERA were really represented by the gastric teeth of the CRUSTACEA, the small central piece of the latter would, of course, be the *incus* of the former: but this cannot be, since the central piece of the latter is affixed to the dorsal wall of the viscus; whereas, in ROTIFERA, the *incus* springs from the ventral or inferior side, its muscles forming the ventral lobe of the bulb.

134. Once more; the gastric teeth are simple masses of calcareous matter, deposited, in the form of tubercles, on the interior of the parietes of the stomach, to grind the food as it passes through the pylorus; whereas, the jaws of the ROTIFERA are complex organs, distinctly articulated, tied by ligaments, moved by their own proper muscles; and their function is, often, to seize prey without the mouth, and always to bruise or lacerate it before it enters the stomach at all.

135. Homology cannot consist with such diversities as these; and, therefore, the gastric teeth of the CRUSTACEA have no true analogy with the jaws of the ROTIFERA.

136. If it should be objected that, in *Floscularia*, &c., we find the jaws affixed to the walls of the digesting stomach, I reply, that these are the most aberrant forms of their class; and that we must not ground our deductions of analogy upon the condition of organs that are just vanishing, or merging into a remotely diverse type; especially when the comparison would tend to unite opposite ends of a series; as, in the present instance, the *Flosculariadæ* are undoubtedly the lowest forms of ROTIFERA,

looking towards the POLYZOA; whereas, it is in the highest forms that we must expect to find affinities with the ARTHROPODA.

137. To the ARTHROPODA I am convinced that the ROTIFERA belong—the humblest members of that great group; though, as my present business is with one system of organs alone, I am precluded, at this time, from adducing the various reasons derived from other parts of their economy, which have guided me to this conviction. I think that they lead, however, to the INSECTA rather than to the CRUSTACEA.

138. The dental organs in ROTIFERA are true *mandibulæ* and *maxillæ*, and the *mastax* is a *mouth*. This is a startling proposition, if we look only at *Floscularia*, where it is situated in the midst of the abdominal cavity, or at the *Philodinadæ*, and the *Brachionidæ*, where it is enclosed in the breast. But I have shown that the apparatus is identical, in these families, with that of *Diglena*, *Furcularia*, and *Synchæta*, in which it is terminal and protrusile. And this latter is the normal condition. I have myself witnessed the protrusion of the jaws from the surface of the body, in the following genera:—*Furcularia, Pleurotrocha, Taphrocampa, Notommata clavulata, N. aurita, N. petromyzon, N. parasita* (all of which represent different genera), *Plagiognatha, Scaridium, Synchæta, Polyarthra, Diglena, Asplanchna, Mastigocerca, Monocerca, Salpina, Monostyla*, and *Anuræa*.

139. The integument in the ROTIFERA is very flexible, and, especially in the frontal regions, is extremely invertible. In those genera in which the buccal apparatus can be brought into contact with the external water, it is ordinarily, to a greater or less degree, retracted within the body, by the inversion of the surrounding parts of the exterior; while, in those genera in which it is permanently enclosed, analogy requires us to consider this condition as induced by a similar inversion, but of permanent duration. If we imagine the head of a soft-bodied Insect-larva retracted to a great degree (as is done partially by many Dipterous larvæ), the skin of the thoracic segments would meet together in front, around a purse-like opening, which would be the orifice of such a buccal funnel as exists in most ROTIFERA. In the latter, it is the normal condition; in the former, it is merely accidental.

140. The delicacy and transparency of the *mastax* are, indeed, unlike the corneous and opaque mouths of Insects, and of most larvæ; but its walls are composed of the same solid materials as the teeth themselves; since they are left after treatment with potash, which dissolves away the muscles*.

141. There is, then, no difficulty, I think, in identifying the *mallei* with the mandibles, and the *rami* of the *incus* with the maxillæ, of Insects. Perhaps the *unci* of the former more strictly represent the mandibles themselves, and the *manubria* the

* "In many Dipterous insects the head is covered with the same flexible membranous skin with the rest of the body, from which it is often scarcely to be distinguished. In these, except that it contains the organs of manducation, it wears no more the appearance of a head than any other segment of the body. The head of these larvæ is also remarkable for another peculiarity; that it is capable of being extended or contracted, and of assuming different forms at the will of the insect" (KIRBY and SPENCE, Int. to Ent. iii. 113).

cheeks, into which they are articulated. As in Insects, their usual form is curved, with the convexity outwards, and the extremity variously notched or dentated. Their motion, as in that great Class, is chiefly a shears-like opening and shutting, in the horizontal plane; combined with a kind of rotatory action, which, according to Mr. KIRBY, the mandibles of some insects possess*. The same excellent entomologist enumerates several instances, in which the mandibles of Insects are not symmetrical, the one being developed more than the other; or, at least, differing from it in form †:—a circumstance in which these organs curiously agree with the unsymmetrical *mallei* in *Mastigocerca*, &c. (§ 89 to 91).

142. A glance at the trophi of *Diglena*, *Albertia*, or *Synchæta*, is sufficient to show that the *incus* with its *rami* is the homologue of the maxillæ, since they are placed, in these genera, nearly in the same plane as the *mallei* (= mandibulæ), and within them. There are not wanting, however, numerous instances, among Insects‡, in which the direction of the maxillæ is not parallel to that of the mandibles, but inclined to it; as is more generally the case in the ROTIFERA§.

143. The maxillæ are much more constantly present, in Insects, than the mandibles; the latter being either greatly deteriorated or entirely wanting in important groups, as in *Lepidoptera*, *Aphaniptera*, and some *Diptera*; and this agrees with the evanescence of the *mallei* in many ROTIFERA, while the *incus* is almost invariably present.

144. The *rami* of the *incus*, then, form the maxillæ proper; but whether the *fulcrum* answers to the two *cardines* (KIRBY), soldered together, or to the *labium*, I am not prepared to decide. I incline, however, to the former hypothesis; since, in *Scaridium*, as we have seen (§ 93), there appears to be a lower lip, though the *fulcrum* is large and distinct.

145. In *Diglena*, and its allies, the manducatory apparatus approaches most nearly to that of a predaceous insect; the *rami*, in particular, being almost the counterparts of the maxillæ of a carnivorous beetle. The series of bristles, with which the inner surface is fringed (§ 80), aids the resemblance. *Asplanchna* also presents an instructive example of resemblance in these organs. Here the extremity is two-lobed (fig. 59), as in many beetles; while the interior edge is, in *A. priodonta*, cut into numerous teeth.

146. The lips (*labrum* and *labium*) seem to be lost in all the species which have the *mastax* permanently inclosed; but in such as can protrude the jaws, the upper and lower margins of the *mastax* form distinct edges, more or less moveable by retractation and protrusion. In some instances I have been able to see these margins (which I must regard as representing the lips), or at least one of them; as in *Synchæta* (§ 75), *Polyarthra* (§ 79), and *Notommata aurita* (§ 52); but, in *Scaridium*,

* KIRBY and SPENCE, Int. to Ent. iii. 431. † Ibid. iii. 432. ‡ Ibid. iii. 439.

§ "The mandibles of the larvæ of Tipulæ, which are transverse and unguiform, do not act against each other, but against two other fixed, internally-concave, and externally-convex, and dentated pieces" (KIRBY and SPENCE, iii. 122).

both are very distinct; the labrum being furnished with stiff arched setæ, the labium plain, but bifid (§ 94).

147. I am not aware that any trace exists, in the ROTIFERA, of any organs, or processes, answering to *palpi*, either labial or maxillary. I was at first disposed to consider the processes at the bases of the *rami*, which I have called *alulæ* (§ 47), as maxillary palpi; but, as muscle-bands, in many cases, are inserted into their points, it is manifest that they will not bear this character.

148. To sum up these observations—we may consider a perfect mouth in the ROTIFERA as consisting of seven elements; viz. a *labrum*, a pair of *mallei*, a pair of *incus-rami*, a *fulcrum*, and a *labium*; corresponding homologically to the *labrum*, the *mandibles*, the *maxillæ* (with their *cardines*), and the *labium*, of Insects.

149. But, if this parallel be truly drawn, it is interesting to trace the same organs to their extreme point of degradation, and to mark where they disappear. Their minimum of development is attained in *Floscularia*, and *Stephanoceros*, which (notwithstanding the opinion of Mr. HUXLEY* to the contrary) do certainly lead to the POLYZOA. The latter, therefore, present the point where the two great divisions of the Animal Kingdom, the MOLLUSCA and the ARTICULATA, unite, in their course towards the true Polypes.

150. Now, in one genus of POLYZOA, there is a structure, which we may compare with the apparatus in ROTIFERA. The oval muscular bulbs in *Bowerbankia densa*, which approach and recede in their action on food, as described by Dr. ARTHUR FARRE, in his admirable memoir on the Ciliobrachiate Polypi†, appear to me to represent the quadranti-globular masses of *Limnias*, *Rotifer*, and *Stephanoceros*, reduced to a lower condition of structure, and deprived of *mallei*.

151. I do not see that this conclusion in the least involves (as Mr. HUXLEY supposes) the denial of either the Molluscous affinities of the POLYZOA, or the relationship of the ROTIFERA with the VERMES; the latter being clearly approached by another road, through the ANNELIDA. It would be easy to show (were this the suitable occasion for it) that the ROTIFERA link with the latter, through *Taphrocampa* and *Chætonotus*; as they do with the ENTOZOA, through *Albertia*.

* Journ. Mic. Sci. i. (Trans.) p. 16.

† Philosophical Transactions, 1837, p. 392.

Explanation of the Plates.

PLATES XVI., XVII., XVIII.

The figures of considerable portions of animals, are magnified 220 diameters; those of the details were taken with a power of 560 diameters, but are not drawn to scale.

The following letters of reference indicate the same organs, or their representatives, in all the figures in which they occur:—

a. Mastax.
b. Malleus.
c. Manubrium.
d. Articulation.
e. Uncus.
f. Incus.
g. Ramus.
h. Fulcrum.
i. Muscle, connecting the uncus with the ramus.
j. Muscle for extending the malleus.
k. Muscle for bending the malleus.
l. Muscle for throwing-in the manubrium.
m. Buccal funnel.
n. Salivary glands.
o. Alula.
p. Œsophagus.
q. Dorsal process of uncus.
r. Labrum.
s. Pancreatic glands.
t., *u.* Muscular bands proceeding from the alulæ, helping to open the rami.
v. Domular structure in funnel.
x. Eye.
y. Ganglion.
z. Nervous chords.

Fig. 1. *Brachionus urceolaris*: mastax and dental apparatus; ventral aspect.
Fig. 2. *Brachionus urceolaris*: jaws; viewed nearly from above.
Fig. 3. *Brachionus urceolaris*: malleus, flattened by pressure.
Fig. 4. *Brachionus urceolaris*: the same; as it appears during life.
Fig. 5. *Brachionus urceolaris*: mastax and dental apparatus; lateral aspect.
Figs. 6, 7, 8. *Brachionus urceolaris*: manubrium; illustrating its changes of position during action.
Fig. 9. *Brachionus urceolaris*: buccal funnel, salivary glands, mastax, and dental apparatus; dorsal aspect.
Fig. 10. *Brachionus rubens*: incus; lateral aspect.
Fig. 11. *Metopidia acuminata*: showing the frontal plate.
Fig. 12. *Euchlanis deflexa*: buccal funnel, mastax, dental apparatus, œsophagus, and salivary glands; dorsal aspect.
Fig. 13. *Euchlanis deflexa*: mastax and dental apparatus, after potash-treatment; lateral aspect.
Fig. 14. *Euchlanis deflexa*: the same; frontal aspect.
Fig. 15. *Euchlanis deflexa*: the same; viewed dorsally, and nearly frontally.

Fig. 16. *Notommata aurita*: mastax, &c.; ventral aspect.
Fig. 17. *Notommata aurita*: the same; lateral aspect.
Fig. 18. *Notommata aurita*: the same; fronto-ventral aspect.
Fig. 19. *Notommata aurita*: the same; frontal aspect.
Fig. 20. *Notommata aurita*: malleus.
Fig. 21. *Notommata aurita*: incus, opened; with labrum.
Fig. 22. *Notommata clavulata*: ganglion, with its eye and nervous chords, mastax, jaws, œsophagus, pancreatic glands, and commencement of stomach; lateral aspect.
Fig. 23. *Notommata clavulata*: mastax, &c.; ventral aspect.
Fig. 24. *Notommata clavulata*: the same, opened.
Fig. 25. *Notommata clavulata*: mallei and incus, after potash-treatment; viewed frontally.
Fig. 26. *Notommata clavulata*: right malleus.
Fig. 27. *Notommata petromyzon*: the head, with the ganglion, mastax, jaws, œsophagus, stomach, and various muscles.
Fig. 28. *Notommata petromyzon*: mastax; ventral aspect.
Fig. 29. *Notommata petromyzon*: the same; fronto-ventral aspect.
Fig. 30. *Notommata petromyzon*: the same; frontal aspect.
Fig. 31. *Notommata petromyzon*: the same; lateral aspect.
Fig. 32. *Notommata lacinulata*: jaws; ventral aspect.
Fig. 33. *Notommata lacinulata*: the same; fronto-ventral aspect.
Fig. 34. *Notommata lacinulata*: mastax, &c.; lateral aspect.
Fig. 35. *Furcularia gibba*: jaws, closed; latero-ventral aspect.
Fig. 36. *Furcularia gibba*: the same, opened.
Fig. 37. *Furcularia gibba*: mastax, &c.; lateral aspect.
Fig. 38. *Notommata gibba*: the head; dorsal aspect.
Fig. 39. *Notommata gibba*: jaws; nearly a ventral aspect.
Fig. 40. *Notommata gibba*: one ramus; designed from study of various aspects.
Fig. 41. *Synchæta tremula*: mastax and jaws; ventral aspect.
Fig. 42. *Synchæta tremula*: the same, after potash-treatment; expanded under pressure.
Fig. 43. *Synchæta tremula*: jaws; lateral aspect.
Fig. 44. *Polyarthra platyptera*: mastax; ventral aspect.
Fig. 45. *Polyarthra platyptera*: the same; dorsal aspect; with the œsophagus, pancreatic glands, and stomach.
Fig. 46. *Polyarthra platyptera*: mastax and jaws; ventral aspect.
Fig. 47. *Polyarthra platyptera*: the same, after potash-treatment; lateral aspect.
Fig. 48. *Polyarthra platyptera*: jaws; viewed obliquely.
Fig. 49. *Polyarthra platyptera*: the same; frontal aspect.
Fig. 50. *Diglena forcipata*: jaws, closed; ventral aspect.

Fig. 51. *Diglena forcipata*: the same, opened.
Fig. 52. *Albertia vermiculus*: jaws; ventral aspect.
Fig. 53. *Albertia vermiculus*: the same; lateral aspect.
Fig. 54. *Furcularia marina*: head, with the jaws protruded.
Fig. 55. *Furcularia marina*: jaws.
Fig. 56. *Asplanchna priodonta*: the anterior third of the animal.
Fig. 57. *Asplanchna priodonta*: jaws, closed; frontal aspect.
Fig. 58. *Asplanchna priodonta*: the same, opened.
Fig. 59. *Asplanchna priodonta*: right jaw, interior surface; lateral aspect.
Fig. 60. *Mastigocerca carinata*: mastax, &c.; dorsal aspect.
Fig. 61. *Mastigocerca carinata*: the same; lateral aspect.
Fig. 62. *Mastigocerca carinata*: jaws, after potash-treatment; ventral aspect.
Fig. 63. *Monocerca porcellus*: jaws; ventral aspect.
Fig. 64. *Scaridium longicaudum*: the head; showing the large ganglion, with the eye pressing on the mastax; lateral aspect.
Fig. 65. *Scaridium longicaudum*: the lips opened, and the rami protruded.
β. Rugæ on mastax.
γ. Setæ of labrum.
δ. Bifid labium.
ε. Projectile setæ of ramus.
Fig. 66. *Limnias ceratophylli*: mastax and jaws; with the termination of the buccal funnel; and the commencement of the œsophagus; dorsal aspect.
Fig. 67. *Limnias ceratophylli*: the same; lateral aspect.
Fig. 68. *Limnias ceratophylli*: jaws, after potash-treatment; lateral aspect.
Fig. 69. *Limnias ceratophylli*: the same; showing the elevation of the mallei.
Fig. 70. *Limnias ceratophylli*: the same; obliquely-ventral aspect.
Fig. 71. *Limnias ceratophylli*: the same; frontal aspect.
Fig. 72. *Rotifer macrurus*: mastax, &c.; dorsal aspect.
Fig. 73. *Rotifer macrurus*: jaws, after potash-treatment; fronto-dorsal aspect.
Fig. 74. *Rotifer macrurus*: the same; frontal aspect.
Fig. 75. *Rotifer macrurus*: the same; lateral aspect.
Figs. 76, 77, 78. *Rotifer macrurus*: diagrams, to illustrate the changes of form in the quadrantic jaws. ζ. Quadrantic masses.
Fig. 79. *Floscularia ornata*: veil, or septum, in crop.
Fig. 80. *Floscularia ornata*: jaws; dorsal aspect.
Fig. 81. *Floscularia ornata*: the same; frontal aspect.
Fig. 82. *Floscularia ornata*: the same, closed.
Fig. 83. *Stephanoceros Eickhornii*: jaws; dorsal aspect.
Fig. 84. *Stephanoceros Eickhornii*: uncus; frontal aspect.
Fig. 85. *Stephanoceros Eickhornii*: the same; oblique aspect.

XX. *Researches on the Action of Sulphuric Acid upon the Amides and Nitriles, together with Remarks upon the Conjugate Sulpho-acids.* By GEORGE B. BUCKTON, *Esq., F.L.S., F.C.S., and* A. W. HOFMANN, *LL.D., Ph.D., F.R.S. &c.*

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EVER since the experiments of DUMAS, MALAGUTI and LEBLANC on the one hand, and those of KOLBE and FRANKLAND on the other, have established the identity of the hydrocyanic ethers with the nitriles, it has been a favourite problem with chemists to reproduce the alcohols from these bodies, the solution of which would afford a passage from an acid, $C_{n2}\,H_{n2}\,O_4$, to an alcohol, $C_{n2-2}\,H_{n2}\,O_2$.

We have been likewise engaged with this question. The deportment of hydrocyanic acid (formonitrile) under the influence of concentrated sulphuric acid, which, as is well known, gives rise to an evolution of carbonic oxide, suggested an examination of the other nitriles in a similar direction. There appeared to be a chance of producing in this manner from a nitrile, the sulphovinic acid of an alcohol containing two equivalents of carbon less than the nitrile. Acetonitrile, for instance, might have been thus converted either into sulphate of methylammonium or into sulphomethylate of ammonium.

$$\underbrace{HC_2N}_{\text{Hydrocyanic acid.}}+2HO+H_2S_2O_8=2CO+\underbrace{\left.\begin{Bmatrix} H \\ NH_4 \end{Bmatrix}S_2O_8\right.}_{\text{Bisulphate of ammonium.}}$$

$$\underbrace{C_2H_3C_2N}_{\text{Acetonitrile, or Cyanide of methyl.}}+2HO+H_2S_2O_8=2CO+\underbrace{\left.\begin{Bmatrix} H \\ N\begin{pmatrix} H_3 \\ C_2H_3 \end{pmatrix} \end{Bmatrix}S_2O_8\right.}_{\text{Bisulphate of methylammonium.}}$$

$$\text{or} \quad =2CO+\underbrace{\left.\begin{Bmatrix} NH_4 \\ C_2H_3 \end{Bmatrix}S_2O_8\right.}_{\text{Methylosulphate of ammonium.}}.$$

Experiment has corroborated neither of these anticipations. This reaction gives rise to the formation of a remarkable conjugate sulpho-acid, which may be considered as the type of a most extensive class of analogous compounds, some isolated terms of which have been met with already in previous investigations.

Before, however, detailing the conditions under which these new bodies are formed, it may not be unacceptable to make a few statements regarding the most efficient

method of procuring the nitrile of the methyl-series in quantity and in a state of purity.

PREPARATION OF ACETONITRILE.

The preparation of cyanide of methyl (acetonitrile), by decomposing sulphomethylate of potassium with cyanide of potassium, is unsatisfactory, both as to quantity and quality of the product obtained. The presence of a minute amount of moisture in the materials employed gives rise to a number of secondary products, the chief of which are, cyanide and carbonate of ammonium, and a gas possessing a highly offensive odour, which imparts to the distillate a smell not at all due to the nitrile itself. By far the most advantageous method of preparing acetonitrile consists in acting upon acetamide with anhydrous phosphoric acid, as proposed by M. DUMAS.

Acetic ether is not immediately soluble in a moderate quantity of aqueous ammonia; but after five or six hours' contact, the layers at first formed disappear, and the liquid becomes homogeneous.

If distillation be now at once commenced, scarcely a trace of acetamide will be obtained. The change, on the other hand, is complete if the mixture be exposed for some hours to a temperature of 120° or 130° C.

We have employed in this operation a wrought-iron cylinder, similar to that used by Dr. FRANKLAND in his researches, which had been constructed for us by Mr. JAMES NASMYTH, who never fails most kindly and liberally to lend the extraordinary resources of his celebrated establishment for scientific purposes. Equal volumes of acetic ether and concentrated ammonia were introduced into the boiler, the brass valve of which was protected from corrosion by a steel screw and leaden washer. After six hours' digestion the acetamide was separated by distillation with the thermometer from the alcohol formed, that part only being reserved as anhydrous which had passed over above 200°. When acetamide is intimately mixed with about an equal volume of anhydrous phosphoric acid in a retort, a powerful action commences immediately, and the nitrile passes over colourless, but contaminated with acetic and hydrocyanic acids. Towards the end of the operation it is necessary to use a strong heat to drive off the last portions. The distillate is now agitated with just sufficient aqueous potash to neutralize the acids, when the nitrile floats on the surface, and may be removed for rectification over a fresh quantity of phosphoric acid, to render it perfectly anhydrous. Pure acetonitrile possesses an ethereal odour, faintly recalling that of cyanogen; its aromatic taste is pungent, but not disagreeable. We observed the boiling-point 77°–78°, which coincides with that observed by DUMAS. It does not appear to have been previously noticed, that acetonitrile burns with a luminous flame, the edges of which are beautifully tinged with peach-blossom colour. The flame and odour of acetonitrile unmistakeably bespeak the cyanic relations of this compound.

ACTION OF SULPHURIC ACID ON ACETONITRILE.

This body, when mixed with its own volume of fuming sulphuric acid, gives rise to a very energetic reaction, a considerable amount of heat being evolved, which causes much of the nitrile to volatilize and thus escape decomposition. In order to avoid this, the operation is best conducted in a retort surrounded with cold water, when a perfect mixture can be effected, scarcely a change of colour becoming perceptible. On application of heat the mixture powerfully intumesces with copious evolution of a gas which on examination over mercury proves to be carbonic acid, without a trace of carbonic oxide; at the same time strong acetic acid passes into the receiver. If the temperature be kept up by the gas-flame until the effervescence almost entirely ceases, the mass when cold forms a brown, tough and transparent solid, readily soluble in water and in alcohol. The aqueous solution, boiled with an excess of carbonate of barium, is then passed through a hot-water filter, when a magnificent salt is deposited, in the form of brilliant colourless rectangular plates which, when gathered in mass, exhibit a nacreous lustre. This salt is remarkably stable. It loses no weight at 100° C., but gives off water of crystallization somewhat below 150°. At a temperature of 220° the substance still remains unaltered, but when heated beyond this point, it commences to turn yellow, disengages water, sulphurous acid and carbonic oxide, whilst sulphur sublimes. When strongly heated it becomes incandescent, leaving a residue of sulphate and sulphite of barium. The new salt may be crystallized unchanged from hydrochloric acid, and may be boiled for hours with concentrated nitric acid, without formation of sulphate of barium. It is quite insoluble in alcohol.

ACTION OF SULPHURIC ACID UPON ACETAMIDE.

As acetamide differs from acetonitrile only in containing two additional equivalents of water, this compound, when heated with fuming sulphuric acid, undergoes a perfectly analogous transformation.

From the comparative facility of its preparation, acetamide offers peculiar advantages for procuring the new substance in large quantity. The chief difficulty which presents itself consists in operating with the right proportions of acid and amide. Distillation of equal volumes of dry acetamide and ordinary Nordhausen acid, for instance, gives scarcely a trace of anything but acetic acid and sulphate of ammonia. The most efficient proportions we found to be two volumes of amide to three volumes of acid. The distillation should be pushed very far, in fact until the product of distillation is accompanied by sulphurous acid. The solution of the solid residue may at once be saturated with carbonate of barium; it will be found, however, more economical to treat the liquid with finely-powdered Carrara marble until the free acid be neutralized, and to reserve the pure carbonate of barium, which is so difficult to wash thoroughly, merely for decomposing the sulphate of ammonium towards the end of the process. The liquid must be boiled with excess of the carbonate until

a filtered sample ceases to give an insoluble precipitate with chloride of barium. The liquid thus obtained contains now in solution chiefly two ammonium-salts, one of which is scarcely crystallizable, whilst the other forms large and regular rhombic crystals, sometimes of an inch diameter. In addition to these ammonium-salts, a certain quantity of barium- and even calcium-salts are present which it is necessary to decompose by carbonate of ammonium in excess before an attempt is made to effect a separation of the two salts.

On adding a saturated solution of either chloride or acetate of barium to a moderately strong solution of the crystallized ammonium-salt, the liquid becomes, in the course of a few minutes, nearly solid from the deposition of a mass of crystals of a barium-compound which may be collected on a filter and well washed with cold water from the chloride of ammonium and excess of the precipitant.

One recrystallization from boiling water furnishes this salt in a state of purity. It in no manner differs from the barium-salt previously described as procured from acetonitrile. The identity is moreover proved by the following analytical results:—

Analysis of the barium-salt resulting from the action of sulphuric acid upon acetonitrile and acetamide.

I. 0·7526 grm. of substance, dried at 190°, gave 0·5678 grm. of sulphate of barium.

II. 0·5998 grm. of substance, dried at 190°, gave 0·4516 grm. of sulphate of barium.

III. 0·6318 grm. of substance, dried at 170°, gave 0·4708 grm. of sulphate of barium.

IV. 0·6383 grm. of substance, dried at 180°, gave 0·4753 grm. of sulphate of barium.

V. 0·5918 grm. of substance, burnt with chromate of lead in a current of oxygen, gave 0·0247 grm. of water and 0·0820 grm. of carbonic acid.

VI. 0·7644 grm. of substance, burnt with chromate of lead in oxygen, gave 0·0440 grm. of water and 0·0975 grm. of carbonic acid.

VII. 0·7612 grm. of substance, burnt with chromate of lead in oxygen, gave 0·0472 grm. of water and 0·0990 grm. of carbonic acid.

VIII. 0·3272 grm. of substance, burnt with chromate of lead in oxygen, gave 0·0224 grm. of water and 0·0502 grm. of carbonic acid.

IX. 0·4638 grm. of substance, ignited with chlorate of potassium, gave 0·7038 grm. of sulphate of barium.

Experiments I., II., V. and IX. refer to salts prepared from acetonitrile, the remainder to preparations from acetamide.

	Percentage composition.								
	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
Barium .	44·36	44·26	43·81	43·78	——	——	——	——	——
Carbon . .	——	——	——	——	3·60	3·52	3·54	4·18	——
Hydrogen .	——	——	——	——	0·46	0·62	0·68	0·76	——
Sulphur .	——	——	——	——	——	——	——	——	20·83

From these data we calculate for the anhydrous salt the formula

$$C\,H\,Ba\,S_2\,O_6,$$

which requires the following numbers:—

	Theory.		Mean of experiments.
1 equiv. of Carbon . . .	6	3·85	3·71
1 equiv. of Hydrogen . .	1	0·67	0·63
1 equiv. of Barium . . .	68·5	44·05	44·05
2 equivs. of Sulphur. . .	32	20·57	20·83
6 equivs. of Oxygen . . .	48	30·86	30·78
	155·5	100·00	100·00

Considerations which we shall develope hereafter, especially the production of similar compounds in analogous series, induce us to double this expression, and to represent the new barium-salt by the formula

$$C_2\,(H_2\,Ba_2)\,S_4\,O_{12}.$$

The salt, as has been stated, retains water of crystallization at 100° C. In determining this water,—

I. 0·8414 grm., dried at 100°, lost 0·0888 grm. at 190° C.

II. 0·5820 grm., dried over sulphuric acid, lost 0·0613 grm. at 210° C.

These results indicate four equivalents of water:

Theory.	Experiment.	
	I.	II.
10·37	10·55	10·53

The formula of the crystallized salt is therefore

$$C_2\,(H_2\,Ba_2)\,S_4\,O_{12}+4aq.$$

We propose for this substance the name *disulphometholate of barium*, and for the acid the term *disulphometholic acid*, without wishing, however, to express by this term any particular view regarding the actual arrangement of the constituent molecules in the substance.

If the formula above given be correct, disulphometholate of barium is the salt of a bibasic acid, the composition of which would be represented by the expression

$$C_2\,H_4\,S_4\,O_{12},$$

furnishing salts of the general formula,

	Acid salt . . .	$C_2(H_3\,M)\,S_4\,O_{12}$
and	Neutral salt . .	$C_2\,(H_2\,M_2)\,S_4\,O_{12}$.

The barium salt which has been described belongs to the second class.

We have not yet succeeded in preparing any acid salts. Disulphometholate of barium crystallizes unchanged from a hydrochloric acid solution.

0·6216 grm. of salt thus crystallized gave 0·4636 of sulphate of barium.

	Acid salt.	Neutral salt.	Experiment.
Percentage of barium . .	28·13	44·05	43·85.

Disulphometholic Acid.

To prepare this acid, the barium-salt is precipitated by a small excess of sulphuric acid which is again removed by means of oxide of lead. Hydrosulphuric acid is then passed through the liquid which becomes thus strongly acid. It may be concentrated to a syrup without decomposition, and even carefully heated until white fumes appear. On cooling the liquid thus concentrated under the receiver of the air-pump, a crystalline mass of fine radiated needles is produced, exceedingly soluble in water and very deliquescent. They have a pure acid taste, with somewhat the flavour of tartaric acid. The attraction of this acid for water is so great, that we have failed in obtaining it in a solid condition by evaporation over sulphuric acid.

It is not changed by boiling with nitric acid, and chlorine gas passed through the aqueous solution does not liberate sulphuric acid.

The corresponding salts are readily obtained by digesting the oxides or the carbonates of the various metals with solutions of the acid. The barium-salt is immediately precipitated by adding chloride of barium to this solution. The salts are all soluble in water, but insoluble in alcohol.

Disulphometholate of Silver

crystallizes either in tufts of flat needles, or in broad tables of considerable size. It is produced when the aqueous solution of the acid is neutralized with carbonate or oxide of silver. We had hoped to obtain crystals of sufficient regularity for measurement of the angles; but although they may be easily produced of sufficient size, the edges are mostly rounded and imperfect.

It bears a temperature of 150° C. without decomposition, but blackens and froths when strongly heated, yielding sulphur, sulphurous acid, and metallic silver. It is insoluble in absolute alcohol, but may be crystallized from spirits of wine.

I. 0·8066 grm. at 120°, by ignition, gave 0·4470 grm. of silver.

II. 0·6045 grm. at 120°, by ignition, gave 0·3345 grm. of silver.

III. 1·0151 grm. at 120°, burnt with chromate of lead, gave 0·1110 grm. of carbonic acid and 0·0534 grm. of water.

	Percentage composition.		
	I.	II.	III.
Silver	55·49	55·33	——
Carbon. . . .	——	——	2·97
Hydrogen. . .	——	——	0·58

leading to the formula

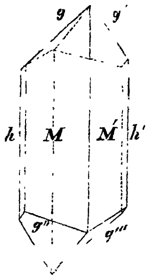
$$C_2 (H_2 Ag_2) S_4 O_{12}.$$

	Theory.		Mean of experiments.
2 equivs. of Carbon . . .	12	3·07	2·97
2 equivs. of Hydrogen . .	2	0·51	0·58
2 equivs. of Silver	216	55·37	55·41
4 equivs. of Sulphur . . .	64	16·41	——
12 equivs. of Oxygen . . .	96	24·64	——
	390	100·00	

Disulphometholate of Ammonium.

The formation of this substance has been already described. It is sparingly soluble in cold, but very soluble in hot water; from which it is again deposited in colourless crystals, often an inch in length. They are anhydrous, decrepitate when heated, and bear a temperature of 190° without being changed.

Through the kindness of Mr. Charles Brooke we are enabled to give the following angular measurements:—



MM'	97° 45'.
M*h*	131° 15'.
M*g*	92° 50'.

The form belongs to the oblique prismatic system, which is in fact nothing more than a hemihedral form of the right prismatic system.

When submitted to analysis this salt gave the following results:—

I. 0·7692 grm. at 190°, ignited with chromate of lead, gave 0·1501 grm. of carbonic acid and 0·3623 grm. of water;

II. 0·5135 grm. at 150°, ignited with chlorate of potassium, gave 1·1425 grm. of sulphate of barium;

III. 0·4508 grm. at 170°, ignited with soda-lime, gave 0·9440 grm. of chloroplatinate of ammonium;

IV. 0·5735 grm. at 170°, ignited with soda-lime, gave 1·1770 grm. of chloroplatinate of ammonium;

these numbers correspond to the formula

$$C_2 [H_2 (NH_4)_2] S_4 O_{12}.$$

	Theory.		Mean of experiments.
2 equivs. of Carbon	12	5·71	5·32
10 equivs. of Hydrogen . . .	10	4·76	5·22
4 equivs. of Sulphur. . . .	64	30·47	30·55
2 equivs. of Nitrogen . . .	28	13·33	12·98*
12 equivs. of Oxygen . . .	96	45·73	45·93
	210	100·00	100·00

Disulphometholate of Potassium

is most readily obtained by adding the crystallized barium-salt by degrees to a boiling solution of carbonate of potassium, avoiding an excess of the former. After filtration, the new substance separates easily from excess of alkali in fine shining needles or brilliant grains, according to the rapid or slow precipitation from its solution. It is not very soluble in cold water, one part requiring fourteen parts of water at 22° C. for solution.

Disulphometholate of Zinc.

When metallic zinc is heated with an aqueous solution of the acid, hydrogen gas is liberated, and on concentration a syrup is formed which crystallizes with great difficulty. Alcohol does not precipitate this salt from its solution.

Disulphometholate of Lead

forms small square nacreous plates not unlike those of the barium-compound. It is very soluble in water.

Disulphometholate of Copper

is produced by careful precipitation of the barium-salt by sulphate of copper. It yields radiated needles or small prisms of a green colour. They are insoluble in absolute alcohol, but soluble in spirits of wine.

We have not succeeded in preparing the ethers of this acid. We anticipated that they would be formed by bringing together iodide of ethyl and disulphometholate of silver. A strong solution of the silver-salt reacts at once on the iodide without application of heat. Iodide of silver is precipitated, but the ether which is formed splits immediately into disulphometholic acid and alcohol, according to the equation

$$C_2(H_2Ag_2)S_4O_{12}+2C_4H_5I+4HO=2AgI+2C_4H_6O_2+C_2H_4S_4O_{12}.$$

* The mean of nitrogen in Experiments III. and IV., viz. 13·11 and 12·85. They were made with specimens of different preparations.

EXAMINATION OF THE MOTHER-LIQUOR OF DISULPHOMETHOLATE OF AMMONIUM.

Sulphacetate of Ammonium and Sulphacetate of Barium.

It has been already noticed that another salt is present in the mother-liquid obtained during the purification of disulphometholate of ammonium. To procure this substance in a state of purity the mother-liquid was concentrated, and the crystals of the disulphometholate removed from the thick syrup as completely as possible.

After a moderate addition of cold water the liquid was mixed with an excess of chloride of barium, and the whole allowed to stand for five or six minutes, when it was filtered, and the filtrate set apart for twenty-four hours. At the expiration of this time a copious granular precipitate had formed which was well washed with cold water and then recrystallized from boiling water, in which, when once deposited, it proved to be very little soluble.

This substance differs both in its crystalline form and in its reactions from the disulphometholate. When strongly heated, it turns black and burns away like tinder. The residuary mass, moistened with hydrochloric acid, evolves according to the time of ignition, hydrosulphuric and sulphurous acid.

By analysis, 0·6266 grm., dried at 220°, gave 0·5287 of sulphate of barium.

The percentage of barium resulting from this experiment agrees well with the amount required by the formula of sulphacetate of barium. We append the theoretical and experimental percentages:—

Formula.	Barium calculated.	Barium found.
$\underbrace{C_4\,(H_2\,Ba_2)\,S_2\,O_{10}}_{\text{Sulphacetate of barium.}}$	49·81	49·60

The quantity of sulphacetate of ammonium produced, mainly depends upon the degree of heat maintained during the reaction of sulphuric acid upon acetamide or acetonitrile. If the mixture be made gradually and cooled after each addition of acid, the elimination of carbonic acid may be entirely prevented, and the chief product is then sulphacetic acid. On the other hand, if the heat be urged as far as is practicable, disulphometholic acid takes the place of the sulphacetic, carbonic acid being evolved at the same time.

Two phases may therefore be traced in the reaction. In the first, nascent acetic acid simply combines with the elements of two equivalents of anhydrous sulphuric acid. In the second phase the acetic molecule undergoes a more thorough transformation, splitting, as it does, into carbonic acid and marsh-gas, the latter of which combines with the elements of four equivalents of anhydrous sulphuric acid.

The new acid may also be regarded as sulphacetic acid, which, losing carbonic acid, has assimilated an equal number of equivalents of anhydrous sulphuric acid.

The two stages in the action of sulphuric acid on acetonitrile may be represented by the following equations:—

$$\underbrace{C_4H_3N}_{\text{Acetonitrile.}}+2HO+2H_2S_2O_8=\underbrace{C_4H_4S_2O_{10}}_{\text{Sulphacetic acid,}}+\left.\begin{matrix}H\\NH_4\end{matrix}\right\}S_2O_8$$

$$\underbrace{C_4H_3N}_{\text{Acetonitrile.}}+3H_2S_2O_8=\underbrace{C_2H_4S_4O_{12}}_{\text{Disulphometholic acid.}}+\left.\begin{matrix}H\\NH_4\end{matrix}\right\}S_2O_8+2CO_2.$$

Acetamide containing but two equivalents of water more than acetonitrile, the production of sulphacetic acid and disulphometholic acid from this substance is intelligible by means of the same equations.

The action then of bases and of acids upon acetic acid presents a remarkable analogy. Under the influence of both agents, we may assume, it splits into marsh-gas and carbonic acid; in the first case, it is the carbonic acid which is fixed by the alkali, whilst in the latter the marsh-gas remains in combination with the acid.

The possibility of assuming the existence of marsh-gas in disulphometholic acid, suggested the idea of endeavouring to combine marsh-gas directly with sulphuric acid. The dry gas obtained by the distillation of acetate of soda with potash-lime was passed into a receiver charged for this purpose with anhydrous sulphuric acid, but we could not detect any combination of the two bodies, either at ordinary temperature or when the receiver was heated to 100° C. The sulphuric acid, after treatment with water and carbonate of barium, furnished no soluble salt whatever.

Although we have thus been unable to convert marsh-gas into disulphometholic acid, we have found that this acid may be readily transformed into marsh-gas. When heated with hydrate of baryta the salts of the acid yield marsh-gas, together with sulphate and sulphite of barium. A method is thus indicated by which probably all the hydrocarbons, $C_{n2-2}\ H_{n2}$, may be prepared from the corresponding sulpho-acids *.

The simplicity of relation which exists between sulphacetic and disulphometholic acid, left no doubt in our mind regarding the convertibility of the former into the latter. We have established this fact moreover experimentally. Sulphacetic acid, prepared by the action of anhydrous sulphuric acid upon glacial acetic acid, furnished without difficulty disulphometholic acid, when again treated with sulphuric acid. A similar observation had in fact been made already by M. Melsens. This chemist, in his researches upon the sulphacetates, appears in some sort to have anticipated the existence of the disulphometholates. He remarks that he once found in the mother-liquor obtained from the preparation of sulphacetate of silver, a crystalline salt the composition of which he represents by the formula

$$C_2H_2Ag_2S_4O_{12}.$$

* We are unable to corroborate M. Aimée's remarks, that marsh-gas is decomposed by sulphuric acid into water, carbon, and sulphurous acid. The acid remains colourless, and the gas after the process exhibits its usual properties unmodified.

It is evident that this was nothing but the disulphometholate of silver, but M. MELSENS does not appear to have investigated the subject further, or to have connected the appearance of this salt with the evolution of carbonic acid gas, which he likewise found among the secondary products of the action of sulphuric acid upon acetic acid.

The chemical character of disulphometholic acid is so marked, and the reaction which produces it so simple and definite, that there could be no doubt regarding the existence of numerous corresponding terms in homologous and even in analogous series of substances. In fact disulphometholic acid may be considered as the type of a very numerous class of bodies of similar composition, some of which are actually known, while a great many remain to be discovered.

We have ourselves traced the formation of the homologues and analogues of disulphometholic acid in several higher series of bodies, but we have been satisfied in establishing the existence of these substances, without entering into a detailed study of their properties.

ETHYL-SERIES.

Preparation of Propionitrile.

The formation of cyanide of ethyl (propionitrile) by means of cyanide and sulphovinate of potassium, has all the disadvantages above enumerated in the case of cyanide of methyl. A large quantity of an inflammable gas of a highly offensive odour is disengaged, the annoyance of which may be avoided by conducting the gas from the receiver of the liquid into the cylinder of a wire gauze burner, in the air-flame of which it is perfectly consumed. In the preparation of cyanide of ethyl by means of cyanide and sulphovinate of potassium, through the intervention of unavoidable moisture, a variety of different reactions appear to proceed side by side. Together with the formation of cyanide of ethyl,

$$\underbrace{C_4 H_5 K, S_2 O_8}_{\substack{\text{Sulphovinate of} \\ \text{potassium.}}} + KCy = \underbrace{C_4 H_5 Cy}_{\substack{\text{Cyanide of} \\ \text{ethyl.}}} + K_2 S_2 O_8$$

considerable quantities of carbonate of ammonium are produced, whilst an inflammable gas is largely generated, which is nothing but hydride of ethyl. This gas is not absorbed by bromine; it therefore cannot contain any ethylene. It combines with chlorine, forming a gaseous compound, burning with a green-edged flame. This is probably the substance obtained by Drs. KOLBE and FRANKLAND when treating ethyl with chlorine gas, isomeric with chloride of ethyl, $C_4 H_5 Cl$. When in contact with excess of chlorine it yields an oily substance, and a crystalline body having the properties and composition of sesquichloride of carbon.

An analysis of the substance gave the following numbers :—

0·1130 grm. gave 0·4100 chloride of silver.

	Theory.	Experiment.
Chlorine in $C_2 Cl_3$	89·87	89·73

It is from this fact chiefly that we infer the identity of the inflammable gas generated in the preparation of propionitrile with hydride of ethyl,

$$\underbrace{C_4H_6}_{\substack{\text{Hydride}\\ \text{of ethyl.}}}+12Cl=\underbrace{C_4Cl_6}_{\substack{\text{Sesqui-}\\ \text{chloride of}\\ \text{carbon.}}}+6HCl.$$

The oily liquid observed obviously consisted of the intermediate substitution-terms.

These substances are evidently secondary products of the decomposition of cyanide of ethyl,

$$\underbrace{C_4H_5Cy}_{\substack{\text{Cyanide of}\\ \text{ethyl.}}}+6HO=\underbrace{C_4H_5H}_{\substack{\text{Hydride of}\\ \text{ethyl.}}}+\underbrace{\left.\begin{matrix}H\\ NH_4\end{matrix}\right\}C_2O_6}_{\substack{\text{Bicarbonate of}\\ \text{ammonium.}}}.$$

Cyanide of ammonium is likewise invariably present, prussic acid being generated by the action of water upon the cyanide of potassium, while alcohol is reproduced to a very considerable extent. Lastly, this process gives rise to a most remarkable body with a most insupportable prussic smell, the formation of which we have traced in a great variety of reactions. We have not completed the study of this compound.

The method finally adopted for obtaining propionitrile in sufficient quantity for us to experiment upon was that recommended by Dr. WILLIAMSON, viz. the digestion of cyanide of potassium with iodide of ethyl in four volumes of alcohol as a vehicle. The presence of so much alcohol, from which the cyanide cannot be separated by rectification, on account of the similarity of their boiling-points, is an evil which unfortunately involves a series of tedious processes, namely, conversion of the nitrile into propionate of potassium, by the action of caustic potassa, separation of the alcohol by distillation, preparation of propionic ether, transformation of the latter into propionamide by the action of ammonia under pressure, action of phosphoric acid upon propionamide to form the nitrile, and ultimately dehydration of the nitrile by means of anhydrous phosphoric acid.

In preparing disulphetholic acid we have almost invariably employed propionamide, on account of its easier preparation. We have however established by experiment that propionitrile exhibits with sulphuric acid exactly the same deportment as propionamide. To effect the transformation of the propionamide with success, it is necessary to employ this substance in a perfectly anhydrous condition. Unless this point be properly attended to, sulphuric acid produces principally sulphate of ammonium and propionic acid; for this purpose the amide should be heated in a retort, and that portion only collected which passes over above the temperature of 210° C. Below this point the amide retains sufficient water to prevent perfect crystallization.

On mixing equal volumes of melted amide and fuming sulphuric acid, very considerable heat is disengaged; the temperature must be regularly maintained until carbonic acid ceases to be evolved and propionic acid no longer distils over. Some

experiments are necessary in order to fix the quantity of acid required for the conversion of the propionamide, since commercial Nordhausen acid varies considerably in composition. If too much acid be employed, the mixture is often perfectly carbonized, whilst too little acid gives rise to the formation of either sulphopropionic or propionic acid.

If care be taken to regulate the heat so as just to keep up the liberation of carbonic acid, the amount of disulphetholic acid may be considerably increased.

The solid residue in the retort, when cold, is dissolved in water, and treated, as in the case of disulphometholic acid, first with carbonate of barium and subsequently with carbonate of ammonium. On evaporation the liquid thus obtained furnishes two salts, one of which is quite uncrystallizable, and must be separated from the other by washing with weak spirit. A crystalline substance remains on the filter, which, after two or three crystallizations from hot water, is the pure

Disulphetholate of Ammonium.

When deposited from an aqueous solution, this salt forms regular cubic crystals or square prisms, perfectly colourless. It is insoluble in alcohol and in ether. In its general behaviour this substance differs but little from the corresponding salt of the methyl-series, the principal feature of difference being a greater solubility and its leaving a carbonaceous residue when strongly heated.

Disulphetholate of Barium

crystallizes with regularity in six-sided plates, which generally arrange themselves from centres. It is easily formed by heating a solution of the ammonium-salt with hydrate of baryta until every trace of ammonia is expelled, and then passing a current of carbonic acid through the solution to remove the excess of baryta. It is very soluble in water, and stable at a temperature of 180°, but blackens when strongly heated in close vessels, empyreumatic vapours being at the same time given off. The black residue burns with a sulphur-flame.

Disulphetholate of barium is insoluble in concentrated nitric acid, which does not decompose it. It crystallizes from the dilute acid without apparent change.

After fusion with chlorate of potassium and carbonate of sodium, the filtrate contains sulphates in solution, proving that there is more sulphur present than is required for saturating the barium of the compound. By this method the disulphetholates can be distinguished from the sulphopropionates presently to be described.

0·4490 grm., at 170° C., gave 0·3266 grm. of sulphate of barium;

0·5045 grm., fused with chlorate of potassium, gave 0·7229 grm. of sulphate of barium;

which numbers accord with the formula

$$C_4\,(H_4\,Ba_2)\,S_4\,O_{12}.$$

	Theory.		Experiment.
4 equivs. of Carbon . . .	24	7·38	——
4 equivs. of Hydrogen . .	4	1·23	——
2 equivs. of Barium . . .	137	42·15	42·36
4 equivs. of Sulphur . . .	64	19·69	19·68
12 equivs. of Oxygen . . .	96	29·55	——
	325	100·00	

For the determination of the water of crystallization, 0·5790 grm., dried at 100°, were heated to 220° C., when they lost 0·0347 grm. Two equivalents of water require 5·24; found, 5·99. It is probable that there is a partial decomposition of the salt at this high temperature.

As has been already stated, the disulphethylates may also be prepared from propionitrile. For this purpose three parts of the propionitrile are distilled with two parts of fuming sulphuric acid. As the action is very energetic, the mixing should be effected by degrees in a moderate-sized retort. A barium-salt was prepared the properties of which agreed in every respect with those of the compound prepared from propionamide.

Disulphetholic Acid

is obtained by decomposing the lead-salt by hydrosulphuric acid. By concentration it forms a thick syrup, which ultimately solidifies into a crystalline mass. By heat it decomposes with deposition of charcoal and production of white fumes.

Disulphetholate of Silver

may be obtained by a process similar to that employed in the case of the disulphometholate. It is crystalline. When dry, it bears a considerable heat without change. The solution blackens on ebullition.

Disulphetholate of Lead

is very soluble in water. When evaporated over sulphuric acid, the salt yields fine prisms or minute quadrangular laminæ. When hastily evaporated it produces a tough gummy mass.

Sulphopropionic Acid.

As might have been anticipated from the results obtained in the methyl-series, disulphetholic acid is by no means the sole product of the action of sulphuric acid upon propionamide and propionitrile. The formation of this acid is preceded by that of another acid richer in carbon and poorer in sulphur, in one word, by that of sulphopropionic acid. The uncrystallizable ammonium-salt, already mentioned, is in fact the sulphopropionate of ammonium. Absolute alcohol throws it down from its solution as a thick treacly mass, with an appearance very unprepossessing to the

analyst. The dilute aqueous solution was therefore boiled with carbonate of barium until entirely decomposed, and the filtered liquid concentrated in a beaker. At a particular point of the evaporation, a quantity of *sulphopropionate of barium* appeared in small cubic grains, which caused so much "bumping" as to make it necessary to evaporate upon the water-bath.

If the concentration be arrested before the deposition of these grains, and the liquid be set aside for twenty-four hours, it yields a plentiful crop of beautiful silky crystals, arranged in spherical groups.

After another crystallization from hot water, in which the substance is moderately soluble, this salt is sufficiently pure. For analysis it was dried at 170°.

0·3636 grm. of substance gave 0·2940 grm. of sulphate of barium.

0·4960 grm. of substance gave 0·2138 grm. of carbonic acid and 0·0672 water.

The results of this analysis agree with the formula of sulphopropionate of barium,

$$C_6(H_4Ba_2)S_2O_{10}.$$

	Theory.		Experiment.
6 equivs. of Carbon . . .	36	12·45	11·75
4 equivs. of Hydrogen . .	4	1·38	1·49
2 equivs. of Barium . . .	137	47·40	47·52
2 equivs. of Sulphur . . .	32	11·07	——
10 equivs. of Oxygen . . .	80	27·70	——
	289	100·00	

PROPYL-SERIES.

From what has been already said, but few words are necessary on the preparation of disulphopropiolic and sulphobutyric acid. It may, however, be stated, that butyramide boils at 216°, and passes over at that temperature in drops which crystallize beautifully. It has the characteristic sweet taste of the other amides of the series, and, like them, it fumes slightly in the air when gently heated.

Three parts of butyramide produce with two parts of fuming sulphuric acid a horny mass, which, after removal of the sulphuric acid and saturation with ammonia, yields a syrup wholly uncrystallizable. The ammonium-salts were converted into barium-salts, which also possess but feeble powers of crystallization.

M. REDTENBACHER has already pointed out how much the solubility in water of the barium-salts of the acids $C_{n2}H_{n2}O_4$ augments as the value of the coefficient n progressively increases. A similar increase of solubility is observed with the barium-salts of the series of conjugated acids which form the subject of this paper. The contrast between the almost insoluble disulphometholate and the very soluble disulphopropiolate of barium is very marked.

From the extreme solubility of these salts, great difficulty was experienced in separating them completely one from the other. The method adopted was that of partial precipitation by alcohol. To solutions moderately concentrated and cold, sufficient absolute alcohol was added to produce a cloud, permanent when rapidly stirred. On allowing the liquid to stand for an hour, a quantity of a granular substance attached itself to the glass vessel, which, after solution in water and reprecipitation, was dried and decomposed by sulphuric acid.

0·2336 grm. at 160° gave 0·1800 grm. of sulphate of barium, which corresponds to the formula for sulphobutyrate of barium,

$$C_8 H_6 Ba_2 S_2 O_{10}.$$

The percentage required is—

	Theory.		Found.
8 equivs. of Carbon . . .	48	15·84	——
6 equivs. of Hydrogen . .	6	1·98	——
2 equivs. of Barium . . .	137	45·21	45·29
2 equivs. of Sulphur . . .	32	10·56	——
10 equivs. of Oxygen . . .	80	26·41	——
	303	100·00	

We have not been able to procure the disulphopropiolate of barium in a state of purity. The alcoholic liquid which remains after deposition of the sulphobutyrate contains a considerable quantity of the salt, but so mixed, that we were unable more than proximately to isolate it. By continuing the process of fractional precipitation, and drying, and by analysing the different samples, the quantity of barium present in the successive specimens was found to decrease. In three consecutive analyses we arrived at the following percentages:—

43·4, 42·9, 41·7.

The formula

$$C_6 (H_6 Ba_2) S_4 O_{12}$$

requires the following values:

	Theory.		Found.
6 equivs. of Carbon . . .	36	10·61	——
6 equivs. of Hydrogen . .	6	1·77	——
2 equivs. of Barium . . .	137	40·41	41·7
4 equivs. of Sulphur . . .	64	18·87	——
12 equivs. of Oxygen . . .	96	28·34	——
	339	100·00	

This salt is not so wholly uncrystallizable as the sulphobutyrate. By slow evaporation it furnishes very minute crystals, which under the microscope show themselves as thin pearly plates. They appear best when slowly thrown down by alcohol from the solution.

Butyl-Series.

Our experiments in this series have been entirely qualitative. We have been satisfied to establish experimentally the analogy of the reactions. These reactions, however, become less and less definite, and, owing to the still greater solubility of the new products, their separation is attended with difficulties almost insurmountable.

The researches detailed in the preceding pages are quite sufficient to establish the general character of the action of sulphuric acid upon the amides and nitriles. Nevertheless, since these experiments exclusively refer to the derivatives of several homologous terms of the series of fatty acids, it appeared desirable to extend the investigation to a nitrile of the analogous group of aromatic acids. No substance appeared to be more appropriate for such an examination than benzonitrile, or cyanide of phenyl.

Phenyl-Series.

Preparation of Benzonitrile.

M. Fehling originally obtained benzonitrile by subjecting benzoate of ammonium to repeated dry distillations, when the four equivalents of water gradually separated. The process is however objectionable, on account of the necessity of repeatedly returning by hand the sublimed salt to the volatilizing vessel. We hoped to have found a more convenient method, in submitting the same salt to the dehydrating action of chloride of zinc; but on rectifying the distillate, we obtained only one-third of the product in the form of benzonitrile, the remaining portion being benzole. The safest, but somewhat lengthened, process for procuring benzonitrile, consists in dehydrating benzamide with anhydrous phosphoric acid, the former body having been produced by the action of carbonate of ammonium upon chloride of benzoyl, as recommended by M. Gerhardt.

Action of Sulphuric Acid upon Benzonitrile.

Sulphuric acid and benzonitrile mix with liberation of far less heat than is observable in the cases previously recounted. No gas is disengaged until the temperature is very considerably raised, when a portion of benzoic acid sublimes in the neck of the retort and carbon is deposited, while sulphurous acid is simultaneously disengaged. The action was continued for some time after the appearance of sulphurous acid, for the purpose of decomposing the sulphobenzoic acid which analogy would lead us to expect among the products of the reaction. On cooling, a semi-transparent hard mass of glassy fracture remained in the retort. The usual treatment with carbonate of barium evolved ammonia, showing the presence of a salt of that base.

Sulphobenzoate of Barium.

As the filtrate was very dark-coloured, the barium-salt was again decomposed by sulphuric acid, and then boiled with excess of oxide of lead, filtered and treated by hydrosulphuric acid. The sulphide of lead was found to carry down with it almost the whole of the colouring matter. The acid liquid was then saturated with carbonate of barium, a portion evaporated nearly to dryness and treated with hydrochloric acid, when a salt crystallized out from the mother-liquor, which, after washing with water, was again crystallized until free from hydrochloric acid.

Dried at 200°, it gave a percentage of barium which coincided with that required by the acid sulphobenzoate of barium analysed by MITSCHERLICH and FEHLING.

0·4092 grm. gave 0·1768 grm. of sulphate of barium.

Formula.	Theory.	Experiment.
$\underbrace{C_{14}\,(H_5\,Ba)\,S_2\,O_{10}}_{\text{Acid sulphobenzoate of barium.}}$	25·41	25·39

It was therefore obvious that the disulphosalt, if present, was the most soluble, and to be found in the filtrate.

The remaining portion of the neutral solution of the barium-salt was therefore dissolved in water, and about one-half of the solid matter precipitated by alcohol, filtered off and recrystallized.

0·3634 grm., at 190°, gave 0·2510 grm. of sulphate of barium,

the percentage of barium in which corresponds to the neutral sulphobenzoate of barium,

Formula.	Theory.	Experiment.
$\underbrace{C_{14}\,(H_4\,Ba_2)\,S_2\,O_{10}}_{\text{Neutral sulphobenzoate of barium.}}$	40·65	40·58

Disulphobenzolate of Barium.

It remained, therefore, to examine the salt last thrown down by alcohol, which, after being purified again by precipitation, gave analytical numbers characterizing the new salt, although still in a state of admixture with sulphobenzoate of barium.

0·4910 grm., at 170°, gave 0·3170 grm. of sulphate of barium,

which percentage approximates to that of the salt suggested by theory.

Formula.	Theory.	Experiment.
$C_{12}\,(H_4\,Ba_2)\,S_4\,O_{12}$	36·72	37·70

This analysis could leave no doubt regarding the nature of the substance analysed, yet it was desirable, if possible, to prepare the new compound in a state of purity; and the question presented itself, whether it could not be more directly formed from sulphobenzolic acid, $C_{12}\,H_6\,S_2O_6$, which is so easily obtained by the action of sulphuric acid upon benzol. Success in its preparation by this means was of consider-

able interest, since in this case it would become highly probable that all the new bodies hitherto described in this memoir might be prepared, from the analogues of sulphobenzolic acid in their respective groups, by a simple assimilation of the elements of sulphuric acid. The method in question is in reality by far the best adapted for procuring the new acid in a state of purity. Sulphobenzolic acid obtained from the lead- or copper-salt is evaporated on the sand bath until the evolution of white fumes proves that the greater part of the water has been volatilized. It is advisable to heat the solution until a slight brown coloration indicates incipient decomposition. The acid is then introduced into a dry retort, together with an equal volume of strong Nordhausen acid, and the whole maintained at the boiling-point for two hours. The liquid is then reduced by evaporation nearly to the original bulk of the sulphobenzolic acid employed. The new acid in this stage has a very dark colour which cannot be removed by boiling with charcoal. Treatment, however, with an excess of oxide of lead, and decomposition of the filtrate by hydrosulphuric acid, furnishes a liquid which is perfectly colourless.

When this liquid is saturated with carbonate of barium and evaporated, an apparently amorphous mass is produced, which, however, under the microscope is distinctly crystalline, showing minute shuttle-shaped forms generally densely grouped together. This salt is very stable. Strongly heated, however, on platinum foil, it burns with evolution of sulphurous acid.

0·3930 grm., at 195°, gave 0·2436 grm. of sulphate of barium;

0·3046 grm., ignited with chlorate of potassium, gave 0·3850 grm. of sulphate of barium,

leading to the formula of disulphobenzolate of barium,

$$C_{12}(H_4Ba_2)S_4O_{12}.$$

	Theory.		Experiment.
12 equivs. of Carbon . . .	72	19·30	——
4 equivs. of Hydrogen . .	4	1·07	——
2 equivs. of Barium . . .	137	36·72	36·43
4 equivs. of Sulphur . . .	64	17·16	17·35
12 equivs. of Oxygen . . .	96	25·75	——
	373	100·00	

For comparison, we subjoin the corresponding number for sulphobenzolate of barium,

$$C_{12}(H_5Ba)S_2O_6.$$

Barium	30·38
Sulphur	14·19

The preceding researches establish in two different groups of bodies the existence of a series of bibasic acids, containing 4 equivalents of sulphur, and which, irrespect-

ively of any special view regarding their molecular arrangement, may be represented as formed by the association of the hydrocarbons (corresponding to marsh-gas) of the various groups with 4 equivalents of anhydrous sulphuric acid:—

Disulphometholic acid	$C_2 H_4 4SO_3$
Disulphetholic acid	$C_4 H_6 4SO_3$
Disulphopropiolic acid	$C_6 H_8 4SO_3$
Disulphobenzolic acid	$C_{12} H_6 4SO_3$

An acid of analogous composition exists in the naphthalin-series:—

Disulphonaphthalic acid . . . . $C_{20} H_8 4SO_3$,

which was discovered by BERZELIUS and subsequently studied by LAURENT.

Many of these substances may actually be produced directly from the hydrocarbons by the action of sulphuric acid. On the other hand, chemists are well acquainted with the deportment of olefiant gas under the influence of anhydrous sulphuric acid. The crystalline compound discovered by MAGNUS, and described by him under the name of Sulphate of Carbyl, whatever its constitution may be, can be considered as a direct combination of olefiant gas with four equivalents of anhydrous sulphuric acid,

Sulphate of Carbyl . . . $C_4 H_4 4SO_3$.

It can scarcely be doubted that all the other hydrocarbons of the series $C_{2n} H_{2n}$, propylene, butylene, amylene, &c., will furnish homologous substances.

Sulphate of carbyl, when submitted to the action of water, assimilates two equivalents, and is converted into a bibasic acid (ethionic), $C_4H_44SO_3+2HO=C_4H_6O_24SO_3$, which accordingly may be viewed as an association of alcohol with four equivalents of anhydrous sulphuric acid. Terms analogous to ethionic acid are sure to be found when the study of the homologues of sulphate of carbyl shall be taken up by chemists.

The production of disulpho-compounds of perfectly similar composition, from substances belonging to such different groups of bodies as the hydrocarbons, homologous and analogous to marsh-gas, as ethylene, and as alcohol, suggested the possibility that the substances observed might be but individual examples of a far more general mode of formation. It became, in fact, probable that all organic bodies, capable of uniting with the elements of two equivalents of anhydrous sulphuric acid, might, under favourable circumstances, be induced to assimilate two additional equivalents of anhydrous sulphuric acid, and thus furnish terms belonging to the class of disulpho-compounds.

The hope of arriving at a more general interpretation of our observations induced us to institute some further inquiries, the result of which we will briefly append.

The first question which naturally suggested itself, was the examination of the deportment of the sulpho-acids derived from acetic, propionic, butyric, benzoic acid, &c., under the influence of an excess of sulphuric acid. To take acetic acid as an illustration, Is sulphacetic acid, $C_4 H_4 S_2 O_{10}=C_4 H_4 O_4 2SO_3$, capable of combining

with two additional equivalents of sulphuric acid in order to furnish the compound $C_4 H_4 S_4 O_{16} = C_4 H_4 O_4 4SO_3$? The experiments hitherto detailed contain no evidence against this assumption. It appeared to us extremely probable that sulphacetic acid, when submitted to the action of sulphuric acid, is converted into disulphacetic acid, which, losing carbonic acid, produces disulphometholic acid, the deportment of disulphacetic under the influence of heat being closely analogous with that of acetic acid,

$$\underbrace{C_4 H_4 O_4}_{\text{Acetic acid.}} - 2CO_2 = \underbrace{C_2 H_4}_{\text{Marsh-gas.}}$$

$$\underset{\substack{\text{Supposed}\\\text{disulphacetic acid.}}}{C_4 H_4 O_4 4SO_3} - 2CO_2 = \underset{\substack{\text{Disulphometholic}\\\text{acid.}}}{C_2 H_4 4SO_3}$$

We have endeavoured to decide this question by experiment; but on account of the greater stability of the benzoic molecule, we have preferred to trace the intermediate acid in the benzoyl-series.

Sulphobenzoic acid, obtained from the acid barium-salt, was evaporated to dryness and heated until it attained the point of quiet fusion. When cold, the acid was coarsely powdered, and mixed with rather more than its own bulk of crystallized sulphuric acid.

It was then heated in a water-bath to a temperature below that at which gases are disengaged. This appeared to be just below 85° C. The substances were kept in contact at this temperature for eight hours. Finally, the mixture was subjected to the heat of boiling water for two hours longer; after which it was treated in the usual way for obtaining the soluble barium-salt.

The salt thus obtained in no manner differed, either in appearance, reaction, or composition, from sulphobenzoate of barium.

0·3838 grm. of salt, dried at 165° C., gave 0·2630 grm. of sulphate of barium,

corresponding to 40·28 per cent. of barium.

For comparison, we append the barium-percentages required for the salts of the two acids, and the amounts found by experiment.

Formula.	Theory.	Formula.	Theory.	Experiment.
$\underbrace{C_{14} (H_4 Ba_2) S_2 O_{10}}_{\text{Sulphobenzoate of barium.}}$	40·65	$\underbrace{C_{14} (H_4 Ba_2) S_4 O_{16}}_{\text{Hypothetical acid.}}$	32·85	40·28

Though the above experiments have failed to realize our hopes of producing disulphobenzoic acid, they by no means disprove the existence of this body, and the possibility of producing it under more favourable circumstances. We insist upon this point, on account of the success which has attended our endeavours to trace the existence of disulpho-acids in a perfectly different group of bodies.

We are indebted to M. Gerhardt for the knowledge of a conjugated sulpho-acid, containing an organic base in the place of hydrocarbons, alcohols, acids, &c., which

are present in the usual sulpho-acids. Sulphanilic acid, $C_{12}H_7NS_2O_6$, may be viewed as formed by the association of one equivalent of aniline and two equivalents of anhydrous sulphuric acid.

We were curious to ascertain whether this acid, by assimilating once more the elements of anhydrous sulphuric acid, was capable of becoming disulphanilic acid, $C_{12}H_7NS_4O_{12}$.

The existence of disulphobenzolic acid, $C_{12}H_6S_4O_{12}$, the barium-salt of which we have described in the preceding pages, together with the formation of an interesting acid lately obtained by M. KILKENKAMP* in the reaction of sulphite of ammonium on dinitrobenzol, appeared to leave no doubt regarding the existence of a disulphanilic acid. In fact, M. KILKENKAMP's dithiobenzic acid, $C_{12}H_8N_2S_4O_{12}$, is most intimately connected with the acid we were in search of, as is obvious by a glance at the following Table, in which we have placed a number of known substances in juxtaposition with bodies, ideal at present, but the existence of which can no longer be doubted.

Hydrocarbons and substitutes.	*Sulpho-acids* and substitutes.	*Disulpho-acids* and substitutes.
$C_{12}H_6$ Benzol.	$C_{12}H_6S_2O_6$ Sulphobenzolic acid.	$C_{12}H_6S_4O_{12}$ Disulphobenzolic acid.
$C_{12}\left\{\begin{matrix}H_5\\NO_4\end{matrix}\right\}$ Nitrobenzol.	$C_{12}\left\{\begin{matrix}H_5\\NO_4\end{matrix}\right\}S_2O_6$ Nitrosulphobenzolic acid.	$C_{12}\left\{\begin{matrix}H_5\\NO_4\end{matrix}\right\}S_4O_{12}$
$C_{12}\left\{\begin{matrix}H_5\\NH_2\end{matrix}\right\}$ Phenylamine. Aniline.	$C_{12}\left\{\begin{matrix}H_5\\NH_2\end{matrix}\right\}S_2O_6$ Phenylsulphamic acid. Sulphanilic acid.	$C_{12}\left\{\begin{matrix}H_5\\NH_2\end{matrix}\right\}S_4O_{12}$ Phenyldisulphamic acid. Disulphanilic acid.
$C_{12}\left\{\begin{matrix}H_4\\NO_4\\NO_4\end{matrix}\right\}$ Dinitrobenzol.	$C_{12}\left\{\begin{matrix}H_4\\NO_4\\NO_4\end{matrix}\right\}S_2O_6$	$C_{12}\left\{\begin{matrix}H_4\\NO_4\\NO_4\end{matrix}\right\}S_4O_{12}$
$C_{12}\left\{\begin{matrix}H_4\\NH_2\\NH_2\end{matrix}\right\}$ Phenyldiamine. (Semaniline ?)	$C_{12}\left\{\begin{matrix}H_4\\NH_2\\NH_2\end{matrix}\right\}S_2O_6$	$C_{12}\left\{\begin{matrix}H_4\\NH_2\\NH_2\end{matrix}\right\}S_4O_{12}$ Dithiobenzic acid. Phenyldisulpho-diamic acid.

The preceding synopsis, whilst it fixes, we believe, the correct position of dithiobenzic acid in the benzol-series, exhibits at the same time its close relation to the disulphanilic acid which we endeavoured to produce, and the formation of which, it may at once be stated, succeeded without much difficulty.

* Annalen der Chemie und Pharmacie, Neue Reihe, B. xix. S. 86.

ACTION OF SULPHURIC ACID UPON ANILINE.

In order to prepare disulphanilic acid, two parts of strong sulphuric acid were mixed with one part of aniline, when heat was freely evolved, sulphate of aniline being thrown down at the same time. On further application of heat the salt redissolved, and as the liquid reached the boiling-point it became very dark in colour, sulphurous acid being freely evolved.

In our first experiment the mixture was kept in full ebullition for ten minutes, and then poured into water after it had partially cooled. A confusedly crystalline mass was produced which was well washed with cold water (in which it is not very soluble), and afterwards recrystallized from hot water.

Sulphanilate of Silver.

A portion of this acid was digested with carbonate of silver. The crystalline silver-salt, obtained by evaporating the solution at a low temperature, was dried at 120° and ignited.

0·3524 grm. gave 0·1360 grm. of reduced silver, which coincides with the theoretical percentage of sulphanilate of silver,

$$C_{12}(H_6Ag)NS_2O_6.$$

	Theory.	Experiment.
Silver . . .	38·57	38·58

Sulphanilate of Barium.

A barium-salt, prepared in the usual way and submitted to analysis by precipitation by sulphuric acid, gave perfectly analogous results.

0·4250 grm., at 160°, gave 0·2060 grm. sulphate of barium,

corresponding to the formula

$$C_{12}(H_6Ba)NS_2O_6.$$

	Theory.	Experiment.
Barium . . .	28·48	28·40

These experiments proved that the acid produced was nothing but M. GERHARDT's well-known sulphanilic acid, the reaction not having gone far enough for the production of the second acid. The treatment with sulphuric acid was therefore resumed. Finely powdered and dry sulphanilic acid, mixed with strong fuming acid to the consistency of a thin paste, was heated in an air-bath to a temperature just approaching that at which sulphurous acid is generated, to 160° and 170° C. This digestion was continued until a portion taken on a glass rod did not solidify on cooling, or give any solid matter when dissolved in a small quantity of water, which happened after the lapse of seven hours, when the mass had the consistency of treacle.

It was dissolved in cold water, and separated from a black, almost insoluble matter, which appeared to be somewhat crystalline.

Disulphanilate of Barium.

After saturating with carbonate of barium the liquid was evaporated to dryness, by which treatment a further separation of the black substance was effected, and the barium-salt was much improved in colour.

If the barium-salt be redissolved and the solution evaporated on the water-bath, a horny substance is formed, which on cooling splits by cracks in all directions; but if the evaporation be completed under the receiver of an air-pump, a mass of microscopic crystals appear, insoluble in alcohol and in ether.

In preparing the new barium-salt, we have found it convenient to precipitate the liquid gradually by alcohol, and to reject, as retaining generally traces of sulphanilate, the precipitate first formed. Under all circumstances it is very difficult entirely to remove all colouring matter from the solutions of this salt, which is usually of a pale rose colour.

0·4679 grm., at 200°, gave 0·2810 grm. of sulphate of barium;
0·3620 grm., at 190°, gave 0·4310 grm. of sulphate of barium,

which numbers agree well with the formula of disulphanilate of barium,

$$C_{12}(H_5Ba_2)NS_4O_{12}.$$

	Theory.		Experiment.
12 equivs. of Carbon	72	18·58	——
5 equivs. of Hydrogen . . .	5	1·28	——
2 equivs. of Barium . . .	137	35·30	35·30
1 equiv. of Nitrogen . . .	14	3·60	——
4 equivs. of Sulphur . . .	64	16·49	16·32
12 equivs. of Oxygen . . .	96	24·75	——
	388	100·00	

Disulphanilate of barium blackens without inflaming when heated on foil, in which respect it differs from the sulphanilate, which burns with a bright but smoky flame. Heated in close vessels it furnishes a vapour, which sublimes in beautiful crystals, probably of sulphite of aniline, obtained under similar circumstances from sulphanilic acid.

Disulphanilate of barium is attacked by concentrated nitric acid and gives a yellow liquid, which furnishes on evaporation crystals of a very bitter taste; sulphate of barium is formed at the same time.

Disulphanilic Acid

is easily produced from a lead-salt. This substance possesses a very acid and pungent taste; it crystallizes with great difficulty, but is insoluble, in alcohol which precipi-

tates strong solutions in white grains. This precipitation is assisted by the addition of a little ether. We have not analysed this body, but we have examined in addition,

Disulphanilate of Silver.

The liquid formed by neutralizing a concentrated solution of the acid with carbonate of silver is precipitated cold by the addition of a mixture of equal volumes of alcohol and ether. The salt subsides in colourless crystalline grains; the deposition is much facilitated by rapidly stirring the contents of the beaker with a glass rod. Disulphanilate of silver crystallizes by spontaneous evaporation of the aqueous solution in small laminæ which blacken and deposit a black powder when boiled with water.

0 4687 grm., dried at 120°, gave 0·2134 grm. of silver.

The formula,

$$C_{12}(H_5 Ag_2) N, S_4 O_{12},$$

requires

	Theory.		Experiment.
12 equivs. of Carbon	72	15·40	——
5 equivs. of Hydrogen . . .	5	1·07	——
2 equivs. of Silver	216	46·25	45·53*
1 equiv. of Nitrogen . . .	14	2·99	——
4 equivs. of Sulphur . . .	64	13·70	——
12 equivs. of Oxygen . . .	96	20·59	——
	467	100·00	

The researches detailed in the preceding paragraphs may serve to characterize more fully a class of compounds of which only a few terms, isolated and scattered in very different groups, had been previously observed. The only disulpho-acids hitherto known, were BERZELIUS and LAURENT's disulphonaphthalic acid and MAGNUS's ethionic (disulphethylic) acid, and lastly, dithiobenzic (phenyl-disulphodiamic) acid, recently discovered by M. KILKENKAMP. To these this memoir adds five new acids belonging to several of the most important series of compounds.

Disulphometholic acid . . .	$C_2 H_4 S_4 O_{12}$.
Disulphetholic acid	$C_4 H_6 S_4 O_{12}$.
Disulphopropiolic acid . . .	$C_6 H_8 S_4 O_{12}$.
Disulphobenzolic acid . . .	$C_{12} H_6 S_4 O_{12}$.
Disulphanilic acid	$C_{12} H_7 N S_4 O_{12}$.

* Although this number is rather low, it marks the composition of the salt sufficiently well, since the acid from which it was formed was known to contain traces of sulphanilic acid; for comparison we append the percentage of silver in sulphanilate of silver, which is 38·57.

Our experiments point out moreover the universal occurrence and the general mode of formation of these substances. All organic molecules, particularly in the nascent state, appear to be capable of assimilating the elements of either two or four equivalents of anhydrous acid. The formation of the two groups of acids which are thus produced, presents a great analogy with the production of the nitro-substitutes generated under the influence of nitric acid. All these compounds are generated with elimination of water. In the action of nitric acid and sulphuric acid upon benzol, for instance, we have

$$\underbrace{C_{12}H_6}_{\text{Benzol.}} + HNO_6 = \underbrace{C_{12}H_5NO_4}_{\text{Nitrobenzol.}} + 2HO.$$

$$\underbrace{C_{12}H_6}_{\text{Benzol.}} + 2HNO_6 = \underbrace{C_{12}H_4N_2O_8}_{\text{Dinitrobenzol.}} + 4HO.$$

$$\underbrace{C_{12}H_6}_{\text{Benzol.}} + H_2S_2O_8 = \underbrace{C_{12}H_6S_2O_6}_{\text{Sulphobenzolic acid.}} + 2HO.$$

$$\underbrace{C_{12}H_6}_{\text{Benzol.}} + 2H_2S_2O_8 = \underbrace{C_{12}H_6S_4O_{12}}_{\text{Disulphobenzolic acid.}} + 4HO.$$

The analogy of these reactions is obvious.

The action of nitric acid upon organic bodies is by no means limited to the production of nitro-compounds, corresponding to nitrobenzol and dinitrobenzol; frequently additional bodies are formed with elimination of 6, 8, and in a few isolated cases, even of 10 equivalents of water. It is possible that analogous sulpho-compounds may exist; hitherto, however, no substances have been observed in which the assimilation of sulphuric acid has gone further than in the disulpho-acids.

Appendix.

During the prosecution of the researches detailed in the preceding paper, our attention was repeatedly called to the substance which M. Liebig has described under the name of *methionic acid*, and which he obtained in the reaction of anhydrous sulphuric acid upon dry ether at low temperatures*. The properties of the barium-salt of this most remarkable acid very closely agree with those which we observed in studying disulphometholate of barium, although both appeared different in composition. We were repeatedly inclined to admit and to doubt the identity of the two substances. We have now arrived at the conclusion, that methionic and disulphometholic acid are actually *identical*.

* Liebig, Annalen der Chemie und Pharm. xiii. 32; xxv. 39. Ann. de Chimie et de Phys. i. 59. 182. Wetherell, Ann. der Chemie und Pharm. lxvi. 122.

The formula adopted by M. LIEBIG for the barium-salt is

$$C\,H_3\,Ba\,S_2\,O_8.$$

If we double this formula we arrive at the expression

$$C_2\,H_6\,Ba_2\,S_4\,O_{16}=C_2(H_2\,Ba_2)\,S_4\,O_{12}+4aq,$$

which represents disulphometholate of barium with 4 equivalents of water of crystallization. Our experiments have actually proved that this salt contains indeed 4 equivalents of water, and that this water is retained at 100° C., the temperature at which M. LIEBIG dried his salt before submitting it to analysis.

More recently M. REDTENBACHER and M. WETHERELL have investigated the same barium-salt, but neither of them appears to have remarked the fact, that this salt loses water between 100° and 200°. M. REDTENBACHER analysed a salt which M. LIEBIG had prepared himself. M. WETHERELL examined a salt which he obtained as a secondary product in his researches on sulphate of ethyl.

Although we have no longer any doubt regarding the identity of methionic acid and disulphometholic acid, we should have liked to offer a direct experimental proof of our opinion by an analysis of the barium-salt obtained either by M. LIEBIG's or M. WETHERELL's process. We have therefore repeatedly endeavoured to prepare the methionate of barium both by M. LIEBIG's and M. WETHERELL's method. By following exactly the plan described by these chemists, we have actually succeeded in obtaining a salt possessing all the characters of disulphometholate of barium. Unfortunately, although we worked upon rather a large scale, we did not succeed in obtaining by either of these processes a quantity of barium-salt sufficient for analysis. In both processes the salt is the product of a secondary reaction.

May 9th, 1856.

XXI. *Elements of a Mathematical Theory of Elasticity.*

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PART I. ON STRESSES AND STRAINS*.

ARTICLE I.—*Initial Definitions and Explanations.*

Def. A STRESS is an equilibrating application of force to a body.

Cor. The stress on any part of a body in equilibrium will thus signify the force which it experiences from the matter touching that part all round, whether entirely homogeneous with itself or only so across a portion of its bounding surface.

Def. A strain is any definite alteration of form or dimensions experienced by a solid.

Examples. Equal and opposite forces acting at the two ends of a wire or rod of any substance constitute a stress upon it. A body pressed equally all round, for instance any mass touched by air on all sides, experiences a stress. A stone in a building experiences stress if it is pressed upon by other stones, or by any parts of the structure, in contact with it. Any part of a continuous solid mass simply resting on a fixed base experiences stress from the surrounding parts in consequence of their weight. The different parts of a ship in a heavy sea experience stresses from which they are exempt when the water is smooth.

If a rod of any substance become either longer or shorter it is said to experience a strain. If a body be uniformly condensed in all directions it experiences a strain. If a stone, a beam, or a mass of metal, in a building, or in a piece of framework, becomes condensed or dilated, in any direction, or bent, or twisted, or distorted in any way, it is said to experience a strain, to become strained, or often in common language, simply "to strain." A ship is said "to strain" if in launching, or when working in a heavy sea, the different parts of it experience relative motions.

ARTICLE II.—*Homogeneous Stresses and Homogeneous Strains.*

Def. A stress is said to be homogeneous throughout a body when equal and similar portions of the body, with corresponding lines parallel, experience equal and parallel pressures or tensions on corresponding elements of their surfaces.

Cor. When a body is subjected to any homogeneous stress, the mutual tension or pressure between the parts of it on two sides of any plane amounts to the same per

* These terms were first definitively introduced into the Theory of Elasticity by RANKINE, and I have found them very valuable in writing on the subject. It will be seen that I have deviated slightly from Mr. RANKINE's definition of the word "stress," as I have applied it to the direct action experienced by a body from the matter around it, and not, as proposed by him, to the elastic reaction of the body equal and opposite to that action.

unit of surface as that between the parts on the two sides of any parallel plane; and the former tension or pressure is parallel to the latter.

A strain is said to be homogeneous throughout a body, or the body is said to be homogeneously strained, when equal and similar portions with corresponding lines parallel, experience equal and similar alterations of dimensions.

Cor. All the particles of the body in parallel planes remain in parallel planes, when the body is homogeneously strained in any way.

Examples. A long uniform rod, if pulled out, will experience a uniform strain, except near its ends. In a pillar bearing a weight in comparison with which its own may be neglected, there will be a sensible heterogeneousness of the strain up to the middle from each end, because of the circumstances that prevent the ends from expanding laterally to the same extent as the middle does.

A piece of cloth held in a plane and distorted so that the warp and woof, instead of being perpendicular to one another, become two sets of parallels cutting one another obliquely, experiences a homogeneous strain. The strain is heterogeneous as to intensity, from the axis to the surface of a cylindrical wire under torsion, and heterogeneous as to direction in different positions in a circle round the axis.

ARTICLE III.—*On the Distribution of Force in a Stress.*

Theorem. In every homogeneous stress there is a system of three rectangular planes, each of which is perpendicular to the direction of the mutual force between the parts of the body on its two sides.

For let P(X), P(Y), P(Z) denote the components, parallel to X, Y, Z, any three rectangular lines of reference, of the force experienced per unit of surface at any portion of the solid bounded by a plane parallel to (Y, Z); Q(X), Q(Y), Q(Z) the corresponding components of the force experienced by any surface of the solid parallel to (Z, X); and R(X), R(Y), R(Z) those of the force at a surface parallel to (X, Y). Now by considering the equilibrium of a cube of the solid with faces parallel to the planes of reference, we see that the couple of forces Q(Z) on its two faces perpendicular to Y is balanced by the couple of forces R(Y) on the faces perpendicular to Z. Hence we must have

$$Q(Z) = R(Y).$$

Similarly, it is seen that

$$R(X) = P(Z),$$

and

$$P(Y) = Q(X).$$

For the sake of brevity, these pairs of equal quantities, being tangential forces respectively perpendicular to X, Y, Z, may be denoted by T(X), T(Y), T(Z).

Consider a tetrahedral portion of the body (surrounded it may be with continuous solid) contained within three planes A, B, C, through a point O parallel to the planes of the pairs of lines of reference, and a third plane K cutting these at angles α, β, γ respectively; so that as regards the areas of the different sides we shall have

$$A = K\cos\alpha, \quad B = K\cos\beta, \quad C = K\cos\gamma.$$

The forces actually experienced by the sides A, B, C have nothing to balance them except the force actually experienced by K. Hence those three forces must have a single resultant, and the force on K must be equal and opposite to it. If, therefore, the force on K per unit of surface be denoted by F, and its direction cosines by l, m, n, we have

$$\begin{aligned} F.K.l &= P(X)A+T(Z)B+T(Y)C, \\ F.K.m &= T(Z)A+Q(Y)B+T(X)C, \\ F.K.n &= T(Y)A+T(X)B+R(Z)C; \end{aligned}$$

and, by the relations between the cases stated above, we deduce

$$\begin{aligned} Fl &= P(X)\cos\alpha+T(Z)\cos\beta+T(Y)\cos\gamma, \\ Fm &= T(Z)\cos\alpha+Q(Y)\cos\beta+T(X)\cos\gamma, \\ Fn &= T(Y)\cos\alpha+T(X)\cos\beta+R(Z)\cos\gamma. \end{aligned}$$

Hence the problem of finding (α, β, γ), so that the force $F(l, m, n)$ may be perpendicular to it, will be solved by substituting $\cos\alpha$, $\cos\beta$, $\cos\gamma$ for l, m, n in these equations. By the elimination of $\cos\alpha$, $\cos\beta$, $\cos\gamma$ from the three equations thus obtained, we have the well-known cubic déterminantal equation, of which the roots, necessarily real, lead, when no two of them are equal, to one and only one system of three rectangular axes having the stated property.

Def. The three lines thus proved to exist for every possible homogeneous stress are called its axes. The planes of their pairs are called its normal planes: the mutual forces between parts of the body separated by these planes, or the forces on portions of the bounding surface parallel to them, are called the principal tensions.

Cor. 1. The Principal Tensions of the stress are the roots of the determinant cubic referred to in the demonstration.

Cor. 2. If a stress be specified by the notation P(X), &c. as explained above, its normal planes are the principal planes of the surface of the second degree whose equation is

$$P(X)X^2+Q(Y)Y^2+R(Z)Z^2+2T(X)YZ+2T(Y)ZX+2T(Z)XY=1;$$

and its Principal Tensions are equal to the reciprocals of the squares of the lengths of the semi principal-axes of the same surface (quantities which are negative of course for the principal axis or axes which do not cut the surface when the surface is a hyperboloid of one or of two sheets).

Cor. 3. The ellipsoid whose equation referred to the Rectangular axes of a stress, is

$$(1-2eF)X^2+(1-2eG)Y^2+(1-2eH)Z^2=1,$$

where F, G, H denote the Principal Tensions, and e any infinitely small quantity, *represents the stress,* in the following manner:—

From any point P in the surface of the ellipsoid draw a line in the tangent plane

half-way to the point where this plane is cut by a perpendicular to it through the centre; and from the end of the first-mentioned line draw a radial line to meet the surface of a sphere of unit radius concentric with the ellipsoid. The tension at this point of the surface of a sphere of the solid is in the line from it to the point P; and its amount per unit of surface is equal to the length of that infinitely small line, divided by e.

Article IV.—*On the Distribution of Displacement in a Strain.*

Prop. In every homogeneous strain any part of the solid bounded by an ellipsoid, remains bounded by an ellipsoid.

For all particles of the solid in a plane remain in a plane, and two parallel planes remain parallel. Consequently every system of conjugate diametral planes of an ellipsoid of the solid retain the property of conjugate diametral planes with reference to the altered curve surface containing the same particles. This altered surface is therefore an ellipsoid.

Prop. There is a single system (and only a single system, except in the cases of symmetry) of three rectangular planes for every homogeneous strain, which remain at right angles to one another in the altered solid.

Def. These three planes are called the normal planes of the strain, or simply the strain-normals. Their lines of intersection are called the axes of the strain.

Remark. The preceding propositions and definitions are applicable, to whatever extent the body may be strained.

Prop. If a body, while experiencing an infinitely small strain, be held with one point fixed and the normal planes of the strain parallel to three fixed rectangular planes through the point, O; a sphere of the solid of unit radius having this point for its centre becomes, when strained, an ellipsoid whose equation, referred to the strain-normals through O, is

$$(1-2x)X^2+(1-2y)Y^2+(1-2z)Z^2=1,$$

if x, y, z denote the elongations of the solid per unit of length, in the directions respectively perpendicular to these three planes; and the position, on the surface of this ellipsoid, attained by any particular point of the solid, is such that if a line be drawn in the tangent plane, half-way to the point of intersection of this plane with a perpendicular from the centre, a radial line drawn through its extremity cuts the primitive spherical surface in the primitive position of that point.

Cor. For every stress, there is a certain infinitely small strain, and conversely, for every infinitely small strain, there is a certain stress, so related that if, while the strain is being acquired, the centre and the strain-normals through it are held fixed, the absolute displacements of particles belonging to a spherical surface of the solid represent, in intensity (according to a definite convention as to units for the representation of force by lines), and in direction, the force (reckoned as to intensity, in

amount per unit of area) experienced by the enclosed sphere of the solid, at the different parts of its surface, when subjected to the stress.

Def. A stress and an infinitely small strain so related are said to be of the same type; and the ellipsoid, by means of which the distribution of force over the surface of a sphere of unit radius is represented in one case and the displacements of particles from the spherical surface are shown in the other, may be called the geometrical type of either.

ARTICLE V.—*Conditions of Perfect Concurrence between Stresses and Strains.*

Def. Two stresses are said to be coincident in direction, or to be perfectly concurrent, when they only differ in absolute magnitude. The same relative designations are applied to two strains differing from one another only in absolute magnitude.

Cor. If two stresses or two strains differ by one being reverse to the other, they may be said to be negatively coincident in direction; or to be directly opposed or directly contrary to one another.

Def. When a homogeneous stress is such that the normal component of the mutual force between the parts of the body on the two sides of any plane whatever through it is proportional to the augmentation of distance between the same plane and another parallel to it and initially at unity of distance, due to a certain strain experienced by the same body, the stress and the strain are said to be perfectly concurrent: also to be coincident in direction. The body is said to be yielding directly to a stress applied to it, when it is acquiring a strain thus related to the stress; and in the same circumstances, the stress is said to be working directly on the body, or to be acting in the same direction as the strain.

Cor. 1. Perfectly concurrent stresses and strains are of the same type.

Cor. 2. If a strain is of the same type as a stress, its reverse will be said to be negatively of the same type, or to be directly opposed to the strain. A body is said to be working directly against a stress applied to it when it is acquiring a strain directly opposed to the stress; and in the same circumstances, the matter round the body is said to be yielding directly to the reactive stress of the body upon it.

ARTICLE VI.—*Orthogonal Stresses and Strains.*

Def. 1. A stress is said to act right across a strain, or to act orthogonally to a strain, or to be orthogonal to a strain, if work is neither done upon nor by the body in virtue of the action of the stress upon it while it is acquiring the strain.

Def. 2. Two stresses are said to be orthogonal when either coincides in direction with a strain orthogonal to the other.

Def. 3. Two strains are said to be orthogonal when either coincides in direction with a stress orthogonal to the other.

Examples.—(1) A uniform cubical compression, and any strain involving no alteration of volume, are orthogonal to one another.

(2) A simple extension or contraction in parallel lines unaccompanied by any transverse extension or contraction, that is, "a simple longitudinal strain," is orthogonal to any similar strain in lines at right angles to those parallels.

(3) A simple longitudinal strain is orthogonal to a "simple tangential strain*" in which the sliding is parallel to its direction or at right angles to it.

(4) Two infinitely small simple tangential strains in the same plane†, with their directions of sliding mutually inclined at an angle of 45°, are orthogonal to one another.

(5) An infinitely small simple tangential strain is orthogonal to every infinitely small simple tangential strain, in a plane either parallel to its plane of sliding or perpendicular to its line of sliding.

ARTICLE VII.—*Composition and Resolution of Stresses and of Strains.*

Any number of simultaneously applied homogeneous stresses are equivalent to a single homogeneous stress which is called their resultant. Any number of superimposed homogeneous strains are equivalent to a single homogeneous resultant strain. Infinitely small strains may be independently superimposed; and in what follows it will be uniformly understood that the strains spoken of are infinitely small, unless the contrary is stated.

Examples.—(1) A strain consisting simply of elongation in one set of parallel lines, and a strain consisting of equal contraction in a direction at right angles to it, applied together, constitute a single strain, of the kind which that described in Example (3) of the preceding article is when infinitely small, and is called a plane distortion, or a simple distortion. It is also sometimes called a simple tangential strain, and when so considered, its plane of sliding may be regarded as either of the planes bisecting the angles between planes normal to the lines of the component longitudinal strains.

(2) Any two simple distortions in one plane may be reduced to a single simple distortion in the same plane.

(3) Two simple distortions not in the same plane have for their resultant a strain which is a distortion unaccompanied by change of volume, and which may be called a compound distortion.

(4) Three equal longitudinal elongations or condensations in three directions at right angles to one another, are equivalent to a single dilatation or condensation equal in all directions. The single stress equivalent to three equal tensions or pressures in directions at right angles to one another is a negative or positive pressure equal in all directions.

(5) If a certain stress or infinitely small strain be defined (Art. III. Cor. 3, or Art. IV.) by the ellipsoid

$$(1+A)X^2+(1+B)Y^2+(1+C)Z^2+DYZ+EZX+FXY=1,$$

and another stress or infinitely small strain by the ellipsoid

$$(1+A')X^2+(1+B')Y^2+(1+C')Z^2+D'YZ+E'ZX+F'XY=1,$$

where A, B, C, D, E, F, &c. are all infinitely small, their resultant stress or strain is that repre-

* That is, a homogeneous strain in which all the particles in one plane remain fixed, and other particles are only displaced parallel to this plane.

† "The plane of a simple tangential strain," or the plane of distortion in a simple tangential strain, is a plane perpendicular to that of the particles supposed to be held fixed, and parallel to the lines of displacement of the others.

sented by the ellipsoid

$$(1+A+A')X^2+(1+B+B')Y^2+(1+C+C')Z^2+(D+D')YZ+(E+E')ZX+(F+F')XY=1.$$

ARTICLE VIII.—*Specification of Strains and Stresses.*

Prop. Six stresses or six strains of six distinct arbitrarily chosen types may be determined to fulfil the condition of having a given stress or a given strain for their resultant, provided those six types are so chosen that a strain belonging to any one of them cannot be the resultant of any strains whatever belonging to the others.

For, just six independent parameters being required to express any stress or strain whatever, the resultant of any set of stresses or strains may be made identical with a given stress or strain by fulfilling six equations among the parameters which they involve; and therefore the magnitudes of six stresses or strains belonging to the six arbitrarily chosen types may be determined, if their resultant be assumed to be identical with the given stress or strain.

Cor. Any stress or strain may be numerically specified in terms of numbers expressing the amounts of six stresses or strains of six arbitrarily chosen types which have it for their resultant.

Types arbitrarily chosen for this purpose will be called types of reference. The specifying elements of a stress or strain will be called its components according to the types of reference. The specifying elements of a strain may also be called its coordinates, with reference to the chosen types.

Examples.—(1) Six strains in each of which one of the six edges of a tetrahedron of the solid is elongated while the others remain unchanged, may be used as types of reference for the specification of any kind of strain or stress. The ellipsoid representing any one of those six types will have its two circular sections parallel to the faces of the tetrahedron which do not contain the stretched side.

(2) Six strains consisting, any one of them, of an infinitely small alteration either of one of the three edges, or of one of the three angles between the faces, of a parallelepiped of the solid, while the other five angles and edges remain unchanged, may be taken as types of reference, for the specification of either stresses or strains. In some cases, as for instance in expressing the probable elastic properties of a crystal of Iceland spar, it may be convenient to use an oblique parallelepiped for such a system of types of reference; but more frequently it will be convenient to adopt a system of types related to the deformations of a cube of the solid, in the manner described.

(3) If

$$AX^2+BY^2+CZ^2+DYZ+EZX+FXY=1$$

be the equation of the surface of a portion of the solid referred to oblique or rectangular coordinates, we may take the six strains, in any one of which the same portion of the solid becomes altered in shape to a surface whose equation differs from the preceding only in having one of the six coefficients altered by an infinitely small quantity, as six types of reference for specifying stresses and strains in general.

ARTICLE IX.—*Orthogonal Types of Reference.*

Def. A normal system of types of reference is one in which the strains or stresses of the different types are all six mutually orthogonal (fifteen conditions). A normal

system of types of reference may also be called an orthogonal system. The elements specifying, with reference to such a system, any stress or strain, will be called orthogonal components or orthogonal coordinates.

Examples.—(1) The six types described in Example (2) of Article VIII. are clearly orthogonal, if the parallelepiped referred to is rectangular. Three of these are simple longitudinal extensions, parallel to the three sets of rectangular edges of the parallelepiped. The remaining three are plane distortions parallel to the faces, their axes bisecting the angles between the edges. They constitute the system of types of reference uniformly used hitherto by writers on the theory of elasticity.

(2) The six strains in which a spherical portion of the solid is changed into ellipsoids having the following equations—

$$(1+A)X^2+Y^2+Z^2=1$$
$$X^2+(1+B)Y^2+Z^2=1$$
$$X^2+Y^2+(1+C)Z^2=1$$
$$X^2+Y^2+Z^2+DYZ=1$$
$$X^2+Y^2+Z^2+EZX=1$$
$$X^2+Y^2+Z^2+FXY=1,$$

are of the same kind as those considered in the preceding example, and therefore constitute a normal system of types of reference. The resultant of the strains specified, according to those equations, by the elements A, B, C, D, E, F, is a strain in which the sphere becomes an ellipsoid whose equation (see above, Art. VII. Ex. (5)) is

$$(1+A)X^2+(1+B)Y^2+(1+C)Z^2+DYZ+EZX+FXY=1.$$

(3)* A uniform cubical compression (I.), three simple distortions having their planes at right angles to one another and their axes† bisecting the angles between the lines of intersection of these planes (II.) (III.) (IV.), any simple or compound distortion consisting of a combination of longitudinal strains parallel to those lines of intersection (V.), and the distortion (VI.), constituted from the same elements which is orthogonal to the last, afford a system of six mutually orthogonal types which will be used as types of reference below in expressing the elasticity of cubically isotropic solids.

ARTICLE X.—*On the Measurement of Strains and Stresses.*

Def. Strains of any types are said to be to one another in the same ratios as stresses of the same types respectively, when any particular plane of the solid acquires relatively to another plane parallel to it, motions in virtue of those strains which are to one another in the same ratios as the normal components of the forces between the parts of the solid on the two sides of either plane due to the respective stresses.

Def. The magnitude of a stress and of a strain of the same type, are quantities which, multiplied one by the other, give the work done on unity of volume of a body acted on by the stress while acquiring the strain.

* This example, as well as (7) of Art. X., (5) of XI., and the example of Art. XII., have been inserted to prepare for an application of the theory of Principal Elasticities to cubically and spherically isotropic bodies, added to the Second Part of this paper since the date of its communication.

† The "axes of a simple distortion" are the lines of its two component longitudinal strains.

Cor. 1. If $x, y, z, \xi, \eta, \zeta$ denote orthogonal components of a certain strain, and if P, Q, R, S, T, U denote components, of the same type respectively, of a stress applied to a body while acquiring that strain, the work done upon it per unit of its volume will be

$$Px+Qy+Rz+S\xi+T\eta+U\zeta.$$

Cor. 2. The condition that two strains or stresses specified by $(x, y, z, \xi, \eta, \zeta)$ and $(x', y', z', \xi', \eta', \zeta')$ in terms of a normal system of types of reference, may be orthogonal to one another, is

$$xx'+yy'+zz'+\xi\xi'+\eta\eta'+\zeta\zeta'=0.$$

Cor. 3. The magnitude of the resultant of two, three, four, five, or six mutually orthogonal strains or stresses is equal to the square root of the sum of their squares. For if P, Q, &c. denote several orthogonal stresses, and F the magnitude of their resultant; and x, y, &c. a set of proportional strains of the same types respectively, and r the magnitude of the single equivalent strain, the resultant stress and strain will be of one type, and therefore the work done by the resultant stress will be Fr. But the amounts done by the several components will be Px, Qy, &c., and therefore

$$Fr=Px+Qy+\&c.$$

Now we have, to express the proportionality of the stresses and strains,

$$\frac{P}{x}=\frac{Q}{y}=\&c.=\frac{F}{r}.$$

Each member must be equal to

$$\frac{P^2+Q^2+\&c.}{Px+Qy+\&c.};$$

and also equal to

$$\frac{Px+Qy+\&c.}{x^2+y^2+\&c.}.$$

Hence $\quad \frac{F}{r}=\frac{P^2+Q^2+\&c.}{Fr}$, which gives $F^2=P^2+Q^2+\&c.$,

and $\quad \frac{F}{r}=\frac{Fr}{x^2+y^2+\&c.}$, which gives $r^2=x^2+y^2+\&c.$

Cor. 4. A definite stress of some particular type chosen arbitrarily may be called unity; and then the numerical reckoning of all strains and stresses becomes perfectly definite.

Def. A uniform pressure or tension in parallel lines, amounting in intensity to the unit of force per unit of area normal to it, will be called a stress of unit magnitude, and will be reckoned as positive when it is tension, and negative when pressure.

Examples.—(1) Hence the magnitude of a simple longitudinal strain, in which lines of the body parallel to a certain direction experience elongation to an extent bearing the ratio x to their original dimensions, must be called x.

(2) The magnitude of the single stress equivalent to three simple pressures in directions at right angles to one another each unity is $-\sqrt{3}$; a uniform compression in all directions of unity per unit of surface, is a negative stress equal to $\sqrt{3}$ in absolute value.

(3) A uniform dilatation in all directions, in which lineal dimensions are augmented in the ratio $1:1+x$, is a strain equal in magnitude to $x\sqrt{3}$; or a uniform "cubic expansion" E is a strain equal to $\frac{E}{\sqrt{3}}$.

(4) A stress compounded of a uniform unit pressure in one direction and an equal tension in a direction at right angles to it, or which is the same thing, a stress compounded of two balancing couples of unit tangential pressures in planes at angles of 45° to the direction of those forces and at right angles to one another, amounts in magnitude to $\sqrt{2}$.

(5) A strain compounded of a simple longitudinal extension, x, and a simple longitudinal condensation of equal absolute value, in a direction perpendicular to it, is a strain of magnitude $x\sqrt{2}$; or, which is the same thing, (if $\sigma=2x$,) a simple distortion such that the relative motion of two planes at unit distances parallel to either of the planes bisecting the angles between the two planes mentioned above is a motion σ, parallel to themselves, is a strain amounting in magnitude to $\frac{\sigma}{\sqrt{2}}$.

(6) If a strain be such that a sphere of unit radius in the body becomes an ellipsoid whose equation is

$$(1-A)X^2+(1-B)Y^2+(1-C)Z^2-DYZ-EZX-FXY=1,$$

the values of the component strains corresponding, as explained in Example (2) of Art. IX., to the different coefficients respectively, are

$$\tfrac{1}{2}A,\ \tfrac{1}{2}B,\ \tfrac{1}{2}C,\ \frac{D}{2\sqrt{2}},\ \frac{E}{2\sqrt{2}},\ \frac{F}{2\sqrt{2}}.$$

For the components corresponding to A, B, C are simple longitudinal strains, in which diameters of the sphere along the axes of coordinates become elongated from 2, to $2+A$, $2+B$, $2+C$ respectively; D is a distortion in which diameters in the plane YOZ, bisecting the angles YOZ and Y′OZ, become respectively elongated and contracted from 2 to $2+\frac{1}{2}D$, and from 2 to $2-\frac{1}{2}D$; and so for the others. Hence, if we take $x, y, z, \xi, \eta, \zeta$ to denote the magnitudes of six component strains, according to the orthogonal system of types described in Examples (1) and (2) of Art. IX., the resultant strain equivalent to them will be one in which a sphere of radius 1 in the solid becomes an ellipsoid whose equation is

$$(1-2x)X^2+(1-2y)Y^2+(1-2z)Z^2-2\sqrt{2}(\xi YZ+\eta ZX+\zeta XY)=1,$$

and its magnitude will be

$$\sqrt{(x^2+y^2+z^2+\xi^2+\eta^2+\zeta^2)}.$$

(7) The specifications, according to the system of reference used in the preceding Example, of unit strains belonging to the six orthogonal types defined in Example (3) of Art. IX., are respectively as follows:—

	x	y	z	ξ	η	ζ
(I.)	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	0	0	0
(II.)	0	0	0	1	0	0
(III.)	0	0	0	0	1	0
(IV.)	0	0	0	0	0	1
(V.)	l	m	n	0	0	0
(VI.)	l'	m'	n'	0	0	0

where l, m, n, l', m', n' denote quantities fulfilling the following conditions:—

$$\begin{aligned} l^2+m^2+n^2&=1,\\ l+m+n&=0,\\ ll'+mm'+nn'&=0,\\ l'^2+m'^2+n'^2&=1,\\ l'+m'+n'&=0. \end{aligned}$$

(8) If $(1-2eP)X^2+(1-2eQ)Y^2+(1-2eR)Z^2-2e\sqrt{2}(SYZ+TZX+UXY)=1$

be the equation of the ellipsoid representing a certain stress, the amount of work done by this stress, if applied to a body while acquiring the strain represented by the equation in the preceding example, will be

$$Px+Qy+Rz+S\xi+T\eta+U\zeta.$$

Cor. Hence, if the variables X, Y, Z be transformed to any other set (X′, Y′, Z′) fulfilling the condition of being the coordinates of the same point, referred to another system of rectangular axes, the coefficients x, y, z, &c., $x_{\prime}, y_{\prime}, z_{\prime}$, &c. in two homogeneous quadratic functions of three variables,

$$(1-2x)X^2+(1-2y)Y^2+(1-2z)Z^2-2\sqrt{2}(\xi YZ+\eta ZX+\zeta XY)$$

and

$$(1-2x_{\prime})X^2+(1-2y_{\prime})Y^2+(1-2z_{\prime})Z^2-2\sqrt{2}(\xi_{\prime} YZ+\eta_{\prime} ZX+\zeta_{\prime} XY),$$

and the corresponding coefficients x', y', z', &c., $x'_{\prime}, y'_{\prime}, z'_{\prime}$, &c. will be so related that

$$x'x'_{\prime}+y'y'_{\prime}+z'z'_{\prime}+\xi'\xi'_{\prime}+\eta'\eta'_{\prime}+\zeta'\zeta'_{\prime}=xx_{\prime}+yy_{\prime}+zz_{\prime}+\xi\xi_{\prime}+\eta\eta_{\prime}+\zeta\zeta_{\prime};$$

or the function $xx_{\prime}+yy_{\prime}+zz_{\prime}+\xi\xi_{\prime}+\eta\eta_{\prime}+\zeta\zeta_{\prime}$ of the coefficients is an "invariant" for linear transformations fulfilling the conditions of transformation from one to another set of rectangular axes. Since $x+y+z$ and $x_{\prime}+y_{\prime}+z_{\prime}$ are clearly invariants also, it follows that $AA_{\prime}+BB_{\prime}+CC_{\prime}+2DD_{\prime}+2EE_{\prime}+2FF_{\prime}$ is an invariant function of the coefficients of the two quadratics

$$AX^2+BY^2+CZ^2+2DYZ+2EZX+2FXY$$

and

$$A_{\prime}X^2+B_{\prime}Y^2+C_{\prime}Z^2+2D_{\prime}YZ+2E_{\prime}ZX+2F_{\prime}XY,$$

which it is easily proved to be by direct transformation.

Article XI.—*On Imperfect Concurrences of two Stress or Strain-types.*

Def. The concurrence of any stresses or strains of two stated types, is the proportion which the work done when a body of unit volume experiences a stress of either type while acquiring a strain of the other, bears to the product of the numbers measuring the stress and strain respectively.

Cor. 1. In orthogonal resolution of a stress or strain, its component of any stated type is equal to its own amount multiplied by its concurrence with that type; or the stress or strain of a stated type which, along with another or others orthogonal to it have a given stress or strain for their resultant, is equal to the amount of the given stress or strain reduced in the ratio of its concurrence with that stated type.

Cor. 2. The concurrence of two coincident stresses or strains is unity; or a perfect concurrence is numerically equal to unity.

Cor. 3. The concurrence of two orthogonal stresses and strains is zero.

Cor. 4. The concurrence of two directly opposed stresses or strains is -1.

Cor. 5. If $x, y, z, \xi, \eta, \zeta$, are orthogonal components of any strain or stress, r, its concurrences with the types of reference are respectively

$$\frac{x}{r}, \frac{y}{r}, \frac{z}{r}, \frac{\xi}{r}, \frac{\eta}{r}, \frac{\zeta}{r},$$

where

$$r=(x^2+y^2+z^2+\xi^2+\eta^2+\zeta^2)^{\frac{1}{2}}.$$

Cor. 6. The mutual concurrence of two stresses or strains is

$$ll'+mm'+nn'+\lambda\lambda'+\mu\mu'+\nu\nu',$$

if $(l, m, n, \lambda, \mu, \nu)$ denote the concurrences of one of them with six orthogonal types of reference, and $(l', m', n', \lambda', \mu', \nu')$ those of the other.

Cor. 7. The most convenient *specification of a type* for strains or stresses, being in general a statement of the components, according to the types of reference, of a unit strain or stress of the type to be specified, becomes a statement of its concurrences with the types of reference when these are orthogonal.

Examples.—(1) The mutual concurrence of two simple longitudinal strains or stresses, inclined to one another at an angle θ, is $\cos^2\theta$.

(2) The mutual concurrence of two simple distortions in the same plane, whose axes are inclined at an angle θ to one another, is $\cos^2\theta-\sin^2\theta$, or $2\sin(45^\circ-\theta)\cos(45^\circ-\theta)$.

Hence the components of a simple distortion, δ, along two rectangular axes in its plane, and two others bisecting the angle between these taken as axes of component simple distortions, are

$$\delta(\cos^2\theta-\sin^2\theta) \text{ and } \delta.2\sin\theta\cos\theta$$

respectively, if θ be the angle between the axis of elongation in the given distortion and in the first component type.

(3) The mutual concurrence of a simple longitudinal strain and a simple distortion is

$$\surd 2.\cos\alpha\cos\beta,$$

if α and β be the angles at which the direction of the longitudinal strain is inclined to the line bisecting the angles between the axes of the distortion; it is also equal to

$$\frac{1}{\surd 2}(\cos^2\phi-\cos^2\psi),$$

if ϕ and ψ denote the angles at which the direction of the longitudinal strain is inclined to the axis of the distortion.

(4) The mutual concurrence of a simple longitudinal strain and of a uniform dilatation is $\frac{1}{\surd 3}$.

(5) The specifying elements exhibited in Example (7) of the preceding article, are the concurrences of the new system of orthogonal types described in Example (3) of Art. IX., with the ordinary system, Examples (1) and (2), Art. IX.

ARTICLE XII.—*On the Transformation of Types of Reference for Stresses or Strains.*

To transform the specification $(x, y, z, \xi, \eta, \zeta)$ of a stress or strain with reference to one system of types into $(x_1, x_2, x_3, x_4, x_5, x_6)$ with reference to another system of types. Let $(a_1, b_1, c_1, e_1, f_1, g_1)$ be the components, according to the original system,

of a unit strain of the first type of the new system; let $(a_2, b_2, c_2, e_2, f_2, g_2)$ be the corresponding specification of the second type of the new system; and so on. Then we have, for the required formulæ of transformation,

$$x = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + a_6x_6,$$
$$y = b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_6x_6,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\zeta = g_1x_1 + g_2x_2 + g_3x_3 + g_4x_4 + g_5x_5 + g_6x_6.$$

Example. The transforming equations to pass from a specification $(x, y, z, \xi, \eta, \zeta)$ in terms of the system of reference used in Examples (6) and (7), Art. X., to a specification $\sigma, \xi, \eta, \zeta, \varpi, \omega$ in terms of the new system described in Example (3) of Art. IX., and specified in Example (7) of Art. X., are as follows:—

$$x = \frac{1}{\sqrt{3}}\sigma + l\varpi + l'\omega,$$

$$y = \frac{1}{\sqrt{3}}\sigma + m\varpi + m'\omega,$$

$$z = \frac{1}{\sqrt{3}}\sigma + n\varpi + n'\omega,$$

$$\xi = \xi, \quad \eta = \eta, \quad \zeta = \zeta;$$

where, as before stated, l, m, n, l', m', n' are quantities fulfilling the conditions

$$l^2 + m^2 + n^2 = 1,$$
$$l + m + n = 0,$$
$$l'^2 + m'^2 + n'^2 = 1,$$
$$l' + m' + n' = 0,$$
$$ll' + mm' + nn' = 0.$$

PART II. ON THE MECHANICAL CONDITIONS OF RELATION BETWEEN STRESSES AND STRAINS, EXPERIENCED BY AN ELASTIC SOLID.

ARTICLE XIII.—*Interpretation of the Differential Equation of Energy.*

In a paper on the Thermo-elastic Properties of Matter, published in the first Number of the Quarterly Mathematical Journal (April 1855), it was proved from general principles in the theory of the Transformation of Energy, that the amount of work (w) required to reduce an elastic solid, kept at a constant temperature, from one stated condition of internal strain to another, depends solely on these two conditions, and not at all on the cycle of varied states through which the body may have been made to pass in effecting the change, provided always there has been no failure in the elasticity under any of the strains it has experienced. Thus for a homogeneous solid homogeneously strained, it appears that w is a function of six independent variables $x, y, z, \xi, \eta, \zeta$, by which the condition of the solid as to strain is specified. Hence to strain the body to the infinitely small extent expressed by the variation from $(x, y, z, \xi, \eta, \zeta)$ to $(x+dx, y+dy, z+dz, \xi+d\xi, \eta+d\eta, \zeta+d\zeta)$, the work required to be done upon it is

$$\frac{dw}{dx}dx + \frac{dw}{dy}dy + \frac{dw}{dz}dz + \frac{dw}{d\xi}d\xi + \frac{dw}{d\eta}d\eta + \frac{dw}{d\zeta}d\zeta.$$

The stress which must be applied to its surface to keep the body in equilibrium in the state $(x, y, z, \xi, \eta, \zeta)$ must therefore be such that it would do this amount of work if the body, under its action, were to acquire the arbitrary strain $dx, dy, dz, d\xi, d\eta, d\zeta$; that is, it must be the resultant of six stresses; one orthogonal to the five strains $dy, dz, d\xi, d\eta, d\zeta$, and of such a magnitude as to do the work $\frac{dw}{dx}dx$ when the body acquires the strain dx; a second orthogonal to $dx, dz, d\xi, d\eta, d\zeta$, and of such a magnitude as to do the work $\frac{dw}{dy}dy$ when the body acquires the strain dy; and so on. If a, b, c, f, g, h denote the respective concurrences of these six stresses, with the types of reference used in the specification $(x, y, z, \xi, \eta, \zeta)$ of the strains, the amounts of the six stresses which fulfil those conditions will (Art. XI.) be given by the equations

$$P=\frac{1}{a}\frac{dw}{dx},\quad Q=\frac{1}{b}\frac{dw}{dy},\quad R=\frac{1}{c}\frac{dw}{dz},$$

$$S=\frac{1}{f}\frac{dw}{d\xi},\quad T=\frac{1}{g}\frac{dw}{d\eta},\quad U=\frac{1}{h}\frac{dw}{d\zeta};$$

and the types of these component stresses are determined by being orthogonal to the fives, of the six strain-types wanting the first, the second, &c. respectively.

Cor. If the types of reference used in expressing the strain of the body constitute an orthogonal system, the types of the component stresses will coincide with them, and each of the concurrences will be unity. Hence the equations of equilibrium of an elastic solid referred to six orthogonal types are simply

$$P=\frac{dw}{dx},\quad Q=\frac{dw}{dy},\quad R=\frac{dw}{dz},$$

$$S=\frac{dw}{d\xi},\quad T=\frac{dw}{d\eta},\quad U=\frac{dw}{d\zeta}.$$

ARTICLE XIV.—*Reduction of the Potential Function, and of the Equations of Equilibrium, of an Elastic Solid to their simplest Forms.*

If the condition of the body from which the work denoted by w is reckoned be that of equilibrium under no stress from without, and if $x, y, z, \xi, \eta, \zeta$ be chosen each zero for this condition, we shall have, by MACLAURIN's theorem,

$$w=H_2(x, y, z, \xi, \eta, \zeta)+H_3(x, y, z, \xi, \eta, \zeta)+ \&c.,$$

where H_2, H_3, &c. denote homogeneous functions of the second order, third order, &c. respectively. Hence $\frac{dw}{dx}$, $\frac{dw}{dy}$, &c. will each be a linear function of the strain-coordinates, together with functions of higher orders derived from H_3, &c. But experience shows that within the elastic limits, the stresses are very nearly if not quite proportional to the strains they are capable of producing; and therefore H_3, &c. may be neglected, and we have simply

$$w=H_2(x, y, z, \xi, \eta, \zeta).$$

Now in general there will be 21 terms, with independent coefficients, in this function; but by a choice of types of reference, that is, by a linear transformation of the independent variables, we may, in an infinite variety of ways, reduce it to the form

$$w=\tfrac{1}{2}(\mathrm{A}x^2+\mathrm{B}y^2+\mathrm{C}z^2+\mathrm{F}\xi^2+\mathrm{G}\eta^2+\mathrm{H}\zeta^2).$$

The equations of equilibrium then become

$$\mathrm{P}=\frac{\mathrm{A}}{a}x,\quad \mathrm{Q}=\frac{\mathrm{B}}{b}y,\quad \mathrm{R}=\frac{\mathrm{C}}{c}z,$$

$$\mathrm{S}=\frac{\mathrm{F}}{f}\xi,\quad \mathrm{T}=\frac{\mathrm{G}}{g}\eta,\quad \mathrm{U}=\frac{\mathrm{H}}{h}\zeta,$$

the simplest possible form under which they can be presented. The interpretation is expressed as follows.

Prop. An infinite number of systems of six types of strains or stresses exist in any given elastic solid such that, if a strain of any one of those types be impressed on the body, the elastic reaction is balanced by a stress orthogonal to the five others of the same system.

ARTICLE XV.—*On the Six Principal Strains of an Elastic Solid.*

To reduce the twenty-one coefficients of the quadratic terms in the expression for the potential energy to six by a linear transformation, we have only fifteen equations to satisfy; while we have thirty disposable transforming coefficients, there being five independent elements to specify a type, and six types to be changed. Any further condition expressible by just fifteen independent equations may be satisfied and makes the transformation determinate. Now the condition that six strains may be mutually orthogonal, is expressible by just as many equations as there are of different pairs of six things; that is fifteen. The well-known algebraic theory of the linear transformation of quadratic functions shows for the case of six variables, (1) that the six coefficients in the reduced form are the roots of a "determinant" of the sixth degree necessarily real; (2) that this multiplicity of roots leads determinately to one, and only one system of six types fulfilling the prescribed conditions unless two or more of the roots are equal to one another, when there will be an infinite number of solutions and definite degrees of isotropy among them; and (3) that there is no equality between any of the six roots of the determinant in general, when there are twenty-one independent coefficients in the given quadratic.

Prop. Hence a single system of six mutually orthogonal types may be determined for any homogeneous elastic solid, so that its potential energy when homogeneously strained in any way is expressed by the sum of the products of the squares of the components of the strain, according to those types, respectively multiplied by six determinate coefficients.

Def. The six strain-types thus determined are called the Six Principal Strain-types of the body.

The concurrences of the stress-components used in interpreting the differential equation of energy with the types of the strain-coordinates in terms of which the potential of elasticity is expressed, being perfect when these constitute an orthogonal system, each of the quantities denoted above by a, b, c, f, g, h, is unity when the six principal strain-types are chosen for the coordinates. The equations of equilibrium of an elastic solid may therefore be expressed as follows:—

$$P=Ax, \quad Q=By, \quad R=Cz,$$
$$S=F\xi, \quad T=G\eta, \quad U=H\zeta,$$

where x, y, z, ξ, η, ζ denote strains belonging to the six Principal Types, and P, Q, R, S, T, U the components according to the same types, of the stress required to hold the body in equilibrium when in the condition of having those strains. The amount of work that must be spent upon it per unit of its volume, to bring it to this state from an unconstrained condition, is given by the equation

$$w=\tfrac{1}{2}(Ax^2+By^2+Cz^2+F\xi^2+G\eta^2+H\zeta^2).$$

Def. The coefficients A, B, C, F, G, H are called the six principal Elasticities of the body.

The equations of equilibrium express the following propositions:—

Prop. If a body be strained according to any one of its six Principal Types, the stress required to hold it so is directly concurrent with the strain.

Examples inserted September 16, 1856.

(1) If a solid be cubically isotropic in its elastic properties, as crystals of the cubical class probably are, any portion of it will, when subjected to a uniform positive or negative normal pressure all round its surface, experience a uniform condensation or dilatation in all directions. Hence a uniform condensation is one of its six Principal Strains. Three plane distortions with axes bisecting the angles between the edges of the cube of symmetry are clearly also principal strains, and since the three corresponding principal elasticities are equal to one another, any strain whatever compounded of these three is a principal strain. Lastly, a plane distortion whose axes coincide with any two edges of the cube, being clearly a principal distortion, and the principal elasticities corresponding to the three distortions of this kind being equal to one another, any distortion compounded of them is also a principal distortion.

Hence the system of orthogonal types treated of in Examples (3) Art. IX., and (7) Art. X., or any system in which, for (II.), (III.), and (IV.), any three orthogonal strains compounded of them are substituted, constitutes a system of six Principal Strains in a solid cubically isotropic. There are only three distinct Principal Elasticities for such a body, and these are (A) its cubic compressibility, (B) its rigidity against diagonal distortion in any of its principal planes (three equal elasticities), and (C) its rigidity against rectangular distortions of a cube of symmetry (two equal elasticities).

(2) In a perfectly isotropic solid, the rigidity against all distortions is equal. Hence the rigidity (B) against diagonal distortion must be equal to the rigidity (C) against rectangular distortion, in a cube; and it is easily seen that if this condition is fulfilled for one set of three rectangular planes for which a substance is isotropic, the isotropy must be complete. The conditions of perfect

or spherical isotropy are therefore expressed in terms of the conditions referred to in the preceding example, with the farther condition, B=C.

A uniform condensation in all directions, and any system whatever of five orthogonal distortions, constitute a system of six Principal Strains in a spherically isotropic solid. Its Principal Elasticities are simply its Cubical Compressibility and its Rigidity.

Prop. Unless some of the six Principal Elasticities be equal to one another, the stress required to keep the body strained otherwise than according to one or other of six distinct types is oblique to the strain.

Prop. The stress required to maintain a given amount of strain is a maximum or minimum if of one of the six Principal Types.

Cor. If A be the greatest and H the least of the six quantities A, B, C, F, G, H, the principal type to which the first corresponds is that of a strain *requiring a greater stress to maintain it* than any other strain of equal amount; and the principal type to which the last corresponds is that of a strain which *is maintained by a less stress* than any other strain of equal amount in the same body. The stresses corresponding to the four other principal strain-types have each the double, maximum and minimum, property in a determinate way.

Prop. If a body be strained in a direction of which the concurrences with the principal strain-types are l, m, n, λ, μ, ν, and to an amount equal to r, the stress required to maintain it in this state will be equal to Ωr, where

$$\Omega=(A^2l^2+B^2m^2+C^2n^2+F^2\lambda^2+G^2\mu^2+H^2\nu^2)^{\frac{1}{2}},$$

and will be of a type of which the concurrences with the principal types are respectively

$$\frac{Al}{\Omega},\ \frac{Bm}{\Omega},\ \frac{Cn}{\Omega},\ \frac{F\lambda}{\Omega},\ \frac{G\mu}{\Omega},\ \frac{H\nu}{\Omega}.$$

Prop. A homogeneously strained elastic solid, crystalline or non-crystalline, subject to magnetic force or free from magnetic force, has neither any right-handed or left-handed, nor any dipolar, properties dependent on elastic forces simply proportional to strains.

Cor. 1. The elastic forces concerned in the luminiferous vibrations of a solid or fluid medium possessing the "right- or left-handed isotropic axial property," or the completely "isotropic rotatory property," (such as quartz crystal, right- or left-handed tartaric acid, solution of sugar,) or the dipolar axial rotatory property discovered by FARADAY in his heavy glass and other transparent bodies, solid and fluid, in the magnetic field, either depend on the heterogeneousness or on the magnitude of the strains experienced.

Hence as they do not depend on the magnitude of the strain, they do depend on its heterogeneousness through the portion of the medium containing a wave.

Cor. 2. There cannot possibly be any characteristic of elastic forces simply proportional to the strains in a homogeneous body, corresponding to certain peculiarities of crystalline form which have been observed; for instance corresponding to the

plagiedral faces, discovered by Sir JOHN HERSCHEL to indicate the optical character, whether right-handed or left-handed, in different specimens of quartz crystal, or corresponding to the distinguishing characteristics of the crystals of the right-handed and left-handed tartaric acids, obtained by M. PASTEUR from racemic acid, or corresponding to the dipolar characteristics of form said to have been discovered in electric crystals.

ARTICLE XVI.—*Application of Conclusions to Natural Crystals.*

In a paper on the Thermo-elastic Properties of Matter, which I hope to be able before long to lay before the Royal Society of Edinburgh, I intend to demonstrate that a body, homogeneous when regarded on a large scale, may be constructed to have twenty-one arbitrarily prescribed values for the coefficients in the expression for its potential energy in terms of any prescribed system of strain coordinates. This proposition was first enunciated in the paper on the Thermo-elastic Properties of Solids, published last April in the Quarterly Mathematical Journal alluded to above. We may infer the following.

Prop. A solid may be constructed to have arbitrarily prescribed values for its six Principal Elasticities and an arbitrary orthogonal system of six strains, specified by fifteen elements, for its principal strain-types; having, for instance, five arbitrarily chosen systems of three rectangular axes, for the normal axes of five of the principal strains, and those of the sixth consequently in general distinct from all the others.

Cor. There is no reason for believing that natural crystals do not exist for which there are six unequal Principal Elasticities, and six distinct strain-types for which the three normal axes constitute six distinct sets of three principal rectangular axes of elasticity.

It would be easy to add arbitrary illustrative examples regarding Principal Elasticities, and to investigate the principal strain-types and the equations of elastic force referred to them or to other natural types, for a body possessing the kind of symmetry as to elastic forces that is possessed by a crystal of Iceland spar or by a crystal of the cubical class (which may be included with the former in an investigation on a very obvious plan).

XXII. *On the Lunar-Diurnal Magnetic Variation at Toronto.*

By Major-General EDWARD SABINE, *R.A., D.C.L., Treas. & V.P.R.S.*

Received June 13,—Read June 19, 1856.

IN fulfilment of an intention expressed to the Royal Society in November 1853 *, I have now the honour to submit to the Society the results of an investigation into the Moon's diurnal influence on the horizontal and vertical components of the magnetic force at Toronto, and the consequent deduction of the Lunar-diurnal Variations of the Inclination and of the Total Force at that Station.

The processes to which the observations of the Bifilar and of the Vertical force Magnetometers, as received from Toronto, were subjected after their arrival at Woolwich, with a view to this and to other investigations, have been already partially described in a communication presented to the Society in a former part of the present Session †. The processes there described had reference particularly to the reduction of the observations to a uniform temperature of the magnets employed to measure the variations of the respective components of the force,—to the elimination of the larger disturbances,—to the formation of normal values (omitting the disturbances) for each of the components at every hour of mean solar time for periods usually of a month's duration,—and to the deduction of the solar-diurnal variation in different years and different months, after the larger disturbances had been eliminated.

The investigation regarding the Moon's influence was commenced by marking every observation in small figures on the face of the monthly tables with the lunar hour to which each observation most nearly corresponded. This was done in the manner described in the second and third pages of Art. XIX. in the Philosophical Transactions of 1853, when treating of a similar process in the case of the Magnetic Declination. A fresh set of monthly tables were then prepared for every month in each of the five years, in which tables were entered the *differences*, each under the lunar hour to which it most nearly corresponded, between the several observations and their corresponding normals. By this proceeding the diurnal and other variations depending on the time of the solar year, and on the hour of the solar day, were in great part at least eliminated. The differences were marked with a + or a − sign, according as the amount of the force at the time of the observation was greater or less than the monthly normal at the same hour. The mean was then taken in every month of every lunar hour (attending to the signs), and the monthly means were collected into yearly means. The lunar hour to which each observation most

* Philosophical Transactions, 1853, Art. XIX.

† Ibid. 1856, Art. XVI.

nearly corresponded was marked by myself, and the differences from the normals and their arrangement in tables exhibiting the lunar-diurnal influence were prepared, under the superintendence of Mr. MAGRATH, by the Non-commissioned Officers of the Royal Artillery employed in the Woolwich Office. The total number of observations in the five years, after the separation of the larger disturbances, was,—of the Bifilar, 34303; and of the Vertical Force Magnetometer, 31773. The lunar influence at the different lunar hours is shown in the tables in decimals of a scale division; the value of a scale division of the Bifilar being ·000087 parts of the horizontal force, and of the vertical force magnetometer ·000065 parts of the vertical force.

Horizontal Force.—Table I. exhibits, in columns 2 to 6, the mean horary variation of the horizontal force at the different lunar hours in each of the five years ending June 30; and in column 7, the mean of the five years:

TABLE I.

Lunar hours.	In the year ending 30th June. 1844.	1845.	1846.	1847.	1848.	Mean of the five years.	Lunar hours.
1.	2. sc. div.	3. sc. div.	4. sc. div.	5. sc. div.	6. sc. div.	7. sc. div.	8.
0	+0·44	−0·05	+0·10	+0·70	−0·07	+0·22	0
1	+0·48	+0·01	+0·09	+0·70	+0·68	+0·39	1
2	+0·68	+0·59	+0·11	+0·29	+0·42	+0·42	2
3	+0·42	+0·48	+0·71	+0·50	+0·08	+0·44	3
4	+0·58	+0·28	+0·22	+0·09	+0·19	+0·27	4
5	+0·16	+0·68	+0·21	+0·43	+0·74	+0·44	5
6	−0·23	+1·11	−0·03	+0·09	+0·43	+0·27	6
7	−0·13	+0·72	−0·20	−0·25	+0·21	+0·07	7
8	−0·16	+0·59	−0·31	−0·24	−0·19	−0·06	8
9	−0·12	+0·28	−0·23	−0·79	+0·24	−0·12	9
10	−0·43	+0·20	−0·10	−0·62	+0·23	−0·14	10
11	−0·02	+0·32	+0·36	+0·39	−0·22	+0·17	11
12	−0·28	+0·37	+0·15	−0·05	−0·31	−0·02	12
13	+0·11	+0·64	+0·11	+0·23	+0·47	+0·31	13
14	+0·06	+0·84	+0·18	−0·09	+0·40	+0·28	14
15	+0·06	+0·04	+0·23	−0·06	+0·73	+0·20	15
16	0·00	−0·01	+0·13	+0·32	+0·28	+0·14	16
17	−0·17	+0·48	−0·47	0·00	+0·33	+0·03	17
18	−0·08	−0·19	−0·13	−0·24	−0·47	−0·22	18
19	−0·21	−0·13	−0·33	−0·54	−0·80	−0·40	19
20	−0·19	−0·70	−0·11	−0·55	−1·33	−0·58	20
21	−0·19	−0·82	−0·31	−0·09	−1·07	−0·50	21
22	−0·28	−0·44	−0·22	+0·09	−0·65	−0·30	22
23	−0·08	−0·46	−0·15	+0·34	−0·18	−0·11	23

We may represent the values in column 7 of this Table (or the variation of the horizontal force at the several lunar hours on the average of the five years of observation) by the first terms of the usual formula for periodical functions, viz.

$$\Delta_x = A_0 + A_1 \cos a + B_1 \sin a + A_2 \cos 2a + B_2 \sin 2a.$$

By substituting in this formula the numerical values of the coefficients obtained from the numbers in column 7, it becomes

$$\Delta_x = +\cdot05 - \cdot024 \cos a + \cdot214 \sin a + \cdot0775 \cos 2a + \cdot323 \sin 2a,$$

or the more convenient equivalent expression,

$$\Delta_x = +\cdot05 + \cdot215 \sin(a + 353^\circ\cdot6) + \cdot3324 \sin(2a + 13^\circ\cdot5),$$

the coefficients being decimals of a scale-division, and a reckoned in hours, multiplied by 15°, from the time of the Moon's superior culmination. By this formula is obtained the curve which is shown by the stronger line in fig. 1 of the accompanying Plate XIX.; and for the purpose of showing the degree of confidence to which this curve is entitled, as an approximate representation of the variation produced in the horizontal force by the moon in the course of a lunar day, the variation in the different years in columns 2 to 6 of Table I. have been so combined as to form two separate means, one representing the columns headed 1844, 1845 and 1846, and a second representing the columns headed 1846, 1847 and 1848; the years 1844 and 1845 having double weight assigned to them in the first mean, and 1847 and 1848 double weight in the second mean. The formulæ representing these separate means are,—

for 1844 to 1846, $\Delta_x = +\cdot088 + \cdot243 \sin(a + 347^\circ\cdot6) + \cdot277 \sin(2a + 4^\circ\cdot4)$,

for 1846 to 1848, $\Delta_x = +\cdot013 + \cdot192 \sin(a + 355^\circ\cdot0) + \cdot395 \sin(2a + 19^\circ\cdot2)$.

The curves respectively computed by these formulæ are shown by the fainter lines in fig. 1, in which the stronger line has been already noticed as derived from the mean of the five years.

Vertical Force.—Table II. exhibits in columns 2 to 6 the mean horary variation of the Vertical Force at the different lunar hours in each of the five years ending June 30; and in column 7, the mean variation in the five years.

TABLE II.

Lunar hours.	In the year ending June 30. 1844.	1845.	1846.	1847.	1848.	Mean of the five years.	Lunar hours.
1.	2.	3.	4.	5.	6.	7.	8.
	sc. div.	sc. div.	sc. div.	sc. div.	sc. div.	sc. div.	
0	+0·08	+0·02	−0·02	−0·03	+0·03	+0·02	0
1	+0·09	−0·03	+0·06	0·00	+0·02	+0·03	1
2	+0·02	−0·07	−0·16	−0·09	−0·18	−0·10	2
3	−0·11	0·00	−0·07	+0·18	−0·06	+0·01	3
4	−0·09	−0·01	−0·01	+0·13	+0·08	+0·02	4
5	+0·01	−0·02	−0·09	+0·07	+0·02	0·00	5
6	−0·12	−0·12	+0·09	+0·01	−0·05	−0·04	6
7	−0·06	−0·24	+0·08	−0·18	−0·02	−0·08	7
8	+0·03	−0·22	+0·03	−0·15	−0·17	−0·10	8
9	−0·04	−0·10	−0·10	+0·02	−0·07	−0·06	9
10	−0·13	−0·10	−0·10	+0·08	−0·11	−0·07	10
11	−0·14	+0·03	−0·03	+0·02	−0·14	−0·05	11
12	−0·08	+0·12	−0·10	+0·01	−0·20	−0·05	12
13	+0·14	+0·03	+0·08	+0·10	−0·16	+0·04	13
14	+0·03	+0·10	+0·04	+0·03	−0·09	+0·02	14
15	+0·08	+0·15	+0·06	+0·02	+0·04	+0·07	15
16	+0·11	+0·18	+0·08	+0·12	+0·06	+0·11	16
17	+0·17	+0·09	+0·04	+0·04	+0·12	+0·09	17
18	+0·19	+0·09	+0·18	−0·03	+0·02	+0·09	18
19	+0·11	+0·05	0·00	0·00	+0·12	+0·06	19
20	+0·14	+0·01	+0·01	−0·14	+0·08	+0·02	20
21	+0·11	−0·07	−0·17	−0·13	+0·05	−0·04	21
22	+0·08	+0·04	−0·21	−0·07	+0·07	−0·02	22
23	+0·11	+0·04	−0·18	−0·13	−0·03	−0·06	23

The curves obtained from the values comprised in this Table are represented to the eye in fig. 2, the fainter lines corresponding to the variations in the separate periods 1844 to 1846, and 1846 to 1848, and the stronger line to the mean of the whole period of five years. The formulæ by which these curves have been computed are as follows:—

1844 to 1846, $\Delta_x = + \cdot 006 - \cdot 092 \sin (a + 0^\circ \cdot 5) + \cdot 036 \sin (2a + 345^\circ \cdot 1)$,

1846 to 1848, $\Delta_x = - \cdot 014 - \cdot 028 \sin (a + 354^\circ \cdot 5) + \cdot 058 \sin (2a + 316 \cdot 75)$,

1844 to 1848, $\Delta_x = - \cdot 005 - \cdot 058 \sin (a + 2^\circ) + \cdot 048 \sin (2a + 330^\circ)$.

The coefficients are decimals of a scale-division, and a is reckoned in hours (multiplied by 15°) from the epoch of the moon's superior culmination.

Inclination and Total Force.—The lunar-diurnal variations of the Inclination and of the Total force are derived from those of the horizontal and vertical components of the force by the formulæ

$$\Delta\theta = \sin\theta \cos\theta \left(\frac{\Delta Y}{Y} - \frac{\Delta X}{X}\right);$$

$$\frac{\Delta\varphi}{\varphi} = \cos^2\theta \frac{\Delta X}{X} + \sin^2\theta \frac{\Delta Y}{Y}$$

θ being the Inclination, φ the Total force, X its horizontal and Y its vertical components. The variation of the Inclination is expressed in seconds of arc, the + sign implying an increase of the dip of the north end of the magnet. The variation of the Total force is expressed in parts of the total force at Toronto, of which force the approximate absolute value is 13·9 in British units.

Table III. presents the variations of the Inclination and of the Total Force produced by the moon at the different hours of the lunar day, derived from the observed variations of the horizontal and vertical components of the force by the formulæ above stated.

TABLE III.

Lunar hours.	Lunar-diurnal variation of the Inclination.	Lunar-diurnal variation of the Total Force.	Lunar hours.	Lunar-diurnal variation of the Inclination.	Lunar-diurnal variation of the Total Force.	Lunar hours.	Lunar-diurnal variation of the Inclination.	Lunar-diurnal variation of the Total Force.
	″	Parts of the Force.		″	Parts of the Force.		″	Parts of the Force.
0	−0·56	−·0000013	8	+0·05	−·0000051	16	−0·22	+·0000063
1	−1·44	+·0000004	9	+0·17	−·0000058	17	+0·57	+·0000053
2	−2·07	+·0000019	10	0·00	−·0000050	18	+1·31	+·0000033
3	−2·26	+·0000026	11	−0·32	−·0000034	19	+1·92	+·0000006
4	−2·05	+·0000021	12	−0·75	−·0000010	20	+2·14	−·0000020
5	−1·61	+·0000008	13	−1·07	+·0000021	21	+1·93	−·0000031
6	−0·97	−·0000012	14	−1·08	+·0000044	22	+1·33	−·0000035
7	−0·33	−·0000034	15	−0·76	+·0000060	23	+0·42	−·0000026

The values contained in this Table are represented to the eye by the stronger lines in figs. 3 and 4: the two fainter lines in each of these figures exhibiting the values

derived from the two half-periods, viz. 1844 to 1846, and 1846 to 1848; the year ending June 30, 1846 having single weight, and the years ending June 30, 1844 and 1845 in the one case, and the years ending June 30, 1847 and 1848 in the other case, having double weight in the respective combinations.

Declination.—To complete the view of the Moon's diurnal influence on the magnetic elements at Toronto, a recalculation has been made of the lunar-diurnal variation of the declination, using the more perfect normals derived by the exclusion of all disturbances equaling or exceeding five minutes of arc*. Table IV. contains the horary variation of the declination at the different hours of the lunar day in each of the six years, from July 1, 1842 to June 30, 1848, and in the eighth column the mean variation in the six years.

TABLE IV.

One scale-division=0'·721.

Lunar hours.	In the year ending June 30. 1843.	1844.	1845.	1846.	1847.	1848.	Mean of the six years.	Lunar hours.
1.	2. sc. div.	3. sc. div.	4. sc. div.	5. sc. div.	6. sc. div.	7. sc. div.	8. sc. div.	9.
0	−0·20	−0·42	−0·45	−0·40	−0·46	−0·37	−0·38	0
1	−0·11	−0·31	−0·18	−0·37	−0·64	−0·29	−0·32	1
2	−0·08	−0·28	+0·04	−0·28	−0·39	−0·31	−0·22	2
3	−0·09	−0·08	−0·07	0·00	−0·36	−0·13	−0·12	3
4	+0·26	+0·09	+0·31	+0·08	+0·13	+0·28	+0·19	4
5	+0·39	+0·09	+0·42	+0·52	+0·20	+0·45	+0·35	5
6	+0·66	+0·40	+0·23	+0·77	+0·15	+0·48	+0·45	6
7	+0·51	+0·29	+0·47	+0·56	+0·31	+0·29	+0·40	7
8	+0·17	+0·26	+0·08	+0·50	+0·09	+0·10	+0·20	8
9	−0·14	+0·21	−0·31	+0·31	−0·23	+0·35	+0·03	9
10	−0·36	−0·24	−0·57	−0·22	−0·40	+0·04	−0·29	10
11	−0·51	−0·33	−0·66	−0·54	−0·24	−0·49	−0·46	11
12	−0·59	−0·48	−0·51	−0·51	−0·52	−0·22	−0·47	12
13	−0·37	−0·27	−0·45	−0·34	−0·29	−0·44	−0·36	13
14	−0·17	−0·31	−0·24	−0·32	+0·07	−0·15	−0·19	14
15	+0·07	−0·14	+0·04	−0·12	+0·52	−0·10	+0·04	15
16	+0·12	+0·22	+0·31	+0·14	+0·43	+0·20	+0·24	16
17	+0·37	+0·45	+0·51	+0·16	+0·55	+0·49	+0·42	17
18	+0·43	+0·74	+0·65	+0·39	+0·47	+0·58	+0·54	18
19	+0·26	+0·25	+0·52	+0·21	+0·55	+0·39	+0·36	19
20	+0·29	+0·15	+0·35	+0·20	+0·22	+0·05	+0·21	20
21	+0·08	+0·15	+0·10	−0·15	−0·21	−0·30	−0·06	21
22	−0·26	−0·05	−0·04	−0·23	−0·42	−0·42	−0·24	22
23	−0·29	−0·37	−0·33	−0·25	−0·31	−0·34	−0·32	23

If we represent the mean of the six years (column 8) by the usual formula of sines and cosines, we have the coefficients of the several terms of the complete formula as follows; the coefficients are expressed in seconds of arc, and a is counted in hours (multiplied by 15°) from the time of the Moon's upper culmination:—

* Philosophical Transactions, 1856, Art. XIX.

Table V.

Arguments ...	Constant	cos a	sin a	cos $2a$	sin $2a$	cos $3a$	sin $3a$	cos $4a$	sin $4a$	cos $5a$	sin $5a$	cos $6a$	sin $6a$
Coefficients	$A_0=$	$A_1=$	$B_1=$	$A_2=$	$B_2=$	$A_3=$	$B_3=$	$A_4=$	$B_4=$	$A_5=$	$B_5=$	$A_6=$	$B_6=$
	0″·0	+0″·20	−1″·03	−19″·18	+0″·45	+1″·84	−0″·32	+1″·21	+0″·59	+0″·61	−0″·48	−0″·22	+0″·57

Arguments ...	cos $7a$	sin $7a$	cos $8a$	sin $8a$	cos $9a$	sin $9a$	cos $10a$	sin $10a$	cos $11a$	sin $11a$	cos $12a$
Coefficients	$A_7=$	$B_7=$	$A_8=$	$B_8=$	$A_9=$	$B_9=$	$A_{10}=$	$B_{10}=$	$A_{11}=$	$B_{11}=$	$A_{12}=$
	−0″·56	+0″·08	+0″·16	−0″·03	−0″·91	−0″·39	−0″·50	−0″·42	+0″·49	+0″·28	−0″·29

The coefficient of principal magnitude is $A_2=-19''\cdot 18$, whose argument is $\cos a$. The same coefficient, calculated for the different years, is as follows:—

Year ending June 30, 1843	−19″·35
Year ending June 30, 1844	−17·61
Year ending June 30, 1845	−21·05
Year ending June 30, 1846	−20·22
Year ending June 30, 1847	−19·04
Year ending June 30, 1848	−18·53
And from the mean of the six years . . .	−19·18

Whence we obtain the probable error of $-19''\cdot 18$, being the value of A_2 derived from the mean of the six years$=\pm 0''\cdot 34$.

With the two first terms of this formula, viz.

$$\Delta_x=0''\cdot 0+0''\cdot 20\cos a-1''\cdot 03\sin a-19''\cdot 18\cos 2a+0''\cdot 45\sin 2a,$$

or its more convenient equivalent,

$$\Delta_x=0''\cdot 0-1''\cdot 05\sin(a+348^\circ\ 52')+19''\cdot 186\sin(2a+271^\circ\ 21'),$$

we obtain the deflections of the north end of the magnet at the several lunar hours as follows:—

Table VI.

Lunar hours.	Deflections.	Lunar hours.	Deflections.	Lunar hours.	Deflections.	Lunar hours.	Deflections.
22	9″·29 to the East	4	9″·19 to the West	10	10″·67 to the East	16	10″·77 to the West
23	15·92	5	15·89	11	17·30	17	17·78
0	18·95	6	18·14	12	19·38	18	20·21
1	16·46	7	15·34	13	16·31	19	17·43
2	9·54	8	8·20	14	8·86	20	10·19
3	0·14	9	0·42 to the East	15	1·04 to the West	21	0·42

Comparing these values with the actual deflections, we find the probable error at each observation hour $\pm 1''\cdot 37$.

In fig. 5 the darker line represents the deflections in Table VI., constituting the lunar-diurnal variation derived from the mean of the six years; and, for the purpose of showing the accordance of the results when the whole period of six years is divided into three portions, each consisting of two years, viz. July 1842 to June 1844, July 1844 to June 1846, and July 1846 to June 1848, the curves for each of those periods have been computed by the subjoined formulæ, obtained from the values in Table IV., and are represented by the fainter lines in fig. 5. The formulæ are,—

$$1842 \text{ to } 1844\ \Delta_x = +0''\cdot41 - 2''\cdot09 \sin(a+291^\circ) - 18''\cdot1 \sin(2a+87^\circ\cdot7),$$

$$1844 \text{ to } 1846\ \Delta_x = +0''\cdot30 + 3''\cdot04 \sin(a+78^\circ\cdot7) + 20''\cdot6 \sin(2a+270^\circ\cdot1),$$

$$1846 \text{ to } 1848\ \Delta_x = -0''\cdot58 - 5''\cdot23 \sin(a+53^\circ\cdot5) + 18''\cdot9 \sin(2a+276^\circ\cdot1).$$

The number of hourly observations employed in the investigation of which this paper contains the results, is, of the Declination 40543; of the Horizontal Force 34303; and of the Vertical Force 31773; making in the whole 106,619 observations.

General Conclusions.—The three magnetic elements concur in showing that the moon exercises a sensible magnetic influence at the surface of the earth, producing in every lunar day a Variation in each of the three elements, which is distinctly appreciable by the instruments employed in the Observatories established to carry out the system of observations recommended by the Royal Society, when due care has been taken in conducting the observations, and suitable methods are adopted for elaborating the results.

The Variation in each of the three elements constitutes a double progression in each lunar day: the declination has two easterly and two westerly maxima in the interval between two successive passages of the moon over the astronomical meridian; and the inclination and the total force have each two maxima and two minima due to the moon's action in the same interval; the variation passing in every case four times through zero in the lunar day. The easterly maxima of the horizontal deflection of the north end of the magnet synchronise with the moon's superior and inferior passages of the meridian; the westerly maxima with the lunar hours of 6 and 18. The maxima of the increased magnetic force due to the moon's action occur about the lunar hours of 3 and 16, and the minima about the hours of 9 and 20. The maxima of the inclination, *i. e.* of the dip of the north end of the magnet, occur about the lunar hours of 3 and 14, and the minima about 9 and 20. The extent of the Variation in the lunar day, or the range between the extremes that are widest apart, is in the Declination 38″·33, in the Inclination 4″·4, and in the total Force ·000012 parts of the whole terrestrial magnetic force at Toronto. These are the values derived from the whole period of observation, *i. e.* from six years of the Declination and from five years of the Inclination and total Force. When the whole period is subdivided into two half-periods, the hours of maxima and minima and the extent of the range accord with the results of the whole period in each of the three elements, with slight

and wholly insignificant exceptions. The reality of the variations is thus attested no less by the accordance of the results when the whole period during which the phenomena were observed is subdivided into separate and independent portions, than by the systematic character which the Variation is seen to possess when the strictly independent results at the several lunar hours are brought together and exhibited continuously.

As it happens that in the declination the variation resulting from the moon's action is greater, relatively to the instrumental means for measuring it, than either in the inclination or in the total force, it is reasonable to conclude that we have a better opportunity of judging of the particular nature and character of the noon's magnetic influence, by studying the effects produced on the declination than those produced on either of the other elements.

Referring to the Table (Table IV. p. 503) which exhibits the coefficients of all the twelve terms in the formula of sines and cosines by which the results of observation are strictly represented, we perceive that the coefficient of the cosine of twice the hour-angle is not only the one of greatest account, but is in fact the only one which we can with confidence regard as possessed of a substantial value. All the other coefficients are, without exception, not only extremely small in comparison with the one above noticed, but are so small, that they may well be supposed to represent such small deviations from a natural law as may well be ascribed to errors which cannot wholly be extinguished in averages derived from not more than six years of observation. On the other hand, the coefficient in the second term has a value far beyond any explanation resting on the supposition of errors of observation. The probable error at any single hour is $1''\cdot37$, whilst the range of the variation is not less than $38''$. Whilst, therefore, the general result of this investigation is to establish conclusively the existence of a lunar-magnetic influence sensible at the surface of the earth, the lunar-diurnal variation which is thus manifested appears to be consistent with the hypothesis that the moon's magnetism may be, in great part at least if not wholly, derived by induction from the magnetism of the earth.

It is further observable, that in the lunar-diurnal variation there is no appearance of the *decennial* period which constitutes so marked a feature in the solar-diurnal variations.

XXIII. *Account of the Construction of a Standard Barometer, and Description of the Apparatus and Processes employed in the Verification of Barometers at the Kew Observatory. By* John Welsh. *Communicated by* J. P. Gassiot, *Esq., F.R.S., Chairman of the Kew Observatory Committee of the British Association.*

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I. *Standard Barometer.*

In the course of the years 1853–54 several attempts were made, under the superintendence of the Kew Committee, to prepare, by the usual method of boiling, a barometer tube of large dimensions. Mr. Negretti, to whom was entrusted the preparation of the tube, succeeded repeatedly in boiling, apparently satisfactorily, tubes of fully 1 inch internal diameter. Many of these however broke spontaneously before they could be mounted, some of them within a few hours and others after an interval of several days. Two or three tubes were ultimately erected, but their condition was not satisfactory. The adhesion of the mercury to the glass was so great, that in a falling barometer the convexity of the top of the column was destroyed, and the surface of the mercury assumed even a concave form. After a few days, rings of dirt or other impurity were formed on the glass near the top of the column, which soon increased to such a degree as entirely to interfere with observation. The mercury employed in filling the tubes had been previously treated for some weeks with dilute nitric acid, and afterwards kept in bottles under strong sulphuric acid, being well washed with water and dried by repeated filtering before use. Dr. W. A. Miller examined specimens of the mercury and could detect no impurity in it.

Suspecting that some injurious effect might have been produced upon the mercury or upon the glass by the great heat to which the tube was necessarily exposed in boiling so large a mass of mercury, it occurred to me that the difficulty might be removed by another method of filling the tube, which I shall now describe.

The tube was in the first place prepared as follows. To its upper end was attached a capillary tube, ADEF (see Plate XX. fig. 1), bent as in the figure, having its bore much contracted at the apex D, with a small bulb blown at E, being finally drawn out to a fine point at F and there sealed. To the lower end of the large tube was attached 10 inches of a smaller tube, BCG, having a bore of 0·3 inch, and to that again was added about 6 inches of capillary tube, GH. A bulb of three-fourths of an inch was blown at G, and the smaller tube finally bent into a syphon form at B. The end H of the capillary tube was now connected with a good air-pump, and the air very slowly extracted, at the same time that the whole tube was strongly heated by passing

a large spirit flame along it. When the air had been as well as possible extracted, and whilst the pump was still in action, and the heat still applied, the capillary tube, GH, was sealed at I by a blowpipe flame. When the tube had cooled, it was placed at a small inclination with the end F in a vessel containing mercury which had been previously boiled. The point F was broken off under the mercury, which then rose in the tube by atmospheric pressure. The mercury continued to rise until the bulb at G was more than half-filled, the remaining space being occupied by the air which the pump had failed to extract. It was estimated from the amount of space thus left unoccupied by the mercury, that the pressure of the residual air in the tube, when cold, must have been less than 0·05 inch. The basin of mercury was then withdrawn from beneath the tube, leaving the point F exposed, the small tube remaining quite filled with mercury. The blowpipe was then applied at F and the opening sealed. When the glass at F had cooled the tube was placed erect, the mercury separating at the contracted part D, leaving the tube from D to F filled, or very nearly so, and from D to A empty. The tube was now sealed at K, the portion KDEF being removed. Finally, the lower end of the tube was broken under mercury at C, leaving about an inch of the syphon.

The earlier tubes filled by this process were not satisfactory; there being, as in those previously prepared by boiling, a considerable adhesion of the mercury to the glass, with the formation, after a few days, of rings of dirt: so similar indeed was the appearance of these tubes to that of the boiled tubes, that I was led to believe that the evil in both cases was due to the same cause. Being satisfied that there was no impurity in the mercury, which, besides having been cleaned with nitric acid, had before these last experiments been redistilled; and suspecting that the evil might have been owing to imperfect cleaning of the tubes, which had only been *wiped* out by the glassblower in the usual way; I had fresh tubes made under my own inspection and sealed at the glass-works immediately after being drawn. Greater care was also taken by the glassblower to prevent the entrance of moisture during the subsequent operations with the blowpipe. These tubes, however, still showed the same imperfection, though in a less degree. About this time I had the advantage of consulting Mr. John Adie of Edinburgh, who informed me that he had also experienced the same inconvenience, and that he had removed it by thoroughly cleaning the tubes by sponging with whiting and spirits of wine. Following his directions, I had the satisfaction of finding the tubes when filled almost wholly free from the imperfections mentioned. A tube of 1·1 inch internal diameter, prepared in July 1855 by the process above described, is at this time in as good condition as when first erected. The top of the column presents a good convexity in all states of the barometer, with only a very slight trace of dirt. No appearance of air-specks can be detected, except a few very minute ones near the lower end of the tube, which have existed since the commencement, and were produced by the temporary entanglement of a small air-bubble at the shoulder L of the tube in the operation of filling. These specks have

not increased in number, nor shown any tendency to rise. A portion of the syphon being retained at the lower end of the tube, it is highly improbable that any air can now enter, the mouth of the syphon being cut off from communication with the external air by the mercury in the cistern. The tube extends to about 9 inches above the mean height of the mercury.

The tube is supported over a glass cistern in a strong brass frame, secured by brackets to the wall of the old mural quadrant of the Observatory, the height of the mercury being measured by a Cathetometer fixed to the same wall, at a distance of 5 feet. A conical point, at the lower end of a short rod of steel, is adjusted by a screw to the surface of the mercury in the cistern (see Plate XX. fig. 2). At the upper end of the steel rod, and above the level of the glass cistern, is a fine mark, whose distance from the conical point has been found, by comparison with the Kew Standard Scale, to be 3·515 inches. When an observation is made, the lower point is adjusted to exact contact with the mercury in the cistern; the telescope of the Cathetometer is then levelled, and its horizontal wire made to bisect the mark on the upper end of the steel rod, the scale reading of the Cathetometer being noted. The telescope is then raised, again levelled, and the wire made a tangent to the surface of the mercury in the tube, the Cathetometer scale reading being again observed: the difference between the two readings of the Cathetometer scale, added to the length of the steel rod, is the height of the column of mercury. Besides the rod terminating in the conical point, a second adjusting rod is provided, whose lower extremity is a straight edge: no difference could be detected between the results from the two methods of adjustment. In order to avoid the inconvenience of light being reflected into the telescope from the surface of the mercury in the tube, a moveable screen is provided, the upper part of which is black and the lower part oiled paper, which is so adjusted as to shut off all light which comes from a higher level than the top of the mercurial column: the surface of the mercury thus presents in the telescope a well-defined dark outline. A window behind the barometer gives a good illumination to the paper screen, a lamp being required at night. A thermometer whose bulb is within the mercury of the cistern gives its temperature, and, the scale of the Cathetometer being of brass, the usual tables can be employed for the temperature correction; the difference in the expansion of steel and brass being insignificant for the length of the short adjusting rod. The variations of the temperature of the room are not rapid, so that no sensible error arises from assuming the temperature of the Cathetometer to be the same as that of the mercury. The cistern of the Standard Barometer is 33·9 feet above the mean level of the sea, being 9·1 feet above the Ordnance bench-mark on the north-east corner of the Observatory, whose elevation is stated by Lieut.-Colonel James to be 24·83 feet.

Observations of this barometer being too troublesome when an extensive series is required, a Standard, by Newman, No. 34, having a tube of 0·55 inch, which has been repeatedly compared with the great Kew Standard, is employed for ordinary use;

its index-correction (which, inclusive of capillary action, $=+0{\cdot}003$ inch) being first applied to the observed readings.

Comparisons, by means of two portable barometers by ADIE, London, were made during last summer between the Kew Standard and that of the Observatory of Paris. The result of these comparisons was, that the Kew Standard reads higher than the Paris Standard by 0·001 inch, no correction being applied to either instrument on account of capillary action.

II. *Verification of Barometers.*

In the best barometers of the present day, a provision is made for adjusting the surface of the mercury in the cistern to the zero of the scale at each observation. Supposing the tube to be in good order, which is easily ascertained by mere inspection, the only source of error in such instruments is to be looked for in the scale. The graduation of the scales of all carefully made barometers is performed by means of a dividing engine, and is not likely to be inaccurate to any sensible extent within the ordinary range of the mercury. If, however, the barometer is intended to be used at considerable elevations, or if it should otherwise be considered desirable to examine the graduation, the error of the divisions can be readily obtained by measurement with the Cathetometer. It frequently happens, however, that the point to which the level of the mercury is adjusted is not the true zero of the scale. The error arising from this source is of course constant for all heights of the barometer. As the capillary action of the tube is also supposed to be constant for the same barometer, and as it is seldom possible to determine its true amount, it is better to consider it in connexion with the zero-error. This is the more advisable, since a reference of the zero-point in a completed barometer to any point of the scale is rendered difficult and uncertain by the circumstance, that it can only be viewed through the glass of the cistern, which, from its irregularity, may considerably affect its apparent position. It is therefore the practice to suspend the barometer to be examined beside the Standard; to make a sufficient number of simultaneous observations of the two instruments; and to adopt the mean difference of their indications as a single constant correction for the combined effects of zero-error and capillary action.

In many portable barometers, and in nearly all marine barometers, there is no means of adjusting the mercury in the cistern to a constant level: it becomes therefore necessary to determine the correction for "Capacity," or the variation in the zero-point corresponding to different heights of the column of mercury. The amount of this correction may be determined during the construction of the instrument; or, by reducing in the required proportion the lengths of the divisions, it may be allowed for in graduating the scale; as has been done in the marine barometers made under the supervision of the Kew Committee by Mr. P. ADIE of London. In order to test the accuracy of this correction, it is necessary to compare the barometer at two con-

siderably different pressures with a standard instrument, that is, with one in which the mercury is adjustable at each observation to a constant zero-point. This is done by placing the barometer and a standard within a receiver provided with the means of altering at pleasure the pressure of the enclosed air.

The Receiver is of cast iron, its horizontal section being rectangular (Plate XX. figs. 3 & 4): it is 39 inches high, 12 inches by $6\frac{1}{2}$ at its lower end, tapering to 10 inches by $4\frac{1}{2}$ at the upper end; there being room for three marine barometers besides the Standard. Windows of strong plate glass, each $11\frac{1}{2}$ inches high and $9\frac{1}{2}$ inches wide, let into both sides of the receiver, admit of the barometers being observed by a Cathetometer. Smaller windows below, each 3 inches square, show the cistern of the Standard Barometer, the mercury in which is adjusted to a constant level by a screw passing through a stuffing-box in the base of the receiver. The barometers to be verified are suspended by a gimbal arrangement from the upper end of the receiver, a massive lid closing the opening at the top by which they are introduced. An opening in the base, furnished with a stopcock, is connected by a flexible tube with a pump which regulates the pressure of the enclosed air. This pump consists of a single barrel and piston: there being openings at both ends of the barrel, the valves are so arranged that, when the flexible tube is attached to the lower opening, air is extracted from the receiver, and when with the upper, air is forced in. The receiver is supported by an iron bracket, securely fixed to the quadrant wall about 10 feet from the great Standard Barometer. The Cathetometer, being between the receiver and the Standard Barometer, can be used at pleasure for either. The adjustable barometer used in the receiver for comparison with the marine barometers, has a tube 0·35 in diameter, there being a contraction in the tube of the same kind and to about the same degree as in the ordinary marine barometers made by Mr. ADIE. This apparatus for the verification of marine barometers has (with the exception of the adjustable barometer, which is by Mr. ADIE) been entirely constructed in the Observatory by Mr. ROBERT BECKLEY the Mechanical Assistant, who has executed the work in a most satisfactory manner, and who has shown much ingenuity in arranging the mechanical details, so as to afford the utmost exactness in observation and convenience in manipulation.

The mode of observation is the following:—Supposing air to have been extracted from the receiver until the barometers stand at about 27 inches, sufficient time having elapsed to allow the mercury to come to a state of rest, and the zero of the Standard having been adjusted, the height of the mercury in each of the barometers is observed by the Cathetometer. Air is then admitted until the mercury stands at about 31 inches, when the same operation is repeated. The length of the graduated scale of the barometer under comparison is then measured by the Cathetometer. If A, a be the Cathetometer readings, at the higher pressure, of the Standard and marine barometers repectively; B, b those for the lower pressure; and if L be the measured length of one inch of the scale of the marine barometer, then the Correction for Capa-

city for one inch $=L-\frac{a-b}{A-B}$. In order to avoid the error which might otherwise arise from the different capillary actions of the Standard tube and that of the marine barometer, it is the practice to make these comparisons only in the forenoon, when the temperature of the room and consequently the pressure of the air within the receiver is slowly increasing.

Besides the determination of the Capacity correction, a series of simultaneous observations are made of the marine barometer and the Standard "NEWMAN 34," for the purpose of obtaining the correction for zero-error. From twenty to thirty comparisons are usually made, care being taken that there shall be as nearly as possible an equal number of observations with the barometer rising and falling; this being necessary in order to eliminate the retardation produced in the movements of the mercury by the contraction of the tube combined with the capillary action. The final corrections at different heights of the mercury are thus deduced, from the data now obtained:—Let H be the height (corrected for zero-error) of NEWMAN 34, h the corresponding height of the marine barometer; T the temperature of NEWMAN 34, and t that of the marine barometer; K being the "Capacity" correction: the correction corresponding to any height h_0 of the marine barometer is

$$H-h+K(h_0-h)+(t-T)\times 0{\cdot}0027.$$

Each barometer, when it leaves the Observatory, is accompanied by a statement of its corrections, of which the following is a specimen:—

"Corrections to the Scale Readings of Marine Barometer, B. T., No. 231, by ADIE, London.

in. At 27·5	in. At 28·0	in. At 28·5	in. At 29·0	in. At 29·5	in. At 30·0	in. At 30·5	in. At 31·0
in. +0·001	in. 0·000	in. −0·001	in. −0·001	in. −0·002	in. −0·003	in. −0·004	in. −0·005

"When the sign of the correction is +, the quantity is to be *added* to the observed reading; and when —, to be *subtracted* from it. The corrections given above include those for Index-error, Capacity, and Capillarity."

III. *Cathetometer.*

The Cathetometer hitherto employed was made by Mr. OERTLING, of London, on the plan of that used in the experiments of M. REGNAULT. It was originally mounted on an independent support, but this was found to be too unsteady for exact observation. It was accordingly removed from its support, and mounted between brackets attached to the quadrant wall. The scale of this instrument has been compared with the Kew Standard Scale, both in the horizontal and vertical positions; in the former, by observation of both scales by fixed micrometer microscopes, and in

the latter position, in measuring by the Cathetometer the divisions of the Standard Scale placed vertically at a distance of five feet. In the horizontal position there appeared to be no appreciable error in the graduation of the Cathetometer, but when vertical its scale was found to be somewhat too long, the measurement of a length of 30 inches requiring a correction of $+0{\cdot}003$ inch. Besides this discrepancy, which is probably due to irregular flexure of the bar and to imperfect fitting of the sliding frame which carries the telescope and level, the manipulation of the instrument is exceedingly inconvenient and troublesome, and requires much care and patience. It is believed, however, that, when the requisite care and time are bestowed, the measurements, after allowing for the correction mentioned, are accurate.

A new Cathetometer is at present being constructed by Mr. Beckley at the Observatory, which promises greater accuracy and convenience. This instrument is very nearly completed, and will be described in a subsequent communication.

XXIV. *On the Presence of Fibrils of Soft Tissue in the Dentinal Tubes.*

By JOHN TOMES, *F.R.S., Surgeon-Dentist to the Middlesex Hospital.*

Received February 21,—Read March 13, 1856.

THE dental tissues, as parts of the human system, have received their full share of attention from anatomists. Papers have from time to time appeared upon this subject, each observer confirming or correcting the views of his predecessor, or adding new facts to those already recorded, until this field of investigation seemed fairly exhausted, at all events of new matter.

Histologists, I think, now agree that dentine is made up of series of tubes, which radiate from one or more cavities, situated within the interior of the tooth. In their way outwards the tubes branch freely, and connect themselves through their branches with each other; thereby establishing a network of communications throughout the whole substance of the dentine. The tubes on the one hand, after running their course, become lost in the anastomosing branches near the outer surface of what has been termed a dentinal system, on the other, terminate by open mouths on the inner surface of the system*, or pulp-cavity. This cavity being occupied by an organ rich in blood-vessels, has led to the opinion generally entertained, that the tubes are canals for the conveyance of nutritive fluid.

M. KÖLLIKER † states, "During life the (dentinal) canals contain a clear fluid, and cannot therefore be readily detected in recent preparations."

In sections which have been dried, the tubes become very distinct, and we may sometimes, on adding a coloured fluid to the preparation when under the microscope, observe the tubes becoming gradually filled.

The foregoing conditions of the dentinal tubes are so easily demonstrated, and appeared to indicate so satisfactorily the offices of these canals, that the subject was regarded as one which had been fully investigated. There are, however, certain physiological conditions observable in teeth, when forming part of the living body, which the recorded knowledge of the histological characters of dentine fails to explain.

If, for instance, a portion of enamel be accidentally broken from the surface of a tooth, so that the dentine becomes exposed, the surface of the latter will be highly sensitive to any variation of temperature from that of the mouth, or to the contact of

* "On the Structure of the Dental Tissue of the Order Rodentia," by JOHN TOMES, Philosophical Transactions, Part 2, 1850.

† Manual of Human Histology, by A. KÖLLIKER. Translated and edited by GEORGE BUSK, F.R.S., and THOMAS HUXLEY, F.R.S. Vol. ii. page 41.

foreign bodies, even pressure from the tongue giving pain. The degree of pain is not, however, increased by increasing the pressure. Then, again, in operating upon the teeth for the removal of carious dentine, it is almost invariably found that the dentine immediately below the enamel is much more sensitive than that situated deeper in the tooth.

If the pulp of a tooth be destroyed, either by an instrument or by an escharotic, the sensitiveness of the whole of the dentine is immediately lost, no pain being afterwards experienced when it is cut either near the enamel or the pulp-cavity. The teeth of young subjects are much more sensitive than those of older people, and this is more especially the case when they are attacked by caries.

The dentine of teeth which are rapidly decaying is much more sensitive than that of teeth in which the destruction progresses more slowly. The former condition is indicated by the light colour of the decomposing part, together with the extent of tissue involved; the latter by the deep brown colour, and the comparative hardness of the affected dentine. In certain cases of caries, the softened tissue appears to be extremely sensitive, so that the patient can scarcely bear its removal; but when the instrument reaches the comparatively healthy dentine, the pain, although present, is much less severe.

In any case, however, the dentine loses its power of feeling pain if the pulp be destroyed; but if, after the destruction, the pulp-cavity be perfectly filled with gold, the tooth, in cases suitable for such an operation, may retain its colour and usefulness for a considerable period. The dentine will not, however, recover its sensitiveness.

These several conditions indicate sufficiently clearly that the sensitiveness of the dentine is dependent upon its connexion with the pulp of the tooth, and that it has no inherent sensibility in its own hard tissue; although the tissue may remain for a considerable period without any manifest change, if the root of the tooth be healthy, and the dentine be protected from the influence of the fluids of the mouth.

After a portion of dentine has been for some time exposed, or if the exposure be brought about gradually by the slow wearing away of the enamel, that acute sensitiveness which has been described is not then found to exist. In parts which have been subject to the foregoing conditions, it will on examination be found that the dentinal tubes, the peripheral extremities of which have been exposed, are more or less obliterated in some part of their course between the surface and the pulp-cavity.

On reviewing the various circumstances under which dentine evinces sensibility, and those under which that sensibility is lost, it is difficult to avoid the conclusion, that the dentinal tubes are in some way the medium through which sensation is distributed through the substance of the tissue. But if the sole office of the tubes be the conveyance of nutrient fluid derived from the pulp, the difficulty of accounting for the sensitiveness of the dentine remains, inasmuch as we have no instance of sensation being manifested in a fluid. We might seem to get out of the difficulty by assuming that the dentinal tubes are constantly filled by fluid, and that pressure made

upon the fluid at the exposed ends of the tubes is felt by the pulp at their inner extremities. This assumption does not, however, account for all the circumstances of the case, failing altogether to explain the greater sensibility of the dentine at one part of the tooth than at another.

The want of accordance between the views usually entertained upon the structure of dentine and the physiological conditions manifested by that tissue when in connexion with the body, has wholly arisen from assuming that the dentinal tubes are solely for the conveyance of fluid, and that they are otherwise empty. With the hope of gaining some further knowledge upon this point, I commenced a series of observations, the results of which it is the main purpose of this paper to communicate. When these investigations were commenced, I had but little expectation of finding that one of the most important parts in dental structure had been overlooked, namely, that each dentinal tube is permanently tenanted by a soft fibril, which, after passing from the pulp into the tubes, follows their ramifications.

With proper care in manipulating, nothing is more easy than to demonstrate the existence of the dentinal fibrils, in any tooth which has been recently extracted. If a thin section be made in the plane of the direction of the tubes, and then placed in dilute hydrochloric acid until the whole or a greater part of the lime is removed, and the section be afterwards torn in a direction transverse to that of the tubes, many of the fibrils will be seen projecting from the torn edges (Plate XXI.* fig. 1). It is desirable, in repeating the experiment, to place the decalcified section upon a slide before tearing, as in moving it from the surface upon which it has been torn, some of the longer fibrils may be folded back upon the body of the specimen and thus become obscured. Where the separation between the torn surfaces has been but slight, we may often see a fibril, unbroken, stretching across from the separated orifices of the tube to which it belongs.

It is not necessary, however, to decalcify dentine in order to show the fibrils. If a similar section to that already described be divided with the edge of a knife, many of these delicate organs will be seen, but they are usually broken off much shorter, many of them scarcely projecting beyond the orifices of the tubes. Again, if a minute portion of dentine be cut with a sharp knife from the surface produced by fracturing a perfectly fresh tooth, the same appearances will be seen, but not with the same certainty and distinctness as in the previous examples.

In order to demonstrate the connexion of the fibrils with the pulp, fine sections should be made with a sharp knife from the edge of the pulp-cavity. In this manner I obtained the specimen from which Mr. De Morgan has been kind enough to draw the accompanying illustration, showing the fibrils stretching from the pulp to the displaced dentine, and some of them passing out on the other side of the fragment (fig. 2). That the fibrils proceed from the pulp may be seen by carefully fracturing a fresh tooth with as little displacement of the fractured parts as possible; and then, by slowly removing the pulp from its place in the tooth, we shall be enabled to examine

the fibrils which have been drawn out from the tubes. By this procedure some of the fibrils will be withdrawn from their normal position in the dentine in the greater part of their length, a few of them retaining short lengths of their branches, but sufficient to show that they have come from the branches of the dentinal tubes.

If a carious tooth be selected in which the diseased part is of a deep brown colour and of tolerably firm consistence (conditions indicating that the disease has been slow in progress), it will be seen, on making a transverse section of the tubes in the affected part, that the fibrils have been consolidated (fig. 3) and their outline lost, the circumference of the tube alone being distinguishable. Indeed the tubes, when in this state and seen in this view, have the appearance of solid rods. But if the section be made in a plane with the tubes, we shall be enabled to trace the calcified fibrils. They appear to have a greater power of resisting decomposition than the surrounding dentine, and hence preserve their rigidity. Some will project from the edge of the specimen, while others may be seen broken within the tubes, and more or less displaced. Were they made of glass the fracture could not be more abrupt and defined (fig. 4), or their outline more distinct. I have on a previous occasion described a zone of consolidation limiting caries*, but I was at that time ignorant of the existence of the tube-fibrils, otherwise I should have more fully understood its import.

Professor Kölliker, in his account of the development of dentine, describes and figures processes extending from the peripheral cells of the dentinal pulp in developing teeth†, but he does not recognize the tube-fibril; indeed he, as before cited, describes the tubes as filled with fluid. M. Lent, in a paper published last year, gives a similar description to that of M. Kölliker, but says that the cell-fibres are best seen in teeth which are but little advanced in development‡. Mr. Huxley states that in a solitary instance he observed a fibre pass a short distance into the dentine§.

Both M. Kölliker and M. Lent regard the process which they observed extending from the peripheral cells of the pulp in forming teeth, as organisms for the development of the dentinal tubes. The latter author, near the conclusion of his article on the development of dentine, states, *the processes of the cells are the dentinal tubes.* He observes further on, that the fact first observed by Muller and then by Kölliker, that the dentinal tubules possess separate walls, which can readily be isolated, is explained by the history of the development; the wall of the dentinal canal is identical with the cellular membrane of the ivory cell.

I do not propose entering upon the subject of dentinal development in the present communication, but shall confine myself to showing that the dentinal tubes are in the

* Lecture on Dental Physiology and Surgery, by John Tomes. Published by Parker, West Strand.

† *Loc. cit.*

‡ Zeitschrift für wissenschaftliche Zoologie, herausgegeben von C. T. Siebold und A. Kölliker, Sechster Band, 1855, p. 121.

§ On the Development of the Teeth, and on the Nature and Imports of Nasmyth's "persistent capsule," by Thomas Huxley, F.R.S., Quarterly Journal of Microscopical Science, No. 3, 1853.

normal condition occupied by fibrils of soft tissue. The above extracts from M. LENT's paper have been made in order to show that he has not recognized the existence of permanent tube contents, although he has probably seen the fibrils themselves.

The nature and office of the dentinal fibrils remain for consideration. If a fibril be examined in its natural condition, by the aid of an eighth of an inch object-glass, it will be found to consist of an almost structureless tissue, transparent, and of a comparatively low refractive power. In glycerine the fibrils are scarcely visible. At present it admits of doubt whether they are tubular or solid. In some cases there is an appearance of tubularity; but being cylindrical this may be a mere optical effect. When accidentally stretched between two fragments of dentine the diameter of the fibril becomes much diminished, and when broken across, a minute globule of transparent but dense fluid may sometimes be seen at the broken end, gathered into a more or less spherical form. These appearances may be explained by assuming that the fibril consists of a sheath containing a semifluid matter, similar to the white fibrillæ of nerves; but whether such a conclusion can be justified admits of doubt. The manner in which the dentinal fibrillæ terminate in the pulp I am at present unable to decide. In favourable specimens they may be traced a short distance into the pulp, but whether they are terminated by cells or in any way connect themselves with nerves, I am unable to determine. The dimensions of the fibrils are the same as those of the interior of the dentinal tubes.

The conditions under which sensation is manifested in dentine have been already stated, together with those under which it is lost, and the difficulty of accounting for these phenomena has been pointed out. The recognition of the fibrils of dentine will, however, I think, remove the difficulty, and enable the physiologist to explain why under certain circumstances that tissue is susceptible of pain, while under other conditions the sensitiveness is lost.

That the dentine owes its sensation to the presence of the dentinal fibrils cannot, I think, be readily doubted, seeing that if their connexion with the pulp be cut off by the destruction of the latter, all sensation is at once lost. It is by no means necessary to assume that the dentinal fibrils are actual nerves before allowing them the power of communicating sensation. Many animals are endowed with sensation which yet possess no demonstrable nervous system; and we may find many points in the human body highly sensitive without our being able to demonstrate nerves in such numbers as would account for the pain uniformly experienced from the puncture of a needle, upon the supposition that the needle had in each case wounded a nerve. Additional evidence in favour of the view that the fibrils possess sensation may be obtained by examining their condition in diseased teeth, and the conditions attendant upon the disease. In those cases in which the fibrils are consolidated in the manner already described, there is perfect absence of pain when the part is removed. but so soon as the instrument reaches the healthy dentine, more or less inconvenience is felt. If, on the other hand, there is no consolidation of the fibrils, but the pulp is yet living, the

operation of removing the carious part is productive of pain, even from the commencement; indeed pressure upon the surface of the softened tissue gives rise to discomfort. If in such cases the softened dentine be examined, fibrils may here and there be found but little altered from their natural appearance.

The greater degree of sensitiveness observable in the dentine immediately below the enamel, that is, at the point of ultimate distribution of the dentinal tubes, and consequently of the fibrils, may be fully accounted for on the supposition that the latter are organs of sensation, just as in nerves of sensation the point of greatest sensibility is that of their ultimate distribution.

The recognition of the dentinal fibrils must lead to a modification of the opinions hitherto entertained as regards the office of the tubes, namely, that they are for the circulation of fluids only. The presence of soft tissue would not, however, hinder the slow passage of fluids; and that fluids do pass through or by the side of the fibrils is rendered probable by the fact, that they are capable of undergoing change at the parts furthest removed from the pulp. When the fibrils become calcified near the surface of the dentine, the hardening material must have been derived from the pulp when the consolidation has taken place in the crown of the tooth.

The foregoing observations wil. I think warrant the conclusion, that the dentinal fibrils are subservient to sensation in the dentine, and are the channels by which nutrition is carried to that tissue.

Further evidence may be adduced in favour of the latter opinion. I have already observed that dentine may remain for a time apparently unaltered if the pulp be destroyed and the cavity filled with gold. After a while many teeth so circumstanced become loose, and when removed it is found that a considerable portion of the dentine has been removed by absorption; a state of things in some respects similar to that which accompanies the loss of teeth in old people. Here we find, that, although the pulp may be living, the tubes of the root of the tooth have become consolidated, and the part rendered translucent. Teeth so circumstanced will on examination exhibit loss of dentine. A similar condition may be found in teeth the crowns of which have been lost; the roots are then diminished by absorption. In each of the instances adduced, the teeth may, however, be retained for a lengthened period in the jaw, but such persistence is always accompanied by the deposition of cementum to an unusual extent upon the roots. These phenomena have been brought forward to show, that the presence of the dentinal fibrils in a state of integrity is necessary to the normal condition of the tooth; that if from any cause they are consolidated or destroyed, nature will coat over the root with cementum, and often to an extent amounting to disease, or will set up a process for its removal. The dentine will be diminished by absorption, the root will be thrown up on the surface of the gum, or the socket will disappear, and the tooth by the one or other process, or by a combination of each, will be cast off as an organ no longer fitted for a place in the living body.

EXPLANATION OF THE PLATE.

PLATE XXI.*

Fig. 1. A section from the crown of the tooth of an adult made in a plane with the direction of the dentinal tubes, and afterwards decalcified and then torn in a line transverse to the direction of the tubes: (*a*) the dentine; (*b*) the torn edge with the dentinal fibril (*c*) extending from the tubes.

Fig. 2. A section made with a knife from the edge of the pulp-cavity of an adult tooth, including a portion of the pulp: (*a*) the dentine; (*b*) the pulp with the peripheral cells arranged in lines; (*c*) the dentinal fibrils drawn out of the displaced dentine; (*d*) fibrils which pass through the fragment of dentine, and appear on the surface furthest removed from the pulp.

Fig. 3. A section from dentine softened by caries, showing the consolidated dentinal tubes and fibrils cut transversely.

Fig. 4. A section in a plane with the tubes, from carious dentine, showing consolidation of the fibrils, some of which are seen projecting from the edge of the specimen, while others have been broken within the tubes and are displaced.

ADDENDUM.

Received June 18, 1856.

SINCE the preceding communication has been in the possession of the Royal Society, the head of a marsupial animal which had been preserved in spirit was placed at my disposal, the teeth of which were in a condition favourable for showing the dentinal fibrils, should such be found to exist.

A paper upon the structure of the dental tissues of Marsupiata will be found in the Philosophical Transactions, Part II. 1849, in which the continuation of the dentinal tubes into the enamel is described and figured; together with those minor differences of structure which are peculiar to the several divisions of this order of Mammalia.

After the discovery of the dentinal fibrils, the examination of a favourable specimen of enamel so peculiarly constituted became a matter of considerable interest, in order to ascertain whether the soft tissue which occupies the dentinal tubes is continued into those of the enamel. I am indebted to my friend Professor QUEKETT for the jaws of *Halmaturus* ——, a member of a genus in which the majority of the dentinal tubes situated in the crown of the tooth are continued into the enamel, and pass to within a short distance of the external surface of that tissue. Thin sections were made both of the incisor and molar teeth by the usual process of grinding.

These were treated with dilute hydrochloric acid, and were examined at short intervals after their immersion in the solvent fluid. The acid acted upon the enamel with great rapidity; and in the course of a few minutes the edge corresponding to the outer surface of the tooth disappeared, leaving in its place a series of fine flexible filaments. More prolonged action of the acid led to a further loss from the surface of the enamel, and also to the solution of the part in contact with the dentine. In this case the fibrils were seen proceeding from the extremities of the dentinal tubes across the space which had been occupied by the enamel, from thence they were continued through that portion of the latter structure which yet remained undissolved, and ultimately formed a delicate fringe floating freely in the fluid by which the preparation was surrounded. If a section presenting the above conditions be again placed in acid the whole of the enamel will be dissolved, leaving the dentine in those parts which have been invested with enamel, bordered by a thick fringe of long delicate fibrils, each one being continued from the peripheral extremity of a dentinal tube. In the dentine the fibrils occupying the tubes were as readily detected as in the human tooth, and presented the same general appearances and relations.

The facility with which the fibrils were demonstrated in the enamel of the teeth of the Kangaroo, induced me to select for examination specimens of human teeth in which the dentinal tubes are continued for a short distance into the enamel. Under similar treatment similar results were obtained. Wherever the dentinal tubes could be traced into the enamel, the presence of the contained fibrils could be demonstrated by the aid of hydrochloric acid.

Cavendish Square,
June 17th, 1856.

XXV. *On the Problem of Three Bodies.* By the *Rev.* J. CHALLIS, *M.A.*, *F.R.S.*, *F.R.A.S.*, *Plumian Professor of Astronomy and Experimental Philosophy in the University of Cambridge.*

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THE determination of the motions of three bodies mutually attracting according to the law of gravity being a problem too complicated for exact solution, mathematicians have employed various methods of solving it approximately. It is well known that of these methods the one which appears to be the most obvious and direct, introduces terms which may increase indefinitely with the time, and render the solution inapplicable to any observed case of motion. This difficulty occurs whether the problem be to find the perturbation of the moon's motion by the sun, or the perturbation of the motion of one planet by another, and the necessity of meeting or evading it has very much determined the courses which the solutions of these problems have taken. In the theory of the moon's motion, LAPLACE, PONTÉCOULANT, and others, have appealed to the results of observations of the motions of the moon's perigee and node, to justify the assumption of a form of solution which is not attended with the above-mentioned difficulty. Although this way of proceeding may lead to correct results, there can be no doubt that it is an abandonment of the principle of determining by analysis alone the form of development which is appropriate to the conditions of the problem. Again, in the theory of the motions of the planets, recourse is had on the same account to the method of the variation of parameters, more especially for determining the secular inequalities. Now it will perhaps be admitted that that method, elegant and exact though it be, is yet not indispensable, and that when it succeeds, there must be some direct method which would be equally successful and conduct to the same results. The discovery of such a method I have long considered to be a desideratum in the theory of gravitation, and having after much labour found out one by which the forms of the expressions for the radius-vector, longitude and latitude, and both the secular and the periodic inequalities, are evolved by the analysis alone, and which is applicable as well to the lunar as the planetary motions, I thought it might deserve the attention of the Royal Society. I propose in this communication to enter at length into the details of the method, and then to add a few remarks on its general principle, and to explain why, in common with the method of the variation of parameters, it succeeds in determining the motion of the apses of a disturbed orbit.

1. Let M represent the mass of the principal body, or its attraction at the unit of distance, m that of the body whose motion it is proposed to investigate, and m' that of the disturbing body. The principal body is supposed to be at rest, and the rectangular coordinates and distances of the other two reckoned from its centre as origin, are respectively x, y, z, r and x', y', z', r', at the time t reckoned from a given epoch. Then if μ be put for $M+m$, and R for the expression

$$\frac{m'(xx'+yy'+zz')}{r'^3}-\frac{m'}{((x'-x)^2+(y'-y)^2+(z'-z)^2)^{\frac{1}{2}}},$$

we have for determining the motion of m the known equations,

$$\frac{d^2x}{dt^2}+\frac{\mu x}{r^3}+\frac{dR}{dx}=0. \quad \text{(1.)}$$

$$\frac{d^2y}{dt^2}+\frac{\mu y}{r^3}+\frac{dR}{dy}=0. \quad \text{(2.)}$$

$$\frac{d^2z}{dt^2}+\frac{\mu z}{r^3}+\frac{dR}{dz}=0. \quad \text{(3.)}$$

Analogous equations apply to the motion of m' as disturbed by m. The directions of the axes of coordinates are entirely arbitrary. Conceive, therefore, to be known at a given instant (T_0) the position of the plane passing through m in the direction of its motion at that instant, and through the centre of M, and let this plane be the plane of xy. Conceive also to be known at a given instant (T'_0) the position of the plane passing through m' in the direction of its motion at that instant, and through the centre of M, and let this plane make with the other the angle ω. Also let the intersection of the two planes be the axis of x

2. Now since according to these suppositions, the body m would continue in the plane xy if the disturbing force of m' ceased at the time T_0, it is clear that the coordinate z at any time $T_0+\tau$ is a small quantity of the order of the disturbing force. By multiplying the equations (1.), (2.), (3.) respectively by $2dx$, $2dy$, and $2dz$, adding, and integrating, we obtain

$$\frac{dx^2}{dt^2}+\frac{dy^2}{dt^2}+\frac{dz^2}{dt^2}-\frac{2\mu}{r}+2\int\frac{(dR)}{dt}dt+C=0, \quad \text{(4.)}$$

$\frac{d(R)}{dt}$ being the differential coefficient of R with respect to x, y, and z considered as functions of the time. But from what is said above, $\frac{dz^2}{dt^2}$ and $\frac{dR}{dz}\cdot\frac{dz}{dt}$ are of the order of the square of the disturbing force. Hence as it is proposed to conduct the approximation according to the powers of the disturbing force, these terms in the first and second approximations must be omitted. Also, if θ be the polar coordinate of m reckoned on the primitive plane of its orbit from the axis of x, and $\frac{z^2}{r^2}$ &c. be neglected, $x=r\cos\theta$, $y=r\sin\theta$, r being now regarded as the projection of the distance on the plane of xy. Consequently

$$dx^2+dy^2=dr^2+r^2d\theta^2,$$
$$\frac{(d\mathrm{R})}{dt}=\frac{d\mathrm{R}}{d\theta}\cdot\frac{d\theta}{dt}+\frac{d\mathrm{R}}{dr}\cdot\frac{dr}{dt};$$

and the equation (4.) becomes

$$\frac{dr^2}{dt^2}+r^2\frac{d\theta^2}{dt^2}-\frac{2\mu}{r}+2\int\left(\frac{d\mathrm{R}}{d\theta}\cdot\frac{d\theta}{dt}+\frac{d\mathrm{R}}{dr}\cdot\frac{dr}{dt}\right)dt+\mathrm{C}=0. \quad \ldots\ldots \quad (5.)$$

Again, the equations (1.) and (2.) give

$$x\frac{d^2y}{dt^2}-y\frac{d^2x}{dt^2}+x\frac{d\mathrm{R}}{dy}-y\frac{d\mathrm{R}}{dx}=0.$$

But $$x\frac{d^2y}{dt^2}-y\frac{d^2x}{dt^2}=\frac{d\,.\,r^2\frac{d\theta}{dt}}{dt},\ \text{and}\ x\frac{d\mathrm{R}}{dy}-y\frac{d\mathrm{R}}{dx}=\frac{d\mathrm{R}}{d\theta}.$$

Hence, by integrating, $$r^2\frac{d\theta}{dt}=h-\int\frac{d\mathrm{R}}{d\theta}\,dt, \quad \ldots\ldots\ldots\ldots \quad (6.)$$

h being an arbitrary constant. Consequently, by substituting for $\frac{d\theta^2}{dt^2}$ in (5.) from (6.), and neglecting the square of the disturbing force,

$$\frac{dr^2}{dt^2}+\frac{h^2}{r^2}-\frac{2\mu}{r}+\mathrm{C}=\frac{2h}{r^2}\int\frac{d\mathrm{R}}{d\theta}\,dt-2\int\left(\frac{d\mathrm{R}}{d\theta}\cdot\frac{d\theta}{dt}+\frac{d\mathrm{R}}{dr}\cdot\frac{dr}{dt}\right)dt.$$

But $$\int\frac{d\mathrm{R}}{d\theta}\cdot\frac{d\theta}{dt}\,dt=\frac{d\theta}{dt}\int\frac{d\mathrm{R}}{d\theta}\,dt-\int\frac{d^2\theta}{dt^2}\left(\int\frac{d\mathrm{R}}{d\theta}\,dt\right)dt,\ \text{and}\ \frac{d\theta}{dt}=\frac{h}{r^2}\ \text{nearly}.$$

Hence it will be seen that to the first power of the disturbing force we have

$$\frac{dr^2}{dt^2}+\frac{h^2}{r^2}-\frac{2\mu}{r}+\mathrm{C}=2\int\left\{\frac{d^2\theta}{dt^2}\left(\int\frac{d\mathrm{R}}{d\theta}\,dt\right)-\frac{d\mathrm{R}}{dr}\cdot\frac{dr}{dt}\right\}dt. \quad \ldots\ldots \quad (7.)$$

The equations (6.) and (7.) are suitable for determining the forms of the developments of r and θ in terms of the time.

The function R becomes, by neglecting $m'z$,

$$\frac{m'(xx'+yy')}{r'^3}-\frac{m'}{(r'^2+r^2-2(xx'+yy'))^{\frac{1}{2}}};$$

and reckoning θ' from the axis of x on the plane of the orbit of m' in its position at the time T'_0, we have to the same approximation,

$$x'=r'\cos\theta',\quad y'=r'\sin\theta'\cos\omega,\quad z'=r'\sin\theta'\sin\omega.$$

Hence

$$\mathrm{R}=\frac{m'r}{r'^2}(\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos\omega)-\frac{m'}{(r'^2+r^2-2rr'(\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos\omega))^{\frac{1}{2}}}.$$

If powers of ω above the second be neglected, the following approximate value of R is obtained:

$$\mathrm{R}=\frac{m'r}{r'^2}\cos(\theta-\theta')-\frac{m'}{(r'^2+r^2-2rr'\cos(\theta-\theta'))^{\frac{1}{2}}}$$
$$-2m'\sin\theta\sin\theta'\sin^2\frac{\omega}{2}\left(\frac{r}{r'^2}-\frac{rr'}{(r'^2+r^2-2rr'\cos(\theta-\theta'))^{\frac{3}{2}}}\right).$$

3. The foregoing preliminaries having been gone through, the order in which the approximate integration is to be effected may now be stated. As the approximation is to proceed according to the disturbing force, the equations (6.) and (7.) must first be integrated omitting the terms involving R. We shall thus obtain values of r and θ as functions of t and arbitrary constants, just as in the case of the problem of two bodies, and these constants may be designated by the letters usually employed in that problem. As the exact values of the functions would be unsuitable for carrying on the approximation, they may be expanded in series proceeding according to the powers of the arbitrary constant e to as many terms as we please. In like manner the functions which express the values of r' and θ' in terms of t, may be expanded according to the powers of e'. When these values of r, θ, r' and θ' have been substituted in the right-hand side of the equation (7.), that side becomes a function of t and constants; and supposing the integrations indicated to have been effected, and the result to be Q, we shall have

$$\frac{dr^2}{dt^2}+\frac{h^2}{r^2}-\frac{2\mu}{r}+\mathrm{C}=2\mathrm{Q},$$

Q being a small quantity of the order of the disturbing force.

Hence
$$dt=dr\cdot\left\{-\frac{h^2}{r^2}+\frac{2\mu}{r}-\mathrm{C}+2\mathrm{Q}\right\}^{-\frac{1}{2}}$$
$$=dr\cdot\left(-\frac{h^2}{r^2}+\frac{2\mu}{r}+\mathrm{C}\right)^{-\frac{1}{2}}-\mathrm{Q}dr\cdot\left(-\frac{h^2}{r^2}+\frac{2\mu}{r}-\mathrm{C}\right)^{-\frac{3}{2}},$$

Q^2 &c. being neglected. In the second term we may substitute for r in terms of t from the first approximation, which gives

$$dr\left(-\frac{h^2}{r^2}+\frac{2\mu}{r}-\mathrm{C}\right)^{-\frac{3}{2}}=dt\left(-\frac{h^2}{r^2}+\frac{2\mu}{r}-\mathrm{C}\right)=\frac{dr^2}{dt^2}\cdot dt.$$

Supposing, therefore, that by the first approximation $\frac{dr^2}{dt^2}=f(t)$, we obtain

$$dt\big(1+\mathrm{Q}f(t)\big)=\frac{dr}{\left(-\frac{h^2}{r^2}+\frac{2\mu}{r}-\mathrm{C}\right)^{\frac{1}{2}}}.$$

This equation being integrated, a relation is found between r, t and arbitrary constants, by means of which r is to be expressed in a series proceeding primarily according to the disturbing force, and subordinately according to the quantities e, e' and ω. This value of r is next to be substituted in the equation (6.), which, being put under the form

$$d\theta=\frac{hdt}{r^2}-\left(\int\frac{d\mathrm{R}}{d\theta}\,dt\right)\frac{dt}{r^2},$$

shows that the right-hand side then becomes a function of t and constants, and that by integration θ may be obtained in a series proceeding according to the same law of arrangement as the series for r.

The plane of xy has hitherto been supposed to be coincident with the plane of m's orbit at the time T_0. On this supposition values of r and θ have been obtained, which fully take into account the first power of the disturbing force and the mutual inclination of the orbits. We are now at liberty to suppose the plane of xy to have any other position making a small angle with the planes of the orbits of m and m' in their positions at the epochs T_0 and T'_0, and the equation (3.), viz.

$$\frac{d^2z}{dt^2}+\frac{\mu z}{r^3}+\frac{dR}{dz}=0,$$

may be employed for finding a series for z in terms of t. In the second term of this equation the value of r given by the second approximation is to be substituted, but in the third term it is only required to substitute the values of r, θ, r' and θ' given by the first approximation. Also for z and z' we may substitute in $\frac{dR}{dz}$ the functions of t which express the values of these quantities on the supposition that the motions are undisturbed, and that they are referred to the new plane of xy. Thus $\frac{dR}{dz}$ becomes a function of t and constants, and the above equation takes the form

$$\frac{d^2z}{dt^2}+\mu\varphi(t)z+\psi(t)=0,$$

which admits of being integrated only by successive approximations.

The process by which it has been shown that r, θ and z are approximated to, gives at the same time, *mutatis mutandis*, the values of r', θ' and z' to the first power of the disturbing force of m. By means of these six quantities the approximation may be carried to terms inclusive of the *square* of the disturbing forces.

Having thus exhibited the general scheme of this approximate solution of the Problem of Three Bodies, I proceed to exemplify its practicability.

First Approximation.

4. The first approximation, which omits the terms involving the disturbing force, and is therefore identical in form with the solution of the problem of two bodies, is obtained by integrating the equations

$$dt=dr\left(-\frac{h^2}{r^2}+\frac{2\mu}{r}-C\right)^{-\frac{1}{2}}, \qquad d\theta=\frac{hdt}{r^2}.$$

The first equation gives, by integration,

$$n(t+T)=\cos^{-1}\frac{a-r}{ae}-\frac{1}{a}\sqrt{a^2e^2-(a-r)^2},$$

where for the sake of brevity a is put for $\frac{\mu}{C}$, e^2 for $1-\frac{h^2C}{\mu^2}$, and n for $\frac{C^{\frac{3}{2}}}{\mu}$ or $\frac{\sqrt{\mu}}{a^{\frac{3}{2}}}$. Let ϵ be the constant introduced by the integration of the second equation, and in order to designate the constants in the present problem by the letters usually employed in

the elliptic theory, let $\varepsilon-\varpi$ be put for n'T. Then substituting p for $nt+\varepsilon-\varpi$, we have, as is known, the following expansions of r and θ to the third power of e:

$$\frac{r}{a}=1-e\cos p+\frac{e^2}{2}(1-\cos 2p)+\frac{3e^3}{8}(\cos p-\cos 3p)$$

$$\theta=\varepsilon+nt+2e\sin p+\frac{5e^2}{4}\sin 2p+\frac{e^3}{4}\left(\frac{13}{3}\sin 3p-\sin p\right).$$

These values of r and θ (excluding the terms involving e^3, for the sake of avoiding long calculations) will be employed in proceeding to the second approximation. It is evident that C, h, ϖ and ε must be regarded as the arbitrary constants of the integration however far the approximation be carried, no other arbitrary quantities being introduced by the process. The quantities a and e are given functions of C and h, and at present they have no other signification.

5. Before proceeding further, an inference may be drawn from the equation (7.) which will be useful hereafter. When the foregoing values of r and θ are substituted in the right-hand side of that equation, the constant e will be a multiplier of that side, independently of any limitation of the orbit of m'. Now let, if possible, $e=0$. Then $1=\frac{h^2\mathrm{C}}{\mu^2}$, and the equation (7.) becomes

$$\frac{dr^2}{dt^2}+\frac{1}{\mathrm{C}}\left(\frac{\mu}{r}-\mathrm{C}\right)^2=0.$$

Since the relation $\mu^2=h^2\mathrm{C}$ shows that C must be positive, it follows from the above equation that $\frac{dr}{dt}=0$ and $\frac{\mu}{r}=\mathrm{C}$, or that the orbit of m is a circle whose radius is equal to $\frac{\mu}{\mathrm{C}}$. But the orbit of m cannot be exactly a circle independently of the form and magnitude of the orbit of m', unless the disturbing force be indefinitely small. Consequently the supposition that $e=0$ draws with it the inference that the disturbing force vanishes. At the same time, the supposition that the disturbing force vanishes must leave e an arbitrary quantity, because on this supposition the problem is that of elliptic motion, and e is the eccentricity of the orbit. These conditions may be analytically expressed by such an equation as $e^2=e_0^2+km'$, k being positive and of fixed value, and e_0 being arbitrary*.

Second Approximation.

6. The first step towards expressing the right-hand side of the equation (7.) as a function of t, is to expand the quantity R in a series proceeding according to cosines of multiples of the arc $\theta-\theta'$. Let

$$\mathrm{R}=\mathrm{R}_0+\mathrm{R}_1\cos(\theta-\theta')+\mathrm{R}_2\cos 2(\theta-\theta')+\ \&\mathrm{c}.$$
$$=\mathrm{R}_0+\Sigma\,.\,\mathrm{R}_s\cos s(\theta-\theta'),$$

the values of s being the integers 1, 2, 3, &c. Also let $r=a(1+u)$, $r'=a'(1+u')$,

* See Note (A) at the end of the paper.

$\theta=nt+\varepsilon+v$, $\theta'=n't+\varepsilon'+v'$; and for the sake of brevity put q for $nt+\varepsilon-(n't'+\varepsilon')$.

Then $$u=-e\cos p+\frac{e^2}{2}-\frac{e^2}{2}\cos 2p, \qquad u'=-e'\cos p',$$

$$\theta-\theta'=q+v-v'=q+2e\sin p-2e'\sin p'.$$

It is not necessary for our present purpose to employ expressions containing higher powers of e and e'.

7. It is next required to obtain an expression for $\int\frac{d\text{R}}{d\theta}dt$ in terms of t. Since each of the factors R_0, R_1, R_2, &c. is a function of r, r' and constants, it follows that

$$\frac{d\text{R}}{d\theta}=-\Sigma\cdot\text{R}_s s\sin s(\theta-\theta').$$

Also, if A_s represent the value of R_s when a is substituted for r and a' for r', we have nearly

$$\text{R}_s=\text{A}_s+\frac{d\text{A}_s}{da}au+\frac{d\text{A}_s}{da'}a'u'$$

$$=\text{A}_s-ae\frac{d\text{A}_s}{da}\cos p-a'e'\frac{d\text{A}_s}{da'}\cos p'.$$

Again, $$s\sin s(\theta-\theta')=s\sin s(q+v-v')$$
$$=s\sin sq\cos s(v-v')+s\cos sq\sin s(v-v').$$

Hence, by substituting the foregoing value of $v-v'$, and retaining only the first power of e and e', it will be found that

$$s\sin s(\theta-\theta')=s\sin sq+es^2\big(\sin(p+sq)+\sin(p-sq)\big)$$
$$-e's^2\big(\sin(p'+sq)+\sin(p'-sq)\big).$$

By supposing s to have all negative as well as positive integer values, this equation may be more briefly written thus:

$$s\sin s(\theta-\theta')=\frac{s}{2}\sin sq+es^2\sin(p+sq)-e's^2\sin(p'+sq).$$

Now observing that $s\sin sq\cos p=s\sin(p+sq)$, because $s=\pm1, \pm2, \pm3$, &c., the following result will be obtained by multiplying the foregoing values of R_s and $s\sin s(\theta-\theta')$:

$$\text{R}_s s\sin s(\theta-\theta')=\frac{s\text{A}_s}{2}\sin sq+\left(s^2\text{A}_s-\frac{as}{2}\cdot\frac{d\text{A}_s}{da}\right)e\sin(p+sq)$$
$$-\left(s^2\text{A}_s+\frac{a's}{2}\cdot\frac{d\text{A}_s}{da'}\right)e'\sin(p'+sq).$$

Consequently, $$2\int\frac{d\text{R}}{d\theta}dt=-\int\Sigma.\text{R}_s s\sin s(\theta-\theta')dt$$

$$=\Sigma.\frac{\text{A}_s}{n-n'}\cos sq+\Sigma.\frac{2s^2\text{A}_s-as\frac{d\text{A}_s}{da}}{n+s(n-n')}e\cos(p+sq)$$

$$-\Sigma.\frac{2s^2\text{A}_s+a's\frac{d\text{A}_s}{da'}}{n'+s(n-n')}e'\cos(p'+sq).$$

Hence, since $\frac{d^2\theta}{dt^2}=-2en^2\sin p-5e^2n^2\sin 2p$, the following equation will be obtained to terms of the second order with respect to e and e':—

$$2\frac{d^2\theta}{dt^2}\int\frac{d\mathrm{R}}{d\theta}\,dt=-\Sigma.\frac{2n^2\mathrm{A}_s}{n-n'}\,e\sin(p+sq)$$

$$+\Sigma.\frac{n^2}{n+s(n-n')}\left(2s^2\mathrm{A}_s-as\frac{d\mathrm{A}_s}{da}\right)e^2\sin sq$$

$$-\Sigma.\left(\frac{n^2}{n+s(n-n')}\left(2s^2\mathrm{A}_s-as\frac{d\mathrm{A}_s}{da}\right)+\frac{5n^2}{n-n'}\,\mathrm{A}_s\right)e^2\sin(2p+sq)$$

$$+\Sigma.\frac{2n^2}{n'+s(n-n')}\left(2s^2\mathrm{A}_s+a's\frac{d\mathrm{A}_s}{da'}\right)ee'\sin p\cos(p'+sq).$$

8. Similarly we have to express $\frac{d\mathrm{R}}{dr}\cdot\frac{dr}{dt}$ as a function of t. Since $\frac{d\mathrm{R}}{dr}$ represents the partial differential coefficient of R with respect to r,

$$\frac{d\mathrm{R}}{dr}\cdot\frac{dr}{dt}=\Sigma.\frac{d\mathrm{R}_s}{dr}\cdot\frac{dr}{dt}\cos s(\theta-\theta'),$$

the values of s being 0, 1, 2, 3, &c.

But $$\mathrm{R}_s=\mathrm{A}_s+\frac{d\mathrm{A}_s}{da}au+\frac{d\mathrm{A}_s}{da'}a'u'+\frac{d^2\mathrm{A}_s}{da^2}\cdot\frac{a^2u^2}{2}+\frac{d^2\mathrm{A}_s}{dada'}\cdot aa'uu'+\frac{d^2\mathrm{A}_s}{da'^2}\,\frac{a'^2u'^2}{2}.$$

Hence $$\frac{d\mathrm{R}_s}{dr}\cdot\frac{dr}{dt}=a\frac{du}{dt}\left(\frac{d\mathrm{A}_s}{da}+\frac{d^2\mathrm{A}_s}{da^2}\,au+\frac{d^2\mathrm{A}_s}{dada'}\,a'u'\right).$$

Also $$\frac{du}{dt}=en\sin p+e^2n\sin 2p \text{ nearly.}$$

Consequently to terms of the second order with respect to e and e',

$$\frac{d\mathrm{R}}{dr}\cdot\frac{dr}{dt}=aen\frac{d\mathrm{A}_s}{da}\sin p+ae^2n\left(\frac{d\mathrm{A}_s}{da}-\frac{a}{2}\,\frac{d^2\mathrm{A}_s}{da^2}\right)\sin 2p-aea'e'n\,\frac{d^2\mathrm{A}_s}{dada'}\sin p\cos p'.$$

Also to the first power of e and e',

$$2\cos s(\theta-\theta')=2\cos sq-2\sin sq.s(v-v')$$

$$=2\cos sq+2es\left(\cos(p+sq)-\cos(p-sq)\right)$$

$$-2e's\left(\cos(p'+sq)-\cos(p'-sq)\right),$$

s being equal to 0, 1, 2, 3, &c. Or, if $s=\pm 0, \pm 1, \pm 2$, &c. on the right-hand side of the equality,

$$2\cos s(\theta-\theta')=\cos sq+2es\cos(p+sq)-2e's\cos(p'+sq).$$

Hence, placing the terms corresponding to $s=\pm 0$ apart from the others, and observing that $\sin(p+sq)\cos p'$ is equivalent to $\sin p\cos(p'+sq)$ when $s=\pm 1, \pm 2$, &c., it

will be found that

$$2\frac{d\mathrm{R}}{dr}\cdot\frac{dr}{dt}=2aen\frac{d\mathrm{A}_0}{da}\sin p+ae^2n\left(2\frac{d\mathrm{A}_0}{da}-a\frac{d^2\mathrm{A}_0}{da^2}\right)\sin 2p-2aea'e'n\frac{d^2\mathrm{A}_0}{dada'}\sin p\cos p'$$

$$+\Sigma.aen\frac{d\mathrm{A}_s}{da}\sin(p+sq)-\Sigma.ae^2ns\frac{d\mathrm{A}_s}{da}\sin sq$$

$$+\Sigma.\left(ae^2n(1+s)\frac{d\mathrm{A}_s}{da}-\frac{a^2e^2n}{2}\frac{d^2\mathrm{A}_s}{da^2}\right)\sin(2p+sq)$$

$$-\Sigma.\left(2aee'ns\frac{d\mathrm{A}_s}{da}+aa'ee'n\frac{d^2\mathrm{A}_s}{dada'}\right)\sin p\cos(p'+sq),$$

the values of s being now ± 1, ± 2, &c.

9. We are now prepared to express the right-hand side of the equation (7.) in terms of t. The results obtained in arts. 7 and 8 give,

$$2\frac{d^2\theta}{dt^2}\int\frac{d\mathrm{R}}{d\theta}dt-2\frac{d\mathrm{R}}{dr}\cdot\frac{dr}{dt}=$$

$$-2aen\frac{d\mathrm{A}_0}{da}\sin p-ae^2n\left(2\frac{d\mathrm{A}_0}{da}-a\frac{d^2\mathrm{A}_0}{da^2}\right)\sin 2p+2aea'e'n\frac{d^2\mathrm{A}_0}{dada'}\sin p\cos p'$$

$$-\Sigma.e\sin(p+sq)\left\{\frac{2n^2}{n-n'}\mathrm{A}_s+an\frac{d\mathrm{A}_s}{da}\right\}-\Sigma.e^2\sin(2p+sq).\left\{\frac{5n^2}{n-n'}\mathrm{A}_s+an\frac{d\mathrm{A}_s}{da}-\frac{a^2n}{2}\frac{d^2\mathrm{A}_s}{da^2}\right\}$$

$$-\Sigma.e^2(\sin(2p+sq)-\sin sq).\left\{\frac{n^2}{n+s(n-n')}\left(2s^2\mathrm{A}_s-as\frac{d\mathrm{A}_s}{da}\right)+ans\frac{d\mathrm{A}_s}{da}\right\}$$

$$+\Sigma.ee'\sin p\cos(p'+sq).\left\{\frac{2n^2}{n'+s(n-n')}\left(2s^2\mathrm{A}_s+a's\frac{d\mathrm{A}_s}{da'}\right)+2ans\frac{d\mathrm{A}_s}{da}+aa'n\frac{d^2\mathrm{A}_s}{dada'}\right\}.$$

For the sake of brevity of expression the following substitutions will be made :—

$$\mathrm{L}=\frac{2n^2}{n-n'}\mathrm{A}_s+an\frac{d\mathrm{A}_s}{da}$$

$$\mathrm{M}=\frac{5n^2}{n-n'}\mathrm{A}_s+an\frac{d\mathrm{A}_s}{da}-\frac{a^2n}{2}\frac{d^2\mathrm{A}_s}{da^2}$$

$$\mathrm{N}=\frac{n^2}{n+s(n-n')}\left(2s^2\mathrm{A}_s-as\frac{d\mathrm{A}_s}{da}\right)+ans\frac{d\mathrm{A}_s}{da}$$

$$\mathrm{P}=\frac{2n^2}{n'+s(n-n')}\left(2s^2\mathrm{A}_s+a's\frac{d\mathrm{A}_s}{da'}\right)+2ans\frac{d\mathrm{A}_s}{da}+aa'n\frac{d^2\mathrm{A}_s}{dada'}.$$

After multiplying the right-hand side of the foregoing equation by dt, integrating, and substituting in the equation (7.), the following will be the result :—

$$\frac{dr^2}{dt^2}+\frac{h^2}{r^2}-\frac{2\mu}{r}+\mathrm{C}=$$

$$2ae\frac{d\mathrm{A}_0}{da}\cos p+ae^2\left(\frac{d\mathrm{A}_0}{da}-\frac{a}{2}\frac{d^2\mathrm{A}_0}{da^2}\right)\cos 2p-\frac{2aa'ee'n}{n^2-n'^2}\frac{d^2\mathrm{A}_0}{dada'}(n\cos p\cos p'+n'\sin p\sin p')$$

$$+\Sigma.\frac{\mathrm{L}e\cos(p+sq)}{n+s(n-n')}+\Sigma.\frac{\mathrm{M}e^2\cos(2p+sq)}{2n+s(n-n')}+\Sigma.\mathrm{N}e^2\left(\frac{\cos(2p+sq)}{2n+s(n-n')}-\frac{\cos sq}{s(n-n')}\right)$$

$$-\Sigma.\frac{\mathrm{P}ee'}{n^2-(n'+s(n-n'))^2}\Big(n\cos p\cos(p'+sq)+\big(n'+s(n-n')\big)\sin p\sin(p'+sq)\Big).$$

Substituting Q for the sum of the terms on the right-hand side of this equation, and neglecting Q^2, &c., we have

$$dt=dr\left\{-\frac{h^2}{r^2}+\frac{2\mu}{r}-C+Q\right\}^{-\frac{1}{2}}$$

$$=dr\left(-\frac{h^2}{r^2}+\frac{2\mu}{r}-C\right)^{-\frac{1}{2}}-\frac{dr}{2}\left(-\frac{h^2}{r^2}+\frac{2\mu}{r}-C\right)^{-\frac{3}{2}}Q$$

$$=\frac{rdr}{\sqrt{-h^2+2\mu r-Cr^2}}-\frac{Qdt}{2a^2e^2n^2\sin^2 p}(1-4e\cos p),$$

because by the first approximation

$$dr\left(-\frac{h^2}{r^2}+\frac{2\mu}{r}-C\right)^{-\frac{3}{2}}=\frac{dt}{\frac{dr^2}{dt^2}}=\frac{dt}{a^2e^2n^2\sin^2 p(1+4e\cos p)}.$$

Consequently

$$nt+\varepsilon-\varpi=\cos^{-1}\frac{a-r}{ae}-\frac{1}{a}\sqrt{a^2e^2-(a-r)^2}-\frac{1}{2a^2e^2n}\int\frac{Qdt}{\sin^2 p}(1-4e\cos p), \quad . \quad . \quad . \quad (8.)$$

the constants a, e, ε and ϖ having the same signification as heretofore*.

10. Before proceeding to effect the integration above indicated, it will be right to remove certain analytical difficulties presented by the form of the equation. First, it may be urged that as Q contains the first power of e, the coefficient of the last term might become infinite if e were indefinitely small, and the equation would no longer hold good. But it has already been proved (art. 5.) that e and the disturbing force vanish together, from which it follows that the quantity, $\frac{1}{e}\times$disturbing force, may approach a finite value or zero as e diminishes. Again, it will be seen, if the quantity to be integrated be put under the form $\chi(t)dt$, that the factor $\chi(t)$ becomes infinite each time $\sin p=0$, and that the development fails for the values of t that satisfy this equation. But it is well known that an analytical circumstance of this kind will not prevent our obtaining in the final analysis the correct development of r, provided the integration above indicated can be effected, and that the failure must admit of some interpretation relative to the proposed problem. Now it is not difficult to point out the significance of the failure in this instance. Let us suppose that $\cos^{-1}\frac{a-r}{ae}=\phi$. Then the arc ϕ can differ from p only by a small quantity, and we have exactly $r=a(1-e\cos\phi)$. Hence as the quantities a and e are absolutely constant, it would seem that the maximum and minimum values of r are $a(1+e)$ and $a(1-e)$ in every revolution of the disturbed body. This inference is manifestly untrue, and the reason that it has not been legitimately deduced is, that the above-mentioned failure occurs when r approaches a maximum or minimum value. The failure has, therefore, an important bearing on the problem, as showing that the maximum and minimum values of the radius-vector are not of constant magnitude. I proceed now with the integration.

* See Note (B) at the end of the paper.

11. In order that our method of solution may be successful, the differential,

$$\frac{\mathrm{Q}dt}{\sin^2 p}(1-4e\cos p),$$

must admit of exact integration to terms inclusive of the second power of e. Now it will be seen by referring to the expression in art. 9 for which Q was substituted, that this integration depends on the following integrals, which are exact. The integration can, therefore, be effected.

$$\int\frac{\cos p}{\sin^2 p}(1-4e\cos p)dt=-\frac{1}{n\sin p}+\frac{4e}{n}\cot p+4et$$

$$\int\frac{\cos 2p}{\sin^2 p}\,dt=-\frac{\cot p}{n}-2t \qquad \int\frac{n\cos p\cos p'+n'\sin p\sin p'}{\sin^2 p}=-\frac{\cos p'}{\sin p}$$

$$\int\Sigma\cdot\frac{\cos(p+sq)}{n+s(n-n')}\cdot\frac{dt}{\sin^2 p}=\int\Sigma\cdot\left(\frac{\cos(p+sq)}{n+s(n-n')}+\frac{\cos(p-sq)}{n-s(n-n')}\right)\frac{dt}{\sin^2 p} \quad [s=1,\ 2,\ \&c.]$$

$$=-\Sigma\cdot\frac{1}{n^2-s^2(n-n')^2}\cdot\frac{\cos sq}{\sin p} \quad [s=\pm 1,\ \pm 2,\ \&c.]$$

$$\int\Sigma\cdot\frac{\cos(p+sq)}{n+s(n-n')}\cdot\frac{\cos p}{\sin^2 p}\,dt=\int\Sigma\cdot\left(\frac{\cos(p+sq)}{n+s(n-n')}+\frac{\cos(p-sq)}{n-s(n-n')}\right)\frac{\cos p}{\sin^2 p}\,dt \quad [s=1,\ 2,\ \&c.]$$

$$=-\Sigma\cdot\frac{1}{n^2-s^2(n-n')^2}\left(\frac{n\sin sq}{s(n-n')}+\cos sq\cot p\right) \quad [s=\pm 1,\ \pm 2,\ \&c.]$$

$$\int\Sigma\cdot\frac{\cos(2p+sq)}{2n+s(n-n')}\cdot\frac{dt}{\sin^2 p}=\int\Sigma\cdot\left(\frac{\cos(2p+sq)}{2n+s(n-n')}+\frac{\cos(2p-sq)}{2n-s(n-n')}\right)\frac{dt}{\sin^2 p} \quad [s=1,\ 2,\ \&c.]$$

$$=-\Sigma\cdot\frac{2}{4n^2-s^2(n-n')^2}\left(\frac{2n\sin sq}{s(n-n')}+\cos sq\cot p\right) \quad [s=\pm 1,\ \pm 2,\ \&c.]$$

$$\int\Sigma\cdot\left(\frac{\cos(2p+sq)}{2n+s(n-n')}-\frac{\cos sq}{s(n-n')}\right)\frac{dt}{\sin^2 p}=\Sigma\cdot\frac{2}{s(n-n')\left(2n+s(n-n')\right)}\cdot\frac{\cos(p+sq)}{\sin p}$$

$$\int\Sigma\cdot\left(n\cos p\cos(p'+sq)+\left(n'+s(n-n')\right)\sin p\sin(p'+sq)\right)\frac{dt}{\sin^2 p}=-\Sigma\cdot\frac{\cos(p'+sq)}{\sin p}.$$

Since the factors which multiply the two last differentials are different according as s is positive or negative, it is important to remark that these integrals have been obtained without giving to s the + and − signs. The factors which multiply the three preceding differentials do not contain s.

From the results of these integrations the following equation is readily obtained:

$$\begin{aligned}\int\frac{\mathrm{Q}dt}{\sin^2 p}(1-4e\cos p)=&\left(6ae^2\frac{d\mathrm{A}_0}{da}+a^2e^2\frac{d^2\mathrm{A}_0}{da^2}\right)t\\ &+\left(\frac{ae}{n}\cdot\frac{d\mathrm{A}_0}{da}(-2+7e\cos p)+\frac{a^2e^2}{2n}\cdot\frac{d^2\mathrm{A}_0}{da^2}\cos p\right)\frac{1}{\sin p}\\ &-\Sigma\cdot\frac{\mathrm{L}e}{n^2-s^2(n-n')^2}\cdot\frac{\cos sq}{\sin p}\\ &+\Sigma\cdot\frac{4\mathrm{L}e^2}{n^2-s^2(n-n')^2}\left(\frac{n}{s(n-n')}\sin sq\sin p+\cos sq\cos p\right)\frac{1}{\sin p}\end{aligned}$$

$$-\Sigma.\frac{2Me^2}{4n^2-s^2(n-n')^2}\left(\frac{2n}{s(n-n')}\sin sq\sin p+\cos sq\cos p\right)\frac{1}{\sin p}$$

$$+\Sigma.\frac{2Ne^2}{s(n-n')\big(2n+s(n-n')\big)}.\frac{\cos(p+sq)}{\sin p}$$

$$+\frac{2aa'ee'n}{n^2-n'^2}.\frac{d^2A_0}{da\,da'}.\frac{\cos p'}{\sin p}$$

$$+\Sigma.\frac{Pee'}{n^2-\big(n'+s(n-n')\big)^2}.\frac{\cos(p'+sq)}{\sin p}.$$

Since $s=\pm 1$, ± 2, &c., the following equalities are true:

$$\Sigma.\cos sq\cos p=\ \Sigma.\cos(p+sq)$$

$$\Sigma.\frac{\sin sq\sin p}{s}=-\Sigma.\frac{\cos(p+sq)}{s}.$$

Hence it will be found that the term in the above equation which involves $\cos(p+sq)$ is

$$\frac{2}{n-n'}\Sigma.\left(-\frac{2Le^2}{s(n+s(n-n'))}+\frac{(M+N)e^2}{s(2n+s(n-n'))}\right)\frac{\cos(p+sq)}{\sin p}.$$

Consequently the equation (8.) becomes

$$\left(n+\frac{3}{na}.\frac{dA_0}{da}+\frac{1}{2n}.\frac{d^2A_0}{da^2}\right)t+\varepsilon-\varpi=\cos^{-1}\frac{a-r}{ae}-\frac{1}{a}\sqrt{a^2e^2-(a-r)^2}$$

$$-\frac{1}{2a^2en\sin p}.\left\{\frac{a}{n}.\frac{dA_0}{da}(-2+7e\cos p)+\frac{a^2e}{2n}.\frac{d^2A_0}{da^2}\cos p+\frac{2aa'e'n}{n^2-n'^2}.\frac{d^2A_0}{da\,da'}\cos p'\right.$$

$$-\Sigma.\frac{L}{n^2-s^2(n-n')^2}\cos sq+\frac{2}{n-n'}.\Sigma.\left(-\frac{2Le}{s\big(n+s(n-n')\big)}+\frac{(M+N)e}{s\big(2n+s(n-n')\big)}\right)\cos(p+sq)$$

$$\left.+\Sigma.\frac{Pe'}{n^2-\big(n'+s(n-n')\big)^2}.\cos(p'+sq)\right\}.$$

12. Before advancing to the next operation, our attention must be directed to the failure of the term containing the denominator $n^2-\big(n'+s(n-n')\big)^2$ in the case of $s=1$. As the denominator vanishes for this value of s, it is necessary to retrace our steps and consider that case separately. Referring to the equation at the beginning of art. 9, it will be seen that if $s=1$ and P_1 represent the consequent value of P, the last term becomes $P_1ee'\sin p\cos(p'+q)$. Also, since

$$p'+q=n't+\varepsilon'-\varpi'+nt+\varepsilon-n't-\varepsilon'=p+\varpi-\varpi',$$

we have $\displaystyle\int\sin p\cos(p'+q)dt=\int\sin p\cos(p+\varpi-\varpi')dt$

$$=-\frac{1}{4n}\cos(2p+\varpi-\varpi')-\frac{t}{2}\sin(\varpi-\varpi').$$

Thus the equation (8.) will contain the term

$$-\frac{1}{2a^2e^2n}\int\frac{P_1ee'dt}{\sin^2 p}\left(-\frac{1}{4n}\cos(2p+\varpi-\varpi')-\frac{t}{2}\sin(\varpi-\varpi')\right),$$

which will give rise in the right-hand side of the equation at the end of art. 11, to the terms

$$-\frac{1}{2a^2en\sin p}\cdot\frac{P_1e'\cos p}{4n^2}\Big(\cos(\varpi-\varpi')+2nt\sin(\varpi-\varpi')\Big)-\frac{P_1e't}{4a^2en^2}\cos(\varpi-\varpi').$$

Hence, taking these terms into account, and putting the equation under the form

$$\frac{a-r}{ae}=\cos\left\{p_{,}+e\sqrt{1-\left(\frac{a-r}{ae}\right)^2}+\frac{1}{2a^2en\sin p_{,}}(\mathrm{U}+e\mathrm{V}+e'\mathrm{W})\right\}, \quad . \quad . \quad . \quad (9.)$$

we shall have

$$p_{,}=\left(n+\frac{3}{na}\frac{d\mathrm{A}_0}{da}+\frac{1}{2n}\frac{d^2\mathrm{A}_0}{da^2}+\frac{P_1e'}{4n^2a^2e}\cos(\varpi-\varpi')\right)t+\varepsilon-\varpi$$

$$\mathrm{U}=-\frac{2a}{n}\frac{d\mathrm{A}_0}{da}-\Sigma\cdot\frac{\mathrm{L}}{n^2-s^2(n-n')^2}\cos sq$$

$$\mathrm{V}=\left(\frac{7a}{n}\frac{d\mathrm{A}_0}{da}+\frac{a^2}{2n}\frac{d^2\mathrm{A}_0}{da^2}\right)\cos p_{,}+\frac{2}{n-n'}\Sigma\cdot\left(\frac{-2\mathrm{L}}{s(n+s(n-n'))}+\frac{\mathrm{M}+\mathrm{N}}{s(2n+s(n-n'))}\right)\cos(p_{,}+sq)$$

$$\mathrm{W}=\frac{P_1}{4n^2}\Big(\cos(\varpi-\varpi')+2nt\sin(\varpi-\varpi')\Big)\cos p_{,}+\frac{2aa'n}{n^2-n'^2}\frac{d^2\mathrm{A}_0}{da\,da'}\cos p'_{,}+\Sigma\cdot\frac{\mathrm{P}}{n^2-(n'+sn-n')^2}\cos(p_{,}+s$$

It may be remarked, that in the terms containing the disturbing force we have put $p_{,}$ for p and $p'_{,}$ for p', which is plainly allowable, because the reasoning might be repeated with these values in the place of p and p'. Also, it appears that the expression for which W is substituted contains a term multiplied by t. This term might be included in $p_{,}$, but it is more convenient to retain it in its present position. I proceed to develope r in terms of t by means of the equation (9.).

13. This equation must give a result of this form,

$$\frac{a-r}{ae}=\mathrm{H}+h,$$

H and h representing respectively the terms which contain, and those which do not contain, the disturbing force. Hence, omitting h^2, &c.,

$$\sqrt{1-\left(\frac{a-r}{ae}\right)^2}=\sqrt{1-\mathrm{H}^2}-\frac{\mathrm{H}h}{\sqrt{1-\mathrm{H}^2}}.$$

Consequently, putting g for the last term within the brackets of equation (9.),

$$\frac{a-r}{ae}=\cos\left\{p_{,}+e\sqrt{1-\mathrm{H}^2}-\frac{e\mathrm{H}h}{\sqrt{1-\mathrm{H}^2}}+g\right\}$$

$$=\cos(p_{,}+e\sqrt{1-\mathrm{H}^2})+\sin(p_{,}+e\sqrt{1-\mathrm{H}^2})\left(\frac{e\mathrm{H}h}{\sqrt{1-\mathrm{H}^2}}-g\right)\text{nearly.}$$

Hence $\quad \mathrm{H}=\cos(p_{,}+e\sqrt{1-\mathrm{H}^2})$

and $\quad h=\sin(p_{,}+e\sqrt{1-\mathrm{H}^2})\left(\frac{e\mathrm{H}h}{\sqrt{1-\mathrm{H}^2}}-g\right).$

By the first of these equations H may be developed in a series proceeding according to the powers of e, which will be found to be identical in form with the analogous

series in the elliptic theory. In the other equation terms involving $e^2\times$ disturbing force are to be omitted, and $\cos p_{,}$ may therefore be put for H. Hence

$$h=\sin(p_{,}+e\sin p_{,})(eh\cot p_{,}-g),$$

or, to the same approximation as before,

$$h=-g\sin p_{,}(1+2e\cos p_{,}).$$

Thus, since $\frac{r}{a}=1-e(\mathrm{H}+h)$, we finally obtain

$$\frac{r}{a}=1-e\mathrm{H}+\frac{1}{2a^2n}\big(\mathrm{U}+e(\mathrm{V}+2\mathrm{U}\cos p_{,})+e'\mathrm{W}\big). \quad \ldots \ldots \quad (10.)$$

It will be seen that in this process $\sin p_{,}$ has disappeared from the denominator.

14. The expressions represented by U, V and W admit of simplifications which will render them more convenient for substitution in the equation (10.). In art. 9 we have

$$\mathrm{L}=\frac{2n^2}{n-n'}\mathrm{A}_s+an\frac{d\mathrm{A}_s}{da}$$

$$\mathrm{N}=\frac{n^2}{n+s(n-n')}\left(2s^2\mathrm{A}_s-as\frac{d\mathrm{A}_s}{da}\right)+ans\frac{d\mathrm{A}_s}{da}.$$

Hence $$\mathrm{N}=\frac{s^2}{n+s(n-n')}\left(2n^2\mathrm{A}_s+an(n-n')\frac{d\mathrm{A}_s}{da}\right)=\frac{s^2(n-n')}{n+s(n-n')}\mathrm{L};$$

also, $$\mathrm{M}=\frac{5n^2}{n-n'}\mathrm{A}_s+an\frac{d\mathrm{A}_s}{da}-\frac{a^2n}{2}\frac{d^2\mathrm{A}_s}{da^2}=\mathrm{L}+\frac{3n^2}{n-n'}\mathrm{A}_s-\frac{a^2n}{2}\frac{d^2\mathrm{A}_s}{da^2}.$$

Using these values, and putting $s(n-n')$ under the form $(s(n-n')+n)-n$, it will be readily found that

$$\frac{2}{n-n'}\left(-\frac{2\mathrm{L}}{s(n+s(n-n'))}+\frac{\mathrm{M}+\mathrm{N}}{s\big(2n+s(n-n')\big)}\right)=$$

$$\frac{2}{(n+s(n-n'))^2-n^2}\left\{\left(\frac{2n^2}{n-n'}\mathrm{A}_s+an\frac{d\mathrm{A}_s}{da}\right)\cdot\frac{(s^2-s)(n-n')-3n}{n+s(n-n')}+\frac{3n^2}{n-n'}\mathrm{A}_s-\frac{a^2n}{2}\frac{d^2\mathrm{A}_s}{da^2}\right\}.$$

Again, since A_s and $\frac{d\mathrm{A}_s}{da}$ are homogeneous functions of a and a' of the dimensions -1 and -2 respectively, by a known theorem we have

$$a'\frac{d\mathrm{A}_s}{da'}=-\mathrm{A}_s-a\frac{d\mathrm{A}_s}{da} \qquad a'\frac{d^2\mathrm{A}_s}{da\,da'}=-2\frac{d\mathrm{A}_s}{da}-a\frac{d^2\mathrm{A}_s}{da^2}.$$

By these equations the differential coefficients of A_s with respect to a' may be eliminated. Thus by substituting in the expression for P in art. 9, and reducing, the following result will be obtained:—

$$\mathrm{P}=\frac{2n}{n'+s(n-n')}\cdot\left\{(2s^2-s)n\mathrm{A}_s+\big((s^2-2s)(n-n')-n'\big)a\frac{d\mathrm{A}_s}{da}\right\}-a^2n\frac{d^2\mathrm{A}_s}{da^2}.$$

Hence, putting $s=1$,

$$\mathrm{P}_1=2n\mathrm{A}_1-2na\frac{d\mathrm{A}_1}{da}-a^2n\frac{d^2\mathrm{A}_1}{da^2}.$$

Also, since
$$U=-\frac{2a}{n}\frac{dA_0}{da}-\Sigma.\frac{L}{n^2-s^2(n-n')^2}\cos sq,$$

$$2eU\cos p_{,}=-\frac{4ae}{n}\,\frac{dA_0}{da}\cos p_{,}-\Sigma.\frac{2Le}{n^2-s^2(n-n')^2}\cos(p_{,}+sq).$$

Consequently, by putting for H its known expression from the elliptic theory, substituting $-2\frac{dA_0}{da}-a\frac{d^2A_0}{da^2}$ for $a'\frac{d^2A_0}{dada'}$, and making use of the foregoing equalities, with the values of V and W given in art. 12, the equation (10.) becomes as follows:—

$$\frac{r}{a}=1-\frac{1}{an^2}\frac{dA_0}{da}-e\cos p_{,}+\frac{e^2}{2}-\frac{e^2}{2}\cos 2p_{,}+\frac{3e^3}{8}(\cos p_{,}-\cos 3p_{,})$$

$$+\left\{\frac{3e}{2an^2}\,\frac{dA_0}{da}+\frac{e}{4n^2}\,\frac{d^2A_0}{da^2}+\frac{e'}{8a^2n^2}\left(2A_1-2a\frac{dA_1}{da}-a^2\,\frac{d^2A_1}{da^2}\right)\left(\cos(\varpi-\varpi')+2nt\sin(\varpi-\varpi')\right)\right\}\cos p_{,}$$

$$-\frac{1}{2a^2}\Sigma.\frac{\frac{2n}{n-n'}A_s+a\frac{dA_s}{da}}{n^2-s^2(n-n')^2}\cos sq$$

$$+\frac{e}{a^2}\Sigma.\left\{-\frac{\frac{2n}{n-n'}A_s+a\frac{dA_s}{da}}{n^2-s^2(n-n')^2}+\frac{1}{\left(n+s(n-n')\right)^2-n^2}.\left[\begin{array}{c}\frac{(s^2-s)(n-n')-3n}{n+s(n-n')}.\left(\frac{2n}{n-n'}A_s+a\frac{dA_s}{da}\right)\\ +\frac{3n}{n-n'}A_s-\frac{a^2}{2}\,\frac{d^2A_s}{da^2}\end{array}\right]\right\}\cos(p_{,}+sq)$$

$$+\frac{e'}{a^2}\Sigma.\frac{1}{n^2-\left(n'+s(n-n')\right)^2}.\left\{\frac{(2s^2-s)nA_s+\left((s^2-2s)(n-n')-n'\right)a\frac{dA_s}{da}}{n'+s(n-n')}-\frac{a^2}{2}\frac{d^2A_s}{da^2}\right\}\cos(p_{,}+sq).$$

In this equation s has the values ± 1, ± 2, ± 3, &c. in the terms containing $\cos sq$ and $\cos(p_{,}+sq)$, and the values ± 0, -1, ± 2, &c. in the term containing $\cos(p_{,}+sq)$.

On comparing this expression for the radius-vector with that obtained by LAPLACE*, terms will be found in the latter identical with all the above, with the exception of those that contain $e'\cos p_{,}$; and there are other terms to which none of the above correspond. These are only differences in form, arising from difference in the processes of integration. It is chiefly important to remark, that in the foregoing expression for r there is no term containing ent as a factor. The signification of that which contains $e'nt$ will be presently considered.

15. Having obtained the development of the radius-vector, it is easy to infer that of the longitude (θ) from the equation (6.), viz.

$$\frac{d\theta}{dt}=\frac{h}{r^2}-\frac{1}{r^2}\int\frac{dR}{d\theta}\,dt,$$

and from the value of $\int\frac{dR}{d\theta}\,dt$ in terms of t, which has been already found. Putting $r_{,}+\delta r$ for r, and taking δr to represent the terms multiplied by the disturbing force,

* Mécanique Cél. part 1. liv. ii. No. 50.

we shall have to the same approximation as before,

$$\frac{d\theta}{dt}=\frac{h}{r_{\prime}^{2}}\left(1-\frac{2\delta r}{r_{\prime}}\right)-\frac{1}{r_{\prime}^{2}}\int\frac{d\mathrm{R}}{d\theta}\,dt$$

$$=\frac{h}{r_{\prime}^{2}}-\frac{2h}{a^{3}}(1+3e\cos p_{\prime})\delta r-\frac{1}{a^{2}}(1+2e\cos p_{\prime})\int\frac{d\mathrm{R}}{d\theta}\,dt\,;$$

$$\therefore\ \theta=\varepsilon+\int\frac{h}{r_{\prime}^{2}}\,dt-\int\left(\frac{2h}{a^{3}}\delta r+\frac{1}{a^{2}}\int\frac{d\mathrm{R}}{d\theta}\,dt\right)dt-\int\left(\frac{6h}{a^{3}}\delta r+\frac{2}{a^{2}}\int\frac{d\mathrm{R}}{d\theta}\,dt\right)e\cos p_{\prime}dt.$$

The development of the term $\int\frac{h}{r_{\prime}^{2}}\,dt$ will be of the same form as in the elliptic theory. An equation obtained in art. 7 gives,

$$\frac{1}{a^{2}}\int\frac{d\mathrm{R}}{d\theta}\,dt=\frac{1}{2a^{2}}\Sigma.\frac{\mathrm{A}_{s}}{n-n^{\prime}}\cos sq+\frac{1}{2a^{2}}\Sigma.\frac{2s^{2}\mathrm{A}_{s}-as\frac{d\mathrm{A}_{s}}{da}}{n+s(n-n^{\prime})}e\cos(p_{\prime}+sq)$$
$$+\frac{1}{2a^{2}}\Sigma.\frac{2s^{2}\mathrm{A}_{s}+a^{\prime}s\frac{d\mathrm{A}_{s}}{da^{\prime}}}{n^{\prime}+s(n-n^{\prime})}e^{\prime}\cos(p_{\prime}^{\prime}+sq)\,;$$

$$\therefore\ \int\left(\frac{1}{a^{2}}\int\frac{d\mathrm{R}}{d\theta}\,dt\right)dt=\frac{1}{2a^{2}}\Sigma.\frac{\mathrm{A}_{s}}{s(n-n^{\prime})^{2}}\sin sq+\frac{1}{2a^{2}}\Sigma.\frac{2s^{2}\mathrm{A}_{s}-as\frac{d\mathrm{A}_{s}}{da}}{\left(n+s(n-n^{\prime})\right)^{2}}e\sin(p_{\prime}+sq)$$
$$+\frac{1}{2a^{2}}\Sigma.\frac{2s^{2}\mathrm{A}_{s}+a^{\prime}s\frac{d\mathrm{A}_{s}}{da^{\prime}}}{\left(n^{\prime}+s(n-n^{\prime})\right)^{2}}e^{\prime}\sin(p_{\prime}^{\prime}+sq).$$

Since the relations of the constants h, a, n and e are expressed in our problem in the same manner as in the elliptic theory, we have $h=na^{2}\sqrt{1-e^{2}}$. Hence we may put na^{2} for h in the terms involving the disturbing force. Consequently, omitting e^{2}, &c.,

$$\int\frac{6h}{a^{3}}\delta re\cos p_{\prime}dt=\int\frac{6h}{a^{3}}\left(-\frac{1}{n^{2}}\cdot\frac{d\mathrm{A}_{0}}{da}-\frac{1}{2a}\Sigma.\frac{\frac{2n}{n-n^{\prime}}\mathrm{A}_{s}+a\frac{d\mathrm{A}_{s}}{da}}{n^{2}-s^{2}(n-n^{\prime})^{2}}\cos sq\right)e\cos p_{\prime}dt$$

$$=-\frac{6e}{an^{2}}\frac{d\mathrm{A}_{0}}{da}\sin p_{\prime}-\frac{3ne}{a^{2}\left(n+s(n-n^{\prime})\right)}\Sigma.\frac{\frac{2n}{n-n^{\prime}}\mathrm{A}_{s}+a\frac{d\mathrm{A}_{s}}{da}}{n^{2}-s^{2}(n-n^{\prime})^{2}}\sin(p_{\prime}+sq).$$

Also, to the same approximation,

$$\int\left(\frac{2}{a^{2}}\int\frac{d\mathrm{R}}{d\theta}\,dt\right)e\cos p_{\prime}dt=\int\frac{e}{a^{2}}\Sigma.\frac{\mathrm{A}_{s}}{n-n^{\prime}}\cos sq\cos p_{\prime}dt$$

$$=\frac{e}{a^{2}}\Sigma.\frac{\mathrm{A}_{s}}{(n-n^{\prime})\left(n+s(n-n^{\prime})\right)}\sin(p_{\prime}+sq).$$

In all these equations the values of s are ±1, ±2, ±3, &c.

Putting now, for the sake of brevity, D for $2\mathrm{A}_{1}-2a\frac{d\mathrm{A}_{1}}{da}-a^{2}\frac{d^{2}\mathrm{A}_{1}}{da^{2}}$, and F, G and H for the coefficients of $\cos sq$, $\cos(p_{\prime}+sq)$, $\cos(p_{\prime}^{\prime}+sq)$ respectively in the expression for $\frac{r}{a}$, and observing that

$$\int t\,dt\cos p_{\prime}=\frac{t\sin p_{\prime}}{n}+\frac{\cos p_{\prime}}{n^{2}},$$

the following equation is readily obtained:

$$\int\frac{2h}{a^3}\delta r dt=-\frac{2}{na}\frac{dA_0}{da}t+\left\{\frac{3e}{an^2}\frac{dA_0}{da}+\frac{e}{2n^2}\frac{d^2A_0}{da^2}-\frac{De'}{4a^2n^2}\cos(\varpi-\varpi')\right\}\sin p_{\prime}$$
$$+\frac{De't}{2a^2n}\sin(\varpi-\varpi')\sin p_{\prime}$$
$$+\frac{De'}{2a^2n^2}\sin(p'_{\prime}+q)$$
$$-\frac{nF}{a^2s(n-n')}\sin sq+\frac{2neG}{a^2\big(n+s(n-n')\big)}\sin(p_{\prime}+sq)+\frac{2ne'H}{a^2\big(n'+s(n-n')\big)}\sin(p'_{\prime}+sq),$$

the value $s=1$ being excluded from the last term.

Substituting the results of these integrations, the following will be found to be the value of θ:

$$\theta=\varepsilon+\left(n+\frac{2}{na}\cdot\frac{dA_0}{da}\right)t+\frac{5e^2}{4}\sin 2p_{\prime}+\frac{e^3}{4}\left(\frac{13}{3}\sin 3p_{\prime}-\sin p_{\prime}\right)$$
$$+\left\{2e+\frac{3e}{an^2}\frac{dA_0}{da}-\frac{e}{2n^2}\frac{d^2A_0}{da^2}+\frac{De'}{4a^2n^2}\big(\cos(\varpi-\varpi')-2nt\sin(\varpi-\varpi')\big)\right\}\sin p_{\prime}$$
$$-\frac{e'}{2a^2n^2}\left(A_1-a\frac{dA_1}{da}+D\right)\sin(p'_{\prime}+q)$$
$$+\frac{1}{2a^2}\Sigma\cdot\left(\frac{2nF}{s(n-n')}-\frac{A_s}{s(n-n')^2}\right)\sin sq$$
$$+\frac{e}{a^2}\Sigma\cdot\frac{1}{n+s(n-n')}\left\{\left[3n+\frac{s}{2}\big(n-s(n-n')\big)\right]F-\frac{(1+s)A_s}{n-n'}-2nG\right\}\sin(p_{\prime}+sq)$$
$$-\frac{e'}{a^2}\Sigma\cdot\frac{1}{n'+s(n-n')}\left\{\frac{(2s^2-s)A_s-as\frac{dA_s}{da}}{2\big(n'+s(n-n')\big)}+2nH\right\}\sin(p'_{\prime}+sq),$$

where s has the values ± 0, -1, ± 2, ± 3, &c. in the last term, and the values ± 1, ± 2, ± 3, &c. in the other terms, and A_{-s} has the same value as A_s.

16. The expressions for r and θ obtained in arts. 14 and 15 may be put under forms more compact, and more convenient for drawing inferences, by making the following substitutions:

$$A=a-\frac{1}{n^2}\cdot\frac{dA_0}{da}+\frac{ae^2}{2}$$
$$E=e-\frac{e}{4n^2}\frac{d^2A_0}{da^2}-\frac{De't}{4a^2n}\sin(\varpi-\varpi')$$
$$\Pi=\varpi-\left(\frac{1}{na}\frac{dA_0}{da}+\frac{1}{2n}\cdot\frac{d^2A_0}{da^2}+\frac{De'}{4na^2e}\cos(\varpi-\varpi')\right)t$$
$$N=n+\frac{2}{na}\frac{dA_0}{da},\text{ so that } p_{\prime}=Nt+\varepsilon-\Pi$$
$$ef=\frac{3e}{2n^2a}\frac{dA_0}{da}+\frac{De'}{8a^2n^2}\cos(\varpi-\varpi')$$
$$e'g=-\frac{e'}{2a^2n^2}\left(A_1-a\frac{dA_1}{da}+D\right).$$

Thus we shall have,

$$r=A-a(E-ef)\cos(Nt+\varepsilon-\Pi)-\frac{ae^2}{2}\cos 2(Nt+\varepsilon-\Pi)+\&c.$$
$$-\frac{1}{2a}\Sigma.F\cos s(Nt+\varepsilon-N't-\varepsilon')$$
$$+\frac{e}{a}\Sigma.G\cos\{s(Nt+\varepsilon-N't-\varepsilon')+Nt+\varepsilon-\Pi\}$$
$$+\frac{e'}{a}\Sigma.H\cos\{(s-1)(Nt+\varepsilon-N't-\varepsilon')+Nt+\varepsilon-\Pi'\}.$$

And putting F′, G′, H′ for the coefficients of $\sin sq$, $\sin(p_{,}+sq)$ and $\sin(p'_{,}+sq)$ in the development of θ, we have

$$\theta=\varepsilon+Nt+2(E+ef)\sin(Nt+\varepsilon-\Pi)+\frac{5e^2}{4}\sin 2(Nt+\varepsilon-\Pi)+\&c.$$
$$+e'g\sin(Nt+\varepsilon-\Pi')+\frac{1}{2a^2}\Sigma.F'\sin s(Nt+\varepsilon-N't-\varepsilon')$$
$$+\frac{e}{a^2}\Sigma.G'\sin\{s(Nt+\varepsilon-N't-\varepsilon')+Nt+\varepsilon-\Pi\}$$
$$+\frac{e'}{a^2}\Sigma.H'\sin\{(s-1)(Nt+\varepsilon-N't-\varepsilon')+Nt+\varepsilon-\Pi'\}.$$

17. I proceed next to draw some conclusions from these values of the radius-vector and longitude.

(1) The quantity A is the non-periodic part of the radius-vector, and being equal to $a-\frac{1}{n^2}\frac{dA_0}{da}+\frac{ae^2}{2}$ is a function of given quantities and arbitrary constants. A is, therefore, invariable. It may also be remarked, that as the value of r may be put to the same approximation under the form

$$a\left(1-\frac{1}{n^2a}\frac{dA_0}{da}\right)\left(1-\frac{ae^2}{2}+\&c.+\text{periodic terms}\right),$$

the quantity $a-\frac{1}{n^2}\frac{dA_0}{da}$ is approximately the mean distance. Thus, so far as this approximation shows, the mean distance is invariable.

(2) The mean motion is necessarily the factor of the non-periodic term Nt in the development of θ. Hence

$$\text{Mean motion}=N=n+\frac{2}{na}\frac{dA_0}{da}.$$

For the reason just adduced, the mean motion is thus proved to be invariable.

As the two quantities A and N are functions of a and e, they are by consequence functions of the arbitrary constants h and C. Hence, if the values of the non-periodic part of the radius-vector and the mean motion be deduced from observation, the constants a and e, or h and C, become known.

(3) The quantity ε, being simply an arbitrary constant, is invariable. Analogous considerations apply to the mean distance, mean motion, and the epoch (ε') of the orbit of m' as disturbed by m.

(4) The expressions for E and Π show that these quantities contain terms which have t for a factor, and may therefore increase indefinitely. This circumstance creates no difficulty with regard to Π, which is always part of a circular arc affected by a sine or cosine. But as E appears as a coefficient, it might seem that the developments of r and θ contain terms which admit of indefinite increase. It must, however, be observed, that according to the remark made at the end of art. 12, the function that has given rise to these terms is really affected by a cosine, and that they have their origin in the development of that function in terms arranged according to the disturbing force.

The following considerations will enable us to obtain, at least approximately, the periodic functions of which Π and E are partial developments. Whatever functions the complete values of Π and E are of t, they may be expanded in series of the form $\alpha+\beta t+\gamma t^2+$&c., the two first terms of which are already determined. Hence

$$\frac{d\Pi}{dt}=\beta+2\gamma t+\text{\&c.}\quad \frac{dE}{dt}=\beta'+2\gamma' t+\text{\&c.}$$

Let t be indefinitely small. Then substituting

$$B \text{ for} -\frac{1}{na}\frac{dA_0}{da}-\frac{1}{2n}\frac{d^2A_0}{da^2},$$

we shall have strictly the values of $\frac{d\Pi}{dt}$ and $\frac{dE}{dt}$ for the epoch at which t commences,

viz. $$\frac{d\Pi}{dt}=B-\frac{De'}{4nea^2}\cos(\varpi-\varpi')\qquad \frac{dE}{dt}=-\frac{De'}{4nea^2}\sin(\varpi-\varpi').$$

Now if t commenced at a different epoch, we should obtain for $\frac{d\Pi}{dt}$ and $\frac{dE}{dt}$ the same expressions as those above, but different in value, because by hypothesis these differential coefficients vary with the time. The changes of value, which in actual cases take place very slowly, are due to changes in the eccentricities, and in the longitudes of the apses, of the two orbits, and will be very approximately taken into account by substituting in the above equations for e, e', ϖ and ϖ', the variable quantities E, E$'$, Π and Π'. Like considerations apply to the values of $\frac{d\Pi'}{dt}$ and $\frac{dE'}{dt}$. Thus we shall have four differential equations, by the simultaneous integration of which the four quantities may be obtained as periodic functions of the time. The arbitrary constants introduced by the integration are determined by the known values of the functions when $t=0$. These periodic functions are to be substituted for E, E$'$, Π and Π', wherever these quantities occur in the developments of r, r', θ and θ'. The four equations just mentioned are identical with those obtained by the method of the variation of parameters for determining the eccentricities and longitudes of the apses. It is worthy of remark, that in both methods the changes of the eccentricities and of the longitudes of the apses which are due to the disturbances, are taken into account in calculating the changes themselves, so that the approximation does in

fact extend beyond the first power of the disturbing force, so far as it relates to these two elements.

If the approximation be made to include generally the square of the disturbing force, and the values of r and θ in art. 16, and the like values of r' and θ', be used for that purpose, terms may arise containing coefficients which have t^2 for a factor. These terms may be converted into periodic functions of the time by the application of the principles exhibited above, but in that case the differential equations by which E, E$'$, Π and Π' are found will be of the second order, and the periodic functions will be more completely determined.

The inferences (1), (2), (3) and (4) respecting the secular variations of the elements, although obtained in a manner quite new, agree exactly with those deduced from previous solutions of the same problem.

18. Having now obtained the developments of r, θ, r' and θ', inclusive of both periodic and secular inequalities, to an extent which is sufficient for most of the applications of the Planetary Theory, I shall reserve for a future opportunity the investigation of the inequalities in latitude, and shall then take occasion to show in detail how this method adapts itself to the determination of the motions of the moon. At present I propose, in concluding this memoir, to make a few general remarks on the Problem of Three Bodies.

It has been already observed, that the solution here adopted introduces no terms containing ent in the coefficients. These terms are to be distinguished from those whose coefficients contain $e'nt$, which, as we have seen, have reference to secular variations of the eccentricity and of the motion of the apse, and would vanish with the eccentricity of the orbit of the disturbing body. The former relate to the motion itself of the apse, and are not peculiar to the Problem of Three Bodies, occurring in fact in cases where the force is directed to a fixed centre. To illustrate this remark, let us suppose the force directed to a fixed centre to be $\frac{\mu}{r^2}-\mu' r$. Then, the differential equation for finding the orbit being

$$\frac{d^2.\frac{1}{r}}{d\theta^2}+\frac{1}{r}-\frac{\mu}{h^2}+\frac{\mu' r^3}{h^2}=0,$$

let this equation be integrated by successive approximations, first neglecting the last term, and then substituting in that term the value of r given by the first approximation. By this process a term containing t in the coefficient will be introduced, and the motion of the apse will fail of being ascertained. But if, instead of this process, the equation

$$\frac{dr^2}{dt^2}+\frac{h^2}{r^2}-\frac{2\mu}{r}-\mu' r^2+\mathrm{C}=0$$

be obtained, and its approximate integration be conducted according to the powers of μ', no such term will arise, and the motion of the apse will be determined. The latter process is exactly analogous to steps employed in the foregoing solution of the

Problem of Three Bodies. The difference of the analytical results of the two methods may be thus explained. The equation obtained by putting $\frac{dr}{dt}=0$, viz. $h^2-2\mu r+Cr^2-\mu' r^4=0$, may be shown to have *three* positive roots if C be positive, so that analytically there are three apsidal distances. The first method, by embracing the third apsidal distance (no step being taken to exclude it), applies to the other two only in an expanded form, the expansion giving rise to terms containing the factor t. The other, by commencing the approximation with the equation

$$\frac{dr^2}{dt^2}+\frac{h^2}{r^2}-\frac{2\mu}{r}+C=0,$$

restricts the application of the solution to the part of the curve which has two apsidal distances, and accordingly finds the function of t which in the other method is expanded. The method of the variation of parameters, by the very nature of the process, restricts the analysis in the Problem of Three Bodies to two apsidal distances, and on this account is successful in determining the motion of the apse.

Again, I think it important to remark that the solution of the Problem of Three Bodies, as here proposed, applies equally to the Lunar and the Planetary Theories. The Problem of the Moon's motion does not differ in the analytical treatment it requires, from that of the motion of a Planet. In the one case as well as the other the approximation ought to be conducted primarily according to the disturbing force, which is assumed to be small compared to the principal force, and secondarily according to the form of the orbit, which is assumed to differ little from a circle. It is not necessary to take account of the ratio of n' to n in arranging the developments, but only in estimating the magnitude and importance of the terms resulting from the integrations. The possibility of effecting the integrations is the proper proof of the correctness of the process, and of its being adequate to give the development which is alone appropriate to the question, and which must result from every process that is in all respects legitimate. After making any assumption respecting the analytical form of the solution (as in the Lunar Theory is done by introducing the constants c and g), there can be no certainty that the solution will not at some stage become empirical. Probably the reason that the process which succeeds for a planet has not been applied to the moon, is the difficulty of extending it to the square and higher powers of the disturbance (which in the Lunar Theory it is necessary to take into account), and of embracing in the same operation both the periodic and the secular inequalities. The method I have exhibited in this communication appears to meet this difficulty by evolving *simultaneously* both kinds of inequalities by a process which obviously may be extended to higher powers of the eccentricity and the disturbing force. Such an extension would require nothing more than great labour in executing the analytical details.

NOTE (A).

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It has been shown in art. 5 from *à priori* considerations, that if the constant $e=0$, the disturbing force vanishes, and if the disturbing force vanishes, e is arbitrary. Hence it appears from the development of the radius-vector in art. 16, that the eccentricity of the disturbed orbit and the disturbing force are related in such a manner that if the eccentricity $=0$, the disturbing force vanishes, and if the disturbing force $=0$, the eccentricity remains arbitrary. The particular relation which satisfies these conditions ought plainly to result from the solution of the Problem of Three Bodies, and it may, therefore, be worth while to inquire how far such a result can be deduced from the integrations effected in the foregoing approximate solution. Now the expressions for the variations of the eccentricity and of the longitude of the apse obtained in art. 17, are identical with those given by the method of the variation of parameters. Hence for the present purpose I may make use of the deductions from those expressions which are usually given in treatises on the Planetary Theory. Referring to Pratt's 'Mechanical Philosophy,' art. 385, we have the equations,

$$Dg=\frac{nam'}{\mu}(2BD+CD') \quad . \quad . \quad . \quad (1.) \qquad Eh=\frac{nam'}{\mu}(2BE+CE') \quad . \quad . \quad . \quad . \quad (2.)$$

$$D'g=\frac{n'a'm}{\mu}(2B'D'+CD) \quad . \quad . \quad . \quad (3.) \qquad E'h=\frac{n'a'm}{\mu}(2B'E'+CE) \quad . \quad . \quad . \quad (4.)$$

$$e_{,}^{2}=D^{2}+E^{2}+2DE\cos\{(g-h)t+k-l\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

$$e_{,}^{\prime 2}=D'^{2}+E'^{2}+2D'E'\cos\{(g-h)t+k-l\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

$$g \text{ or } h=\frac{nam'B+n'a'mB'}{\mu}\pm\frac{1}{\mu}\{(nam'B-n'a'mB')^{2}+nn'aa'mm'C^{2}\}^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

In these equations B, B′ and C are known quantities independent of the eccentricities and longitudes of the apses, $e_{,}$ and $e'_{,}$ are respectively the eccentricities of the orbits of the disturbed and disturbing bodies, k and l are arbitrary quantities, and D, D′, E, E′ are also arbitrary, excepting so far as they are connected by the first four equations. Let, if possible, $e_{,}=0$ independently of the time. Then it follows from (5.) that $D=0$ and $E=0$. Hence, since $e'_{,}$ does not consequently vanish, it appears by (6.) that D′ and E′ do not on this supposition both vanish, and, therefore, by the first or second equation, that $m'=0$. Again, let $m'=0$. Then by (1.) and (2.), $Dg=0$ and $Eh=0$, and by (7.), one of the quantities g and h vanishes. Hence one of the quantities D and E vanishes and the other remains arbitrary. Hence also $e_{,}$ is arbitrary. These results confirm the reasoning in art. 5.

NOTE (B).

The following method of obtaining an expression for dt equivalent to that in art. 9, was communicated to me by Sir JOHN LUBBOCK after the reading of my Paper, and appears to be well worthy of being recorded in connection with the process of solution I have adopted, as, on a resumption of the reasoning for the purpose of carrying the approximation farther, it might considerably abbreviate the analytical details.

"If $dt=\sqrt{\frac{a}{\mu}}\,rdv$, and v be taken for the independent variable, the equation

$$\frac{d^2r^2}{2dt^2}-\frac{\mu}{r}+\frac{\mu}{a}+2\int d\mathrm{R}+\frac{rd\mathrm{R}}{dr}=0$$

becomes

$$\frac{d^2r}{dv^2}-a+r+\frac{ar}{\mu}\left(2\int d\mathrm{R}+\frac{rd\mathrm{R}}{dr}\right)=0\,;$$

and if

$$\frac{ar}{\mu}\left(2\int d\mathrm{R}+\frac{rd\mathrm{R}}{dr}\right)=\mathrm{Q},$$

$$r=a-ae\cos v+\cos v\int \mathrm{Q}\sin vdv-\sin v\int \mathrm{Q}\cos vdv,$$

and

$$v=\cos^{-1}\left\{\frac{a-r}{ae}+\frac{\cos v}{ae}\int \mathrm{Q}\sin vdv-\frac{\sin v}{ae}\int \mathrm{Q}\cos vdv\right\}.$$

Hence

$$dt=\sqrt{\frac{a}{\mu}}\,rdv=\sqrt{\frac{a}{\mu}}\cdot\frac{rdr+r\sin vdv\int \mathrm{Q}\sin vdv+r\cos vdv\int \mathrm{Q}\cos vdv}{\{a^2e^2-a-r+(\cos v\int \mathrm{Q}\sin vdv-\sin v\int \mathrm{Q}\cos vdv)^2\}^{\frac{1}{2}}}.$$

Neglecting powers of Q above the first,

$$dt=\sqrt{\frac{a}{\mu}}\cdot\left\{\frac{rdr}{(a^2e^2-(a-r)^2)^{\frac{1}{2}}}+\frac{rdv(\sin v\int \mathrm{Q}\sin vdv+\cos v\int \mathrm{Q}\cos vdv)}{(a^2e^2-(a-r)^2)^{\frac{1}{2}}}\right.$$
$$\left.+\frac{r(a-r)dr(\cos v\int \mathrm{Q}\sin vdv-\sin v\int \mathrm{Q}\cos vdv)}{(a^2e^2-(a-r)^2)^{\frac{3}{2}}}\right\}$$

$$=\sqrt{\frac{a}{\mu}}\cdot\left\{\frac{rdr}{(a^2e^2-(a-r)^2)^{\frac{1}{2}}}+\frac{rdv}{ae\sin v}\left(\sin v\int \mathrm{Q}\sin vdv+\cos v\int \mathrm{Q}\cos vdv\right)\right.$$
$$\left.+\frac{r\cos vdv}{ae\sin^2 v}\left(\cos v\int \mathrm{Q}\sin vdv-\sin v\int \mathrm{Q}\cos vdv\right)\right\}$$

$$=\sqrt{\frac{a}{\mu}}\cdot\left\{\frac{rdr}{(a^2e^2-(a-r)^2)^{\frac{1}{2}}}+\frac{r}{ae\sin^2 v}\int \mathrm{Q}\sin vdv\right\},$$

which equation is true to all powers of the eccentricities and inclinations, v being the eccentric anomaly."

XXVI. *Researches on the Foraminifera.*

By WILLIAM B. CARPENTER, *M.D., F.R.S., F.G.S. &c.*

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PART II.

ON THE GENERA ORBICULINA, ALVEOLINA, CYCLOCLYPEUS AND HETEROSTEGINA.

Genus ORBICULINA.

78. *History.*—THE interesting group of organisms belonging to this type seems to have early attracted notice, probably on account of the great abundance in which it presents itself on the sands of many of the West Indian shores. Three species are described and figured by FICHTEL and MOLL*, under the names of *Nautilus orbiculus, N. angulatus,* and *N. aduncus.* LAMARCK, however, in his first systematic treatise†, separated them from *Nautilus,* and raised them to the rank of an independent genus, to which he gave the name of *Orbiculina*; and he also changed two of the specific names, the three standing respectively as *O. numismalis, O. angulata,* and *O. uncinata.* By DENYS DE MONTFORT‡, these species were raised to the rank of independent genera, under the names of *Helenis, Archaias,* and *Ilotes*; but these genera have not been adopted by any other systematist. M. D'ORBIGNY§, in his first classification of the Foraminifera, not merely adopted LAMARCK's generic designation, but affirmed that the three reputed species were really nothing else than one and the same organism in different phases of growth, *O. angulata* being the youngest, *O. numismalis* the next in age, and *O. uncinata* the adult. He arrived at this result, of the truth of which I am myself well assured, by the comparison of a great number of specimens,—a process which it would have been well for science if he had more constantly adopted. The name of the adult form should of course stand as that of the species; but the organism in question is more commonly known under the designation *Orbiculina adunca,* which seems to have been conferred upon it by M. DESLONGSCHAMPS‖.

79. A considerable number of figures of this species are given by M. D'ORBIGNY in his treatise on the Foraminifera of Cuba, which forms part of the great work of M. RAMON DE LA SAGRA on the Natural History, &c. of that island¶. These figures,

* Testacea Microscopica, Vindob. 1798.

† Système des Animaux sans Vertèbres. Paris, 1801.

‡ Conchyliologie Systematique. Paris, 1808.

§ Tableau Méthodique de la Classe des Cephalopodes. Paris, 1825.

‖ Encyclopédie Méthodique, Zoophytes. Paris, 1824.

¶ Histoire Physique, Politique, et Naturelle de l'Ile de Cuba. Paris, 1840.

however, serve only to give a general idea of the diversities of external conformation which had presented themselves to him; and notwithstanding their number and variety, they do not include some of the most important among the protean shapes of these bodies, nor do they throw any light upon their internal structure.

80. The memoir of Professor EHRENBERG*, in which his group of Bryozoa was originally constituted, contains the first recognition of the relationship between *Orbitolites* (¶ 4) and *Orbiculina*; of which the first had previously been ranked among the Zoophytes; while the second (until the Rhizopodous nature of the whole group of Foraminifera was made known by M. DUJARDIN in 1835) had been associated with the Cephalopods. Professor EHRENBERG's description and figures of *Orbiculina*, however, being just as inaccurate as I have shown those of *Orbitolites* to be (partly, it seems likely, through his unacquaintance with the mode of making thin sections), there is no occasion for me to make further reference to them.

81. The excellent memoir of Professor WILLIAMSON† "On the minute structure of the Calcareous Shells of some recent species of Foraminifera," contains the first approach to a correct description of the internal conformation of *Orbiculina*; and the fact was fully recognized by him, that in advanced age, when the spiral type of growth has given place to the cyclical, there is no other difference between the structure of *Orbitolites* and that of *Orbiculina*, than that which arises from the dissimilarity of their earlier mode of development. Although I am satisfied that, as to one or two points of minute structure, Professor WILLIAMSON has fallen into error, I am disposed to attribute this to the want of a sufficient number of well-preserved specimens for examination; and for having it in my power to correct and extend his description, I am chiefly indebted to Mr. HUGH CUMING, the specimens of this type included in his Philippine collection being remarkable both for their high development, and for their very beautiful state of preservation.

82. The investigations of Professor WILLIAMSON are entirely unnoticed by M. D'ORBIGNY in his latest classification of the Foraminifera‡; *Orbiculina* being ranked in the order *Helicostègues*, and being defined as follows,—"Coquille nautiloide, comprimée, formée de loges divisées intérieurement en compartiments réguliers, percées de nombreuses ouvertures en lignes longitudinales à l'enroulement spécial;"—whilst *Orbitolites* (his definition of which has been already cited, ¶ 5) is placed in the order *Cyclostègues*. The separation of the two genera by so wide an interval, is grounded, therefore, on the assumption that the type of growth in *Orbiculina* is spiral, whilst that of *Orbitolites* is cyclical. I have already shown that this assumption is incorrect as regards *Orbitolites*, the early plan of whose growth is frequently spiral (¶ 54); and I shall presently show that it is equally incorrect as regards *Orbiculina*, whose later plan of growth is typically cyclical. And after having closely compared these and

* Transactions of the Royal Academy of Berlin, 1839.

† Transactions of the Microscopical Society, 1st series, vol. iii. p. 120.

‡ Cours Élémentaire de Paléontologie, tom. ii. Paris, 1852.

other types, we shall be in a position to inquire what value is really to be attached to such a character as a basis for classification.

83. *Organization.*—Among the diversified forms that are presented by the species before us, it is of course necessary to select some particular type as that with which others may be compared; and this I consider to be the form delineated in Plate XXVIII. fig. 5; since by far the larger proportion of the very numerous specimens I have examined show such a degree of approximation to it, that their differences may fairly be set-down to the account of incomplete development. The general aspect of a typical *Orbiculina*, then, differs but little from that of *Orbitolites*, except in the prominence of its nucleus, and in the peculiar spiral disposition of the bands by which the surface of the nucleus is marked. The species never attains, so far as I am aware, the dimensions of *Orbitolites*; the diameter of the largest recent disk in my possession (one of Mr. CUMING's Philippine specimens) being ·20 of an inch; whilst of the specimens which I have seen from the West Indian and Ægean seas, none exceed ·12 of an inch. Mr. CARTER*, however, describes under the name of *Orbitolites Malabaricus* a fossil *Orbiculina* (¶ 90), whose specific identity with the type before us I see no reason to doubt, as attaining a diameter of from 7 to 8 lines. The thickness of the disk is usually less in proportion to its diameter, than it is in *Orbitolites*; but this is by no means a constant difference, depending as it does merely on the relative increase of the columns of sarcode in the vertical direction, as compared with the extension of the surface by the addition of new annular bands.

84. On more closely comparing the marginal portion of such a disk with the corresponding portion of one of those forms of *Orbitolites*, in which the concentric annulation is strongly marked externally whilst the transverse divisions of each annulus are scarcely indicated (¶ 49), no essential difference is perceptible between them; and although these transverse divisions are usually but very faintly indicated in *Orbiculina* (Plate XXVIII. figs. 13, 14), yet they are sometimes very obvious, as is seen in figs. 15, 16. In the fossil *Orbiculina* just adverted to, the surface-markings (figs. 21, 22) are entirely conformable to the ordinary type of *Orbitolites*. The margin itself exhibits one or more ranges of pores (figs. 6, 7, 18, 19) arranged after precisely the same fashion as those of *Orbitolites*: these ranges are more numerous in the fossil than in the recent forms of this type.

85. The internal structure of *Orbiculina* presents such a general conformity to that of *Orbitolites*, that it will not be requisite here to do more than specify the points of agreement and of difference. The general aspect of the horizontal section of such a typical specimen as is shown in Plate XXVIII. fig. 11, does not differ from that of a corresponding section of *Orbitolites* in any other essential particular than the disposition of the central portion, in which the successive additions to the first-formed chambers are so made as to produce a spiral, instead of a concentric disk. When we examine this central portion more closely, we see that its plan of growth is exactly the

* Annals of Natural History, New Series, vol. xi. p. 425.

same as that which is presented by those aberrant forms of *Orbitolites*, in which the central cell buds-out only on one side, instead of all around (see ¶ 54, and Plate IX. fig. 4). The spiral mode of increase is usually carried-on much further in *Orbiculina*, than it ever is in *Orbitolites*; several turns being made before it gives place to the cyclical plan. But as this is a mere question of degree, such a difference would scarcely alone afford a valid distinction between these two types. There is, however, this very definite positive distinction,—that in *Orbiculina* each turn of the spire not only surrounds the preceding, but completely invests it above and beneath, every band of new chambers being continued to the very centre; so that, whilst the spiral mode of growth continues, the thickness of the shell increases with each turn; and after this has given place to the cyclical, the central nucleus, thus augmented in thickness, projects above the plane of the disk. This peculiarity in *Orbiculina*, which I have never found to be wanting, and to which there is never the least approach in *Orbitolites*, is best seen in vertical sections. Thus in Plate XXIX. fig. 3 is shown that part of a vertical section of a large disk, which has passed through its nucleus; the innermost and therefore first-formed portion of which is seen to be invested above and below by three layers, formed by three turns of the spiral. The same peculiarity is shown in Plate XXVIII. fig. 17, which represents a portion of a fossil disk near the nucleus.

86. The transition from the spiral to the cyclical mode of growth is effected in *Orbiculina*, exactly as in the aberrant forms of *Orbitolites* just referred-to, by the opening-out (so to speak) of the mouth of the spire; the successive rows extending themselves more and more on either side, until they meet around the previously-formed portion; after which each new row forms a complete zone or annulus. The commencement of this change is seen in Plate XXVIII. fig. 1; and its subsequent stages are shown in figs. 2–5, a comparison of which will show that the specimens which they represent are in different phases of this transitional state.

87. But this transition by no means constantly occurs; for the original spiral plan of growth is not unfrequently maintained, apparently through the whole of life; so that specimens are often met-with, which are not inferior in size or in number of rows to the larger disks, but which retain the *aduncal* form. Such a series of specimens is shown in figs. 8–10; the first of which represents a very young *Orbiculina*, in that stage which is common to both types of growth; whilst it is obvious from a comparison of this with the two following, that *their* increase has continued to take place upon the same plan, each row that is put-forth from its predecessor terminating abruptly like it at its free extremity, without any such disposition to extend itself as would carry it round the nucleus so as to form a complete annulus. A horizontal section of such a specimen as is represented in fig. 10, is shown in fig. 12.—It may, of course, be urged that such a difference ought to be accounted sufficient to separate the *spiral* and the *discoidal* types of Orbiculina, as two distinct species; but the following reasons appear to me quite sufficient to negative such a mode of viewing

them:—First, they so closely resemble one another, as to be undistinguishable, in their early condition. Second, they correspond in every particular, so far as regards the structure of their minute parts. Third, the assumption of the cyclical plan of growth does not take-place at any one fixed epoch of development, but may occur at various periods. Fourth, the persistence of the original plan of growth throughout life, cannot be fairly regarded as anything else than an arrest of development, such as we shall presently see to be a common occurrence in *Orbiculina*, as in *Orbitolites*, in regard to other particulars (¶ 90).

88. Turning now from the general plan of growth to the minute structure of the individual parts of *Orbiculina*, we continue to find a very close conformity to the type of *Orbitolites*. The texture of the shell is precisely the same; and it exhibits no other peculiarity than a minute punctation of the superficial layers (Plate XXVIII. fig. 13), which at first suggests the idea of apertures*, but which is found on careful examination of transparent sections (Plate XXIX. fig. 2) to be due to a mere thinning of the shell at certain points, so as to give an appearance of cellular areolation closely resembling what is seen in *Orbitolites* (Plate VI. fig. 5).

89. Not having had the good fortune to obtain specimens in which the animal body has been preserved, I cannot speak as confidently on the subject of its conformation, as I could in regard to that of *Orbitolites*; but a comparison of the features which present themselves in the structure of the calcareous skeleton of the two types, can leave no reasonable doubt that the general arrangement of its segments of sarcode must have been precisely the same. For the conformation of the chambers and passages, as displayed by a horizontal section (Plate XXIX. fig. 1), shows that the soft body must consist of a succession of bands of sarcode, each band swelling at intervals into larger segments; that the segments of each band usually alternate with those of the bands internal and external to it, so as to be opposite to the intervals between them; and that the stolons which connect one band with another pass-forth from pores in the intervals between the segments of one, into the segments of the next, those of the outermost band emerging from the margin as pseudopodia. By the coalescence of these a new band would originate, which would become thickened into segments opposite the pores of the preceding, and would give-off its own pseudopodia from the intervals between the segments.

90. Again, by an examination of the natural margins of *Orbiculinæ* (Plate XXVIII. figs. 6, 7, 18, 19), and by a comparison of these with vertical sections (Plate XXIX. fig. 3), it becomes evident that the same variety exists as in *Orbitolites*, in regard to the increase of the disk in thickness by the vertical elongation of the segments of sarcode. For some specimens are altogether conformable to the 'simple type' of *Orbitolites*, in having but a single floor of chambers (so to speak), with a single row of marginal pores; whilst others correspond with the complex type of *Orbitolites*, in

* Professor WILLIAMSON (*loc. cit.*) has described these punctations as *perforations* for the passage of pseudopodia; but I am quite certain that such is not the case.

having many such floors, with numerous rows of marginal pores. In the former, the segments of sarcode, with their single annular stolon, would resemble rounded beads strung at short distances on a cord. In the latter the segments would be columnar, with constrictions at intervals, and would communicate with each other by two or more annular stolons.—It is true that we seldom find any such complete differentiation of the superficial cells, as the fully-developed type of *Orbitolites* presents; but we have seen that such differentiation is by no means a constant character in that genus (¶ 58); and the structure of the most developed specimens I have examined among the recent *Orbiculinæ* closely corresponds in this respect with that of the fossil *Orbitolites* of the Paris basin, as will be seen on comparing the left-hand portion of Plate XXIX. fig. 3 with Plate VI. fig. 11 of my former memoir. But it is not a little remarkable, that while the fossil *Orbitolites* of the Paris tertiaries are *less* developed in this respect than their existing representatives in the South Seas, the fossil *Orbiculinæ* of the Malabar tertiaries should be *more* developed than existing specimens of the same type; for in them we find the superficial cells differentiated from the intermediate layers (Plate XXVIII. fig. 20), precisely as in *Orbitolites**. Looking to the far larger dimensions, as well as to the higher development, which these fossils present (¶ 83), as compared with the existing specimens of *Orbiculina*, I am disposed to believe that this type attained its highest evolution in a period long since passed, and that we now have, so to speak, only the degenerate descendants of an ancestry of higher rank; whilst in the case of *Orbitolites*, I am inclined to think that the type is most fully evolved at the present time.

91. *General conclusion.*—From the foregoing details it is obvious that the relationship between *Orbitolites* and *Orbiculina* is extremely close; the only essential point of difference between them being that which is furnished by the structure of the nucleus. Whether or not they ought to rank as types of *distinct genera*, or whether they ought (as Professor WILLIAMSON maintains) to rank as *cognate species* of the same genus, is a point as to which it is impossible to arrive at a satisfactory conclusion, until the characters which should serve for the distinction of genera and species in this class shall have been determined on a physiological basis. This much I think myself entitled to assert with confidence,—that even if they are to be regarded as distinct genera, they must be ranked in the *same family*, and in immediate proximity to each other; and that no classification can have any claim to be considered as *natural*, in which they shall be widely separated.

Genus ALVEOLINA.

92. *History.*—Although the form and aspect of the Foraminifera which are referable to this genus, would seem to remove them altogether from proximity to the pre-

* Although Mr. CARTER has described this fossil as a species of *Orbitolites*, yet it is really an *Orbiculina*, as is shown by the spiral conformation of its central portion (Plate XXVIII. fig. 17), and by the investment afforded by each turn of the spire to its predecessor, as shown in fig. 18.

ceding, yet the two really bear a very close relationship in all essential points of minute structure, as will appear from the particulars I shall presently have to detail. The following outline of what has been previously ascertained respecting it (for which I am indebted to M. D'ORBIGNY*), will show how little the nature of its organization has hitherto been understood.—Most of the species at present known are fossils, occurring in association with *Nummulites*, *Orbitolites*, &c. in the Nummulitic limestone, or in other formations which represent it; and the examples first described (by FORTIS) were confounded with Nummulites and Orbitolites under the term *Discolites*. By FICHTEL and MOLL, they were ranked as a sub-type of their comprehensive genus *Nautilus*. The designation *Alveolites* was first given to this type by BOSC†; but it was not generally adopted; and MONTFORT, according to his wont, raised three of Bosc's species to the rank of genera, under the names of *Borelia*, *Clausalia*, and *Meliolites*. LAMARCK did not adopt either Bosc's or MONTFORT'S generic designations, but substituted a new one, *Melonia*; and this was adopted by CUVIER and FERUSSAC. DEFRANCE proposed yet another name, *Orizaria*. And finally M. D'ORBIGNY, in 1825, adopted Bosc's name, with a slight alteration in its termination, which served at the same time to mark the continued existence of the type, and to distinguish it from a genus of Corals which also had received the name of *Alveolites*. The name *Alveolina* was soon afterwards adopted by M. DESHAYES; and it may now be considered as the established designation of the genus. The following is the latest definition given of this type by M. D'ORBIGNY:—"Coquille allongée dans le sens de l'axe d'enroulement, formée de loges divisées par des canaux capillaires, ronds, percées de nombreuses ouvertures placées en lignes transversales à l'enroulement spiral." Not the least idea seems to me to be conveyed by this definition of the real structure of these bodies, such as is brought into view by thin sections; and no one, so far as I am aware, has previously attempted thus to elucidate it.

93. *Organization*.—My investigations have been made upon specimens which were tolerably abundant both in Mr. JUKES'S Australian dredgings, and in Mr. CUMING'S Philippine Collection. These were obviously identical specifically, but the latter considerably exceeded the former in average size. The length of the longest complete specimen in my possession is ·35 of an inch; but I have a specimen whose shape is somewhat abnormal—indicating that it has increased with unusual rapidity in length, as proportioned to its diameter,—which, though incomplete at one end, measures ·50 of an inch. The ordinary form, from which any considerable departure is very rare, is that which is exhibited in Plate XXVIII. fig. 23; and it is obviously produced, as correctly stated by M. D'ORBIGNY, by the involution of a spiral around an elongated axis. The surface is marked-out by longitudinal furrows into a succession of bands of tolerably uniform breadth; and each of these is crossed by secondary furrows,

* Foram. Foss. de Vienne, p. 140.

† Bulletin des Séances de la Société Philomathique, No. 61.

which lie so closely together as to mark-out each band into a series of very elongated cells, which remind us of the oblong superficial cells of the complex type of *Orbitolites*, but are much narrower in proportion to their length. The mouth of the spire is closed by a solid wall, the surface of which is nearly flat, and which is perforated by three, four, or five rows of rounded pores, bearing a very close resemblance to those at the margin of *Orbitolites* and *Orbiculina*.

94. The resemblance suggested by external configuration is fully borne-out by the examination of the internal structure of this genus, as brought into view by longitudinal and transverse sections. For it is shown in Plate XXVIII. fig. 24, that the whole organism has originated from a single large nearly-globular cell, around which are successive layers of chambers freely communicating with each other, each layer completely enveloping the preceding, and adding much more to its length than to its diameter. A portion of such a section, more highly magnified, is shown in Plate XXIX. fig. 9. The formation of the successive layers by spiral involution, is well displayed by a series of transverse sections taken at different points, such as are delineated in Plate XXIX. figs. 4 to 7; the first (fig. 4) having crossed near the narrow termination of the fusiform shell, and having consequently traversed only the two last-formed turns; the last having crossed near the middle of its length, and having traversed all the six turns by which it was formed. Each whorl seems to be, as to all essential particulars, a repetition of the rest; and hence it will be sufficient to make a detailed examination of only a small part of a section, such as is shown on a larger scale in fig. 8. It is there observed that the spaces occupied by the sarcode-body and bounded by the shell, though far from regular, conform to a certain general plan. Each lamina is made-up (as the surface-furrowing and the marginal pores indicate) of a succession of bands of superimposed cells, of elongated form; each band answering to one of the annuli of *Orbitolites*. At each point corresponding with the external furrow, the superficial layer of shell sends a prolongation inwards (*a, a, a*), which thus marks-off the superficial cells more completely from each other than is the case with the subjacent cells; just below this projection is a rounded perforation (*b, b, b*), which marks the passage of a longitudinal channel; and at a little distance beneath this is another (*c, c, c*), which is of very considerable dimensions. Sometimes there are more than two such channels; but this is comparatively rare. These channels must have given passage to longitudinal bands of sarcode, running from one end of the lamina to the other, and freely connecting together all the segments occupying the piles of elongated cells of which it is composed. The marginal pores are the orifices of the chambers of the last-formed band; and it will be observed in Plate XXVIII. fig. 23, that one row of them is contiguous with the preceding whorl of the spire, along which, as is shown in Plate XXIX. fig. 8, there is always a free passage. Thus we may consider the sarcode-body, as in the case of Orbitolite, to be composed of a mass of closely-connected segments, between which a calcareous skeleton grows-up according to a certain tolerably-regular type. It seems to me

probable from the conformation of the shell, that the segments forming successive bands do not communicate directly with each other so much as with the great longitudinal stolons; just as the successive annuli of superficial cells in Orbitolites communicate with each other, not immediately, but through the annular stolons (¶ 28). The pseudopodial prolongations issuing from the marginal orifices, probably first coalesce into a longitudinal cord of sarcode, when a new band of cells is to be formed; and from this another series of segments is then budded-off.—In the texture of the shell, in the relations of the different chambers, in their mode of communication with the exterior, in all (to speak concisely) which marks the physiological condition of this organism, its conformity to the types previously described is so close, that, notwithstanding the marked difference in their mode of increase (on which depends their form), they must rank, in any natural classification, in very close proximity with these*.

Genus Cycloclypeus.

95. The organisms which I have now to describe, and to which I shall give the generic designation *Cycloclypeus* (suggested to me by Dr. J. E. Gray), are amongst the most interesting of all the Foraminifera at present existing; on account both of the large dimensions which they sometimes attain, and also of the complexity of their structure. The only specimens of them yet known, were dredged by Sir Edward Belcher from a considerable depth of water off the Coast of Borneo. Two of these, which are now in the British Museum†, are complete disks measuring no less than $2\frac{1}{4}$ inches in diameter; and by the kindness of Dr. J. E. Gray, I have had the opportunity of making microscopic sections of a fragment of a disk, which, when entire, must have nearly equalled these in size. Smaller disks of various dimensions presented themselves in the same dredgings.

96. *Organization.*—The external aspect of these disks is sufficiently like that of *Orbitolites*, to prevent the two genera from being readily distinguished by a superficial examination, especially when young specimens of *Cycloclypeus* are compared with *Orbitolites* of the complex type; since, on the two snrfaces of the former (Plate XXX. fig. 1), there can be distinguished concentric rings of oblong chambers, which are not at all unlike the similarly-disposed superficial cells of the latter. The peculiarly-compact texture of the shell of *Cycloclypeus*, however, gives to its surface a smooth and glistening appearance, which is very different from that of *Orbitolites*. And further, the forms of the two disks ordinarily differ in this,—that whilst the centre of *Orbitolite* is usually rather depressed than elevated, and the thickness

* I have confined myself to an account of the existing species, as I have not had the opportunity of making a similar examination of any large number of fossil forms of this type. It is well, however, for me to mention, that the existing species seems to me to be certainly identical with the *A. Boscii* of the Paris tertiaries.

† These are the disks referred-to by Professor Williamson in his Memoir on Orbitolites, &c., Trans. of Microsc. Soc. ser. 1. vol. iii. p. 127. He appears to have considered them as gigantic Orbitolites, not being acquainted with their peculiarities of internal structure.

of the disk generally increases towards the periphery, the central portion of *Cycloclypeus* always presents a knobby elevation, on the surface of which the oblong boundaries of the chambers are superseded by rounded 'punctations,' whilst the thickness of its disk gradually diminishes towards its margin, where it is so reduced as to come to a sharp edge. In older specimens of *Cycloclypeus*, the boundary-markings of the chambers are scarcely distinguishable, save near the margin; their concentric annuli are marked-out, however, by rows of 'punctations,' similar in appearance to those of the central eminence. In either case, it is usually observable that the breadth of the annuli is far from constant; and that the annuli are not unfrequently incomplete, extending round only a portion of the disk. This irregularity has been noticed in *Orbitolites* (¶ 20) as of rare occurrence; in *Cycloclypeus* it is so common that I have not yet met with specimens which are entirely free from it.

97. Whilst agreeing with *Orbitolites* in those external features which result from the *cyclical* mode of growth that is common to both forms, *Cycloclypeus* presents as wide a contrast to it in every other feature of its organization, as is anywhere known to exist within the limits of the Foraminiferous group. For whilst, as we have seen, the general plan of structure of *Orbitolites* removes it but little from the grade of Sponges—the several segments of its aggregate body being but very imperfectly separated one from the other, and the shell which grows-up in the midst of them having no discoverable organization,—that of *Cycloclypeus* closely approximates to the Nummulitic type, in which the successive segments are as completely isolated as they can be without entire disconnection, and in which, by the peculiarly-elaborate construction of the shelly covering, a special provision is made for their independent nutrition.

98. On making horizontal and vertical sections of the *Cycloclypeus*-disk, its central plane is found to be occupied by chambers, disposed (ordinarily in a single layer) in concentric annuli; these being covered-in above and beneath by compact plates of shell, which are thicker towards the centre, thinner towards the circumference (Plate XXX. fig. 1). The typical form of these chambers seems to be a parallelogram with its angles rounded off, whose sides are to each other as $1\frac{1}{2}$ to 1, or as 2 or even 3 to 1, the longest side lying in the direction of the radius of the disk; but owing to the variation in the *length* of the chambers which results from the before-mentioned irregularity in the breadth of the annuli (¶ 96), the *breadth* of the chambers remaining more constant, their proportions vary greatly in different parts of the same annulus, or in adjacent parts of different annuli, as shown in Plate XXIX. fig. 12. I have occasionally met with chambers whose length was to their breadth as 4 to 1 (Plate XXXI. fig. 3). The vertical thickness or depth of the chambers, seems usually to be pretty constant in different parts of the disk, except near its centre; the thinning-away towards its margin being due, not so much to a diminution in the vertical height of the chambers, as to the reduction of the thickness of the shelly plates that enclose them above and below.

99. But although the existence of only a single layer of chambers is obviously the rule in this species, yet exceptions to it are not unfrequent; a subdivision of the entire stratum into two or three presenting itself when its thickness is above the average, as is shown in Plate XXXI. fig. 8, *a*. Occasionally one or two chambers only are thus subdivided, as shown in figs. 4, 5. The cavity of each chamber is surrounded by a proper wall of its own, quite distinct from that of the chambers which it adjoins; and hence the septum by which each chamber is divided from the adjacent one on either side, is formed of at least two lamellæ (Plate XXIX. fig. 12). These come into close contact with each other at the junction of the vertical septum with the horizontal roof and floor of the chamber, as shown in Plate XXXI. figs. 2, 4, 5; but elsewhere they diverge from one another, leaving an interseptal space, which is partly filled-up by an interposed lamina of shell-substance, but is partly occupied by the *interseptal canals* to be presently described. A thicker space of the same kind is in like manner left between the proper walls of the chambers forming one annulus, and those of the chambers forming the annuli internal and external to it: this space is almost entirely filled-up by a shelly deposit, the interseptal canals which pass between the successive annuli being less numerous than those which run between the chambers of the same annulus (¶ 105).

100. As in *Orbitolites*, the chambers of each annulus usually alternate in position with those of the annuli internal and external to it. But this is by no means constantly the case; since additional chambers are 'interpolated' here and there, so as to increase the number according to the augmented diameter of the annulus; and such an interpolation disturbs the regular arrangement of the neighbouring chambers.

101. The adjacent chambers of the same annulus have not, so far as I have been able to ascertain, any direct communication with each other; an indirect communication, however, is perhaps established through the system of interseptal canals. But each chamber normally communicates with two chambers of the annulus within it, and also with two of that which surrounds it, by large passages (shown in horizontal section in Plate XXIX. fig. 12, and in vertical section in Plate XXXI. fig. 2, *c*, *c*, and represented in perspective view in Plate XXX. fig. 4, *ff* and *g g*), which traverse the annular septa; of these passages there seems to be normally but a single one for each pair of chambers thus to be brought into communication; but I have frequently met with two, and occasionally three, one placed directly or obliquely above the other. Thus at each extremity of the oblong chamber, there are normally two passages leading to two chambers of the annulus next internal or external to it; but since to each of these chambers there may be two or even three passages, the total number at each end may be three, four, five, or six. The variations as to this point of structure which are presented in adjacent chambers, are shown in Plate XXXI. fig. 2. But since, on the other hand, each of the 'interpolated' chambers communicates with only one chamber in the annulus next internal to it, there may be but a single passage, in place of two or more.

102. The shelly plates which bound the chambered plane above and below, are formed of a succession of superimposed lamellæ (Plate XXXI. fig. 10). These lamellæ, which are of tolerably uniform thickness, are most numerous in the older or more central portions of the disk, and diminish in number towards the marginal or last-formed portions; so that it seems pretty certain that new lamellæ must be added from time to time, as the disk is augmented by the formation of new annuli. I have often met with appearances, which might seem to indicate that the formation of a new lamella over the entire surface of the disk, and the addition of a new annulus at its margin, were parts of one and the same act of growth, the new lamella being continued into the annular septum; but if this were constantly the case, the number of lamellæ which form the ceiling or floor of any chamber, would always correspond with the number of annuli external to it, which I do not find to hold-good.

103. Each of these lamellæ is perforated by an assemblage of parallel tubuli very closely set-together, which pass from its inner towards its outer surface (Plate XXXI. figs. 9, 10); and there is such a continuity between the tubuli of successive lamellæ, that a communication is thus established between the cavity of the thickest-walled chamber, and the external surface of the disk. These tubuli, however, are very minute, their diameter being not above $\frac{1}{10,000}$th of an inch. They are wanting in certain parts of the shell, which then presents a transparence that contrasts strikingly with the semi-opacity produced by the tubular perforations. By the comparison of vertical with horizontal sections taken in different planes, it appears that these transparent portions of the shell have a conical form, the base of each being on the surface of the shell, and its apex pointing to one of the angles at the outer margin of a chamber (Plate XXX. fig. 4, *c c*, *d d*). Their gradual widening towards the surface causes the diameter of their bases to increase with every addition to the thickness of the shell; and thus it is on the older portion of the shell, and especially on its central protuberance, that they become most conspicuous as rounded 'punctations' (§ 96). In horizontal sections of the superficial lamellæ, they form a large proportion of the area (Plate XXXI. fig. 6); whilst in similar sections near the chambered plane (fig. 9), they become blended with angular projections of the annular partitions, that fill-up the spaces left between the proper walls of the chambers by the rounding-off of their angles.

104. The lamellated structure is seen in these conical pillars (Plate XXXI. fig. 10, *b*), the lamellæ being continuous with those of the tubular part of the shell; so that at each increase in thickness, a tubular and a non-tubular portion must be superimposed upon the corresponding parts of the preceding lamella. Both in the tubular structure of the shell, and in the presence of these non-tubular columns, there is an exact conformity to the structure of *Nummulite* and its congeners*.

* I avail myself of this opportunity of correcting a mistake into which I fell in my original description of the structure of *Nummulite* (Quart. Journ. of Geol. Soc., 1850, p. 26), in regarding the non-tubular *columns* of the shell as having been *passages* which had become filled-up by the infiltration of carbonate of lime

105. I have now to speak of another feature in the structure of this organism, which most strikingly differentiates it from *Orbitolites* and its congeners, and at the same time furnishes an additional proof of its close approximation to *Nummulites*, notwithstanding the difference in its plan of increase. I allude to the system of *interseptal canals*, which establish a direct communication between the external surface, and the parts of the interior most removed from it. Such *radial* canals are seen both in horizontal and vertical sections (Plate XXIX. fig. 10, Plate XXXI. fig. 4, *c*) excavated in that shelly substance, which occupies part of the space that intervenes in the radiating partitions between the proper walls of adjacent chambers of the same annulus. When the canal reaches the end of the radial septum, it usually subdivides into two, which diverge at a considerable angle from each other, so as, by traversing the annular septum, to reach the two alternating radial partitions of the next annulus; and as each branch, before entering the partition towards which it runs, unites with another branch that inclines towards it from the radial canal next adjacent, it follows that just as every chamber communicates (normally) with the two alternating chambers in the annuli internal and external to it, so do the interseptal canals of every radiating partition communicate with those of the partitions alternating with it in the internal and external annuli. This arrangement, which cannot be described verbally without some complexity, will be readily comprehended by an inspection of Plate XXIX. fig. 11. In each radial partition there are at least two, and very commonly three tiers of such canals, as is best seen in vertical sections that cross the radial partitions transversely (Plate XXXI. figs. 4, 5). Short transverse branches, apparently communicating with the cavity of the chambers (Plate XXIX. fig. 11), are sometimes seen to proceed from the longitudinal canals; in regard to these communications I would not speak with confidence from what I have seen in *Cycloclypeus*; but that they exist in other organisms, hereafter to be described, is unquestionable. There can be no doubt, moreover, that the horizontal radiating canals communicate with *vertical* canals which pass directly towards the two surfaces of the disk, whereon they open (Plate XXXI. fig. 5, *c*); these canals are best seen in horizontal sections taken near the upper or under surfaces of the chambers (Plate XXXI. figs. 3, 9), in which they present themselves in regular rows, *d*, *d*, corresponding to the radial partitions; whilst in similar sections taken nearer the surface, they are seen to be less regularly disposed, in consequence of their following a somewhat oblique

in the process of fossilization. I was led to this by the very marked contrast which exists in *Nummulite* between the tubular, and non-tubular portions of the shell, and the peculiarly inorganic semi-crystalline appearance of the latter, closely resembling that of the calcareous infiltration which usually occupies the interior of the chambers. Subsequent examination, however, of *Nonionina* and other recent forms most closely allied to *Nummulite*, has satisfied me that these columns were part of the original shell, as my friend Professor WILLIAMSON maintained from the first. It is not a little curious, however, that in certain other species of *Nummulite* described by MM. D'ARCHIAC and HAIME, a system of passages should exist, very analogous to those which I thought I had discovered in *N. lævigata*.

direction. Still their continuity is maintained through all the successive layers of which even the thickest part of the shelly disk may be composed.

106. Besides the radial and vertical systems of canals, there is an *annular* system, which traverses the thick band of shell-substance that usually intervenes between the successive annuli, and which is continually brought into view in horizontal sections (Plate XXIX. figs. 10, 12). It appears from vertical sections traversing the annular septa, that several tiers of these annular canals may exist. I have frequently traced them running continuously for a considerable distance, without appearing either to give off any branches, or to communicate with the radial canals; but I have occasionally seen appearances which indicate that such a communication is established by means of canals passing vertically downwards at the angles of the chambers, so as to unite the three sets of canals into one continuous system, furnished with a multitude of orifices upon the surface of the disk. A representation of the whole canal-system, as I believe it to exist in this organism, is given in Plate XXX. fig. 4.

107. The uses of this canal-system can only be a matter of speculation. Not having had the opportunity of examining specimens in which the soft animal substance had been preserved, I am unable to affirm whether the interseptal canals of *Cycloclypeus* are occupied in the living state by a portion of the sarcode-body, or whether they are empty; but as I have unquestionable evidence that the former is the case in *Polystomella*, I should think there can be little doubt that it is also true of this genus. Now if we come to examine the purpose of this canal-system, we are at once struck with the fact, that it can scarcely be requisite for the nutrition of the segments of the sarcode-body enclosed within the chambers; since the mutual communication which these segments have with each other, seems fully as adequate for the purpose in *Cycloclypeus*, as it is in *Orbitolites*, *Orbiculina*, or *Alveolina*. If we examine wherein this organism so differs from the foregoing as to require such an additional system, we may find a not improbable answer in the possession of that additional skeleton which intervenes between the proper walls of the chambers; for the canal-system, excavated in the very substance of this, would seem to furnish the appropriate channel for its nutrition. And that such is its object, will be shown in a future memoir to be almost certainly proved, by the comparison of facts then to be adduced from the structure of other genera.

108. *Monstrosities.*—Although the number of specimens of this type which I have had the opportunity of examining is but small, yet two among them exhibited the same kind of monstrosity as that which is common in *Orbitolites*; namely, the superposition of a vertical plate upon the horizontal disk (Plate XXX. fig. 3). And in each it is sufficiently apparent that this plate has originated from the central cell, and that its increase has taken place *pari passu* with that of the horizontal disk.

109. *General Summary.*—If, now, we review the principal facts relating to the structure of *Cycloclypeus*, and compare them with those furnished by *Orbitolites* on the one hand and by *Nummulites* on the other, we shall see that, notwithstanding

its resemblance to the former in external aspect and plan of growth, it is far more closely allied to the latter in those features of its organization which indicate its physiological condition. It has been shown that in *Orbitolites* the communication between the different portions of the sarcode-body is so free, that the whole may be regarded as a continuous mass in which the segmental division is but imperfectly indicated (¶ 67); and this view is in complete harmony with the fact, that every addition made to the shelly disk forms (save in a few rare cases) an entire annulus. In *Cycloclypeus*, on the other hand, the chambers are so completely separated from each other laterally, that no other communication exists between them than such as may be established by the interseptal canals; and the communications between the chambers forming successive annuli, are only large enough to allow the passage of narrow bands of sarcode. Hence we see that there is here as much segmental independence as is consistent with the existence of these animals, which involves the maintenance of a communication between the innermost and outermost chambers, for the transmission of nutriment to the segments of the sarcode-body contained within the former. And this independence is strikingly manifested, by the frequency with which incomplete annuli are added to the previous margin of the disk, extending (it may be) along not more than a third, a half, or two-thirds of the entire circumference. The want of constancy in the number and position of the communications between the chambers (¶ 101), even in this high type of Foraminiferous structure, is a point of fundamental importance in the determination of the value of the shape of the aperture, as a character of discrimination between genera and species in this group of organisms. I have elsewhere shown that a like want of constancy exists in *Nummulites* (*op. cit.* p. 24), the different septa of one and the same specimen having apertures of very varied forms. We are not in a condition to assign a positive function to the minute tubuli that traverse the shelly layers intervening between the chambers and the two surfaces of the disk. But it is quite possible that these tubuli may give passage to pseudopodial prolongations of extreme minuteness, which may spread themselves forth from the whole surface of the disk, as we know that they do from the larger pores of *Rotalia*, and may coalesce so as to form upon it a continuous layer of sarcode, by whose instrumentality a lamina of shell is added from time to time to those previously existing. That either by giving passage to threads of sarcode, or by conveying organizable fluid, they furnish the means for progressive increase of the shell in thickness, would seem a very probable account of their use. Whether this hypothesis, however, be correct or not, there can be no reasonable doubt that the minute organization of these shelly layers in *Cycloclypeus* and *Nummulites*,—an organization as high as that of dentine,—is a feature of high elevation, as compared with the simple concretionary condition of the calcareous skeleton in the three genera previously examined.

110. Thus, then, in the almost complete isolation of the segments, in the enclosure of each of them in its own proper wall, in the interposition of an intermediate skele-

ton and of a canal-system between the contiguous walls of adjacent chambers, and in the minutely-tubular structure of the shell,—all of them points of high physiological importance,—*Cycloclypeus* differs entirely from *Orbitolites*, and agrees with *Nummulites*; whilst it agrees with *Orbitolites*, and differs from *Nummulites*, in the single circumstance that its mode of increase is cyclical instead of helical,—a difference which we have seen to present itself at two different periods of life of the very same specimens of *Orbitolites* and *Orbiculina*, and which must, therefore, be a character of quite subordinate importance.

Genus Heterostegina.

111. The correctness of the views just advanced is fully borne-out by the occurrence of a type, which bears precisely the same relation to *Cycloclypeus*, that *Orbiculina* bears to *Orbitolites*; one, namely, in which—the form and connexions of the individual chambers, the minute structure of the shell, and the distribution of the canal-system, being essentially the same—the plan of growth is *helical*, at least during the earlier period of life. This is the case with the genus *Heterostegina*, which was established by M. d'Orbigny in his memoir of 1825, but the essential structure of which he has altogether misapprehended. In his latest classification of the Foraminifera*, he ranks this genus in his order *Entomostègues*, which is composed of Foraminifera, whose segments are disposed in a spiral, but in two different planes alternating with each other, so as to render the entire shell inequilateral. Of the genus *Heterostegina*, which he ranks in close approximation to *Amphistegina*, he gives this definition:—"Coquille à spire embrassante, dont les loges sont séparées intérieurement par des cloisons transversales." Now in the first place, I am quite satisfied that the chambers of *Heterostegina* do not alternate one with another, but are arranged in one plane about the same axis, as is shown in figs. 1, 7, Plate XXXI.; and secondly, it gives by no means a correct idea of its structure, to liken its chambers to those of *Amphistegina* save for their division by transverse partitions.

112. I have had the opportunity of examining, by the kindness of Mr. Cuming, a very extensive series of specimens of this genus (belonging, apparently, to the species *H. costata*, D'Orb.†), from the Philippine islands; many of these are of large size, attaining as much as half an inch in diameter; and the appearance of the adult specimens scarcely differs less from that of the young (which latter are alone figured by M. d'Orbigny), than it does in the case of *Orbiculina*. The dredgings of Mr. Jukes have furnished me with numerous specimens of *Heterostegina* from the Australian coast; these closely correspond with the figures of M. d'Orbigny, being of comparatively small size, and not exhibiting that peculiar mode of development which is characteristic of the adult. As the Australian forms correspond precisely with the young of the Philippine, there can be no doubt of their specific identity. I recognise the shells of the same species as almost the sole components of a fossilized deposit,

* Cours élémentaire de Paléontologie, tom. ii. p. 201.

† Foram. Foss. de Vienne, p. 212.

which I understand to be very commonly met-with in Malta in fissures of the rocks, but of which the age is uncertain.

113. *Organization.*—The older specimens of *Heterostegina* (Plate XXX. fig. 2) present a form which, when regular, may be characterized as discoidal. There is, however, a knobby elevation or nucleus, which is usually somewhat excentric; and from this the turns of a spire are seen to commence, the last of which usually becomes continuous with one part of the margin of the disk (*a b c*), which there possesses a thick and defined border. As this spire opens-out, however, it becomes thinner and flatter; and this thinning is especially noticeable at that part of the margin of the disk (*a d c*) which corresponds with the opening of the spire. An examination of this portion of the disk shows that it precisely corresponds in structure with *Cycloclypeus*; the form and disposition of the chambers, their mode of communication, the structure of their shelly walls, and the interposition of the intermediate skeleton and of its canal-system, being all points of such close resemblance, that, as there is positively no other point of difference than a somewhat inferior thickness of the intermediate skeleton between the successive rows of chambers in *Heterostegina*, a fragment of this marginal portion of the spirally-formed disk of *Heterostegina* might be taken for a fragment of the cyclical disk of *Cycloclypeus*, without the possibility of certainly distinguishing them, and *vice versâ*. It is interesting to observe, moreover, how close is the conformity of these two types, even as regards their irregularities; for it will be seen, on an inspection of the figure, how little uniformity there is in the breadth of the successive rows of chambers, and how frequently it happens that a row is incomplete, just as in *Cycloclypeus* (¶ 96).

114. If, now, we examine the structure and arrangement of that spirally-coiled portion of the disk, which constitutes its nucleus, and which is best shown in younger specimens, we see that, as in the other cases, the first chamber (Plate XXXI. fig. 1, *a*) is globular, that the second (*b*) buds-forth from one side of this, and each successive chamber from the outer side of the preceding, just as in *Nummulite* or any other simple helical form. But before one turn of the spire is completed, each newly-formed chamber is seen to be double (*c c*) instead of single, a small portion being divided-off (as it were) near the marginal part of the whorl; and just about the part where the second turn is completed, the gradual opening-out of the spire gives room for the interposition of a third chamber in each row (*d*); and the number is soon further augmented, in accordance with the progressive increase in the breadth of the spire, the dimensions of the individual chambers retaining a pretty close conformity to a constant average. An examination of this figure will further show, that the increase in the number of chambers in successive rows always takes-place at the inner margin of the spire; some of those nearest the outer margin dying-out, as it were, without giving origin to new chambers in the next row. This may, I think, be connected with the fact, that there is always a large free opening (*e*, *e*) between one row of chambers and the next, at the inner margin of each spire (the situation of the open-

ing in *Nummulite*), and that the chamber of the row abutting on the preceding whorl is nearly always much larger than the rest, and gives origin to two or even three chambers in the next row. Further, it is shown by vertical sections (Plate XXXI. fig. 7), that the innermost chambers of the whorl are not only broader but thicker, their upper and under walls diverging from each other where they are to be continued over the spire they invest. Hence it is pretty obvious, that this portion of the whorl is that wherein the most active nutrition takes place; and it is here that the marked accession to the number of chambers occurs, which tends to carry the later rows around the whole circumference of the disk.

115. Each of the early turns of the spire not only surrounds, but completely invests its predecessor; as is best shown by a vertical section, such as that represented in fig. 7. The investing whorl does not, in the younger part of the spire, come into immediate contact with the two surfaces of that which it includes, but is separated from it by the prolongation of the chambers and of their septa, very much as in ordinary *Nummulites*. But between the later whorls, there are no such interspaces. The successive layers come into absolute continuity with one another; both the tubuli and the cones of non-tubular substance being continued from each into the one external to it. From the time that the rapid thinning-away and opening-out of the spire commences, the investment of the previously-formed whorls seems to discontinue. It is at the margin of each whorl, that we find the intermediate or additional skeleton most remarkably developed; and the canal-system sometimes forms quite a network in its substance (fig. 11).

116. *General Summary.*—It is obvious, from the foregoing details, that the physiological condition of each individual segment of the animal of *Heterostegina* must be essentially the same as that of each segment of *Cycloclypeus*; and that the only difference in the condition of the two organisms arises out of the mode in which these segments are increased in number. In *Cycloclypeus*, in which each row of segments is (normally at least) a complete annulus, a new annulus is formed around its predecessor by gemmation from its several segments along the entire circumference; and this mode of increase may be traced-back to the central cell, which buds-out equally on all sides. But in *Heterostegina*, each row is limited by the breadth of the spire; and while most of its chambers are formed by the like kind of gemmation from their predecessors, there is a special provision for an augmentation in the number of chambers at the end of the row nearest the previous whorl. Tracing-back the spire to its origin, we find that it commences in the one-sided gemmation of the central cell, just as we found it to do in *Orbiculina* (¶ 85) and in those forms of *Orbitolites* which have a spiral commencement (¶ 54). Hence there is nothing but the plan of increase, which separates *Heterostegina* from *Cycloclypeus*; and their relation is exactly the same as that of *Orbiculina* and *Orbitolites*. For in *Heterostegina*, as in *Orbiculina*, the first-formed portion is a spire, of which each turn invests its predecessors; but after three or four turns have been made, the spire spreads-out,

tends to surround the whole disk with its mouth or growing margin, and thenceforward the growth is cyclical, as in *Cycloclypeus* and *Orbitolites.*

Concluding Remarks.

117. Looking now to the general organization of the five genera which I have described, and to the peculiarities by which I have shown that each is characterized, I think that we are in a position to inquire into the value of the system of classification which has been erected by M. D'ORBIGNY on the exclusive basis of plan of growth; on which inquiry, the facts which I have now brought together have an obvious and direct bearing.

I. The very close physiological relationship which has been shown to exist between *Orbitolites* and *Orbiculina*, requires that they should be associated in the same Family, if not in the same Genus. In the classification of M. D'ORBIGNY they are ranked under different Orders (*Cyclostègues* and *Helicostègues*).

II. In like manner, the close physiological relationship which has been shown to exist between *Cycloclypeus* and *Heterostegina* requires that they should be associated in the same Family, if not in the same Genus. In the classification of M. D'ORBIGNY they would be ranked under different Orders (*Cyclostègues* and *Entomostègues*).

III. Again, the strongly-marked physiological difference which has been shown to exist between *Orbitolites* and *Cycloclypeus,* would seem to require that they should be separated by the widest possible interval; yet the system of classification adopted by M. D'ORBIGNY would have forced him to associate them (if he had been acquainted with the last-named type) side by side in the same Order (*Cyclostègues*).

IV. In like manner, the corresponding difference which has been shown to exist between *Orbiculina* and *Heterostegina,* would seem to require that they should be separated by the widest possible interval; yet the system of classification adopted by M. D'ORBIGNY would have forced him to associate them (if he had been acquainted with the real plan of structure of the last-named type) side by side in the same Order (*Helicostègues*).

V. The doctrine which I base on the foregoing facts,—that physiological conformity in the condition of each individual segment, as indicated by the structure of its shelly investment, is a character of primary importance, whilst the plan of growth, that is, the mode of increase in the number of chambers, is a character of subordinate importance,—is further borne-out by the following considerations:

1. In *Orbitolites,* the general plan being cyclical, the early plan of growth is frequently spiral.
2. In *Orbiculina,* while the early plan of growth is uniformly spiral, and this is sometimes continued throughout life, it is very commonly exchanged in adult age for the cyclical.
3. In *Alveolina,* whose physiological approximation to *Orbitolites* and *Orbiculina* is unquestionable, a plan of growth is followed, which differs more from

that of either of them, than the plans of the two latter differ from each other.

4. In *Heterostegina*, as in *Orbiculina*, the early plan of growth being uniformly spiral, there is a tendency in adult age to the assumption of the cyclical.

118. I think myself justified, therefore, by the foregoing comparisons, in asserting that the system of classification proposed by M. D'ORBIGNY is founded on an estimation of the value of characters, which is entirely erroneous; and that any classification which shall be really *natural*, must be based on an order of facts relating to the economy of the animal, of which his imperfect methods of observation have left him in entire ignorance*. It is not my intention, in this stage of the inquiry, to propose the erection of any new system; my sole aim, at present, being to establish the fundamental principles upon which alone can a natural arrangement be securely built-up.

EXPLANATION OF THE PLATES.

PLATE XXVIII.

Figs. 1–5. Successive stages of growth of the ordinary type of *Orbiculina adunca*, showing the change from the *spiral* to the *cyclical* plan of development:—16 diam.

Fig. 6. Edge of a disk of *Orbiculina adunca*, showing but a single row of apertures, as in the simple type of *Orbitolites*:—50 diam.

Fig. 7. Edge of a disk of *Orbiculina adunca*, showing three rows of apertures, as in the complex type of *Orbitolites*:—50 diam.

Figs. 8–10. Successive stages of growth of the less common type of *Orbiculina adunca*, in which the spiral plan of development is retained throughout life:—16 diam.

Fig. 11. Horizontal section of a disk resembling fig. 5:—16 diam.

Fig. 12. Horizontal section of a spiral resembling fig. 10:—16 diam.

Figs. 13–16. Portions of the superficies of *Orbiculina adunca*, showing varieties in the surface-markings:—50 diam.

Fig. 17. Central portion of a disk of a fossil *Orbiculina* (*Orbitolites Malabaricus*, CARTER), showing its spiral commencement on the plan of fig. 8:—16 diam.

Fig. 18. Marginal portion of a similar disk, showing the investment of the early whorl by the later, with the characters of the surfaces and edge:—50 diam.

Fig. 19. Marginal portion from a similar fossil disk, which had extended itself like fig. 5:—50 diam.

* I am constrained to make a similar remark respecting the classification proposed by Professor SCHULTZE (Über den Organismus der Polythalamien); which, although in many respects an improvement upon that of M. D'ORBIGNY, is almost equally far from representing the natural affinities of these organisms, as revealed by minute investigation of the structure of their *testæ*.

Fig. 20. Inner surface of one of the annuli of a similar fossil disk, showing the differentiation of the superficial from the intermediate layers of cells:—50 diam.

Figs. 21, 22. Portions of the superficies from similar fossil disks, showing varieties in the surface-markings:—50 diam.

Fig. 23. External aspect of *Alveolina Boscii* (recent); *a, a,* growing margin, showing multiple apertures resembling those at the margins of *Orbitolites* and *Orbiculina*:—40 diam.

Fig. 24. Longitudinal section of *Alveolina Boscii*, showing its internal structure and the successive stages of its growth:—40 diam.

PLATE XXIX.

Fig. 1. Section of disk of *Orbiculina adunca* parallel to the surface, showing the cells and their communications:—100 diam.

Fig. 2. Surface-layer of disk of *Orbiculina adunca,* showing its punctuated appearance:—100 diam.

Fig. 3. Vertical section of disk of *Orbiculina adunca,* passing through its central nucleus, and showing columnar arrangement of its cells, and the manner in which the earlier whorls of the spire are invested by the later:—100 diam.

Figs. 4–7. Transverse sections of *Alveolina Boscii,* showing the increase in the number of turns of the spire from its terminal to its central portion:—40 diam.

Fig. 8. Portion of a similar transverse section enlarged; showing—*a, a, a,* internal prolongations of the surface-layer; *b, b, b,* outer longitudinal canals; *c, c, c,* inner longitudinal canals:—80 diam.

Fig. 9. Portion of a longitudinal section (Plate XXVIII. fig. 24) similarly enlarged; showing 1, 2, 3, 4 successive layers formed by the involution of the spire; and in each the passages *a a, b b,* and *c c,* between one band and the next. (N.B. The variation in the appearances presented by the other layers, depends upon the difference of relative direction in which the section traverses each of them respectively):—80 diam.

Fig. 10. Thin section of *Cycloclypeus,* taken parallel to the surface, and close to the covering of the chambers; showing part of the system of interseptal canals:—50 diam.

Fig. 11. Diagram of a single chamber of *Cycloclypeus,* showing its relations to other chambers, and to the interseptal system of canals: *a,* cavity of chamber; *b, b′,* adjacent chambers of the same annulus, each separated from *a* by a double septum; *c c′* and *d d′,* chambers of internal and external annuli, separated from *a* by the annular partitions *e e, e′ e′,* but communicating with it by the passages *f, f′, f, f′*; in the septa between *a* and *b, b′* are seen the interseptal canals, each of which sends two oblique branches across the annular septa, to communicate with corresponding canals in the septa

dividing $c\,c'$ and $d\,d'$; these interseptal canals seem to communicate by short lateral twigs with the cavities of the adjacent chambers, whilst at *g, g, g, g* they become connected with vertical branches, which unite them with those of other planes: at *h h*, $h'\,h'$ are seen the canals proper to the annular septa.

Fig. 12. Thin section of *Cycloclypeus*, passing through its central plane, showing portions of four annuli, 1 1, 2 2, 3 3, 4 4, with the general relations and connexions of their chambers, the double septa by which they are separated, and (in parts) the interseptal system of canals:—36 diam.

PLATE XXX.

Fig. 1. General view of a disk of *Cycloclypeus*, showing the aspect of its surface, and the appearances presented by horizontal and vertical sections:—12 diam.

Fig. 2. Surface-view of a full-grown specimen of *Heterostegina*, showing its tendency to assume the discoidal form by the opening-out of the spire: *a b c*, the thickened margin of the spire; *c d a*, its growing edge or mouth:—10 diam.

Fig. 3. Monstrous specimen of *Cycloclypeus*, having a vertical plate superimposed upon the horizontal disk:—5 diam.

Fig. 4. Ideal figure of a portion of a *Cycloclypeus*-disk laid-open to show the details of its structure: *a, a, a*, upper stratum, consisting of superimposed tubular laminæ; *b, b, b*, portion of lower stratum; *c, c, c*, cones of non-tubular substance, sometimes perforated by larger canals; *d, d, d*, their bases projecting on the surface; *e, e*, plates of non-tubular substance, continuous with the septa between the chambers; *f, f*, passages of communication between the chambers, through the inter-annular partitions, as seen in section; *g, g*, the same as seen from the interior of the chambers; *h, h*, interseptal canals cut across; *i*, a chamber on the walls of which the system of interseptal canals is represented as fully displayed; *k, k*, passage of the principal canals along the line of junction between the roof of the chambers and the vertical septa:—60 diam.

PLATE XXXI.

Fig. 1. Section of a young specimen of *Heterostegina*, taken parallel to the surface, partly through the chambered plane and partly (owing to a slight inequality of the specimen) through the shelly investment of the chambers, the boundaries of which, however, are still distinguishable, owing to the difference of texture between the shell that covers the chambers and that which unites with the septa: *a*, first cell; *b*, second cell; *c c*, first subdivision; *d*, second subdivision; *e, e*, large passages connecting the chambers that abut on the pre-formed whorl:—60 diam.

Fig. 2. Vertical section of *Cycloclypeus*, taken in the direction of one of the annuli, showing the upper and under layers *a b*, *a' b'*, with the septa between the chambers, each of which is seen to be composed of two layers with an interposed lamella, pierced by two or more canals whose orifices are shown; on the left side of the figure, the chambers are seen to be closed by the inter-annular septum, which is pierced by several irregularly-disposed passages, *c*, *c*, that establish communications between each chamber and those of the annuli internal and external to it:—60 diam.

Fig. 3. Section of *Cycloclypeus*, taken horizontally (or parallel to the surface of the disk) through the shelly covering of the chambers, showing its minutely tubular structure, and its junction with the two interannular non-tubular septa *a b*, *a' b'*, and with the intercameral septa *c c*, *c' c'*, along the course of which last are seen the divided ends, *d*, of vertical interseptal canals: the chamber thus inclosed is remarkable for its great length in comparison with its breadth:—100 diam.

Figs. 4 & 5. Vertical sections of *Cycloclypeus*, corresponding with fig. 2, and showing irregularities in the division of the chambers: at *c c*, fig. 4, are seen the orifices of the interseptal canals laid open; and at *c*, fig. 5, is shown a part of the vertical canal-system, passing upwards and downwards towards the two surfaces of the shell, through its transparent portions:—60 diam.

Fig. 6. Section of *Cycloclypeus*, taken in the same direction as fig. 3, but nearer to the surface, showing the relation of the tubular and non-tubular portions of the shell, and the orifices of the canals:—100 diam.

Fig. 7. Vertical section of young specimen of *Heterostegina*, showing the investment of the first-formed whorls by those which succeed them, and (on the right) the commencement of the thinning-out: *a*, *a*, *a*, passages of communication between the successive bands of which each whorl is composed:—35 diam.

Fig. 8. Vertical section of young disk of *Cycloclypeus*, showing at *a* an irregular subdivision into two or three layers of chambers:—22 diam.

Fig. 9. Horizontal section of a portion of *Cycloclypeus*, corresponding with fig. 3, but more highly magnified:—150 diam.

Fig. 10. Vertical section of the floor of one of the chambers, showing its lamellated arrangement, the minutely-tubular structure of one portion of it, *a a*, and one of the non-tubular cones, *b b*, which is continuous with the annular septum *c*:—150 diam.

Fig. 11. Horizontal section of a portion of *Heterostegina*, taken near the margin, showing the disposition of the chambers, the communications *a a* between those of successive bands, the minutely-tubular structure of their roof, the interseptal canals, and the high development of the canal-system in the thickened non-tubular margin, *b b*:—150 diam.

XXVII. *On the* Megatherium (Megatherium Americanum, Cuvier *and* Blumenbach). Part III.—*The Skull. By Professor* Owen, *F.R.S. &c.*

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THE vertebral segments which coalesce more or less completely to form the important and characteristic part of the skeleton called the 'skull,' constitute, in the Megatherium, one which is remarkable for its small relative size to the rest of the body, for the slight diminution of the transverse diameter from the occiput forward to the nasal bones, for the length and strength of the ascending and descending processes from the orbital end of the zygomatic arch, and for the peculiar depth of the lower jaw, especially at its middle part, where it lodges the molar teeth.

The widest part of the long and narrow cranium is formed by the zygomatic arches, Plate XXII. fig. 2, $_{26}$, $_{27}$, and the deepest part by the dentigerous portion of the lower jaw, Plate XXI. fig. 2, *d*.

In the skulls and portions of skulls of the Megatherium which have come under my observation, all the cranial and most of the facial sutures, save those that unite the tympanic to the rest of the temporal bone, had been obliterated, and the originally complex assemblage of bones forming the cranium and upper jaw had been reduced to a continuous whole.

Viewed from behind (Plate XXIII. $_1$), the skull shows the unusual degree to which the pterygoid, $_{24}$, palatine, and maxillary portions descend below the level of the true 'basis cranii' ($_1$): in consequence of which the foramen magnum (*o*) and occipital condyles ($_2$) appear to be situated in the upper half of the direct back view.

The basioccipital ($_1$) is a broad depressed plate with a thin, smoothly rounded, concave posterior border: it slightly increases in thickness as it advances forward to blend with the basisphenoid (Plate XXIV. $_5$), but developes on each side a rough low triangular protuberance, *z*, for muscular attachments, anterior to the occipital condyles. In a side view (Plate XXI. fig. 1), these condyles ($_2$) form the most prominent parts of the occipital region, which, as it rises above them, slopes forward, giving a low character of intelligence to the cranium: the plane of the posterior surface of the skull forms with the basal plane an angle of 105°. Each condyle (Plate XXIII. fig. 1, $_2$, $_2$) is a convexity of a subtriangular form, with the base straight and the sides curving to an obtuse apex: the extent of their convex curvature in the vertical or antero-posterior direction (Plate XXI. fig. 1, $_2$) equals that of a semicircle, and indicates that the Megatherium possessed considerable freedom and extent of motion of the head. The parallel bases of the condyles are turned toward each other and flank the sides of the foramen magnum, Plate XXIII. fig. 1, *o*. This

foramen is a wide or full ellipse with the longer axis transverse, and the plane, through the greater prominence of the upper border, inclined backward and downward at an angle of 135° with the basioccipital. The occipital region (*ib.* $_{1, 2, 3, 8}$) is semicircular, its contour being formed by a thick rugged ridge continued from the superoccipital ($_3$) on each side to the mastoid, $_8$: the osseous wall included between this arched ridge and the condyles ($_2$), is divided into four shallow depressions, by a sharp median vertical ridge terminating below at a venous foramen just above the upper border of the foramen magnum, and by a pair of thick obtuse ridges (*r*) extending from the sides of the arched ridge obliquely inwards to near the apices of the occipital condyles, and thence to the paroccipitals ($_4$). The depressions so defined are roughened by muscular implantations.

The paroccipital (Plate XXIII. fig. 1, $_4$) is a moderately developed triangular tuberosity abutting against the back part of the articular cavity for the stylohyal (Plate XXIV. $_{16}$), and separated by the outer occipital depression from the mastoid ($_8$). The precondyloid canals (Plate XXIV. *p*), or those that transmit the motory-lingual or ninth pair of nerves, begin by a large oblique aperture at the middle of the inner side of the base of the condyles, and extend along a course of $2\frac{1}{2}$ inches in extent forward and outward to open upon the back part of the rough fossa between the paroccipital ($_4$) and petrosal ($_{16}$) externally, and the basioccipital and basisphenoid internally, which fossa answers to the 'foramen lacerum in basi cranii' of ordinary mammals. The carotid foramina (*c*) open upon the fore-part of this fossa. The external precondyloid foramina (*p*, *p*) are situated on the inner side of the paroccipitals, are each an inch in diameter, and thus indicate the considerable muscular development of the tongue, and its great use in stripping off the leaves and smaller branches of the trees affording the nourishment of the Megatherium.

The basisphenoid (Plate XXIV. $_5$) presents on each of its hinder angles a low rugged subcircular protuberance (*x*) for muscular insertions; in advance of which it slightly expands, the lateral margins inclining downwards so as to form the beginning of the smooth channel that contracts and deepens as it advances towards the posterior aperture of the bony nostrils. The broad subvertical pterygoid plates ($_{24}$) are continued insensibly from the deflected margins of the basisphenoid; they are smooth on the inside, irregularly channelled for muscular attachments on the outside. The posterior aperture of the bony nasal canal (Plate XXIII. fig. 1, *n*) is a long and narrow vertical oval, bounded below by a low transverse ridge (Plate XXIV. *r*), which extends from the hinder boder of the bony palate about two inches backwards upon the smooth surface of the pterygoid plates, where it gradually subsides: this ridge forms the angular termination of the bony palate (*ib.* $_{20}$) behind. The substance of the basisphenoid is excavated by air-sinuses, continued backward from the ordinary position of the sphenoidal sinuses, and extending into the basioccipital as far back as the jugular foramina, *j*. They are exposed by fracture of the walls in the protuberances marked *x*, *x*, in Plate XXIV.

From the state of the sutures already alluded to, the limits of the superoccipital, parietal, and frontal bones cannot be defined. Above the arched superoccipital ridge (Plate XXII. fig. 2, 3) there are two semielliptical rough depressions (*a*, *a*) for muscular attachments; and, in advance of these, the upper surface of the cranium shows two other similar but shallower muscular impressions, *b*, *b*. The smooth surface of the parietal, gradually narrowed to an inch in width (7) between the temporal ridges (*t*), again as gradually expands into the frontal region (11), and it is perforated, a little anterior to the middle of the temporal fossa, by a submedian vascular (venous?) foramen (*v*), about half an inch in diameter. The temporal fossæ (Plates XXI. & XXII. 27, *t*, 12) are remarkable for their great antero-posterior extent, and for the encroachment upon them by the peculiar process (*c*) sent upward and backward from the malar bone, 26. They are partially defined anteriorly by the extension of a postorbital process (Plate XXI. fig. 1, 12, Plate XXII. fig. 2, 12) downward to the malar bone (26); but, beneath and within this slender process, they communicate freely with the orbits, *o*. The posterior boundary ridge, continued from that on the parietal bone, curves forward below and is continued into the sharp upper border of the zygoma, Plate XXI. fig. 1, 27. The surface of the temporal fossæ is grooved and perforated posteriorly by large vessels, and is everywhere strongly impressed by the attachments of muscular fasciculi. The base of the zygomatic process of the temporal bone has an extensive origin (Plate XXII. fig. 2, *t*, 27), not less than 6 inches in antero-posterior extent; its free portion, to where it joins the malar, being 3 inches in length. It is a strong trihedral bar of bone, rather concave on the upper and outer sides, and forming on the underside the glenoid articular cavity for the lower jaw. This cavity (Plate XXIV. *g*), of an oval form with the long axis transverse, measures 3 inches by $2\frac{2}{3}$ inches, and is half an inch in depth. Behind this cavity the base of the zygoma (Plate XXI. fig. 1, 27) has coalesced with the mastoid (8) and petrosal (16) elements of the temporal, which combine to form the meatus auditorius externus, *e*. This canal is subcircular, about 10 lines in diameter at the deeper part, where it is formed by the above elements, but doubtless wider at its outer part, where it was completed by the tympanic bone. This bone is wanting in the skulls of the Megatherium hitherto transmitted to England: the absence of any fractured surface upon the contour of the orifice of the auditory canal indicates, however, that the bone was a free element of the temporal in the Megatherium as in the Mylodon* and Glossotherium†. The mastoid (Plate XXII. fig. 1, 8) forms a rugged process, in depth or length not exceeding the paroccipital (4), but of greater breadth and thickness; above it, externally, and probably in the line of the primitive suture with the squamosal, is a venous foramen, Plate XXI. fig. 1, 27. The petromastoid—probably the petrosal part in a greater degree—forms the hemispheric articular cavity (Plate XXIV. 16) for the stylohyal (38), the anterior rugged wall of which cavity extends downwards farther than any

* Description of the Skeleton of an Extinct Gigantic Sloth (*Mylodon robustus*, OWEN), 4to, 1842, p. 28.

† Zoology of the Voyage of the Beagle, 'Fossil Mammalia,' 4to, 1840, p. 59. pl. 16. fig. 4.

other part of the proper basis cranii, Plate XXI. fig. 1, 16: the petrosal, anterior to this, sends down a shorter rough pyramidal process. The carotid foramen (Plate XXIV. *c*), a full ellipse with diameters of 5 lines and 4 lines, is situated between the petrosal and basisphenoid at the fore-part of that oblong depression which is terminated behind by the large precondyloid foramen.

The stylohyal (38) has the form of a hammer, with a long, slightly bent handle, terminated by an obliquely truncated rough surface for syndesmosis with the ceratohyal. At the opposite end the handle is subcompressed, and the head is formed by a sudden expansion in the vertical direction, terminated posteriorly by a straight but rugged margin, and with the upper end produced, thickened, and forming a smooth convexity, or condyle, adapted to the cavity above described in the petromastoid. The lower end of the head or expanded part of the hammer-shaped bone is more produced, more rugged. and terminates obtusely. The outer surface has a wide depression at the middle, which is rough, with several short and well-marked ridges. The length of the specimen described is 8 inches, the breadth or depth of the expanded end is 3 inches and a half.

The upper part of the coalesced frontals (Plate XXII. fig. 2, 11) forms a smooth triangular plate, rapidly expanding to the postorbital processes (12) and very slightly convex. Some indistinct traces of the fronto-nasal suture seem to show that the nasal bones (Plate XXII. fig. 2, 15) extended backward beyond the transverse parallel of the postorbital processes: more distinct traces of the naso-maxillary sutures (21), show that the coalesced nasals were 2 inches 9 lines across at their narrow posterior part, where they are flat above: at first slightly contracting, they then gradually expand, and become more and more convex transversely to their anterior extremity. Here the nasal bones are also thickened, are rugged for the firmer attachment of the cartilaginous parts of the nose, and their under surface, being excavated by two longitudinal grooves, the thickened terminal surface is divided into a middle (Plate XXIII. fig. 2, *m*) and two lateral (*n*, *n*) parts, the latter being convex and subangular, and the middle expansion slightly excavated. As in the Two-toed Sloth (*Cholœpus didactylus*), the under surface of each nasal bone sends off a terminal plate or process for the attachment of a turbinal cartilage or ossicle. A narrow median groove indicates the original suture between the nasal bones along their anterior half.

The cranial cavity of the Megatherium is considerably smaller than the cranial part of the skull, the outer wall or plate of bone being separated by large irregular air-cells from the vitreous plate, or that case of bone which immediately invested the brain and its membranes. The vertical diameter of the cranial cavity is 4 inches 8 lines; its transverse diameter, which is greatest at the posterior third part of the cavity, corresponding with the posterior part of the cerebrum, is 6 inches. The brain of the Megatherium, to judge from its bony case, must have been less, by nearly one half, than that of the Elephant; but with the cerebellum relatively larger and situated more posteriorly to the cerebral hemispheres: whence it may be inferred

that the Megatherium was a beast of less intelligence, and with the command of fewer resources, or less varied instincts, than the Elephant.

The 'maxilla superior,' or maxillary bone, may be divided into a palatal, alveolar, and facial portion: the latter (Plate XXI. fig. 1, 21) is remarkable for the excess of its vertical over its antero-posterior extent: it forms, with the coalesced lacrymal (*l*), the anterior and part of the inferior boundary of the orbit by a strong subvertical outstanding plate, curved with the convexity forward, perforated at the middle part of its base by the antorbital canal (*r*), which is double on the left side, and near the upper part of its thick obtuse margin by the lacrymal canal (*l*): it is smooth behind, or next the orbit, rather rough and irregular in front: a rough, shallow depression (Plate XXIII. fig. 2, *s*) near the upper part of this surface indicates the origin of a strong labial muscle. The outer surface of the facial plate of the maxillary is smooth and slightly undulated; it evidently extends as far as the postorbital process upwards and backwards, in connexion with the nasal bone: its anterior border (Plate XXI. fig. 1, *n*), terminating the side of the nostril, is vertical, slightly concave and sharp, and is smoothly excavated on the inner side or towards the nasal cavity. The lower part of this nasal wall presents a deep and rough sutural notch for articulation with the premaxillary bone.

The alveolar part of the maxillary (Plate XXIV. *i*, *v*) extends about an inch below the suborbital process. The extent of the alveolar tract is 10 inches; its greatest breadth is 2 inches 4 lines, viz. between the second and third teeth. The number of alveoli is five. The first (*i*) has a subtriangular transverse section, with the apex very obtusely rounded off and turned forward; the borders of this alveolus are sharp and somewhat produced below the level of the surrounding bone. The second alveolus (*ii*) is close to the first, and the corresponding teeth are nearly in contact; its transverse section is quadrate, the hinder side being the broadest, the outer side the narrowest; the fore-side is more curved than the back one. The partition between this and the third alveolus is thicker than the preceding one, and the teeth stand further apart. The third and fourth sockets are most nearly of a square form, but the transverse diameter predominates; the fifth socket (*v*) is suddenly reduced in size, and resembles most the first in form, but with the rounded apex of the triangle turned backwards.

No trace of the suture between the maxillary (21) and palatine (20) bones remains: the alveolar border beyond the fifth socket (*v*) rapidly contracts to the thin vertical pterygoid plate (24).

The bony palate terminates behind in an angular notch, formed by the ridges (*r*, *r*) before described. The bony palate forms a narrow tract, with parallel lateral borders gently diverging at the fore and back part of the tract, which is very slightly concave transversely: it is perforated by numerous foramina; two long ones, like fissures (Plate XXIV. *v*, *v*), opposite the interspace between the third and fourth molars, seem to represent the *post-palatal foramina*; there are, also, some large foramina (*u*, *u*)

between the first alveoli. The extent of the palatal part of the maxillary in advance of these alveoli is about 1 inch to the hindmost part of the premaxillary, 22, and 2½ inches to the apex of the process (21) articulating with that bone.

The premaxillaries (Plates XXIV., XXI. fig. 1, and XXII. figs. 1 & 2, 22) have coalesced along the major part of their extent, leaving only a median fissure on their upper surface (Plate XXII. fig. 2, 22), of about 1½ inch in length, at about the same distance from their slightly expanded anterior ends; at their under surface (Plate XXIV.) the same fissure is more advanced, and contracts to a few foramina. They form a slender, elongated, subdepressed, four-sided portion of bone, and constitute a singular anterior termination of the skull.

At the base or back part this portion of bone measures 4½ inches across; the fore-end is 2 inches 9 lines across; the narrowest part, near this end, is 2 inches 4 lines across; the vertical diameter is pretty nearly throughout 1 inch 6 lines, but decreases anteriorly. The posterior third of the bone sends upward from its median line a ridge, which enlarges as it approaches the corresponding ridge from the maxillaries, and there presents a smooth and gradually expanding groove at its upper part, for the support of the vomer or its cartilaginous septal prolongation (Plate XXIII. fig. 2). Anterior to the median ridge begins the groove which sinks into the fissure, and is then again continued forward as a groove to within an inch of the fore-end of the bone: this part (Plate XXIV. 22) seems crossed by a rough plate or cap of bone, flat, and about an inch in breadth at its upper part, and there terminating behind, as it does below, in a free margin.

The under surface of the premaxillary mass (Plate XXIV. 22) is rather convex antero-posteriorly, as also transversely along its middle third: the groove indicating the primitive suture runs along the whole of this surface, and sinking into its fore-part, opens by two or three foramina into the fissure which is seen on the upper surface. The back part of the under surface of each premaxillary is notched to receive a triangular process of the palatine part of the maxillary (21): the more slender median parts of the notches partly divide the prepalatal or incisive fissure (*s*), which thus presents the form of a chevron.

The malar (Plate XXI. fig. 1, 26) is a singularly developed mass of bone, and has always attracted attention as one of the most remarkable features of the skull, from the period of the earliest notices of the Megatherium. Its bulk and complex shape appear to relate to the unusual share which a modified and largely developed masseter muscle must have taken in the act of mastication.

Firmly articulated by extensive reciprocally indented sutures, at one end with the maxillary (21), at the other end with the zygomatic bones (27), and giving an extensive surface of attachment, by a peculiar upward prolongation, to fasciculi of the temporal muscle, it afforded the requisite fixity for the origins of the large and complex masseter.

The suture with the maxillary is in great part obliterated in the skull under descrip-

tion; but a portion remaining on both sides shows that the malar ascended to the level of the antorbital foramen, Plate XXIII. fig. 2, *r*: it forms the lower and a great part of the hinder boundary of the orbit; the latter by a triangular, slightly bent postorbital process (Plate XXI. fig. 1, *a*), which almost touches the corresponding more slender process of the frontal (ib. 12). The ascending process (ib. *c*) is a long, narrow, unequal-sided triangle with an obtuse apex; the descending process (*d*) is a longer and stronger one, extending, when the mouth is shut, outside and for three inches below the alveolar border of the lower jaw: its extremity is obtuse and recurved. The fourth process (ib. *b*), which may be called the 'zygomatic' one, extends beneath the end of the corresponding process of the temporal bone, but the obliteration of the suture in the present skull prevents a precise definition of its limits. The whole outer surface of the malar is slightly convex, moderately smooth, with a defined surface for muscular attachment near the back part of the base of the descending process. The inner surface shows, by its well-marked ridges and depressions, the vigorous action of the muscular fasciculi which derived their origin from that part.

The orbit (Plate XXI. fig. 1, *o*), of proportionally small size, as in all large mammalian quadrupeds, presents a long vertically oval form; or rather, by the convex border of the malar (*a*), is reniform. Its peripheral contour is almost completed by the descending postorbital process of the frontal (ib. 12) in the present skull; anterior to which the prominent boundary is effaced by a broad smooth channel, where the orbital surface is more directly continued upon the facial surface of the maxillary: this part answers to the supraciliary notch in quadrupeds. The lacrymal bone being completely coalescent, if not connate, with the maxillary, is recognisable only by the lacrymal foramen (ib. *l*), which is just within or behind the obtuse anterior border of the orbit. Admitting the essential presence of the lacrymal by this character, it then combines with the frontal, maxillary and malar bones, to form the contour of the orbit. Within this frame, the orbit, as already remarked, communicates extensively with the temporal fossa.

The anterior aperture of the bony nasal canal (Plate XXIII. fig. 2, *m*, *n*, 22) is subcircular, and is formed by the nasals, maxillaries and premaxillaries; the deep vertical sides being contributed wholly by the maxillaries.

The formation of the external bony aperture of the organ of hearing has already been described.

Mandible.—The chief characteristic of the mandible or lower jaw is the near equality of its vertical to its horizontal or longitudinal extent, due to the height of the coronoid process, and more especially to the depth of the dentigerous part of the bone. The latter dimension relates to the interesting modification of the principle of maintenance of the efficiency of the masticating machinery, as contrasted with that in the great proboscidian quadrupeds with a similar diet to the Megatherium.

The condyle of the jaw (Plate XXV. fig. 1, *a*) is transversely elliptical, 3 inches in

the long, 1 inch 10 lines in the short, diameter: it is moderately convex, and least so from behind forwards: it seems but a small surface for the articulation of so massive a bone, laden with large teeth, to the cranium; but the adequate firmness of suspension was afforded by the enormous muscles which seem to have embraced every other part of the ascending ramus of the mandible. The coronoid process (Plates XXI. and XXV. fig. 2, *b*) was lofty compared with its antero-posterior diameter: it is mutilated in the present skull, but seems to be entire in that of the skeleton at Madrid; and its form and extent may be appreciated in the figures published by BRU * and PANDER †. It is much compressed, begins to curve upward immediately anterior to the neck of the condyle, being continued from the middle of that part. The angular process (ib. *c*) of the lower jaw curves backward $4\frac{1}{2}$ inches below the condyle: it is a broad triangular plate, moderately convex externally, concave internally and chiefly by a slight inward bending of the lower margin, Plate XXV. fig. 2, *c*. A few ridges on the comparatively smooth outer surface indicate the insertions of muscles; but the inner surface is strongly sculptured by pits and grooves with strong intervening bony crests. The oblique beginning of the dentary canal (*e*) is situated 6 inches below the condyle, and the foramen is 2 inches from the last alveolus, but above its level. The anterior border of the base of the coronoid process is below the interspace between the fourth and fifth alveolus; on its inner side is a large elliptical outlet of a division of the dentary canal, Plate XXI. fig. 2, *f*. The outer and inner surfaces of the coronoid process present characters analogous to those on the same surfaces of the angular process, in regard to muscular traces, but the concavity is on the outer side of the coronoid.

The contour line, which is usually continued forward, straight, or with a gentle curve or undulation in ordinary quadrupeds, is interrupted in the Megatherium about one foot from the apex of the angular process by a notch, from which the contour line describes an abrupt deep convex curve below the molar teeth, and then as suddenly rises and passes by a concave curve to the under side of the long and slender symphysis, *d*, *d*. The depth of the dentigerous part of the horizontal ramus is 9 inches 6 lines; it is slightly convex externally, and forms a flat deep vertical wall internally, Plate XXV. fig. 2, *d*, *iii*.

The antero-posterior extent of the alveolar border is 9 inches (Plate XXV. fig. 1). The first socket is irregularly four-sided, the front side being the shortest, slightly convex, and with the angles rounded off between it and the outer and inner sides: the outer side forms rather an acute angle with the hinder side. The area of the second socket is more regularly quadrate, with the transverse diameter the longest; that of the third socket is nearly a true square; the fourth and last is similar to, but

* GARRIGA, J. Descripcion del esqueletto de un quadrupedo muy corpulento y raro, &c. fol. Madrid, 1796, tab. i. and ii.

† Das Riesen-Faulthier (*Bradypus giganteus*), fol. trans., Bonn, 1821, tab. iii. See also CUVIER, 'Ossemens Fossiles,' 4to, tom. v. part 1. pl. 16. figs. 1 and 2.

smaller than, the first, with the shortest and most curved side at the back part, and with the antero-posterior diameter a little exceeding the transverse one. The intervals between the alveoli are narrow and subequal.

The rami of the jaw are blended together at the symphysis, which is of great extent, Plate XXI. fig. 2, Plate XXV. fig. 1, *d, d*: it begins posteriorly at the fore-part of the mandibular convexity, opposite the second alveolus, whence the symphysis rapidly contracts to the shape of a scoop or spout, which is prolonged $8\frac{1}{2}$ inches from the alveolar part, and terminates in a thick, rough, rounded and emarginate extremity: the canal at the upper part of this spout-like symphysis is semicylindrical, slightly bent down at the end, and 3 inches in diameter; it becomes roughened by numerous small vascular impressions near the end, but elsewhere is smooth, and has obviously served for the support, during acts of protrusion and retraction, of a long cylindrical tongue. The margins of the canal are thick and rounded. The 'mental foramina,' or anterior outlets of the dental canal (Plate XXV. *g*), are two on the right side and three on the left, from 4 lines to 8 lines in diameter.

The teeth are of one kind, molars, five on each side of the upper jaw (Plate XXIV. and Plate XXVI. fig. 2, *i, ii, iii, iv, v*), four on each side of the lower jaw (Plate XXV. fig. 1, Plate XXI. fig. 2, *i, ii, iii, iv*), eighteen in total number.

In the upper jaw, the first or anterior molar (*i*) is the second in point of size, the last (*v*) being the least. The first molar (Plate XXVI. fig. 2, *i*) is $8\frac{1}{2}$ inches in length; the pulp-cavity extends six inches from the base; it presents two slight curvatures, one having the convexity turned forward, and the other inward. The transverse section (Plate XXIV. *i*) gives an irregular semicircle, with the convexity turned forward, and the flat side next the second tooth; the angles at which this side joins the curve are rounded; the outer angle is somewhat produced, and the outer side of the curve is flattened. The central axis of the tooth, formed by the vaso-dentine*, is irregularly tetragonal; the cement is thick on the anterior and posterior surfaces, thin on the sides of the tooth.

The second molar (Plate XXIV., Plate XXVI. fig. 1 & fig. 2, *ii*) is the largest of the upper series; it exceeds 9 inches in length, is of a tetragonal form, with two slight curvatures, as in the first molar. The posterior and broadest side is nearly flat, the anterior side somewhat convex, the outer and narrowest side is concave, the inner side is sinuous, having a median longitudinal eminence between two longitudinal concavities. The central axis of vaso-dentine (Plate XXVI. fig. 2, *v*) is more compressed from before backwards than in the preceding tooth, and its posterior surface is concave; the two transverse ridges of the grinding surface of the tooth formed by the dentine (ib. *d, d*) are nearly equal; but the sloping side formed by the vaso-dentine is larger than that formed by the cement (ib. *c*).

The third tooth (Plate XXIV. *iii*, Plate XXVI. fig. 2, *iii*) is of nearly the same size

* See 'Odontography,' and Art. TEETH in 'Cyclopædia of Anatomy,' vol. iv. for the definition of the different dental tissues.

and form as the second, but is somewhat narrower; the anterior and outer angle is less rounded off, and the external longitudinal depression is deeper: it is further removed from the second tooth than this is from the first.

The fourth molar (Plate XXIV. *iv*, Plate XXVI. fig. 2, *iv*) is smaller than the two preceding, but of nearly equal length, viz. 8½ inches, and is distinguished from the other teeth by being curved in only one direction, and that in a very slight degree, the concavity looking, as in the other teeth, outward: the central axis of the tooth, in reference to the anterior and posterior planes of the skull, is straight: the anterior and posterior layers of cement decrease in thickness as they approach the base of the tooth, so as to describe a slight curve, the convexity of which is turned, on both sides, towards the adjoining tooth. The fourth molar is tetragonal, and with more equal sides than the two preceding teeth; the outer and inner sides are concave, the anterior and posterior ones convex; the angles are rounded, but the anterior and inner angle is more produced than the rest. The grinding surface presents two equal transverse ridges, the contiguous sides of which are the longest.

The fifth molar (Plate XXIII. fig. 1, Plate XXIV. *v*, Plate XXVI. fig. 2, *s*) is 5 inches in length, 1 inch 2 lines in transverse, and 10½ lines in antero-posterior diameter: its principal curvature presents its concavity forward, or toward that of the anterior tooth; the curve in the transverse axis of the skull is scarcely appreciable. The transverse section of this tooth is rhomboidal, with the angles rounded, and with the longest diameter intersecting the antero-internal and the postero-external angles. The dentinal axis is transversely quadrilateral, with the posterior angles entire, and the posterior surface concave: the layer of cement which covers this surface is the thickest, and its posterior surface is convex: the layers which cover the outer and inner sides of the tooth are, as in the rest, the thinnest; the anterior layer is less than one-third the thickness of the posterior layer. The anterior ridge of dentine is slightly prominent, and the posterior alone forms the summit of a transverse eminence with sloping sides, but these diverge at a more open angle than in the preceding teeth.

At the date of the publication of my 'Odontography,' no specimen of the lower jaw of the Megatherium had reached England, and certain detached teeth with slight differences from those known to belong to the upper jaw were conjecturally referred to the lower one *. The entire bone, with the dental series complete (Plate XXV., Plate XXI. fig. 2, *i, ii, iii, iv*), shows that three of those teeth were rightly so referred; but that the small molar alluded to at p. 342, *op. cit.*, does not belong to the lower jaw, which has only four teeth in each ramus.

The first molar (Plate XXV. and XXI. fig. 2, *i*) is 8 inches in length, with a pulp-cavity of 5 inches in depth; it presents a curve, with the convexity forwards, which is more marked than in any of the upper molars. The anterior surface is so much less convex transversely than in the first upper molar, that the transverse sec-

* Odontography, p. 341.

tion presents a tetragonal rather than a semicylindrical figure; the anterior side, however, being only three-fourths the breadth of the posterior one, by which the first lower molar may be distinguished from all the tetragonal teeth of the upper jaw. Both the inner and outer sides are slightly concave transversely, the posterior side is moderately convex. The posterior ridge has a base twice as thick as the shorter anterior ridge. The greatest transverse breadth of the crown is 2 inches, the greatest fore and aft breadth is 1 inch 7 lines.

The second molar (Plates XXV. and XXI. fig. 2, *ii*) is the largest, at least the broadest transversely, of those of the lower jaw. It is 9 inches in length, with a minor curvature, convex forward. The anterior side is the broadest, being more extended inward than the posterior side: its transverse diameter is 2 inches 3 lines, the fore and aft diameter of the crown is 1 inch 10 lines: the base of the hinder eminence in the latter diameter exceeds that of the front eminence chiefly by the greater extent of dentine exposed.

The third molar (Plates XXV. and XXI. fig. 2, *iii*) is of the same length as the second, but has its two diameters more nearly equal, the transverse section being nearly square, the anterior division being rather the broadest transversely, and of equal thickness from before backwards Both this and the preceding tooth are convex transversely before and behind, concave at the sides.

The last lower molar (Plates XXV. and XXI. fig. 2, *iv*), with an equal antero-posterior diameter to the preceding, is shorter and narrower transversely, especially in regard to its posterior division, which is more rounded, or convex transversely, behind, than in any of the antecedent teeth. The hinder slope of the hinder ridge is more nearly horizontal, and those towards the middle of the tooth are less deep: the modification of the grinding surface of this tooth relating to the flatter surface of the fifth molar above, and its greater antero-posterior extent as compared with its breadth compensating for the absence of a fifth molar in the lower jaw. The grinding surface of the four lower molars equals that included between the anterior ridge of the first molar and the posterior ridge of the last molar in the upper jaw.

Each molar has its base undivided, but excavated by a deep conical pulp-cavity (Plate XXVI. fig. 2, *p*, *p*), from the apex of which cavity a fissure is continued to the middle of the grinding surface of the tooth, where it is more conspicuous in the upper (Plate XXIV.) than in the lower molars. Plate XXVI. fig. 2, exhibits a longitudinal section of the five molars of the upper jaw, *in situ*. The central axis of vaso-dentine (*v*) is surrounded by a thin layer of true or hard dentine (*d*), and this is coated by cement (*c*, *c*), which is of great thickness on the fore and hind surfaces, but is thin where it covers the outer and inner sides of the tooth.

As the outer layer of the vaso-dentine is first formed by the centripetal calcification of the pulp, the thin crust of that substance at the open base of the tooth includes a space equal to the vaso-dentine at the crown of the tooth: the contraction of the base of the tooth is due to the progressively-diminishing thickness of the cement as

it approaches that part; the intervening vacancy (*m, m*) in the socket indicating the primitive thickness of the vascular capsule, by the ossification of which the cement is formed.

The vaso-dentine (Plate XXVI. fig. 3, *v*) is traversed throughout by medullary canals, $\frac{1}{500}$th of an inch in diameter, which are continued from the pulp-cavity and proceed, at an angle of 50° to the plane of the hard dentine, parallel to each other, with a slightly undulating course, having regular interspaces, equal to one diameter and a half of their own area, generally anastomosing in pairs by a loop (ib. *l, l*), the convexity of which is turned toward the origin of the tubes of the hard dentine, forming a continuous reflected canal.

The loops are situated near, and for the most part close, to the hard dentine. In a few places one of the medullary canals may be observed to extend across the hard dentine, and to anastomose with a corresponding canal in the cement. The interspaces of the medullary canals of the vaso-dentine are principally occupied by dentinal tubes, which have an irregular course, form reticulate anastomoses, and terminate in very minute cells, at least one hundred times smaller than the calcigerous radiated cells of the cement.

The more regular and parallel tubes, which traverse the thin layer of unvascular dentine (ib. *d*), are given off from the convexity of the terminal loops of the medullary canals. The course of these tubes is more directly transverse to the axis of the tooth than is that of the medullary canals from which they are continued. They run parallel with each other, but with fine undulations throughout their course. They have a diameter of $\frac{1}{10,000}$th of an inch, and have interspaces of about twice that diameter. As the dentinal tubes approach the cement they divide and subdivide, and become more wavy and irregular; their terminal branches take on a bent direction and form anastomoses, dilate into small cells, and many are seen to become continuous with the radiating tubes of the cells of the contiguous cement.

The cement (ib. *c*), which enters so largely into the composition of the grinders of the Megatherium, is characterized in that extinct animal by the size, number, and regularity of the vascular or medullary canals (ib. *m, m*) which traverse it. They present the diameter of $\frac{1}{1200}$th of an inch, and are separated by intervals equal to from four to six of their own diameters. Commencing at the outer surface of the cement, they traverse it in a direction slightly inclined from the transverse axis towards the crown of the tooth, running parallel with each other; they divide a few times dichotomously in their course, and finally anastomose in loops, the convexity of which is directed towards, and in most cases is in close contiguity with, the layer of hard dentine. Fine tubules are sent off, generally at right angles, from the medullary canals, which quickly divide and subdivide, form anastomosing reticulations, and communicate freely with the similar tubules that radiate from the lacunæ or calcigerous cells, ib. *r, r*. These cells are dispersed throughout the dentine, and present an oblong form, with the long axis transverse to that of the tooth, measuring $\frac{1}{2000}$th

of an inch in diameter. The cavity of the cell, which is not quite occupied by their opake contents, is often very clearly demonstrated. The tubes, which radiate from the cells nearest the hard dentine, and from the terminal loops of the vascular canals, intercommunicate freely with the tubes of the hard dentine. The tooth of the Megatherium thus offers an unequivocal example of a course of nutriment from the dentine to the cement, and reciprocally.

In the structure which the fossil teeth of the Megatherium and its extinct congeners clearly demonstrate, we have striking evidence of the rich organization of those once-deemed extravascular parts, and that they were pervaded by vital activity. All the constituents of the blood freely circulated through the vascular dentine and the cement, and the vessels of each substance intercommunicated by a few canals continued across the hard or unvascular dentine.

With respect to those minuter tubes, the more important as being more immediately engaged in nutrition, which pervade every part of the tooth, characterizing by their difference of length and course the three constituent substances, they form one continuous and freely intercommunicating system of strengthening and reparative vessels, by which the plasma of the blood was distributed throughout the entire tooth for its nutrition and maintenance in a healthy state*.

The grinding surface of the molars of the Megatherium differs, on account of the greater thickness of the cement on their anterior and posterior surfaces, from those of all the smaller Megatherioids, in presenting two transverse ridges; one of the sloping sides of each ridge being formed by the cement, the other by the vascular dentine; whilst the unvascular dentine, as the hardest constituent, forms the summit of the ridge, like the plate of enamel between the dentine and cement in the Elephant's grinder. The great length of the teeth and concomitant depth of the jaws, the close-set series of the teeth, and the narrow palate, are also strong features of resemblance between the Megatherium and Elephant in their dental and maxillary organization. In both these gigantic phyllophagous quadrupeds provision has likewise been made for the maintenance of the grinding machinery in an effective state; but the fertility of the Creative resources is well displayed by the different modes in which this provision has been effected: in the Elephant, it is by the formation of new teeth to supply the place of the old when worn out; in the Megatherium, by the constant repair of the teeth in use, to the base of which new matter is added in proportion as the old is worn away from the crown. Thus the extinct Megatherium had

* The first statement of the continuation of filamentary processes of the pulp into the tissue of the growing tooth was published in the 'Comptes Rendus de l'Académie des Sciences,' Paris, 1839, p. 787, and the earliest observation of their continuation into the dentinal tubuli was, I believe, recorded in the following passage:—"I had the tusk and pulp of the great Elephant at the Zoological Gardens longitudinally divided, soon after the death of that animal in the summer of 1847. Although the pulp could be easily detached from the inner surface of the pulp-cavity, it was not without a certain resistance; and when the edges of the co-adapted pulp and tooth were examined by a strong lens, the filamentary processes from the outer surface of the pulp could be seen stretching as they were withdrawn from the dentinal tubes before they broke."—Art. TEETH, Cyclopædia of Anatomy, vol. iv. p. 929.

both the same structure and mode of growth and renovation of the molar teeth, as are manifested in the present day by the diminutive Sloths.

Comparison of the Skull and Dentition.

The important affinity indicated by the dentition is confirmed by the characters of the skull. In no other edentate family, save the *Bradypodidæ,* is the cheek-bone so nearly developed to the megatherioid proportions of that bone; in no other does it ascend above the zygoma into the temporal fossa or descend below the level of the molar teeth. The large and complex malar bone is also associated, in the Sloths, with a terminal position of the great anterior and posterior orifices of the cranium, with terminal occipital condyles, and in the Ai (*Bradypus tridactylus*) with a sloping occipital region. The cranial division of the skull is relatively as great in the Sloths as in the Megatherium; and the actual capacity of the cerebral cavity is masked by a similar expansion of the air-cells, which almost everywhere surround that cavity, and raise the outer plate of its bony parietes above the inner one. The occiput presents the same expanded proportions, the same broad depressed basilar plate, and the anterior condyloid foramina are of large relative size. The tympanic is small; it nearly completes a circular frame for the ear-drum, to which function it is limited, and it long remains a separate bone. The detached and, in the skull described, lost tympanics of the Megatherium have been evidently restricted to the same office. The temporal fossa, in the Sloths, is long and large, and communicates freely with the orbit, the outer boundary of which, however, is not completed in any living species of Sloth. The nasals become confluent in old Sloths, and develope turbinal laminæ from their under surface. The premaxillaries are edentulous and without any ascending process. The rami of the lower jaw expand and branch out behind into a coronoid, a condyloid, and a long and deep angular process, and they are anchylosed anteriorly at a broad sloping symphysis. Only in the genus *Bradypus,* amongst known existing quadrupeds, do the alveoli of both jaws correspond in number, simplicity, relative depth and position with those of the Megatherium. The still more important agreement between these existing and extinct *Bruta,* in the peculiar structure of the teeth, yields the crowning proof that it is to the diminutive arboreal Sloths that the gigantic Megatherium and its less bulky though larger extinct congeners have the closest natural affinity.

The chief differences observable in the cranial anatomy of the Sloths, as compared with that of the Megatherium, are the greater relative depth and breadth, and the more convex outline of the coronal aspect of the skull; but this difference would be, doubtless, much less marked in the immature than in the adult Megatherium. The zygomatic process, in the Sloths, is relatively shorter, and does not attain the malar bone; this, therefore, has not the middle process for supporting the zygoma, and is two-pronged, instead of being, as in the Megatherium, four-pronged. The chief characters by which the Megatherium deviates, in its cranial structure, from the bradypodal and approaches to the myrmecophagal type, are the elongation of the slender edentulous

fore-part of the upper jaw, and of the corresponding grooved slender edentulous part of the lower one: but the prolongation of the upper jaw is due to relatively longer premaxillaries than are developed in any of the true Edentata. The zygomatic arches, moreover, are more defective in the Anteaters and Pangolins than in the Sloths; the malar part especially being minute or obsolete. Only in the Orycterope and Armadillos, amongst the existing *Bruta*, is the zygomatic arch complete, but it is simple, without ascending or descending processes. The great Glyptodon, indeed, exemplifies that tendency to community of characters so often presented by extinct species, in an inferior prolongation of the malar bone analogous to that of the Sloths and Megatherium. With other existing mammals than those of the Edentate order, it would be lost time to pursue the present comparison with a view to the elucidation of the affinities of the Megathere. It needs only to place the skull of this animal by the side of those of the Elephant, Rhinoceros, Sivatherium, Ox, Elk, Horse, Dugong, or other vegetable-feeding mammal of corresponding or approximate size, to be struck with the peculiarities of the fossil, and to be convinced that the habits and mode of feeding of the Megatherium had been such as are no longer manifested by the larger Herbivora of the present day.

It remains then to inquire whether, among the extinct forms of the mammalian class to which was assigned the office of restraining the too luxuriant vegetation of a former world, there be any that, from their cranial or dental characters, may be concluded to have resembled the Megatherium in the mode of performing that task.

The skull of the Mylodon, while it presents all the essential resemblances to that of the Megatherium which have been pointed out in the skull of the Sloth, as, *e.g.* the long cranium, the terminal position of the occipital condyles, and of the occipital and nasal apertures, and the large and complicated malar bones, approximates still more closely to the Megatherium in the junction of the malar with the zygomatic process of the temporal, and in the relative depression and flatness of the elongated cranium. But in thus receding from the existing Sloths, the Mylodon does not approach any other existing genus, but only another member of its own peculiar extinct family.

The most marked differences in the skulls of the Mylodon and Megatherium depend on the minor length of the teeth and consequent depth of their sockets in the smaller species, which require a less vertical extent of the maxillary bone in the molar region and of the corresponding part of the lower jaw, the lower border of which is consequently nearly straight in the Mylodon, as it is in the existing Sloths. So great a proportional extent of the descending process of the malar bone is consequently not required, and this process is more oblique in direction, and is relatively broader and thinner than in the Megatherium. In these differences also the Mylodon shows its closer resemblance to the Sloths. The basioccipital is relatively broader, and the occipital condyles are wider apart in the Mylodon; the occipital plane is more inclined and the zygomatic process proportionally weaker than in the

Megatherium. The minor depth of the lower jaw and the lighter grinding instruments call for a less extensive origin of the temporal muscles, and accordingly the superior boundaries of their fossæ are separated in the adult Mylodon by a wide and smooth parietal tract*, as in the Sloths. The postfrontal process is rudimentary in the Mylodon, and the postorbital process of the malar bone is obsolete; the orbit is consequently without any bony boundary behind; the malar, accordingly, has but three processes, and is thus less complicated than in the Megatherium. The interorbital part of the skull is relatively narrower, the maxillary part relatively broader, than in the Megathere. No trace of premaxillaries was present in the skull of the *Mylodon robustus* described by me, and the broad truncated symphysis of the lower jaw indicates that they must have been very small if they existed: the peculiar length of the premaxillaries in the Megatherium, and the corresponding prolongation of the long and narrow symphysis mandibulæ, offer the most conspicuous differences in the conformation of the skull, and proportionally remove that genus from the existing Sloths. The bony palate is absolutely broader in the much smaller skull of the Mylodon than in the Megatherium, and it gradually contracts from the first to the fifth molar: it is, *e. g.* 5 inches in breadth between the first molars in the *Mylodon robustus*, and only 2 inches in breadth between the same teeth in the Megatherium.

The opportunity of a comparison of the skull of the Megalonyx with that of the Megatherium is yet a desideratum: it would probably demonstrate some intermediate modifications between the latter and the Mylodon.

The extinct megatherioid animal of which, after the Mylodon and the Megatherium, the most complete cranium has hitherto been obtained, is the Scelidotherium†. In many respects the skull of this animal resembles that of the Megatherium more than does that of the Mylodon. The plane of the occiput is rather less inclined from below forward than in the Megatherium, but more resembles that part than in the Mylodon: the upper boundaries of the temporal fossæ more nearly approximate than in the Mylodon: the bony palate is narrower, and its sides more parallel than in the Mylodon; but instead of being concave transversely, as in the Megatherium, it is convex: the alveoli are nearer together than in the Mylodon, and the first is not separated by a wider diastema than the rest. The symphysis of the lower jaw is much prolonged, but is less deeply channelled above than in the Megatherium, and is not so distinctly defined by the abrupt increase of depth of the ramus behind it which characterizes the Megatherium; the molar part of the mandible makes, however, a greater convexity below than in the Mylodon.

With these marks of approximation to the Megatherium there are, however, the same differences as in the Mylodon in regard to the widely open orbit, the more simple, trifurcate, malar bone, the minor depth of the alveolar portions of the jaws, and the straighter outline of the lower border of the mandible. In both the Mylodon and

* Memoir on the Mylodon, 4to, 1842, pl. 3.

† Fossil Mammalia of the 'Voyage of the Beagle,' p. 73. pls. 20, 21, & 22.

Scelidothere the coronoid process is relatively shorter than in the Megathere, and the foramen near the fore-part of the base of that process is outside and below that base, not on the inside of it as in the Megatherium. The mastoid process is relatively shorter and the stylohyal pit is shallower; the lacrymal bone is more distinct, and the foramen is larger in the Scelidothere than in the Mylodon or Megatherium. In all the essential characters of the lower jaw, as in the number, structure and kind of teeth, the extinct megatherioid quadrupeds more closely resemble each other, and the existing Sloths, than any other known existing or extinct animals.

The number of the teeth, their length, equable breadth and thickness, and absence of fangs, their deeply excavated base, and the unlimited growth resulting from the persistent matrix, together with their composition of cement, dentine and vaso-dentine, without any true enamel, are characters common to the Megatherioids and Sloths. The form of the teeth differs in, and characterizes, each genus. It would seem that the Megalonyx, in the elliptical or subcylindrical shape of such of its teeth as are known, more closely resembled the existing Sloths than do the other Megatherioids. The Unau (*Cholœpus didactylus*) resembles the Mylodon in the distance between the first and the second molars of the upper jaw; but the advanced molar assumes, in that existing Sloth, the form and proportions as well as the position of a canine, and the corresponding tooth of the lower jaw is similarly developed and separated from the other three teeth by a nearly equal interval. In the Ai (*Bradypus tridactylus*) the first molar in both jaws presents nearly the same proportionate size to the rest, as in the Megatherium, and is not separated from them by a wider interval than the rest, as it is in the Unau and Mylodon. In both species of existing Sloth, the last molar of the lower jaw is as simple in form as in the Megatherium: in the Mylodon and Scelidotherium it is larger and more complex in shape, the grinding surface being divided into two lobes by two oblique channels, which traverse longitudinally one the outer the other the inner side of the tooth: these grooves are more shallow in the Scelidothere than in the Mylodon, and the lobes of the tooth are more equal and more compressed. The grinding surface itself, in all the molars of both Mylodon and Scelidothere, resembles that in the Sloths; the two transverse ridges are developed only in the teeth of the Megatherium, which are longer in proportion to their thickness than in the Mylodon, Megalonyx, or Scelidothere. These modifications, with the narrow palate, the close-set series of teeth, their great length, and the concomitant depth of the jaws, are features of resemblance to the maxillary and dental characters of the Elephant; but the fundamental structure of the teeth, not only of the Megatherium, but of all its extinct congeners, is manifested in the present day exclusively by the restricted and diminutive family of the *Bradypodidæ*. I conclude, therefore, the present section of this memoir by repeating the remark which I was led to make in a former memoir*, relative to the existing Sloths:—"These Mammals

* On the *Mylodon robustus*, 4to, p. 45.

present to the zoologist, conversant only with living species, a singular exception in their dental characters to the rest of their class; but there has been a time when this peculiar dentition was manifested under as various modifications as may now be traced in some of the more common dental types in existing orders of Mammalia."

Comparative Table of Dimensions of the Skull of MEGATHERE, MYLODON, and SCELIDOTHERE.

	Megatherium Americanum.			*Mylodon robustus.*			*Scelidotherium leptocephalum.*		
CRANIUM.	ft.	in.	lin.	ft.	in.	lin.	ft.	in.	lin.
Length from the occipital condyles to the fore-end of the upper jaw	2	7	0	1	6	6	1	8	4
Length from the occipital condyles to the fore-part of the malar bone	1	8	0	1	2	0	0	11	6
Length from the fore-part of the malar bone to the fore-end of the upper jaw	0	10	6	0	5	0	0	8	8
Breadth across the widest part of the zygomatic arches				0	10	9	0	7	0
Least breadth at the interspace of those arches	0	6	0	0	5	4	0	3	7
Breadth of the fore-part of the nasal bones	0	4	6	0	3	3	0	2	8
MANDIBLE.									
Length	2	3	0	1	3	6	1	6	6
Breadth between the hinder ends (angles) of the rami	0	7	0	0	6	3	0	4	7
Breadth between the condyles	0	4	0	0	4	2	0	1	8
Breadth between the posterior sockets of the teeth	0	2	3	0	2	10	0	1	6
Breadth between the anterior sockets of the teeth	0	1	9	0	4	0	0	1	8
Breadth across the fore-part of the symphysis	0	4	5	0	5	4	0	2	5
Depth of ascending ramus from the upper part of the condyle	1	5	0	0	5	7	0	5	8
Depth of ascending ramus at the fore-part of the base of the coronoid process	0	9	0	0	3	7	0	3	7
Depth of horizontal ramus at the fore-part of the first socket	0	8	0	0	3	0	0	2	7
Length of the symphysis following the outer curve	1	1	0	0	4	3	0	7	3
Fore and aft extent of base of coronoid process	0	9	0	0	3	8	0	3	9
From the back part of the condyle to the end of the angular process	0	6	0	0	3	2	0	3	6
From the end of the angular process to the last socket	0	9	7	0	6	6	0	6	5
From the first socket to the anterior margin of the jaw	0	9	0	0	3	6	0	7	10
Extent of the alveolar series	0	9	0	0	5	4	0	4	4
Breadth of the condyle	0	3	0	0	2	6	0	1	10

Description of the Plates.

All the figures, save where otherwise expressed, are one-fourth the natural size.

PLATE XXI.

Fig. 1. Side view of the skull of the Megatherium.
Fig. 2. Inner side view of the mandible and of the section of the symphysis.

PLATE XXII.

Fig. 1. The opposite and more mutilated side view of the same skull, showing the surface of the temporal fossa.
Fig. 2. Upper view of the same skull.

PLATE XXIII.

Fig. 1. Back view of the same skull.
Fig. 2. Front view of the same skull.
Fig. 3. Back view of the mandible.
Fig. 4. Front view of the same mandible.

PLATE XXIV.

Base view, natural size, of the same skull, and grinding surface of the upper teeth.

PLATE XXV.

Fig. 1. Upper view of the mandible, and grinding surface of the lower teeth.
Fig. 2. Side view of the mandible.

PLATE XXVI.

Fig. 1. The second upper molar tooth:—natural size.
Fig. 2. Longitudinal section of the alveolar part of the upper jaw and of the five upper molars, *in situ*:—three-fourths of the natural size.
Fig. 3. Transverse section of a portion of a molar tooth of the Megatherium:—magnified 500 linear diameters.

The letters and ciphers on the several figures are explained in the text.

XXVIII. *On the Deflection of the Plumb-line at Arthur's Seat, and the Mean Specific Gravity of the Earth. Communicated by Lieutenant-Colonel* James, *R.E., F.R.S., M.R.I.A. &c., Superintendent of the Ordnance Survey.*

Received February 11,—Read February 21, 1856.

The Royal Society has, from the very commencement of the Ordnance Survey of the United Kingdom, taken a deep interest in its progress. I have therefore great pleasure in announcing to the Society the completion of all the computations connected with the Primary Triangulation, the measurement of the Arcs of Meridians, and the figure and dimensions of the Earth.

The account of all the operations and calculations which have been undertaken and executed is now in the press, and will shortly be in the hands of the public.

After determining the most probable spheroid from all the astronomical and geodetic amplitudes in Great Britain, we find that the plumb-line is considerably deflected at several of our principal Trigonometrical Stations, and at almost every station the cause of the deflection is apparent in the configuration of the surrounding country.

The deflection of the plumb-line at Arthur's Seat is $5''\cdot25$, and at the Royal Observatory at Edinburgh it amounts to $5''\cdot63$ to the South. The unequal distribution of matter here, the great trough of the Firth of Forth being on the North, and the range of the Pentland on the South, presents a tangible cause for the deflection; but as the contoured plans of the county of Edinburgh are published, and we have the most perfect data that it is possible to obtain for estimating the amount of local attraction at Arthur's Seat and the Calton Hill, and as it appeared to me that an investigation of this matter was not only necessary to confirm and establish the results arrived at from the previous investigation of all the observed latitudes, but would also prove highly interesting to science, I decided on having observations taken with Airy's Zenith Sector on the summit of Arthur's Seat, and at points near the meridian on the North and South of that mountain, at about one-third of its altitude above the surrounding country.

The observations were made by Serjeant-Major Steel of the Royal Sappers and Miners, during the months of September and October last; 220 double observations of stars were taken at each Station, and the results have justified my confidence in him as an observer.

To Captain Clarke, R.E., I entrusted all the reductions and computations connected with these observations, as well as the computations of the local attraction at

Calton Hill. The following communication has been drawn up by him, and I trust it will prove acceptable to the Royal Society, and do him credit as a mathematician.

I have myself examined the geological structure of Arthur's Seat and the whole of the county of Edinburgh, and have had the specific gravity of all the rocks ascertained, with the view of estimating the mean specific gravity of the whole mass; but although the geological structure of Arthur's Seat is well exposed, and we have deduced from its mean specific gravity (2·75) the mean specific gravity of the earth, viz. 5·316, it is not such a mountain as I should have selected for this special object.

Since these observations were made, on examining the correspondence connected with the Survey, with the view of drawing up an historical sketch of its progress for publication, I was agreeably surprised to find that the late Dr. MACCULLOCH had been employed for six years, from 1814 to 1819, in examining the whole of Scotland for the purpose of selecting a mountain which, from its homogeneous structure, size, and form, would be best suited for observations for the purpose of determining the mean specific gravity of the earth, and that he considered the Stack Mountain in Sutherlandshire admirably suited for the purpose. The transfer of the whole force of the Survey from the North of England and Scotland to Ireland, prevented the late General COLBY from undertaking this investigation; but as the Survey of Scotland is now in full progress, I purpose early in the spring to go down to the Stack Mountain, to have it and the surrounding country surveyed and contoured, and to have the observations taken for determining the attraction of its mass, and I trust at the close of the present year to lay the results before the Royal Society.

I forward herewith a model of Arthur's Seat, made from the contoured plan on the scale of 6 inches to a mile, and also an impression of the plan itself, with sections showing the geological structure of Arthur's Seat, and a table of the specific gravity of the rock of which it is composed.

HENRY JAMES,
Lieut.-Col. R.E.

Feb. 7, 1856.

In deducing from the observations made at the three stations on Arthur's Seat, with the zenith sector, the latitudes of those stations, if we assign to the resulting latitude given by any one star a weight equal to the number of observations of that star, the final latitudes of the three stations will stand thus:—

Stations.	Designated.	Latitude.	Number of observations.
South Station	S	55° 56′ 26″·69	427
Arthur's Seat (summit *).........	A	55 56 43·95	425
North Station	N	55 57 9·50	411

* The station on the summit of the hill was 14 feet from the Ordnance Trigonometrical Station, and bearing 18° North-west; the former is therefore 0″·13 North of the latter.

The latitudes thus obtained being affected by the errors of the assumed declinations of the stars, the amplitudes to be adopted as final are obtained in the following manner. Let $\varphi_1\,\varphi_2$ be the values of the amplitudes SA, AN to be determined, and let the stars observed at S and A only, give these values—

$$\varphi_1=a,\quad \varphi_1=a',\quad \varphi_1=a''\ldots..$$

Let stars observed at A and N only, give the values—

$$\varphi_2=b,\quad \varphi_2=b',\quad \varphi_2=b''\ldots..$$

Let stars observed at S and N only, give the values—

$$\varphi_1+\varphi_2=c,\quad \varphi_1+\varphi_2=c',\quad \varphi_1+\varphi_2=c''\ldots..$$

And let stars observed at S, A, and N give the values—

$$\varphi_1=a_1,\quad \varphi_1=a_1',\quad \varphi_1=a_1''\ldots.$$
$$\varphi_2=b_1,\quad \varphi_2=b_1',\quad \varphi_2=b_1''\ldots.$$

Let d, e, and the same letters accented, be taken to denote the number of times the stars of the first set are observed at S and A respectively. Let f, g and h, k represent the same quantities for the stars of the second and third set; and let n, p, q, and the same letters accented, be taken to denote the numbers of times the stars of the fourth and last set are observed at S, A, N respectively.

The values of $\varphi_1\,\varphi_2$ adopted are those which render the quantity

$$\Sigma\left\{\frac{de}{d+e}(\varphi_1-a)^2\right\}+\Sigma\left\{\frac{fg}{f+g}(\varphi_2-b)^2\right\}+\Sigma\left\{\frac{hk}{h+k}(\varphi_1+\varphi_2-c)^2\right\}$$
$$+\Sigma\left\{\frac{np}{n+p}(\varphi_1-a_1)^2\right\}+\Sigma\left\{\frac{pq}{p+q}(\varphi_2-b_1)^2\right\}$$

a minimum. Making the differential coefficients of this quantity with respect to φ_1 and φ_2 respectively $=0$, we obtain

$$H\varphi_1+K\varphi_2-L=0$$
$$K\varphi_1+M\varphi_2-N=0,$$

in which equations

$$H=\Sigma\left(\frac{de}{d+e}\right)+\Sigma\left(\frac{hk}{h+k}\right)+\Sigma\left(\frac{np}{n+p}\right)\qquad M=\Sigma\left(\frac{fg}{f+g}\right)+\Sigma\left(\frac{hk}{h+k}\right)+\Sigma\left(\frac{pq}{p+q}\right)$$

$$L=\Sigma\left(\frac{de}{d+e}\cdot a\right)+\Sigma\left(\frac{hk}{h+k}\cdot c\right)+\Sigma\left(\frac{np}{n+p}a_1\right)\quad N=\Sigma\left(\frac{fg}{f+g}b\right)+\Sigma\left(\frac{hk}{k+k}c\right)+\Sigma\left(\frac{pq}{p+q}b_1\right)$$

$$K=\Sigma\left(\frac{hk}{h+k}\right).$$

If μ be any number, the value of $\varphi_1+\mu\varphi_2$ is

$$\varphi_1+\mu\varphi_2=\frac{(-\mu K+M)L+(-K+\mu H)N}{HM-K^2},$$

hence the error of $\varphi_1+\mu\varphi_2$ depends upon the manner in which the errors of the quantities $a\,a_1\ldots b\,b_1\ldots c\ldots..$ enter into this expression.

Let (γ_a) and (γ_b) be the sums of the errors of observation at S and A, of a star of the first set, the same quantities being accented for other stars. Let (α_b), (α_c) represent corresponding quantities for stars of the second set, (β_a), (β_c) and (ϵ_a), (ϵ_b), (ϵ_c) the same quantities for the third and fourth sets of stars.

Then L and N are affected with the errors

$$\text{L} \quad \ldots \Sigma\frac{d(\gamma_b)-e(\gamma_a)}{d+e}+\Sigma\frac{k(\beta_c)-h(\beta_a)}{h+k}+\Sigma\frac{n(\epsilon_b)-p(\epsilon_a)}{n+p}$$

$$\text{N} \quad \ldots \Sigma\frac{g(\alpha_c)-f(\alpha_b)}{f+g}+\Sigma\frac{k(\beta_c)-h(\beta_a)}{h+k}+\Sigma\frac{p(\epsilon_c)-q(\epsilon_b)}{p+q}.$$

From these expressions we may derive, finally, the following: if E be the probable error of an observation, the probable error of $\phi_1+\mu\phi_2$ is

$$\frac{\text{E}}{\text{MH}-\text{K}^2}\Big\{\text{M}(\text{MH}-\text{K}^2)+2\text{PMK}-2\mu\big(\text{K}(\text{HM}-\text{K}^2)+\text{P}(\text{HM}+\text{K}^2)\big)+\mu^2(\text{H}(\text{HM}-\text{K}^2)+2\text{PKH})\Big\}^{\frac{1}{2}},$$

where

$$\text{P}=\Sigma\frac{npq}{(n+p)(p+q)}.$$

The values of H, M, K, P, L and N are found to be

$$\text{H}=168{\cdot}93 \quad \text{M}=168{\cdot}52 \quad \text{K}=46{\cdot}06$$

$$\text{L}=362{\cdot}40 \quad \text{N}=182{\cdot}20 \quad \text{P}=49{\cdot}34,$$

whence we obtain

$$\phi_1=17''{\cdot}00 \quad \phi_2=25''{\cdot}53 \quad \phi_1+\phi_2=42''{\cdot}53.$$

Now the value of E is to be deduced from the differences between the individual and mean results given by the different stars. The sum of the squares of these errors is found from the whole of the observations to be 712·1, hence the mean square of an error of observation (1263 obs.) is 0·56, and the probable error of an observation consequently $=0''{\cdot}50$ $(={\cdot}67\sqrt{0{\cdot}56})$.

We have therefore the probable error of $\phi_1+\mu\phi_2$ equal to

$$\frac{0''{\cdot}50}{263{\cdot}4}\Big\{520{\cdot}48-544{\cdot}66\,\mu+522{\cdot}04\,\mu^2\Big\}^{\frac{1}{2}}=0''{\cdot}043\Big\{1-1{\cdot}046\,\mu+\mu^2\Big\}^{\frac{1}{2}},$$

so that the probable errors of ϕ_1 and ϕ_2 are each equal $0''{\cdot}043$.

As the differences of latitude are the quantities principally required, we may append these amplitudes to any one of the observed latitudes. Thus making use of the observed latitude of the South Station, namely 55° 56′ 26″·69, there will result by applying the above most probable amplitudes the following latitudes:—

Latitude of S = 55° 56′ 26″·69

" " A = 55° 56′ 43″·69

" " N = 55° 57′ 9″·22.

The last two latitudes differ from those in the first table by about a quarter of a second each.

The amplitudes derived from the latitudes in the first table, when compared with those we have considered as most probable, show the following differences:—

$$A-S \ldots. +0''\cdot26$$
$$N-A \ldots. +0''\cdot02$$
$$N-S \ldots. +0''\cdot28.$$

Geodetical Amplitudes.

By means of a small network of triangulation connected with the secondary triangulation of the Ordnance Survey in the county of Edinburgh, the following results were obtained:—

From	To	Distance.	Bearing.
		ft.	° ′ ″
Arthur's Seat, Trigonometrical Station	S.	1426·7	179 42 7
	N.	2490·0	6 0 17

The bearings being reckoned from North round by East. The corresponding amplitudes are $14''\cdot06$ and $24''\cdot40$: in order, however, to the comparison of these with the amplitudes before considered, the quantity $0''\cdot13$ must be added to the first of the geodetical amplitudes and deducted from the second for the difference of the two stations on the summit of the hill. The geodetical amplitudes are therefore

$$A-S = 14''\cdot19$$
$$N-A = 24''\cdot27$$
$$N-S = 38''\cdot46.$$

By comparing these amplitudes with the actual astronomical amplitudes we find the following results:—

(1) Between the vertex of the hill and the South Station, the astronomical amplitude exceeds the geodetical by $2''\cdot81$.

(2) Between the vertex of the hill and the North Station, the astronomical amplitude exceeds the geodetical by $1''\cdot26$.

(3) Between the North Station and the South Station, the astronomical amplitude exceeds the geodetical by $4''\cdot07$.

Geodetical Latitudes.

The latitude of the Trigonometrical Station on the summit of Arthur's Seat, is, when referred to, or projected on that spheroidal surface which best represents all the astronomically determined points in Great Britain,

$$55° \ 56' \ 38''\cdot31,$$

from which, by the application of the geodetical amplitudes, we obtain the latitudes

of the other two points, shown in the following Table, in contrast with the observed latitudes:—

Station.	Astronomical Latitude.	Geodetical Latitude.	A—G.
S	55° 56′ 26″·69	55° 56′ 24″·25	2″·44
A	55 56 43·69	55 56 38·44	5·25
N	55 57 9·22	55 57 2·71	6·51

It might have been anticipated, that, on account of the attraction of the hill at the South Station, the deflection of the plumb-line would have been to the north, which by throwing the zenith to the south would have caused the observed latitude to be less than its true value. The contrary, however, takes place, for the observed latitude is greater than the geodetical. On proceeding next to the second station, namely, that on the summit of the hill, a similar anomaly is observed; there is an attraction or deflection to the south of more than five seconds, which can by no means be attributed to the hill, as its attraction upon any object at its vertex is very nearly equal north or south. A similar anomaly is visible at the North Station; there is a deflection to the south of $6''\cdot5$, which is considerably more than that due to the mass of the hill, as will appear hereafter.

It is clear, therefore, that there is some *other* disturbing force acting at each of these stations besides the attraction of Arthur's Seat, and which appears to produce a *general deflection* to the south of about five seconds.

The comparison of the observed and calculated latitudes of the observatory on the Calton Hill serves to corroborate this fact. The latitude of this observatory, as determined by observation, is

$$55^\circ\ 57'\ 23''\cdot20.$$

The latitude of the Trigonometrical Station on this hill, when referred to the same spheroidal surface we have before mentioned, namely, that agreeing most nearly with all the astronomically determined points in Great Britain, is $55^\circ\ 57'\ 17''\cdot51$: the difference of latitude of these two points (taking the centre of the Altitude and Azimuth instrument of the observatory as the point whose latitude is above given) is $0''\cdot06$, so that the calculated latitude of the Calton Hill observatory is

$$55^\circ\ 57'\ 17''\cdot57,$$

which is less than the astronomical by $5''\cdot63$; showing a deflection to the south of that amount in existence at the Calton Hill. Now the attraction of the mass of Arthur's Seat upon the Calton Hill is easily calculated to be between $0''\cdot1$ and $0''\cdot2$, consequently the deflection here visible is certainly not due to Arthur's Seat.

It seems therefore very probable that the general deflection of five seconds to the south, brought out at all these stations, is due to one and the same cause.

An explanation of this phenomenon immediately offers itself in the existence of the hollow of the Forth to the north, and the Pentland Hills and other high ground to

the south, but whether these may be sufficient to produce the effect observed will be considered hereafter.

Deflection caused by an Attracting Mass.

Let it be required to find the attraction exercised by a given mass placed on the surface of the earth upon a given point on the surface, the distance being supposed so small that the sphericity of the earth need not be considered. Let the position of any point of the attracting mass be determined by the coordinates r, θ, z; r and θ originating in the attracted point and being measured in the horizontal plane passing through that point, z being measured perpendicular to this plane. Let also the value of $\theta=0$ correspond to the meridian line, then the volume of an indefinitely small element of the attracting mass being $rd\theta.dr.dz$, if ϱ be its density, its attraction will be

$$\frac{r.\varrho dr.d\theta.dz}{r^2+z^2},$$

and therefore its attraction in the direction of the meridian is equal to this quantity multiplied by $r.(r^2+z^2)^{-\frac{1}{2}}.\cos\theta$; so that the attraction of the whole mass is equal to

$$\iiint\varrho\cos\theta d\theta\frac{r^2dr.dz}{(r^2+z^2)^{\frac{3}{2}}}.$$

In order to perform the integrations here indicated, the equation of the surface of the attracting mass is required to determine the limits; this cannot be expressed, nor can ϱ, which is also a function of $r\theta z$. But it is easy to find the attraction of a mass of uniform density included within the following surfaces:—The horizontal planes $z=0$, $z=h$, the two cylindrical surfaces defined by the equations $r=r_1$, $r=r_2$, r_1r_2 being constants, and two vertical planes determined by the equations $\theta=\theta_1$, $\theta=\theta_2$, $\theta_1\theta_2$ being constants; ϱ being supposed also constant. Integrating between these limits, the attraction of the mass under consideration is found to be

$$A=\varrho h(\sin\theta_2-\sin\theta_1)\log\frac{r_2+\sqrt{r_2^2+h^2}}{r_1+\sqrt{r_1^2+h^2}},$$

which being expanded is equal to (putting $r_1+r_2=2r$)

$$A=\varrho(r_2-r_1)(\sin\theta_2-\sin\theta_1)\frac{h}{\sqrt{r^2+h^2}}\left\{1+\frac{(r_2-r_1)^2}{24}\frac{2r^2-h^2}{(r^2+h^2)^2}+\dots\right\}.$$

Hence, by taking r_2-r_1 sufficiently small,

$$A=\varrho(r_2-r_1)(\sin\theta_2-\sin\theta_1)\frac{h}{(r^2+h^2)^{\frac{1}{2}}},$$

or if ε be the angle of elevation of the centre point of the upper horizontal surface of the mass in question, at the attracted point

$$A=\varrho(r_2-r_1)(\sin\theta_2-\sin\theta_1)\sin\varepsilon.$$

If h be small, so that its square may be neglected,

$$A=\varrho(\sin\theta_2-\sin\theta_1)\left(\log\frac{r_2}{r_1}\right)h.$$

The angle of deflection produced by any horizontal attracting force acting on the plumb-line is measured by the ratio of the attracting force to the force of gravity or the attraction of the earth.

The attraction of the earth upon any point on its surface in latitude λ is*

$$\frac{M}{b^2}\left(1-e-\frac{3m}{2}+\left(\frac{5m}{2}-e\right)\sin^2\lambda\right),$$

where b is the polar semiaxis, e the ellipticity of the surface, and m the ratio of the centrifugal force at the equator to the equatorial gravity; if we put a for the radius of the equator, the attraction may also be expressed thus:

$$\frac{M}{ab}\left(1-\frac{3m}{2}+\left(\frac{5m}{2}-e\right)\sin^2\lambda\right);$$

here $m=\frac{1}{289}$, $e=\frac{1}{300}$, $\sin^2\lambda=\frac{69}{100}$ nearly; whence it will follow that the term within the bracket will only influence the attraction by less than a six-hundredth part of its amount, and will therefore only affect the calculated deflection in that ratio. Therefore it is sufficiently exact to assume the attraction equal to that of a sphere whose radius is equal to the mean of the principal semidiameters of the earth, or 3956·1 miles: hence the attraction on any point on its surface $=\frac{4}{3}\pi.\delta(3956{\cdot}1)$, taking the mile as the unit of measure linear. The *deflection*, therefore (expressed in seconds), caused by any attracting force A on the surface of the earth may be taken as

$$\frac{A}{\frac{4}{3}\pi.\delta.(3956{\cdot}1)\sin 1''}=\frac{A}{\delta}\times 12''{\cdot}447,$$

δ being the mean density of the earth. Consequently the deflection caused by such a mass as we have been considering at the origin of coordinates or attracted point, is

$$D=\frac{\varrho}{\delta}(r_2-r_1)s\sin\varepsilon\times 12''{\cdot}447,$$

or

$$D=\frac{\varrho}{\delta}sh\log\frac{r_2}{r_1}\times 12''{\cdot}447,$$

where s is put for the difference of the sines.

The calculation of the attraction or deflection of the plumb-line at any point of the hill is easily effected by means of these formulæ. If through any one of the stations observed from, we draw on a contoured plan of the hill and surrounding country, a number of lines the sines of whose azimuth are successively $0, \frac{1}{12}, \frac{2}{12}, \ldots \frac{11}{12}, \frac{12}{12}$, counting from the south meridian in either direction, and from the north meridian in either direction; and draw also a number of circles whose radii are 500, 1000, 1500, 2000 .... feet, being in arithmetical progression with a common difference of 500 feet; the hill will be thus divided into a number of prisms, the deflection caused by any

* Airy's Mathematical Tracts, pp. 167, 173.

one of which will be, putting x for the unknown ratio of the density of the hill to the density of the earth,

$$D=x\frac{500}{5280}\times 12''\cdot 447\times\frac{1}{12}\sin\varepsilon,$$

so that the total deflection is equal to

$$0''\cdot 0982\ \Sigma(\sin\varepsilon)x.$$

At each of the three stations, the first ring of 500 feet was subdivided by rings at the distance of 100 feet; the result is shown in the following Table:—

South Station.

Sums of Sines.	1st Ring, 100 feet.	2nd Ring, 200 feet.	3rd Ring, 300 feet.	4th Ring, 400 feet.	5th Ring, 500 feet.
$\Sigma(\sin\varepsilon)$ for Prisms South of station	−1·796	−11·294	−11·808	−11·360	−10·001
$\Sigma(\sin\varepsilon)$ for Prisms North of station	+1·689	+ 5·448	+ 7·705	+ 9·038	+ 8·845
$\Sigma(\sin\varepsilon)=78\cdot 984$ Deflect North.					
Arthur's Seat.					
$\Sigma(\sin\varepsilon)$ for Prisms North of station	−7·118	−11·281	−11·845	−12·014	−11·719
$\Sigma(\sin\varepsilon)$ for Prisms South of station	−3·524	− 9·164	− 8·603	− 7·436	− 6·421
$\Sigma(\sin\varepsilon)=18\cdot 829$ Deflect South.					
North Station.					
$\Sigma(\sin\varepsilon)$ for Prisms South of station	+4·867	+4·347	+3·159	+2·179	+1·932
$\Sigma(\sin\varepsilon)$ for Prisms North of station	−2·002	−8·038	−8·856	−8·424	−6·973
$\Sigma(\sin\varepsilon)=50\cdot 777$ Deflect South.					

By drawing twelve rings at 500 feet apart round the centre station, and sixteen rings round each of the other two stations, the results contained in the following Table are obtained:—

Stations.	Σ (sin ϵ).	2nd Ring.	3rd Ring.	4th Ring.	5th Ring.	6th Ring.	7th Ring.	8th Ring.	9th Ring.	10th Ring.	11th Ring.	12th Ring.	13th Ring.	14th Ring.	15th Ring.	16th Ring.
South Station.	Σ_n	+ 6·614	+3·187	+1·088	+0·025	−0·095	−0·088	−0 418	−0·926	−1·004	−1·037	−1·004	−0·908	−0·843	−0·828	−0·806
	Σ_s	− 6·299	−3·531	−2·536	−2·026	−1·713	−1·488	−1·309	−1·149	−0·985	−0·844	−0·729	−0·625	−0·539	−0·472	−0·434
Arthur's Seat.	Σ_n	−10·560	−7·996	−5·850	−4·656	−4·474	−4·570	−4·209	−3·839	−3·467	−3·119	−2·883				
	Σ_s	− 5 148	−6·634	−7·715	−6·425	−5·295	−4·549	−3·994	−3·587	−3·234	−2·931	−2·667				
North Station.	Σ_n	− 4·644	−3·094	−2·859	−2·361	−1·954	−1·623	−1·422	−1·258	−1·176	−1·117	−1·048	−0·996	−0·941	−0·935	−0·894
	Σ_s	+ 2·009	+1·695	+0·978	+1·028	+1·195	+0·745	−0·361	−0·747	−0·671	−0·604	−0·575	−0·558	−0·525	−0·488	−0·449

where Σ_n signifies $\Sigma(\sin\varepsilon)$ for the prisms north of the station,

Σ_s signifies $\Sigma(\sin\varepsilon)$ for the prisms south of the station.

Hence we obtain—

South Station.	Arthur's Seat.	North Station.
$\Sigma_n - \Sigma_s = +27{\cdot}636$	$\Sigma_s - \Sigma_n = +3{\cdot}441$	$\Sigma_s - \Sigma_n = +28{\cdot}994.$

In order to obtain the whole effect at each station, we must add to these the fifth part of the sum of the sines in the first ring of 500 feet at each of these stations: these are, respectively, 15·797, 3·766, 10·155; so that we have—

At South Station . . . $\Sigma(\sin \varepsilon) = 43{\cdot}433$

At Arthur's Seat . . . $\Sigma(\sin \varepsilon) = 7{\cdot}207$

At North Station . . . $\Sigma(\sin \varepsilon) = 39{\cdot}149$

Consequently,

Deflection at South Station $= 4''{\cdot}265\, x$ North

Deflection at Arthur's Seat $= 0''{\cdot}708\, x$ South

Deflection at North Station $= 3''{\cdot}845\, x$ South.

Comparison of observed and calculated Deflection.

We may now determine a value of x by the comparison of the observed effects of the action of the hill upon the amplitudes, with the calculated effects in terms of x. The equations thus obtained are

$$4{\cdot}973\, x = 2{\cdot}81$$

$$8{\cdot}110\, x = 4{\cdot}07,$$

whence

$$90{\cdot}503\, x = 46{\cdot}982$$

$$x = {\cdot}5191.$$

This solution contains tacitly the assumption that the effect of the *general* south deflection is equal at each of the three stations; if we put y for this quantity expressed in seconds, then the following equations will result from the comparison of the observed and geodetical latitudes, together with the calculated but unknown deflections in x,

$$y - 4{\cdot}265\, x - 2{\cdot}44 = 0$$

$$y + 0{\cdot}708\, x - 5{\cdot}25 = 0$$

$$y + 3{\cdot}845\, x - 6{\cdot}51 = 0,$$

which give $y = 4''{\cdot}68$ and $x = {\cdot}5076$.

These quantities give, when supplied in the equations, the following errors:

$$+0''{\cdot}08;\ -0''{\cdot}21;\ +0''{\cdot}13;$$

so nearly are the observations represented by these values of x and y.

Extension of the Calculation of Deflection.

The result just obtained, namely, that the ratio of the density of Arthur's Seat to

the mean density of the earth is equal to ·5076, is somewhat arbitrary, from this cause, that it is slightly dependent on the extent to which the calculation of the attraction is carried out. Had there not existed a marked difference in the mean height of the ground on the north side and on the south side of the hill, a smaller number of circles would have been sufficient. The existence of this attracting mass forbids our limiting the calculation to the visible extent of the hill; we must, therefore, in order to compare with what we have already obtained, extend the calculation to include a circle of about nine miles diameter round each station.

We shall now, instead of drawing the circles at 500 feet apart, make the radius of the $(n+1)$th circle equal to $\frac{7}{6}$ of the radius of the nth circle, so that they shall be in geometric progression. We have already drawn twelve circles round Arthur's Seat; the radius of the 13th circle will therefore be $\frac{7}{6}\cdot 500\cdot 12=(\frac{7}{6})\,6000$ feet, that of the 14th will be $(\frac{7}{6})^2\,6000$ feet, and so on. Around each of the other two stations sixteen circles have been drawn, the radii of the 17th and 18th will therefore be $(\frac{7}{6})\cdot 8000$ and $(\frac{7}{6})^2\cdot 8000$ feet, and so on.

Now if h be the height in feet of any one of the compartments thus formed, we have shown that the resulting deflection in seconds is

$$x.\frac{h}{12}\log_e(\tfrac{7}{6})\,12\cdot 447\,\frac{1}{5280}$$
$$=0\cdot 00003027xh.$$

The following Table contains the sums of the heights of the surface for each of the additional rings:—

Station.	$\Sigma(h)$.	17th.	18th.	19th.	20th.	21st.	22nd.	23rd.			Total.
South Station.	$\Sigma(h)$ north	+2385	+1685	+ 935	+ 630	+ 465	+ 180	− 10	......	......	+ 6270
	$\Sigma(h)$ south	+6060	+6870	+8295	+9010	+8925	+8175	+10060	......	......	+57935
North Station.	$\Sigma(h)$ north	+ 520	+ 385	+ 277	− 15	− 200	− 320	− 375	......	......	+ 272
	$\Sigma(h)$ south	+4495	+5045	+5640	+6475	+8000	+8175	+ 8135	......	......	+45965
		13th.	14th.	15th.	16th.	17th.	18th.	19th.	20th.	21st.	
Arthur's Seat.	$\Sigma(h)$ north	+2643	+2245	+1624	+ 904	+ 548	+ 427	+ 148	− 92	− 132	+ 8315
	$\Sigma(h)$ south	+4645	+5025	+5460	+6080	+7460	+8485	+ 8845	+8550	+9595	+64125

Consequently the effective sums of the heights are,—

South Station . . . . . $\Sigma(h)=51665$

Arthur's Seat . . . . . $\Sigma(h)=55810$

North Station . . . . . $\Sigma(h)=45693$

And therefore, multiplying by $\cdot 00003027x$, the resulting deflections are,—

South Station . . . . . $1\cdot 565\,x$

Arthur's Seat . . . . . $1\cdot 691\,x$

North Station . . . . . $1\cdot 393\,x$

We see from this that the assumption of y being constant for the three stations was not very erroneous, though the difference is perceptible.

We shall now form the equations for x and y, remarking that y is not now the same quantity that was before represented by that symbol, and that the assumption of its being constant for the three stations is now almost unobjectionable. Taking into consideration the deflections before obtained, the total deflections *south* at each of the stations will be—

South Station	$(1{\cdot}565-4{\cdot}265)x=-2{\cdot}700x$
Arthur's Seat	$(0{\cdot}708+1{\cdot}691)x=\ \ 2{\cdot}399x$
North Station	$(3{\cdot}845+1{\cdot}393)x=\ \ 5{\cdot}238x$

Hence the equations are,—

$$y-2{\cdot}700x-2{\cdot}44=0$$
$$y+2{\cdot}399x-5{\cdot}25=0$$
$$y+5{\cdot}238x-6{\cdot}51=0,$$

which give for the most probable values of x and y,

$$x={\cdot}5173 \qquad y=3{\cdot}8820.$$

By substituting these values in the equations, they show the errors

$$+0''{\cdot}04; \quad -0''{\cdot}13; \quad +0''{\cdot}08,$$

showing that the above values agree very well with the observations. From a comparison of the errors of these equations with those previously solved, it would appear that the probable error of this value of x is considerably less than that of the value (${\cdot}5076$) then obtained. The two values, however, are as close as could be expected. We shall adopt, therefore, as most probable, so far as resulting from these observations, $x={\cdot}5173.$

We may estimate the probable error of this quantity dependent upon the probable errors of the observed amplitudes thus; writing the three equations in the form

$$y+ax+a'=0$$
$$y+bx+b'=0$$
$$y+cx+c'=0,$$

we have
$$x=-\frac{(a-b)(a'-b')+(b-c)(b'-c')+(c-a)(c'-a')}{(a-b)^2+(b-c)^2+(c-a)^2}.$$

If now λ be the observed latitude of the South station, λ_1, λ_2, λ_3 the geodetic latitudes of the three stations,—

$$a'=\lambda-\lambda_1 \qquad b'=\lambda+\phi_1-\lambda_2 \qquad c'=\lambda+\phi_1+\phi_2-\lambda_3$$
$$a'-b'=\lambda_2-\lambda_1-\phi_1$$
$$b'-c'=\lambda_3-\lambda_2-\phi_2$$
$$c'-a'=\lambda_1-\lambda_3+\phi_1+\phi_2.$$

The probable error of x depends on the probable errors of $a'-b'$, $b'-c'$, and $c'-a'$, that is supposing the geodetic amplitudes to be free of error, on the probable errors of ϕ_1 and ϕ_2. The part of x involving ϕ_1 and ϕ_2 is $\frac{1}{97{\cdot}07}\times(13{\cdot}037\phi_1+11{\cdot}070\phi_2)$: consequently the probable error of x is equal to the probable error of $\cdot1343(\phi_1+0{\cdot}85\phi_2)$, which, by means of the expression given for the probable error of $\phi_1+\mu\phi_2$, becomes (making $\mu=0{\cdot}85$)

$$\text{probable error of } x=\pm0{\cdot}0053.$$

Mean Density of the Earth.

Having now ascertained the *ratio* of the mean density of Arthur's Seat to the mean density of the earth, the knowledge of the latter results immediately from the knowledge of the former. Assuming as the result of observation 2·75 for the mean density of Arthur's Seat, it follows that

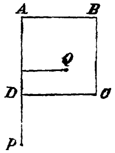
$$\text{Mean density of earth }=\frac{2{\cdot}75}{\cdot5173}=5{\cdot}316.$$

The probable error of the divisor ·5173 being ·0053, the probable error of the resulting mean density is $\pm\cdot054$, so that, considering no other cause of error than those of the zenith sector observations, we have

$$\text{Mean density of earth }=5{\cdot}316\pm\cdot054.$$

The General Deflection.

We proceed now to examine into the question of the sufficiency of the cause before mentioned, namely, the defect of matter to the north of Edinburgh and the accumulation of matter to the south, to produce the general deflection that is observed to the amount of 5″, or rather more. In the first place, let it be required to find the attraction of a rectangular film ABCD, whose thickness is h and density ϱ, upon a point P in the production of one of its sides, AD. Measure x along PA and y perpendicular to it in the plane of the rectangle, then the mass of a small element is $\varrho h dx dy$, and therefore its attraction in the direction AP is



$$\frac{\varrho h x dx dy}{(x^2+y^2)^{\frac{3}{2}}};$$

the integral with respect to x between the limits aa' is

$$\varrho h\left\{\frac{dy}{(a^2+y^2)^{\frac{1}{2}}}-\frac{dy}{(a'^2+y')^{\frac{1}{2}}}\right\},$$

which being integrated from $y=0$ to $y=b$, is

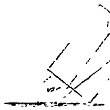
$$\varrho h\log_e\frac{\frac{y}{a}+\sqrt{1+\left(\frac{y}{a}\right)^2}}{\frac{y}{a'}+\sqrt{1+\left(\frac{y}{a}\right)}}.$$

If now we put the angles DCP$=\varphi$, ABP$=\varphi'$, we shall get, since

$$\frac{y}{a}=\cot\varphi \quad \frac{y}{a'}=\cot\varphi,$$

$$\text{Attraction } =\varrho h \log_e\left(\frac{\cot\frac{1}{2}\varphi}{\cot\frac{1}{2}\varphi'}\right).$$

We may thus determine a sufficiently close approximation to the effect of the hollow of the Forth. An examination of a map of Scotland, on a sufficiently large scale, will show that a rectangle of eighteen miles by twelve, having its longest side inclined 40° to the meridian, may be placed so as to cover the greater part of the Forth with some exactness, having also Edinburgh opposite to the middle point of the south side and Arthur's Seat nearly two miles from this side, as in the accompanying diagram. The angles φ and φ' will be found to be 73° and 18°, and therefore the attraction of a rectangular stratum of these dimensions with thickness h and density ϱ will be, (2·3025 being the reciprocal of the modulus of the common system of logarithms)



$$=2\varrho\frac{h}{5280}(2\cdot3025)\log\frac{\cot 9^\circ}{\cot(36^\circ\ 30')}$$

$$=2\varrho h\frac{2\cdot3025\times\cdot6695}{5280},$$

and therefore the corresponding or resulting deflection is

$$2\frac{2\cdot3025\times\cdot6695\times12\cdot44\cdot7}{5280}\frac{\varrho}{\delta}h=0''\cdot00727\frac{\varrho}{\delta}h$$

in seconds, h to be expressed in feet, δ the mean density of the earth.

An inspection of a chart of the Forth will show that the depth may be taken at a very even average of 30 feet below mean water-level; so that the attraction of the water ($\varrho=1$) upon Arthur's Seat causes a deflection $=0''\cdot04$ to the north-east at mean water; the latitude of points in the neighbourhood is consequently variable to the amount of about $0''\cdot02$, depending on the tide.

We may now suppose the water to be removed and the hollow filled up with rock to a mean level of 70 feet. Then taking 2·5 for the mean density of the rock, the attraction of this stratum would be $0''\cdot36$, or resolved in the direction of the meridian, the deflection north would be $0''\cdot28$. If the hollow were filled up to a mean level of 150 feet, the deflection north would be $0''\cdot50$.

From this we conclude that the existence of the hollow of the Forth will account for but a small portion of the deflection of $5''$.

To the south of Edinburgh the country gradually rises, until at the southern boundary of the country the mean level is about 1000 feet with peaks rising to 1750 feet. The contours for the county of Peebles are not yet sufficiently advanced to permit the calculation of the attraction of the hills in the north of that county. We may however extend the calculation to the southern borders of Edinburghshire.

The number of circles already drawn round Arthur's Seat is 21, the last nine being drawn according to the law $r_{n+1}=\frac{7}{6}r_n$: if we draw seven more according to the same law, this will carry us slightly beyond the boundary of the county. The sums of the heights in the different rings will then be as follows:—

22nd.	23rd.	24th.	25th.	26th.	27th.	28th.
11850	12050	12850	15950	17100	18750	16800

the sum of which is 105350. The consequent deflection will be, using the same value of x, namely ·5173,

$$0''\cdot00003027\times105350\times\cdot5173=1''\cdot64.$$

To this we have to add the quantity obtained for the preceding nine rings, namely, $1''\cdot69x$ or $0''\cdot88$, making altogether $2''\cdot52$ due to the high ground to the south within the county of Edinburgh.

From the height of the country in the north of Peebleshire, it seems probable that when the calculation can be carried into that county, a sensible addition to the quantity above determined will be obtained, and that the whole of the $5''$ may *possibly* be accounted for.

In conclusion, the principal results arrived at are these:—

1st. The effect of the attraction of the Pentland Hills is observed in nearly equal amount at each of the three stations on Arthur's Seat.

2nd. The calculated attractions of the mass of Arthur's Seat at the three stations are,

South Station.	Arthur's Seat.	North Station.
$2''\cdot21$ North.	$0''\cdot37$ South.	$2''\cdot00$ South.

and, since the observed deflection at Arthur's Seat is $5''\cdot25$, the apparent effect of the Pentlands is $4''\cdot88$ at the summit of the hill.

3rd. Of this deflection of $4''\cdot88$, the computed attraction due to the configuration of the ground within a radius of fifteen miles accounts for about $2''\cdot5$; and, inasmuch as we know that the igneous rocks of Arthur's Seat and the Pentland Hills have an origin at a great depth below the surface of the earth, the difference between the observed and computed attraction is probably owing in part to the high specific gravity of the mass of rock beneath them.

4th. The deflection at the Royal Observatory, Calton Hill, being $5''\cdot63$ South, exceeds that at Arthur's Seat by $0''\cdot70$. Of this deflection, $0''\cdot60$ is due to the configuration of the ground comprised within a circle of a mile and a quarter round the Observatory.

5th. The latitude of Arthur's Seat or points in the neighbourhood varies to the amount of about $0''\cdot02$ between high and low water.

6th. The mean density of the earth determined from the observations at the three

stations on Arthur's Seat, is 5·316, with a probable error of ±·054 (or one hundredth part) due to the probable errors of the astronomical amplitudes. If δ be the probable error of the assumed mean density of Arthur's Seat, the probable error of this determination of the mean density of the earth is

$$\pm\sqrt{3{\cdot}725\,\delta^2+{\cdot}003}.$$

Remarks.

In the original paper as read at the Meeting of the Royal Society on 21st February, the mean density was given as 5·14 with a probable error of ±·07. In a subsequent revision of the calculations, the astronomical amplitudes and their probable errors were determined as herein explained. These amplitudes exceed those previously used by $0''{\cdot}02$, $0''{\cdot}01$, $0''{\cdot}03$, tending to increase the density. The attraction due to the ground within 100 feet round each of the stations, originally omitted, is now included, also tending to increase the density.

Explanation of the Plates.

PLATE XXXII.

Is the contoured plan of Arthur's Seat, on the scale of six inches to a mile: this is part of one of the sheets of the plan of Edinburghshire which has been published on that scale. The zenith sector stations and the lines of sections are marked on this plan. The contours furnish sufficient data to make a model.

PLATE XXXIII.

Contains geological sections taken on the three lines which are drawn on the plan, and also a table of the specific gravity of the rocks.

These plates have been engraved and electrotyped at the Ordnance Survey Office, and form part of the series of plates made to illustrate the account of the Trigonometrical Survey of Great Britain which is now in the press.

XXIX. *On the Figure, Dimensions, and Mean Specific Gravity of the Earth, as derived from the Ordnance Trigonometrical Survey of Great Britain and Ireland. Communicated by Lieut.-Colonel* JAMES, *R.E., F.R.S. &c., Superintendent of the Ordnance Survey.*

Received April 30,—Read May 8, 1856.

THE Trigonometrical Survey of the United Kingdom commenced in the year 1784, under the immediate auspices of the Royal Society; the first base was traced by General ROY, of the Royal Engineers, on the 16th of April of that year, on Hounslow Heath, in presence of Sir JOSEPH BANKS, the then President of the Society, and some of its most distinguished Fellows.

The principal object which the Government had then in view, was the connexion of the Observatories of Paris and Greenwich by means of a triangulation, for the purpose of determining the difference of longitude between the two observatories.

A detailed account of the operations then carried on is given in the first volume of the 'Trigonometrical Survey,' which is a revised account of that which was first published in the 'Philosophical Transactions' for 1785 and three following years.

At the time when these operations were in progress, the Survey of several counties in the south-east of England, including Kent, Sussex, Surrey, and Hampshire, was also in progress, under the direction of the Master-General of the Ordnance, for the purpose of making military maps of the most important parts of the kingdom in a military point of view; and it was then decided to make the triangulation which extended from Hounslow to Dover the basis of a triangulation for these surveys.

It is extremely to be regretted that a more enlarged view of the subject had not then been taken, and a proper geometrical projection made for the map of the whole kingdom. As it is, the south-eastern counties were first drawn and published in reference to the meridian of Greenwich, then Devonshire in reference to the meridian of Butterton in that county, and thirdly the northern counties in reference to the meridian of Delamere in Cheshire; but there is a large intermediate space, the maps of which are made of various sizes to accommodate them to the convergence of the meridian.

In 1799 the Royal Society gave further proof of the interest it took in the progress of the Survey, by lending to the Ordnance its great 3-foot Theodolite, made by RAMSDEN, for the purpose of expediting the work of the Survey; and I have great pleasure in stating, that although this instrument has been in almost constant use for the last sixty-seven years, during which time it has been placed on the highest

church towers and the loftiest mountains in the kingdom, from the Shetlands to the Scilly Islands, it is at this day in perfect working order, and probably one of the very best instruments that was ever made.

The great Trigonometrical operations of the Survey have been carried on under so many officers, from the time of their commencement under General Roy down to the present time, that it would be quite impossible, in this short notice, to mention more than the names of several Superintendents who have succeeded General Roy, viz. Colonel Williams, Major-General Mudge, Major-General Colby, and Colonel Hall; but in justice to the highly meritorious body of non-commissioned officers of the Corps of Royal Sappers and Miners, I should state, that whilst in the early part of the Survey the most important and delicate observations were entrusted solely to the commissioned officers, these duties have of late years been performed by the non-commissioned officers with the greatest skill and accuracy.

In the Historical sketch of the Survey which I purpose publishing shortly, I hope to be able to do justice to the individual merits of all employed.

The computations connected with the corrections of the observed angles, to make the whole triangulation as nearly as possible perfectly consistent, have been most voluminous, and have been made under the direction of Lieut.-Colonel Yolland, Captain Cameron, and Captain Alexander R. Clarke; but I gladly avail myself of this opportunity to acknowledge the great and important assistance and advice which, both as regards the instruments and the calculations, we have at all times received from the Astronomer Royal.

The triangulation by the methods which will be explained, is now made consistent in every part, so that any side of any triangle being taken as a base, the same distance will be reproduced when it is computed through any portion or the whole series of triangles; and when the five measured bases on which we rely are incorporated in this triangulation, the greatest difference between their measured and computed lengths is not as much as 3 inches, and yet some of the bases are upwards of 400 miles apart.

Several bases of from five to seven miles long have been measured, but those upon which the chief reliance has been placed are the Lough Foyle and Salisbury Plain bases, which were measured with General Colby's compensation bars. The difference between the measured and computed length of the one base from the other through the triangulation is 0·4178 ft., or about 5 inches.

This difference has been divided in proportion to the square root of the lengths of the measured bases, by which we have obtained the mean base which has been used in the triangulation; there is therefore a difference of + or −0·2 ft., or $2\frac{1}{2}$ inches between the measured and computed length of these bases from the mean base.

The Hounslow Heath base was measured with Ramsden's 100-ft. steel chains, and only differs 0·173 ft., or about 2 inches, from its computed length from the mean base.

The Belhelvie base in Aberdeenshire, also measured with the steel chains, differs only 0·24 ft., or less than 3 inches, from the computed length.

The difference between the measured and computed length of the Misterton Carr base, near Doncaster, also measured with the steel chains, is only 0·191 ft., or about 2 inches; and it will be observed that the difference between the computed and measured lengths of these three bases (measured with chains) is not greater than the difference between the measured and computed length of the Lough Foyle and Salisbury Plain bases (measured with the compensation bars), from which it may be inferred that bases measured with steel chains are deserving of the greatest confidence; and when the great simplicity, portability, and cheapness of the chains is compared with the complex, heavy and expensive apparatus of the compensation bars, I should anticipate that they would be more generally employed than they have been of late years, especially in the colonies, and in countries where the transport of heavy articles is effected with difficulty.

The length of the base on Rhuddlan Marsh in North Wales, which was measured with steel chains, differs 1·596 ft. from the computed length; but from the circumstance that the extremities of the base are very badly situated with reference to the surrounding Trigonometrical stations, the angles being very acute, and not well observed, we have placed little confidence in the result of the comparison of its computed and measured length.

One of the first practical results arising from the completion of the triangulation is, that we are now able to engrave the latitude and longitude on the marginal lines of the old sheets of the 1-inch map of England, and this is now being done.

The following account of the Trigonometrical operations and calculations has been drawn up by Captain Alexander R. Clarke, R.E.: this account may be considered as an abridgement of that more detailed account which is now in the press, and will be shortly published.

It will be seen that the equatorial diameter of the earth, as derived from the Ordnance Survey, is 7926·610 miles, or about one mile greater than it is given by the Astronomer Royal in his 'Figure of the Earth,' and that the ellipticity is $\frac{1}{299{,}33}$, or as the Astronomer Royal conjectured, something "greater than $\frac{1}{300}$," which he gives in the same paper.

The mean specific gravity of the earth, as derived from the observations at Arthur's Seat, was stated in a former paper to be 5·14; the calculations have since been revised, and we now find it to be 5·316.

The mean specific gravity of the earth, as derived from the only other observations on the attraction of mountain masses on which any reliance has been placed, viz. the Schehallien observations, give, as finally corrected by Hutton, $\frac{99}{20}$, or almost 5·0.

From the experiments with balls we have the following results :—

By Cavendish, as corrected by Baily	5·448
By Baily	5·67
By Reich	5·44

From the pendulum experiments at a great depth and on the surface, the Astronomer Royal obtained 6·566.

I have recently received, through the Astronomer Royal, two copies of the new National Standard Yard: it is obviously necessary that our geodetic measures should be given in reference to the standard; but not knowing from what scale the standard has been taken, I am unable to say at present in what way the reduction is to be made; that is, whether by reference to the comparison of the old standards which have been already made, or by the mechanical process of a direct comparison of the Ordnance Standard with the new National Standard.

H. JAMES, *Lieut.-Colonel R.E.*

The Principal Triangulation of the Ordnance Survey of Great Britain and Ireland, extending from the Scilly Isles, in latitude 49° 53′, to the Shetland Isles, in latitude 60° 50′, and embracing at its widest extent about 12° of longitude, consists of about 250 stations.

The observations for the connexion of the trigonometrical stations have been made with four large theodolites, two of 3 feet, one of 2 feet, and the other of 18 inches in diameter. The first two instruments (one of which is the property of the Royal Society), and the 18-inch theodolite, were constructed by RAMSDEN at the commencement of the trigonometrical operations in England in 1798: the 2-feet theodolite was constructed by Messrs. TROUGHTON and SIMMS at the commencement of the Irish Survey in 1824.

The latitudes of thirty-two of the stations of the principal triangulation have been determined by observations made with RAMSDEN's zenith sector, and since the destruction of that instrument in the great fire at the Tower of London, with AIRY's zenith sector. All the observations made with these instruments have been published in detail*.

The mode of observing with the theodolite may be shortly described as follows:—The instrument being first placed very carefully over the precise centre of the station, an object having a fine vertical line of light, with a breadth of about 10″, is set up in a convenient position within a mile or two of the station; this object, called the "referring-object," serves as a point of reference from which all angles are measured. The lower limb of the instrument being clamped, the observer intersects the referring-object and then each of the principal points in succession, concluding with a second observation of the referring-object, which should be identical, within the limits of errors of observation, with the first reading of that object: the instrument is then unclamped and the bearings read again on different parts of the divided circle. The method by which these observations are reduced to the most probable

* 'Astronomical Observations made with RAMSDEN's Zenith Sector,' 1842. 'Astronomical Observations made with AIRY's Zenith Sector,' 1852.

results, is an approximate solution of the equations resulting from the method of least squares.

The direction of the meridian has been determined by observations of the elongations of α, δ, λ Urs. min. and 51 Cephei; at six of the stations at which these observations have been made, the probable error of the result is under $0''.40$, at twelve under $0''.50$, at thirty-four under $0''.70$, and at fifty-one under $1''.00$.

Measured Base Lines.

The account of the measurement with RAMSDEN's steel chain of the base lines on Hounslow Heath in 1791, on Salisbury Plain in 1794, on Misterton Carr in 1801, and on Rhuddlan Marsh in 1806, will be found in the 'Account of the Trigonometrical Survey.' These base lines are all expressed in terms of RAMSDEN's brass scale at the temperature of 62° FAHRENHEIT. The chains were compared with RAMSDEN's prismatic bar (20 feet in length), which was laid off from the brass scale at the temperature of 54° FAHRENHEIT. By a series of comparisons of the Ordnance 10-foot standard iron bar (designated O_1) with RAMSDEN's 20-foot standard, made at Southampton, it was found that

$$\text{RAMSDEN's bar} = 20.0007656 \text{ feet of } O_1,$$

so that any measurement expressed in terms of feet of RAMSDEN's bar at 62° must be multiplied by 1·0000383 to give feet of O_1, at the same temperature. Also to reduce a measurement expressed in terms of RAMSDEN's brass scale to the same in terms of RAMSDEN's bar, it must be increased by a quantity corresponding to the difference of the expansions of brass and iron for 8°; and taking these quantities as used in the reduction of the bases, it will be found that the multiplier is 1·0000328, and hence to reduce the old bases to feet of O_1, they must be multiplied by 1·0000711.

In 1816 a base line of five miles in length was measured by Major-General COLBY on Belhelvie Sands, Aberdeenshire: the measurement was effected with RAMSDEN's steel chains, and in precisely the same manner as the previous bases. The chains were compared with RAMSDEN's bar by Mr. BERGE both before and after the measurement.

In 1826–27 the Lough Foyle Base was measured in the north of Ireland with Major-General COLBY's compensation bars. Of this measurement a description in detail has been published*.

In 1849 the old base line on Salisbury Plain was remeasured. This measurement exceeded the old measure when reduced to the same standard by a foot. The guns marking the extremities of the old line were found imbedded very firmly in the earth, and in all probability in exactly their original positions.

By a series of comparisons instituted in 1834† between the Royal Astronomical

* 'Account of the Measurement of the Lough Foyle Base,' by Captain YOLLAND, R.E., 1847.

† The observations which were made by the late Lieut. MURPHY, Royal Engineers, are given in the 'Account of the Measurement of the Lough Foyle Base.'

Society's Scale and the Ordnance Standard O_2, it was determined that

Ord. Standard O_1 = 119·997508 mean inches of the centre yard of the Royal Astronomical Society's Scale;

also from Mr. Baily's comparison of this scale with the standard metre he determined*

Standard Metre = 39·369678 mean inches of the centre yard of the Royal Astronomical Society's Scale.

From more recent observations, it appears that the Royal Astronomical Society's scale has undergone a permanent alteration of length; the interval however between the two series of observations above quoted was not sufficiently long to vitiate the connexion thus established between the Ordnance Standard and the metre. The resulting value of the metre in terms of O_1 is therefore

Standard Metre = 3·2808746 mean feet of O_1,

and hence since the metre = 443·296 lines of the toise of Peru,

Toise = 6·3945438 mean feet of O_1.

Reduction of the Triangulation.

If u represent the true ratio of the distance between any two points in a network of triangulation to the base line; A B C...A'... the true angles, whose observed values are $A+\alpha$, $B+\beta$, $C+\gamma$...$A'+\alpha'$..., then if u_1 be the calculated value of u obtained by using the series of observed angles ABC..., u'_1 the value obtained by using the series of observed angles A'B'C'....

$$u = f(ABC\ldots) = f(A'B'C'\ldots) = \ldots.$$
$$u_1 = u + a\alpha + b\beta + c\gamma + \ldots.$$
$$u'_1 = u + a'\alpha' + b'\beta' + c'\gamma' + \ldots.$$

and so on. Each different calculation of u will therefore give a different value for that quantity.

In the necessary existence of these discrepancies among the calculated values of u, it becomes of much importance to obtain the most probable value. In ordinary calculations this has been generally effected by assuming it to be the mean of all the calculated values $u_1 u'_1 u''_1$..... This might be improved upon by assigning to each value of u its proper weight by means of the weights of the observed angles, but the method would still be imperfect and discrepancies would still exist in other parts of the work.

From the above equations, though we cannot determine the precise value of u, yet we can obtain some precise information respecting the errors of observation; for we have evidently, since the quantities $abc\ldots a'b'c'\ldots$ are numerical, certain equations of condition between the unknown errors.

But the number of such equations of condition for the whole figure being necessarily less than the actual number of errors, an indefinite number of systems of corrections

* Memoirs of the Royal Astronomical Society, vol. ix.

might be obtained that would satisfy all the geometrical relations of the triangulation. The question then is to determine that system which is the most probable, and the solution derived from the theory of probabilities is, that the most probable system of corrections $x\,x'\ldots$ is that which makes the function $\Sigma(wx^2)$* a minimum.

If n be the number of observed angles in a network of triangulation, m the number of points, then $2(m-2)$ will be the number of angles absolutely required to fix all the points, consequently the geometrical figure must supply $n-2m+4$ equations of condition amongst the true angles or amongst the corrections to the observed angles: we have therefore $n-2+4$ equations of the form

$$0=a_1+b_1x+c_1x'+d_1x''+\ldots..$$
$$0=a_2+b_2x+c_2x'+d_2x''+\ldots..,$$

which are to be determined so that the quantity

$$U=wx^2+w'x'^2+w''x''^2+\ldots..$$

shall be a minimum. Multiplying the equations of condition by unknown quantities $\lambda_1\lambda_2\lambda_3\ldots.$, we obtain by the theory of maxima and minima of functions of many variables,

$$\begin{aligned} wx\ &=b_1\lambda_1+b_2\lambda_2+b_3\lambda_3+\ldots. \\ w'x'\ &=c_1\lambda_1+c_2\lambda_2+c_3\lambda_3+\ldots. \\ w''x''&=d_1\lambda_1+d_2\lambda_2+d_3\lambda_3+\ldots. \end{aligned}$$

Substituting the values of $xx'\ldots$ in terms of $\lambda_1\lambda_2\ldots.$ as obtained from these equations, in the equations of condition we get a system of equations from which $\lambda_1\lambda_2\ldots$ may be determined; and having obtained the numerical values of these quantities, the last set of equations will give the numerical values of the required corrections $xx'\ldots$

In order that the results of the triangulation as applied to the determination of the figure of the earth might have the greatest weight possible, the most probable system of corrections has been calculated according to BESSEL's method, shortly described above. The principal and only objection to the application of this method of obtaining the most trustworthy results, is the extremely voluminous and tedious nature of the calculations. The total number of equations of condition for the triangulation is 920; if therefore the whole were to be reduced in one mass, it would involve as a small part of the work the solution of an equation of 920 unknown quantities. The following method of approximation therefore was adopted: the triangulation was divided into twenty-one parts or figures, each affording a not unmanageable number of equations of condition; four of these parts, not adjacent, were first adjusted by the method just explained. The corrections determined in these figures were substituted, so far as they entered, in the equations of condition of the adjacent figure, and the sum of the squares of the remaining corrections in that figure made a minimum. Thus each division of the triangulation, with the exception of the four specified above, is dependent upon one or more of the figures to which it is adjacent.

* Where w is the weight of the observation corresponding to the error x.

The average number of equations in each figure is about 44; the greatest number of equations in any one figure is 77. Each figure was worked by two independent computers. This of itself alone would have been insufficient to secure freedom from error, but the final working of every possible triangle, after the corrections were applied to the observed angles, secured perfect accuracy.

Corrections to all the observed bearings having been obtained in the manner explained, it is clear that since all the geometrical relations of the figure are satisfied, no discrepancies can present themselves between the calculated values of the distance between any two points by whatever series of angles it may be obtained. The triangles are calculated by LEGENDRE's theorem. This theorem may be applied to triangles of any magnitude, up to two or three hundred miles, without fear of error. The greatest errors that can result in the values of the sides a, b of a spheroidal triangle as calculated from the side c by spherical trigonometry, using the geometric mean of the principal radii of curvature of the surface for the mean latitude of the triangle as the radius of the sphere, are (the position of the triangle in *azimuth* being the variable quantity with respect to which the errors are the greatest possible),

$$\varepsilon_a=\mu(4l^2+m^2+4n^2)^{\frac{1}{2}}$$

$$\varepsilon_b=\mu(l^2+4m^2+4n^2)^{\frac{1}{2}}$$

$$\mu=\frac{e^2\cos^2\lambda . abc}{12\mathrm{R}^2\sqrt{1-n^2}},$$

where R is the radius of the sphere, e the eccentricity of the earth's surface, lmn the cosines of the angles of the triangle opposite abc respectively, and λ its mean latitude.

Comparison of Bases.

The absolute length of any side, or the linear scale of the triangulation, is made to depend on the bases measured with the compensation bars at Lough Foyle and on Salisbury Plain. The discrepancy between the measured and calculated length of these bases is about 5 inches; this discrepancy is divided so that each of the two bases shall exhibit an error proportional to the square root of its length: the comparison of all the bases is then as follows:—

Date.	Bases.	Length in terms of RAMSDEN's Scale.	Length in terms of Ordnance Standard.	Length in Triangulation.	Difference.
		ft.	ft.	ft.	ft.
1791.	Hounslow Heath............	27404·24	27406·190	27406·363	+0·173
1794.	Salisbury Plain	36574·23	36576·830	36577·656	+0·826
1801.	Misterton Carr	26342·19	26344·060	26343·869	−0·191
1806.	Rhuddlan Marsh............	24514·26	24516·000	24517·596	+1·596
1817.	Belhelvie......................	26515·65	26517·530	26517·770	+0·240
1827.	Lough Foyle	....	41640·887	41641·103	+0·216
1849.	Salisbury Plain	....	36577·858	36577·656	−0·202

Latitudes and Longitudes.

For short distances the ordinary formulæ are sufficient, but in the case of distances above 80 or 100 miles the following formulæ are used, A being the given point, and B that whose latitude and longitude are required:—

Let s =distance AB measured on the surface of the earth

ν =normal to minor axis at A

θ =angle subtended at the foot of this normal by the curve s

α =azimuth of B at A

α'=azimuth of A at B, both measured from the north

λ =latitude of A; $\varkappa=90-\lambda$

λ'=latitude of B

ω =difference of longitude

ϱ =radius of curvature of the meridian for the latitude $\frac{1}{2}(\lambda+\lambda')$

$$\tan\tfrac{1}{2}(\alpha'+\zeta-\omega)=\frac{\sin\frac{1}{2}(\varkappa-\theta)}{\sin\frac{1}{2}(\varkappa+\theta)}\cot\tfrac{1}{2}\alpha$$

$$\tan\tfrac{1}{2}(\alpha'+\zeta+\omega)=\frac{\cos\frac{1}{2}(\varkappa-\theta)}{\cos\frac{1}{2}(\varkappa+\theta)}\cot\tfrac{1}{2}\alpha$$

$$\lambda'-\lambda=\frac{s}{\varrho}\,\frac{\sin\frac{1}{2}(\alpha'+\zeta-\alpha)}{\sin\frac{1}{2}(\alpha'+\zeta+\alpha)}\left(1+\frac{\theta^2}{12}\cos^2\tfrac{1}{2}(\alpha'-\alpha)\right)$$

$$\theta=\frac{s}{\nu}+\frac{e^2\theta^3}{6(1-e^2)}\cos^2\lambda\cos^2\alpha$$

$$\zeta=\frac{e^2\theta^2}{4(1-e^2)}\cos^2\lambda\sin 2\alpha,$$

ζ being a minute angular correction here expressed, as also θ, in angular measure.

In the calculation of latitudes and longitudes we must suppose all the points to be projected on a regular spheroidal surface (A), very nearly agreeing with the actual surface covered by the triangulation, then by equations of condition between the observed and calculated latitudes, longitudes and azimuths, small alterations to the elements of the assumed spheroid and its position must be determined: this new spheroid (B) will be that most nearly representing the actual surface under consideration.

For the first spheroid, that determined by the Astronomer Royal as most nearly representing the earth's surface; namely,

$$a=20923713 \text{ feet}$$
$$b=20853810 \text{ feet}$$

was used as the spheroid of reference (A).

If we resolve the inclination of the actual surface at any point to that of the spheroid (B) at the same point, or rather its projection, into two inclinations, one north and the other east, and call these inclinations ξ_n and η_n, these quantities being positive when the actual surface, as compared with that of the regular surface, rises to the

north and to the east, and if we put ξ and η for the same quantities at Greenwich, from which point the calculations of latitude and longitude and azimuths had their commencement, then each point at which the latitude has been observed will give an equation of the form

$$\xi_n = a + b\xi + c\eta + m\delta a + n\delta e,$$

and each point at which the longitude or direction of the meridian has been determined will give an equation of the form

$$\eta_n = a' + b'\xi + c'\eta + m'\delta a + n'\delta e,$$

in which δa and δe are the increments to the semiaxis major and eccentricity of the spheroid (A).

In consequence of the smallness of the coefficients $n\,n'$ in the latitude of Great Britain, the quantity δe would have very little weight as determined from these equations.

Surface of Great Britain.

The approximate results derived from the above equations are, assuming $\delta e = 0$,

$$\begin{aligned} \delta a &= 2536 \\ \eta &= 0 \\ \xi &= +1''\!\cdot 4, \end{aligned}$$

so that the semiaxes of the spheroid most nearly representing the surface of Great Britain are

$$\left.\begin{aligned} a &= 20926249 \\ b &= 20856337 \end{aligned}\right\} \text{feet of Ordnance Standard } O_1.$$

$$\frac{a-b}{a} = \frac{1}{299{\cdot}33}$$

Most probable Deflections.

The last column of the following Table contains the most probable deflections at the various stations as resulting from a comparison of the actual observed latitudes with those of spheroid (B), or of the *apparent* and *mean* latitudes.

Stations.	Latitudes.				
	Observed.	Spheroid (A).	Diff. Obs.—Cal.	Spheroid (B).	Diff. Obs.—Cal.
	° ′ ″	″	″	″	″
Saint Agnes	49 53 33·94	30·78	−3·16	32·93	−1·01
Goonhilly	50 2 50·07	45·35	−4·72	47·42	−2·65
Hensbarrow	50 22 61·84	58·81	−3·03	60·73	−1·11
Port Valley	50 35 43·20	43·17	−0·03	44·95	+1·75
Week Down	50 35 51·42	50·28	−1·14	52·06	+0·64
Boniface Down	50 36 10·55	9·63	−0·92	11·41	+0·86
Dunnose	50 37 7·15	3·75	−3·40	5·53	−1·62
Black Down	50 41 8·89	10·29	+1·40	12·04	+3·15
Southampton	50 54 46·97	47·23	+0·26	48·88	+1·91
Greenwich	51 28 38·30	38·30	0·00	39·70	+1·40
Hungry Hill	51 41 10·26	11·47	+1·21	12·94	+2·68
Feaghmaan	51 55 22·85	20·30	−2·55	21·68	−1·17
Precelly	51 56 45·18	44·76	−0·42	45·99	+0·81
Arbury Hill	52 13 27·01	27·29	+0·28	28·36	+1·35
Forth Mountain	52 18 57·91	56·83	−1·08	57·93	+0·02
Delamere	53 13 18·56	17·27	−1·29	17·93	−0·63
Clifton Beacon	53 27 30·27	27·02	−3·25	27·56	−2·71
South Berule	54 8 56·40	57·60	+1·20	57·87	+1·47
Tawnaghmore	54 17 41·34	39·61	−1·73	39·93	−1·41
Burleigh Moor	54 34 19·75	15·74	−4·01	15·80	−3·95
S. end of Lough Foyle Base	55 2 38·74	33·93	−4·81	33·86	−4·88
Ben Lomond	56 11 26·27	25·26	−1·01	24·64	−1·63
Kellie Law	56 14 51·43	53·69	+2·26	53·02	+1·59
Ben Heynish	56 27 16·88	19·64	+2·76	18·95	+2·07
Great Stirling	57 27 49·12	50·14	+1·02	48·94	−0·18
Cowhythe	57 40 68·92	60·49	−8·43	59·20	−9·72
Monach	58 21 20·84	23·54	+2·70	22·01	+1·17
Ben Hutig	58 33 6·47	5·12	−1·35	3·47	−3·00
North Rona	59 7 15·19	17·67	+2·48	15·80	+0·61
Balta	60 45 1·75	7·16	+5·41	4·51	+2·76
Gerth of Scaw	60 48 56·43	61·82	+5·39	59·15	+2·72
Saxavord	60 49 38·58	41·99	+3·41	39·31	+0·73

Arcs of Meridian.

The two longest meridional lines in the triangulation are from Dunnose, in the Isle of Wight, to Saxavord in the Shetland Isles, and from St. Agnes' Lighthouse, in the Scilly Islands, to North Rona; the lengths and amplitudes are as follows:—

	Length.	Astronomical amplitude.
	feet	° ′ ″
Dunnose to Saxavord	3729335·8	10 12 31·43
St. Agnes to North Rona	3370394·2	9 13 41·25

Arc of Parallel.

The volume of the Greenwich Observations for 1845 contains the account of the determination of the longitude of Feaghmaan, a station in the Island of Valencia on the west coast of Ireland, by the transmission of chronometers.

The whole arc was subdivided by two intermediate stations, of which one was the

Liverpool Observatory, the other a temporary observatory erected at Kingstown. The following Table contains the comparison of the astronomical and geodetical determinations:—

Stations.	Observed Longitude W.	Geodetic Longitude.		Difference.	
		Spheroid (A).	Spheroid (B).	(A).	(B).
	m s	m s	m s	s	s
Liverpool	12 0·05	12 0·35	12 0·26	+0·30	+0·21
Kingstown	24 31·20	24 31·48	24 31·26	+0·28	+0·06
Feaghmaan.....................	41 23·23	41 23·02	41 22·74	−0·21	−0·49

Observatories.

The positions of the principal observatories, as calculated by their connexion with the Triangulation of the Ordnance Survey, are contained in the following Table:—

Names.	Latitudes			Longitudes		
	On Spheroid (B).	As observed.	Diff.	On Spheroid (B).	As observed.	Diff.
	° ′ ″	° ′ ″	″	m s	m s	s
Greenwich	51 28 39·70	51 28 38·30	+1·40	0 0·00	0 0·00	0·00
Edinburgh	55 57 17·57	55 57 23·20	−5·63	12 43·61	12 43·00	+0·61
Dublin	53 23 14·21	53 23 13·46	+0·75	25 20·87	25 22·00	−1·13
Cambridge	52 12 51·90	52 12 51·63	+0·27	0 23·26	0 22·69	+0·57
Oxford	51 45 38·56	51 45 36·00	+2·56	5 2·91	5 2·60	−0·31
Durham	54 46 5·27	54 46 6·20	−0·93	6 20·25	6 19·75	+0·50
Liverpool	53 24 47·06	53 24 47·80	−0·74	12 0·26	12 0·05	+0·21
Makerstown	55 49 1·83	55 49 6·00	−4·17	19 26·20	19 28·00	−1·80
Armagh	54 21 10·76	54 21 12·67	−1·91	26 35·52	26 35·50	+0·02

Figure of the Earth.

In obtaining the spheroid most nearly representing the measured arcs of meridian, we shall follow the method given by Bessel in his determination of the figure of the earth in Nos. 333 and 438 of the 'Astronomische Nachrichten;' substituting for the English arc as used by him, the data of the present results, and for the Indian arc as used by him, the data contained at page 427 of Colonel Everest's 'Account of the Measurement of Two Sections of the Meridional Arc of India.'

Colonel Everest's measurements are expressed in terms of his standard 10-foot iron bar A, and the standard 6-inch scale A, twenty parts of any linear result being in terms of feet of the iron standard and one part in terms of feet of the 6-inch scale. By means of the comparisons contained at page 436 of Colonel Everest's work and at pages 101 and [40] of the 'Account of the Measurement of the Lough Foyle Base,' it will be found, that to reduce the results contained in the former work to feet of O_1, they must be multiplied by ·99999026. There is, however, some uncertainty in the unit of measure of the earlier portion of the second East Indian Arc.

Peruvian Arc.—The data are a mean between the reduced results obtained by Delambre and Zach (Base du Syst. Metr. iii. 133, and Mon. Corresp. xxvi. 52).

Indian Arc.—The data for the first Indian Arc are given in vol. viii. of the Asiatic Researches, page 137.

French Arc.—The data for this arc will be found in the Base du Syst. Metr. ii. 565, 615, and iii. 548, 549.

Hanoverian Arc.—From GAUSS' Breitenunterscheid, &c., p. 71.

Danish and Russian Arcs, as used by BESSEL, Astronomische Nachrichten, No. 333.

Prussian Arc.—Gradmessung in Ostpreussen: BESSEL, 1838.

Swedish Arc.—Exposition des Opérations faites en Lapponie: SVANBERG.

	Latitude.	Amplitude.	Distance between the parallels.
1. Peruvian Arc.			
Tarqui	−3° 4′ 32″·068		toise.
Cotchesqui	+0 2 31·387	3° 7′ 3″·455	176875·5
2. First East Indian Arc.			
Trivandeporum	11 44 52·590		feet.
Pandree	13 19 49·018	1 34 56·428	574327·9
3. Second East Indian Arc.			
Punnæ	8 9 31·722		feet.
Damargida	18 3 15·292	9 53 43·570	3591744·06
Kalianpur	24 7 11·262	15 57 39·540	5794648·82
Kaliana	29 30 48·322	21 21 16·600	7755786·84
4. French Arc.			
Formentera	38 39 56·11		toise.
Mountjouy	41 21 44·96	2 41 48·85	153673·61
Barcelona	41 22 47·90	2 42 51·79	154616·74
Carcassonne	43 12 54·30	4 32 58·19	259172·61
Evaux	46 10 42·54	7 30 46·43	428019·31
Panthéon	48 50 49·37	10 10 53·26	580312·41
Dunkirk	51 2 8·85	12 22 12·74	705257·21
5. English Arc.			
Dunnose	50 37 7·15		feet.
Southampton	50 54 46·97	0 17 39·82	107807·19
Greenwich	51 28 38·30	0 51 31·15	313716·97
Arbury Hill	52 13 27·01	1 36 19·86	586356·34
Clifton	53 27 30·27	2 50 23·12	1036581·55
Kellie Law	56 14 51·43	5 37 44·28	2055737·20
Great Stirling	57 27 49·12	6 50 41·97	2499839·45
Saxavord	60 49 38·58	10 12 31·43	3729335·78
6. Hanoverian Arc.			
Göttingen	51 31 47·85		toise
Altona	53 32 45·27	2 0 57·42	115163·725

	Latitude.	Amplitude.	Distance between the parallels.
7. Danish Arc.			
	° ′ ″	° ′ ″	toise.
Lauenburg	53 22 17·046		
Lyssabbel	54 54 10·352	1 31 53·306	87436·538
8. Prussian Arc.			
			toise.
Trunz	54 13 11·466		
Königsberg	54 42 50·500	0 29 39·034	28211·629
Memel	55 43 40·446	1 30 28·980	86176·975
9. Russian Arc.			
			toise.
Bélin	52 2 40·864		
Nemesch	54 39 4·519	2 36 23·655	148811·418
Jacobstadt	56 30 4·562	4 27 23·698	254543·454
Bristen	56 34 51·550	4 32 10·686	259110·085
Dorpat	58 22 47·280	6 20 6·416	361824·461
Hochland	60 5 9·771	8 2 28·907	459363·008
10. Swedish Arc.			
			toise.
Malörn	65 31 30·265		
Pahtawara	67 8 49·830	1 37 19·565	92777·981

If ρ be the radius of curvature of the meridian at a point whose latitude is λ,

$$\rho=\frac{a(1-e^2)}{(1-e^2\sin^2\lambda)^{\frac{3}{2}}};$$

or if we make

$$\frac{a-b}{a+b}=n,\quad e^2=\frac{4n}{(1+n)^2},$$

$$\therefore\quad \rho=\frac{a(1-n)(1-n^2)}{(1+2n\cos 2\lambda+n^2)^{\frac{3}{2}}},$$

which being expanded becomes

$$\rho=a(1-n)(1-n^2)\mathrm{N}\{1-2\alpha\cos 2\lambda+2\alpha'\cos 4\lambda-\ldots..\}$$

$$\mathrm{N}\ =1+\left(\frac{3}{2}\right)^2n^2+\left(\frac{3.5}{2\cdot 2}\right)^2n^4+\ldots.$$

$$\mathrm{N}\alpha=\frac{3}{2}n+\frac{3.5}{2.4}\cdot\frac{3}{2}n^3+\ldots...$$

$$\mathrm{N}\alpha'=\frac{3.5}{2.4}n^2+\frac{3.5.7}{2.4.6}\cdot\frac{3}{2}n^4+\ldots..$$

If s be the meridian distance between the points whose latitudes are $\lambda_1\lambda_2$

$$s=\int_{\lambda_1}^{\lambda_2}\rho.d\lambda;$$

and putting $\lambda_2-\lambda_1=\phi$, $2\lambda=\lambda_1+\lambda_2$,

$$s=\frac{180g}{\pi}\left\{\phi-2\alpha\sin\phi\cos 2\lambda+\alpha'\sin 2\phi\cos 4\lambda-\ldots.\right\},$$

where g is the length of a mean degree of the meridian determined by the relation

$$g=\frac{\pi}{180}a(1-n)(1-n^2)\mathrm{N}.$$

Hence if s be the meridian distance of two points whose latitudes are λ_1+x_1 and λ_2+x_2, we must substitute in this equation $\phi+x_2-x_1$ for ϕ, neglecting the influence of the small quantities x on the mean latitude of the arc; after this substitution we obtain

$$x_2-x_1=\mu\left(\frac{s\pi}{180g}-\phi+2\alpha\sin\phi\cos2\lambda-\alpha'\sin2\phi\cos4\lambda\right)$$

$$\mu=1+2\alpha\cos\phi\cos2\lambda.$$

Now let g_1 α_1 be approximate values of g and α, so that

$$\frac{1}{g}=\frac{1+i}{g_1},\quad \alpha=\alpha_1(1+k).$$

Then substituting in the preceding equation, we have finally

$$x_2-x_1=\mu\left\{\frac{3600}{g_1}s-\phi+\mathrm{P}_1-\frac{1}{6}\mathrm{P}_2\right\}$$
$$+\mu\left\{\frac{3600}{g_1}s\right\}i+\mu\left\{\mathrm{P}_1-\frac{1}{3}\mathrm{P}_2\right\}k,$$

where x_1 x_2 and ϕ are expressed in seconds, and

$$\mathrm{P}_1=\frac{2\alpha_1}{\sin1''}\sin\phi\cos2\lambda$$

$$\mathrm{P}_2=\frac{5\alpha_1^2}{\sin1''}\sin2\phi\cos4\lambda.$$

This equation contains the relation between the corrections to the terminal latitudes of a measured line s required to bring them into accordance with the measured distance, the elements of the spheroid of reference being as expressed above in g_1 α_1 i k. An arc in which there are n observed latitudes will therefore afford $n-1$ equations of the form

$$x_2-x_1=m+ai+bk;$$

the quantities ik must then be determined so as to make the function

$$x_1^2+x_2^2+x_3^2+\ldots.$$

a minimum.

The final equations thus deduced are

$$0=\mathrm{M}+\mathrm{A}i+\mathrm{B}k$$
$$0=\mathrm{M}'+\mathrm{B}i+\mathrm{B}'k$$

$$\mathrm{M}=\Sigma\left\{(am)-\frac{(a)(m)}{r}\right\}\quad \mathrm{M}'=\Sigma\left\{(bm)-\frac{(b)(m)}{r}\right\}$$

$$\mathrm{A}=\Sigma\left\{(a^2)-\frac{(a)^2}{r}\right\}\qquad \mathrm{B}=\Sigma\left\{(ab)-\frac{(a)(b)}{r}\right\}\quad \mathrm{B}'=\Sigma\left\{(b^2)-\frac{(b)^2}{r}\right\},$$

where r is the number of observed latitudes in any arc, the symbol Σ signifying summation with respect to the different arcs.

BESSEL adopts the approximate elements $g_1=57008$ toises $\alpha_1=\frac{1}{400}$; the equations for the different arcs are then as follows, putting $10000\,i=p$ and $10\,k=q$.

1. Peruvian Arc.

$$x_1^{(1)}-x_1=+1{\cdot}966+1{\cdot}1225\,p+5{\cdot}6059\,q.$$

2. First East Indian Arc.

$$x_2^{(1)}-x_2=+0{\cdot}937+0{\cdot}5697\,p+2{\cdot}5835\,q.$$

3. Second East Indian Arc.

$$x_3^{(1)}-x_3=+\ 5{\cdot}346+3{\cdot}5628\,p+15{\cdot}9269\,q.$$
$$x_3^{(2)}-x_3=+\ 4{\cdot}801+5{\cdot}7458\,p+24{\cdot}0257\,q.$$
$$x_3^{(3)}-x_3=+12{\cdot}440+7{\cdot}6875\,p+29{\cdot}7981\,q.$$

4. French Arc.

$$x_4^{(1)}-x_4=+3{\cdot}991+0{\cdot}9713\,p+0{\cdot}8601\,q.$$
$$x_4^{(2)}-x_4=+0{\cdot}646+0{\cdot}9772\,p+0{\cdot}8642\,q.$$
$$x_4^{(3)}-x_4=+0{\cdot}026+1{\cdot}6378\,p+1{\cdot}1889\,q.$$
$$x_4^{(4)}-x_4=-5{\cdot}035+2{\cdot}7041\,p+1{\cdot}2671\,q.$$
$$x_4^{(5)}-x_4=+1{\cdot}191+3{\cdot}6655\,p+0{\cdot}8659\,q.$$
$$x_4^{(6)}-x_4=+5{\cdot}171+4{\cdot}4537\,p+0{\cdot}2051\,q.$$

5. English Arc.

$$x_5^{(1)}-x_5=+\ 3{\cdot}772+0{\cdot}1064\,p-0{\cdot}1038\,q.$$
$$x_5^{(2)}-x_5=+\ 3{\cdot}735+0{\cdot}3095\,p-0{\cdot}3176\,q.$$
$$x_5^{(3)}-x_5=+\ 4{\cdot}302+0{\cdot}5784\,p-0{\cdot}6308\,q.$$
$$x_5^{(4)}-x_5=+\ 1{\cdot}258+1{\cdot}0224\,p-1{\cdot}2226\,q.$$
$$x_5^{(5)}-x_5=+\ 7{\cdot}874+2{\cdot}0272\,p-2{\cdot}8959\,q.$$
$$x_5^{(6)}-x_5=+\ 7{\cdot}127+2{\cdot}4649\,p-3{\cdot}7683\,q.$$
$$x_5^{(7)}-x_5=+10{\cdot}883+3{\cdot}6763\,p-6{\cdot}6163\,q.$$

6. Hanoverian Arc.

$$x_6^{(1)}-x_6=+5{\cdot}679+0{\cdot}7263\,p-0{\cdot}9224\,q.$$

7. Danish Arc.

$$x_7^{(1)}-x_7=-0{\cdot}369+0{\cdot}5513\,p-0{\cdot}8537\,q.$$

8. Prussian Arc.

$$x_8^{(1)}-x_8=-0{\cdot}368+0{\cdot}1779\,p-0{\cdot}2852\,q.$$

$$x_8^{(2)}-x_8=+3{\cdot}790+0{\cdot}5433\,p-0{\cdot}9157\,q.$$

9. Russian Arc.

$$x_9^{(1)}-x_9=+0{\cdot}248+0{\cdot}9384\,p-1{\cdot}3293\,q.$$

$$x_9^{(2)}-x_9=+5{\cdot}110+1{\cdot}6049\,p-2{\cdot}5184\,q.$$

$$x_9^{(3)}-x_9=+5{\cdot}939+1{\cdot}6337\,p-2{\cdot}5741\,q.$$

$$x_9^{(4)}-x_9=+2{\cdot}909+2{\cdot}2809\,p-3{\cdot}9289\,q.$$

$$x_9^{(5)}-x_9=+5{\cdot}276+2{\cdot}8953\,p-5{\cdot}3824\,q.$$

10. Swedish Arc.

$$x_{10}^{(1)}-x_{10}=-0{\cdot}507+0{\cdot}5839\,p-1{\cdot}9711\,q.$$

From which we obtain the following quantities for the different arcs:—

No. of Arc.	(m)	(a)	(b)	(am)	(aa)	(ab)	(bm)	(bb)
1.	+ 1·996	+1·1225	+ 5·6059	+ 2·2068	1·2600	+ 6·2926	+ 11·0211	31·4261
2.	+ 0·937	0·5697	+ 2·5835	+ 0·5338	0·3246	+ 1·4718	+ 2·4207	6·6745
3.	+22·587	16·9961	+69·7507	+142·2645	104·8052	+423·8645	+581·1810	1718·8272
4.	+ 5·990	14·4096	+ 5·2513	+ 18·3309	45·1641	+ 11·1409	− 0·2661	5·2976
5.	+38·951	10·1851	−15·5553	+ 78·8502	25·1874	− 41·2067	−127·4937	68·3661
6.	+ 5·679	0·7263	− 0·9294	+ 4·1247	0·5275	− 0·6750	− 5·2780	0·8638
7.	− 0·369	0·5513	− 0·8537	− 0·2034	0·3039	− 0·4706	+ 0·3150	0·7288
8.	+ 3·422	0·7212	− 1·2009	+ 1·9936	0·3268	− 0·5482	− 3·3655	0·9198
9.	+19·482	9·3532	−15·7331	+ 40·0469	19·7106	− 34·0396	− 68·3130	59·1418
10.	− 0·507	+ 0·5839	− 1·9711	− 0·2960	0·3409	− 1·1509	+ 0·9994	3·8852

No. of Arc.	M.	A.	B.	M′.	B′.
1.	+ 1·1034	0·6300	+ 3·1463	+ 5·5106	15·7131
2.	+ 0·2669	0·1623	+ 0·7359	+ 1·2104	3·3373
3.	+46·2918	32·5885	+127·4920	+177·3163	502·5376
4.	+ 6·0003	15·5021	+ 0·3313	− 4·7597	1·3583
5.	+29·2602	12·2204	− 21·4027	− 51·7569	38·1202
6.	+ 2·0624	0·2638	− 0·3375	− 2·6390	0·4319
7.	− 0·1017	0·1519	− 0·2353	+ 0·1575	0·3644
8.	+ 1·1710	0·1534	− 0·2595	− 1·9957	0·4391
9.	+ 9·6768	5·1302	− 9·5138	− 17·2270	17·8868
10.	− 0·1480	0·1705	− 0·5755	+ 0·4997	1·9426
Sums	+95·5831	66·9731	+ 99·3812	+106·3162	582·1313

The final equations are therefore—

$$0=+\ 95{\cdot}5831+66{\cdot}9731\,p+\ 99{\cdot}3812\,q$$
$$0=+106{\cdot}3162+99{\cdot}3812\,p+582{\cdot}1313\,q,$$

from which

$$p=-1{\cdot}548450$$
$$q=+0{\cdot}081718.$$

These quantities, when substituted in the equations of condition, give the following values of the corrections required:—

1. Peruvian Arc.	
Tarqui	$-0''{\cdot}360$
Cotchesqui	$+0{\cdot}360$
2. First Indian Arc.	
Trivandeporum	$-0{\cdot}134$
Pandree	$+0{\cdot}134$
3. Second Indian Arc.	
Punnæ	$-0{\cdot}492$
Damargida	$+0{\cdot}638$
Kalianpur	$-2{\cdot}625$
Kaliana	$+2{\cdot}479$
4. French Arc.	
Formentera	$+2{\cdot}270$
Mountjouy	$+4{\cdot}828$
Barcelona	$+1{\cdot}474$
Carcassonne	$-0{\cdot}142$
Evaux	$-6{\cdot}848$
Panthéon	$-2{\cdot}144$
Dunkirk	$+0{\cdot}562$
5. English Arc.	
Dunnose	$-2{\cdot}738$
Southampton	$+0{\cdot}860$
Greenwich	$+0{\cdot}491$
Arbury Hill	$+0{\cdot}616$
Clifton	$-3{\cdot}164$
Kellie Law	$+1{\cdot}760$
Great Stirling	$+0{\cdot}264$
Saxavord	$+1{\cdot}911$
6. Hanoverian Arc.	
Göttingen	$-2''{\cdot}239$
Altona	$+2{\cdot}239$
7. Danish Arc.	
Lauenburg	$+0{\cdot}646$
Lyssabbel	$-0{\cdot}646$
8. Prussian Arc.	
Trunz	$-0{\cdot}736$
Königsberg	$-1{\cdot}402$
Memel	$+2{\cdot}138$
9. Russian Arc.	
Bélin	$-0{\cdot}619$
Nemesch	$-1{\cdot}932$
Jacobstadt	$+1{\cdot}800$
Bristen	$+2{\cdot}579$
Dorpat	$-1{\cdot}563$
Hochland	$-0{\cdot}260$
10. Swedish Arc.	
Malörn	$+0{\cdot}786$
Pahtawara	$-0{\cdot}786$

The values of the axes are—

$$a=\frac{180g}{\pi N(1-n^2)^2}(1+n)$$

$$b=\frac{180g}{\pi N(1-n^2)^2}(1-n),$$

which in terms of g_1 p and q, are—

$$a=\frac{601}{600}\cdot\frac{180g_1}{\pi}\left(1-\frac{n^2}{4}\right)-\frac{180g_1}{10000\pi}\left(p-\frac{10}{6}q\right)+\cdots$$

$$b=\frac{599}{600}\cdot\frac{180g_1}{\pi}\left(1-\frac{n^2}{4}\right)-\frac{180g_1}{10000\pi}\left(p+\frac{10}{6}q\right)+\cdots$$

If we put ε for the mean error of an equation,

$$\text{Mean error of } p\pm\lambda q=\frac{\varepsilon}{291\cdot07}\sqrt{1267\cdot2\mp258\cdot7\lambda+115\cdot3\lambda^2}.$$

Now the sum of the squares of the errors, or quantities x, is 160·26;

$$\therefore\ \varepsilon=\sqrt{\frac{160\cdot26}{38-12}}=\pm2\cdot48.$$

The values of a and b, and their mean errors, are consequently

$a=20924933$; mean error ±800

$b=20854731$; mean error ±606.

The ratio of the axes is expressed by the relation

$$a:b::\frac{1}{2n}+\frac{1}{2}:\frac{1}{2n}-\frac{1}{2}$$

$$\frac{1}{2n}=300-30q+3q^2.$$

Consequently the compression is

$$\frac{a-b}{a}=\frac{1}{298\cdot07};\ \text{mean error of denominator } \pm2\cdot70.$$

The length of a degree of the meridian whose mean latitude is λ, is consequently

$$=364596\cdot61-1837\cdot79\cos2\lambda+3\cdot85\cos4\lambda,$$

and the length of a degree of longitude in latitude λ

$$=365515\cdot56\cos\lambda-306\cdot96\cos3\lambda+0\cdot39\cos5\lambda.$$

Had the point Evaux in the French Arc, at which there is obviously some peculiar local disturbance, been omitted, we should have obtained $p=-1\cdot65000$, $q=+\cdot09341$: these values would have increased the values of the semiaxes as obtained above by about 200 feet each, and increased the compression to

$$\frac{a-b}{a}=\frac{1}{297\cdot72}.$$

The corrections in the French Arc would then stand thus:—

Formentera	+1·319
Mountjouy	+3·788
Barcelona	+0·434
Carcassonne	−1·246
Panthéon	−3·457
Dunkirk	−0·839

and the correction for Evaux is increased to 8·059. The corrections for the other arcs are not materially altered, but are in general diminished: the mean error of the

equations is $\pm 2{\cdot}05$, which would diminish the probable error of the results in the proportion of 5 : 4.

Summary.

We may state the results of this paper briefly as follows:—

1st. The four bases of verification, when their measured lengths are compared with their lengths as calculated from a mean of the Lough Foyle and Salisbury Plain bases, show the following discrepancies, expressed in feet:

Hounslow.	Misterton Carr.	Rhuddlan Marsh.	Belhelvie.
$+0{\cdot}173$	$-0{\cdot}191$	$+1{\cdot}596$	$+0{\cdot}240$

2nd. The elements of the spheroid (B) most nearly representing the surface of Great Britain are—

$$\begin{array}{lll} & \text{feet } O_1. & \text{miles.} \\ \text{Equatorial semidiameter} & =20926249 & =3963{\cdot}305 \\ \text{Polar semidiameter} \ldots\ldots & =20856337 & =3950{\cdot}064 \end{array}\Bigg\}\text{compression}=\frac{1}{299{\cdot}33}.$$

3rd. The elements of the spheroid (C) most nearly representing the whole of the measured arcs considered in this paper are—

$$\begin{array}{lll} & \text{feet } O_1. & \text{miles.} \\ \text{Equatorial semidiameter} & =20924933 & =3963{\cdot}057 \\ \text{Polar semidiameter} \ldots\ldots & =20854731 & =3949{\cdot}760 \end{array}\Bigg\}\text{compression}=\frac{1}{298{\cdot}07}.$$

4th. The lengths of the degrees of latitude and longitude in Great Britain are a in the following Table:—

Mean latitude.	From Ord. Survey, Spheroid (B).		From Spheroid (C).	
	Length in feet of 1° of latitude.	Length in feet of 1° of longitude.	Length in feet of 1° of latitude.	Length in feet of 1° of longitude.
50°	364936·33	235227·42	364912·12	235214·58
51	364999·14	230312·27	364975·19	230299·77
52	365061·50	225326·39	365037·81	225314·19
53	365123·34	220271·15	365099·92	220259·23
54	365184·58	215148·11	365161·41	215136·58
55	365245·15	209958·83	365222·23	209947·61
56	365304·96	204704·93	365282·29	204694·04
57	365363·96	199387·90	365341·53	199377·33
58	365422·06	194009·37	365399·88	193999·13
59	365479·20	188571·00	365457·26	188561·08
60	365535·30	183074·50	365513·59	183064·93

Explanation of the Plate.

PLATE XXXIV.

Is a diagram of the triangulation of the United Kingdom, and is one of the plates which has been engraved at the Ordnance Survey Office to illustrate the Account of the Trigonometrical Survey of the United Kingdom which is now in the press.

XXX. *A Third Memoir upon Quantics.* By ARTHUR CAYLEY, *Esq.*

Received March 13,—Read April 10, 1856.

MY object in the present memoir is chiefly to collect together and put upon record various results useful in the theories of the particular quantics to which they relate. The tables at the commencement relate to binary quantics, and are a direct sequel to the tables in my Second Memoir upon Quantics, vol. cxlvi. (1856) p. 101. The definitions and explanations in the next part of the present memoir are given here for the sake of convenience, the further development of the subjects to which they relate being reserved for another occasion. The remainder of the memoir consists of tables and explanations relating to ternary quadrics and cubics.

Covariant and other Tables, Nos. 27 to 50 (Nos. 1 to 50 binary quantics)*.

Nos. 27 to 29 are a continuation of the tables relating to the quintic

$$(a, b, c, d, e, f \between x, y)^5.$$

No. 27 gives the values of the different determinants of the matrix

$$\left(\begin{array}{cccccc} & a, & 4b, & 6c, & 4d, & e \\ a, & 4b, & 6c, & 4d, & e & \\ & b, & 4c, & 6d, & 4e, & f \\ b, & 4c, & 6d, & 4e, & f & \end{array}\right)$$

determinants which are represented by 1234, 1235, &c., where the numbers refer to the different columns of the matrix. No. 28 gives the values of certain linear functions of these determinants, viz.

$$\begin{aligned}
L &= 1256 + 2345 - 2.1346 \\
L' &= 3.1256 - 1346 \\
8M &= -1345 + 2.1246 \\
8M' &= -2346 + 2.1356 \\
8N &= -1245 + 3.1236 \\
8N' &= -2356 + 3.1456 \\
80P &= L' - 3L = 5.1346 - 3.2345 \\
16P' &= -5L' - L = -16.1256 - 3.1346 - 2356.
\end{aligned}$$

At the end of the two tables there are given certain relations which exist between the terms of Tables 14, 16, 25, 26, 27 and 28.

* The Tables 49 and 50 were inserted October 6, 1856.—A. C.

No. 27.

1234.	1235.	1236.	1245.	1246.	1345.	1256.	2345.	1346.	2346	1356.	2356.	1456.	2456.	3456.
$\wp\ a^2bf$	$-4\ a^2cf$	$+6\ a^2df$	$-6\ a^2df$	$+4\ a^2ef$	$\wp\ a^2ef$	$+1\ a^2f^2$	$\wp\ a^2f^2$	$\wp\ a^2f^2$	$\wp\ abf^2$	$+4\ abf^2$	$-6\ acf^2$	$+6\ acf^2$	$-4\ adf^2$	$\wp\ aef^2$
$-16\ a^2ce$	$+24\ a^2de$	$\wp\ a^2e^2$	$+16\ a^2e^2$	$-4\ abdf$	$-24\ abdf$	$-2\ abef$	$\wp\ abef$	$+16\ abef$	$-24\ acef$	$-4\ acef$	$+6\ adef$	$-22\ adef$	$+4\ ae^2f$	$-16\ bdf^2$
$+36\ a^2d^2$	$+4\ ab^2f$	$-22\ abcf$	$+6\ abcf$	$-4\ abe^2$	$+64\ abe^2$	$-16\ acdf$	$+20\ acdf$	$-36\ acdf$	$+24\ ad^2f$	$-24\ ad^2f$	$\wp\ ae^3$	$+16\ ae^3$	$+24\ bcf^2$	$+16\ be^2f$
$+16\ ab^2e$	$-84\ abce$	$-6\ abde$	$-26\ abde$	$-24\ ac^2f$	$+24\ ac^2f$	$+16\ ace^2$	$-80\ ace^2$	$-16\ ace^2$	$\wp\ ade^2$	$+24\ ade^2$	$+16\ b^2f^2$	$\wp\ b^2f^2$	$-84\ bdef$	$+36\ c^2f^2$
$-152\ abcd$	$-24\ abd^2$	$+16\ ac^2e$	$-96\ ac^2e$	$+24\ acde$	$-208\ acde$	$+16\ ad^2e$	$+60\ ad^2e$	$+36\ ad^2e$	$+64\ b^2ef$	$-4\ b^2ef$	$-26\ bcef$	$-6\ bcef$	$+60\ be^3$	$-152\ cdef$
$+96\ ac^3$	$+64\ ac^2d$	$\wp\ acd^2$	$+96\ acd^2$	$\wp\ ad^3$	$+144\ ad^3$	$-15\ b^2df$	$-80\ b^2df$	$-16\ b^2df$	$-208\ bcdf$	$+24\ bcdf$	$-96\ b^2df$	$+16\ bd^2f$	$-24\ c^2ef$	$+80\ ce^3$
$+80\ b^3d$	$+60\ b^3e$	$+16\ b^3f$	$\wp\ b^3f$	$+24\ b^2cf$	$\wp\ b^2cf$	$\wp\ b^2e^2$	$+240\ b^2e^2$	$\wp\ b^2e^2$	$-40\ bce^2$	$-20\ bce^2$	$+90\ bde^2$	$-10\ bde^2$	$+64\ cd^2f$	$+96\ d^3f$
$-60\ b^2c^2$	$-40\ b^2cd$	$-10\ b^2ce$	$+90\ b^2ce$	$-20\ b^2de$	$-40\ b^2de$	$\wp\ bc^2f$	$+60\ bc^2f$	$+36\ bc^2f$	$+60\ bd^2e$	$\wp\ bd^2e$	$+96\ c^2df$	$\wp\ c^2df$	$-40\ cde^2$	$-60\ d^2e^2$
	$\wp\ bc^3$	$\wp\ b^2d^2$	$-80\ b^2d^2$	$\wp\ bc^2e$	$+60\ bc^2e$	$\wp\ bcde$	$-860\ bcde$	$-20\ bcde$	$+144\ c^3f$	$\wp\ c^3f$	$-80\ c^2e^2$	$\wp\ c^2e^2$	$\wp\ d^3e$	
		$\wp\ bc^2d$	$\wp\ bc^2d$	$\wp\ bcd^2$	$-40\ bcd^2$	$\wp\ bd^3$	$+960\ bd^3$	$\wp\ bd^3$	$-40\ c^2de$	$\wp\ c^2de$	$\wp\ cd^2e$	$\wp\ cd^2e$		
		$\wp\ c^4$	$\wp\ c^4$	$\wp\ c^3d$	$\wp\ c^3d$	$\wp\ c^3e$	$+960\ c^3e$	$\wp\ c^3e$	$\wp\ cd^3$	$\wp\ cd^3$	$\wp\ d^4$	$\wp\ d^4$		
						$\wp\ c^2d^2$	$-320\ c^2d^2$	$\wp\ c^2d^2$						

No. 28.

N.	M.	L.	L′.	P.	P′.	M′.	N′.
$+3\ a^2df$	$+1\ a^2ef$	$+1\ a^2f^2$	$+3\ a^2f^2$	$\wp\ a^2f^2$	$-1\ a^2f^2$	$+1\ abf^2$	$+3\ acf^2$
$-2\ a^2e^2$	$+2\ abdf$	$-34\ abef$	$-22\ abef$	$+1\ abef$	$+9\ abef$	$+2\ acef$	$-9\ adef$
$-9\ abcf$	$-9\ abe^2$	$+76\ acdf$	$-12\ acdf$	$-3\ acdf$	$-1\ acdf$	$-9\ ad^2f$	$+6\ ae^3$
$+1\ abde$	$-9\ ac^2f$	$-32\ ace^2$	$+64\ ace^2$	$+2\ ace^2$	$-18\ ace^2$	$+6\ ade^2$	$-2\ b^2f^2$
$+18\ ac^2e$	$+32\ acde$	$-12\ ad^2e$	$-36\ ad^2e$	$\wp\ ad^2e$	$+12\ ad^2e$	$-9\ b^2ef$	$+1\ bcef$
$-12\ acd^2$	$-18\ ad^3$	$-32\ b^2df$	$+64\ b^2df$	$+2\ b^2df$	$-18\ b^2df$	$+32\ bcdf$	$+18\ bd^2f$
$+6\ b^3f$	$+6\ b^2cf$	$+225\ b^2e^2$	$-45\ b^2e^2$	$-9\ b^2e^2$	$\wp\ b^2e^2$	$\wp\ bce^2$	$-15\ bde^2$
$-15\ b^2ce$	$\wp\ b^2de$	$-12\ bc^2f$	$-36\ bc^2f$	$\wp\ bc^2f$	$+12\ bc^2f$	$-15\ bd^2e$	$-12\ c^2df$
$+10\ b^2d^2$	$-15\ bc^2e$	$-820\ bcde$	$+20\ bcde$	$+31\ bcde$	$+45\ bcde$	$-18\ c^3f$	$+10\ c^2e^2$
$\wp\ bc^2d$	$+10\ bcd^2$	$+480\ bd^3$	$\wp\ bd^3$	$-18\ bd^3$	$-30\ bd^3$	$+10\ c^2de$	$\wp\ cd^2e$
$\wp\ c^4$	$\wp\ c^3d$	$+480\ c^3e$	$\wp\ c^3e$	$-18\ c^3e$	$-30\ c^3e$	$\wp\ cd^3$	$\wp\ d^4$
		$-320\ c^2d^2$	$\wp\ c^2d^2$	$+12\ c^2d^2$	$+20\ c^2d^2$		

If the coefficients of the table 14 are represented by $\tfrac{1}{2}$A, B, $\tfrac{1}{2}$C, viz.

$$\begin{aligned} A&=2(ae-4bd+3c^2)\\ B&=\ af-3be+2cd\\ C&=2(bf-4ce+3d^2), \end{aligned}$$

then we have the following relations between 1234, &c. and A, B, C, viz.

	C×	+B×	+A×
1234=	$+6\ a^2$	$-12\ ab$	$+16\ ac\ -10\ b^2$
1235=	$+6\ ab$	$-2\ ac\ -10\ b^2$	$+6\ ad$
1236=	$-2\ ac+8\ b^2$	$+6\ ad\ -18\ bc$	$-2\ df+8\ e^2$
1245=	$+18\ ac$	$-6\ ad\ -30\ bc$	$+8\ ae\ +10\ bd$
1246=	$+12\ bc$	$+4\ ae-4\ bd-24\ c^2$	$+4\ be+8\ cd$
1345=	$+24\ ad$	$-8\ ae\ -40\ bd$	$+4\ af\ +20\ be$
1256=	$-1\ ae+4\ bd+3\ c^2$	$+1\ af+5\ be-18\ cd$	$-1\ bf+4\ ce+3\ d^2$
2345=	$+20\ ae+40\ bd-30\ c^2$	$-80\ be\ +20\ cd$	$+20\ bf\ +40\ ce-30\ d^2$
1346=	$+4\ ae+8\ bd+6\ c^2$	$-36\ cd$	$+4\ bf+8\ ce+6\ d^2$
2346=	$+4\ af\ +20\ be$	$-8\ bf-4\ ce$	$+24\ cf$
1356=	$+4\ be+8\ cd$	$+4\ bf-4\ ce-24\ d^2$	$+12\ de$
2356=	$+8\ bf\ +10\ ce$	$-6\ cf\ -30\ de$	$+18\ df$
1456=	$+6\ ce$	$+6\ cf\ -18\ de$	$-2\ df+8\ e^2$
2456=	$+6\ cf$	$-2\ df\ -10\ e^2$	$+6\ ef$
3456=	$+16\ df\ -10\ e^2$	$-12\ ef$	$+6\ f^2$

and the following relations between L, L′, &c. and A, B, C, viz.

	C×	+B×	+A×
N =	$-3\ ac+3\ b^2$	$+3\ ad-3\ bc$	$-1\ ae+1\ bd$
M =	$-3\ ad+3bc$	$+3\ ae-3\ c^2$	$-1\ af+1\ cd$
L =	$+11\ ae+28\ bd-39\ c^2$	$+1\ af-75\ be+74\ cd$	$+11\ bf+28\ ce-39\ d^2$
L′ =	$-7\ ae+4\ bd+3\ c^2$	$+3\ af+15\ be-18\ cd$	$-7\ bf+4\ ce+3\ d^2$
2P =	$-1\ ae-2\ bd+3\ c^2$	$+3\ be-3\ cd$	$+1\ bf+2\ ce-3\ d^2$
P′ =	$+3\ ae-6\ bd+3\ c^2$	$-1\ af+1\ cd$	$+3\ bf-6\ ce+3\ d^2$
M′=	$-1\ af+1\ cd$	$+3\ bf-3\ d^2$	$-3\ cf+3\ de$
N′=	$-1\ bf+1\ ce$	$+3\ cf-3\ de$	$-3\ df+3\ e^2$

We have also the following relations between L, L′, &c. and a, b, c, d, e, f, viz.

$$\begin{array}{llllll} aP & -bM & +cN & & & =0\\ aM'+bP' & -2cM+3dN & & & & =0\\ aN'+2bM'- & cL' & . & +3eN & & =0\\ \quad +3bN' & . & - dL' & +2eM & +fN & =0\\ & +3cN'- & 2dM'+ & eP' & +fM & =0\\ & & + dN'- & eM' & +fP & =0. \end{array}$$

The quartinvariant No. 19 is equal to

$$-AC+B^2,$$

i. e. it is in fact equal to -4 into the discriminant of the quintic No. 14.

The octinvariant No. 25 is expressible in terms of the coefficients of Nos. 14 and

16, viz. A, B, C, as before, and $\frac{1}{3}\alpha$, β, γ, $\frac{1}{3}\delta$ the coefficients of No. 16, *i. e.*

$$\begin{aligned}
\alpha &= 3(ace-ad^2-\ b^2e+2bcd-c^3)\\
\beta &= \ acf-ade-\ b^2f+\ bd^2+bce-c^2d\\
\gamma &= \ adf-ae^2-\ bcf+\ bde+c^2e-cd^2\\
\delta &= 3(bdf-be^2+2cde-\ c^2f-d^3),
\end{aligned}$$

then No. 25 is equal to

$$\begin{vmatrix} A, & B, & C \\ \alpha, & \beta, & \gamma \\ \beta, & \gamma, & \delta \end{vmatrix}$$

The value of the discriminant No. 26 is

$$(\text{No. } 19)^2-128 \text{ No. } 25.$$

We have also an expression for the discriminant in terms of L, L′, &c., viz. three times the discriminant No. 26 is equal to

$$LL'+64MM'-64NN',$$

a remarkable formula, the discovery of which is due to Mr. SALMON.

It may be noticed, that in the particular case in which the quintic has two square factors, if we write

$$(a, b, c, d, e, f \between x, y)^5=5\{(p, q, r \between x, y)^2\}^2(\lambda, \mu \between x, y),$$

then

$$\begin{aligned}
&a=5\lambda p^2, \quad b=4pq\lambda+p^2\mu, \quad c=(2q^2+pr)\lambda+2pq\mu,\\
&f=5r^2\mu, \quad e=r^2\lambda+4qr\mu, \quad d=2qr\lambda+(2q^2+pr)\mu\,;
\end{aligned}$$

and these values give

$$\begin{aligned}
P&=K(6q^2-pr) & P'&=K(10q^2-15pr)\\
M&=K\,.\,10pq & M'&=K\,.\,10qr\\
N&=K\,.\,5p^2 & N'&=K\,.\,5r^2,
\end{aligned}$$

where the value of K is

$$8(p\mu^2-2q\mu\lambda+r\lambda^2)^2(pr-q^2)^2.$$

The table No. 29 is the invariant of the twelfth degree of the quintic, given in its simplest form, *i. e.* in a form not containing any power higher than the fourth of the leading coefficient a: this invariant was first calculated by M. FAA DE BRUNO.

No. 29.

$-1\ a^4c^2d^2f^4$	$+2\ a^2b^4de^2f^3$	$-324\ a^2c^3d^5ef$	$+18\ abc^7f^2$	$+194\ b^4c^2d^4f^2$
$+2\ a^4c^2de^2f^3$	$-1\ a^2b^4e^4f^2$	$-440\ a^2c^3d^4e^3$	$+242\ abc^6def^2$	$-652\ b^4c^2d^3e^2f$
$-1\ a^4c^2e^4f^2$	$-16\ a^2b^3c^2df^4$	$+78\ a^2c^2d^7f$	$-128\ abc^6e^3f$	$+713\ b^4c^2d^2e^4$
$+6\ a^4cd^3ef^3$	$+16\ a^2b^3c^2e^2f^3$	$+428\ a^2c^2d^6e^2$	$-324\ abc^5d^3f^2$	$+136\ b^4cd^5ef$
$-16\ a^4cd^2e^3f^2$	$+82\ a^2b^3cd^2ef^3$	$-180\ a^2cd^8e$	$-498\ abc^5d^2e^2f$	$-246\ b^4cd^4e^3$
$+14\ a^4cde^5f$	$-132\ a^2b^3cde^3f^2$	$+27\ a^2d^{10}$	$+136\ abc^5de^4$	$+16\ b^4d^7f$
$-4\ a^4ce^7$	$+50\ a^2b^3ce^5f$	$+14\ ab^5cdf^4$	$+1078\ abc^4d^4ef$	$+4\ b^4d^6e^2$
$-4\ a^4d^5f^3$	$-16\ a^2b^3d^4f^3$	$-14\ ab^5ce^2f^3$	$+206\ abc^4d^3e^3$	$-14\ b^3c^6f^3$
$+11\ a^4d^4e^2f^2$	$-14\ a^2b^3d^3e^2f^2$	$-32\ ab^5d^2ef^3$	$-342\ abc^3d^6f$	$-294\ b^3c^5def^2$
$-10\ a^4d^3e^4f$	$+60\ a^2b^3d^2e^4f$	$+50\ ab^5de^3f^2$	$-804\ abc^3d^5e^2$	$+138\ b^3c^5e^3f$
$+3\ a^4d^2e^6$	$-30\ a^2b^3de^6$	$-18\ ab^5e^5f$	$+506\ abc^2d^7e$	$-440\ b^3c^4d^3f^2$
$+2\ a^3b^2cd^2f^4$	$+11\ a^2b^2c^4f^4$	$-10\ ab^4c^3f^4$	$-90\ abcd^9$	$+1246\ b^3c^4d^2e^2f$
$-4\ a^3b^2cde^2f^3$	$-30\ a^2b^2c^3def^3$	$-30\ ab^4c^2def^3$	$-72\ ac^8ef^2$	$-246\ b^3c^4de^4$
$+2\ a^3b^2ce^4f^2$	$-14\ a^2b^2c^3e^3f^2$	$+60\ ab^4c^2e^3f^2$	$+78\ ac^7d^2f^2$	$+206\ b^3c^3d^4ef$
$-6\ a^3b^2d^3ef^3$	$-50\ a^2b^2c^2d^3f^3$	$-48\ ab^4cd^2e^2f^2$	$+224\ ac^7de^2f$	$-866\ b^3c^3d^3e^3$
$+16\ a^3b^2d^2e^3f^2$	$+168\ a^2b^2c^2d^2e^2f^2$	$+16\ ab^4cd^3f^3$	$+16\ ac^7e^4$	$-220\ b^3c^2d^6f$
$-14\ a^3b^2de^5f$	$-48\ a^2b^2c^2de^4f$	$+38\ ab^4cde^4f$	$-342\ ac^6d^3ef$	$+550\ b^3c^2d^5e^2$
$+4\ a^3b^2e^7$	$-4\ a^2b^2c^2e^6$	$-36\ ab^4ce^6$	$-220\ ac^6d^2e^3$	$-56\ b^3cd^7e$
$+6\ a^3bc^3df^4$	$-48\ a^2b^2cd^4ef^2$	$+112\ ab^4d^4ef^2$	$+106\ ac^5d^5f$	$-4\ b^3d^9$
$-6\ a^3bc^3e^2f^3$	$-2\ a^2b^2cd^3e^3f$	$-204\ ab^4d^3e^3f$	$+392\ ac^5d^4e^2$	$+78\ b^2c^7ef^2$
$-50\ a^3bc^2d^2ef^3$	$+6\ a^2b^2cd^2e^5$	$+102\ ab^4d^2e^5f$	$-222\ ac^4d^6e$	$+428\ b^2c^6d^2f^2$
$+82\ a^3bc^2de^3f^2$	$+62\ a^2b^2d^6f^2$	$+50\ ab^3c^4ef^3$	$+40\ ac^3d^8$	$-516\ b^2c^6de^2f$
$-32\ a^3bc^2e^5f$	$-90\ a^2b^2d^5e^2f$	$+46\ ab^3c^3d^2f^3$	$-4\ b^7df^4$	$+4\ b^2c^6e^4$
$+36\ a^3bcd^4f^3$	$+39\ a^2b^2d^4e^4$	$-2\ ab^3c^3de^2f^2$	$+4\ b^7e^2f^3$	$-804\ b^2c^5d^3ef$
$-30\ a^3bcd^3e^2f^2$	$-28\ a^2bc^5ef^3$	$-204\ ab^3c^3e^4f$	$+3\ b^6c^2f^4$	$+550\ b^2c^5d^2e^3$
$-30\ a^3bcd^2e^4f$	$+54\ a^2bc^4d^2f^3$	$-170\ ab^3c^2d^3ef^2$	$+24\ b^6cdef^3$	$+392\ b^2c^4d^5f$
$+24\ a^3bcde^6$	$-48\ a^2bc^4de^2f^2$	$+308\ ab^3c^2de^5$	$-30\ b^6ce^3f^2$	$+139\ b^2c^4d^4e^2$
$-28\ a^3bd^5ef^2$	$+112\ a^2bc^4e^4f$	$+42\ ab^3c^2d^2e^3f$	$+16\ b^6d^3f^3$	$-354\ b^2c^3d^6e$
$+50\ a^3bd^4e^3f$	$+82\ a^2bc^3d^3ef^2$	$-164\ ab^3cd^5f^2$	$-4\ b^6d^2e^2f^2$	$+83\ b^2c^2d^8$
$-22\ a^3bd^3e^5$	$-170\ a^2bc^3d^2e^3f$	$+674\ ab^3cd^4e^2f$	$-36\ b^6de^4f$	$-180\ bc^8df^2$
$-4\ a^3c^5f^4$	$-104\ a^2bc^3de^5$	$-590\ ab^3cd^3e^4$	$+27\ b^6e^6$	$+48\ bc^8e^2f$
$+36\ a^3c^4def^3$	$-108\ a^2bc^2d^5f^2$	$-128\ ab^3d^6ef$	$-104\ b^5cd^3ef^2$	$+506\ bc^7d^2ef$
$-16\ a^3c^4e^3f^2$	$-42\ a^2bc^2d^4e^2f$	$+138\ ab^3d^5e^3$	$-22\ b^5c^3ef^3$	$-56\ bc^7de^3$
$-22\ a^3c^3d^3f^3$	$+298\ a^2bc^2d^3e^4$	$-70\ ab^2c^5df^3$	$-60\ b^5c^2d^2f^3$	$-222\ bc^6d^4f$
$-50\ a^3c^3d^2e^2f^2$	$+242\ a^2bcd^6ef$	$-90\ ab^2c^5e^2f^2$	$+6\ b^5c^2de^2f^2$	$-354\ bc^6d^3e^2$
$+16\ a^3c^3de^4f$	$-294\ a^2bcd^5e^3$	$-42\ ab^2c^4d^2ef^2$	$+102\ b^5c^2e^4f$	$+330\ bc^5d^5e$
$+16\ a^3c^3e^6$	$-72\ a^2bd^8f$	$+674\ ab^2c^4de^3f$	$+308\ b^5cd^2e^3f$	$-72\ bc^4d^7$
$+54\ a^3c^2d^4ef^2$	$+78\ a^2bd^7e^2$	$-4\ ab^2c^4e^5$	$-234\ b^5cde^5$	$+27\ c^{10}f^2$
$+46\ a^3c^2d^3e^3f$	$-6\ a^2c^6df^3$	$+394\ ab^2c^3d^4f^2$	$-24\ b^5d^5f^2$	$-90\ c^9def$
$-60\ a^3c^2d^2e^5$	$+62\ a^2c^6e^2f^2$	$-652\ ab^2c^3d^2e^4$	$-4\ b^5d^4e^2f$	$-4\ c^9e^3$
$-6\ a^3cd^6f^2$	$-108\ a^2c^5d^2ef^2$	$-714\ ab^2c^3d^3e^2f$	$+32\ b^5d^3e^4$	$+40\ c^8d^3f$
$-70\ a^3cd^5e^2f$	$-164\ a^2c^5de^3f$	$-498\ ab^2c^2d^5ef$	$+56\ b^4c^4df^3$	$+83\ c^8d^2e^2$
$+56\ a^3cd^4e^4$	$-24\ a^2c^5e^5$	$+1246\ ab^2c^2d^4e^3$	$+39\ b^4c^4e^2f^2$	$-72\ c^7d^4e$
$+18\ a^3d^7ef$	$+63\ a^2c^4d^4f^2$	$+224\ ab^2cd^7f$	$+298\ b^4c^3d^2ef^2$	$+16\ c^6d^6$
$-14\ a^3d^6e^3$	$+394\ a^2c^4d^3e^2f$	$-516\ ab^2cd^6e^2$	$-590\ b^4c^3de^3f$	
$-1\ a^2b^4d^2f^4$	$+194\ a^2c^4d^2e^4$	$+48\ ab^2d^8e$	$+32\ b^4c^3e^5$	

The tables Nos. 30 to 35 relate to a sextic. No. 30 is the sextic itself; No. 31 the quadrinvariant; Nos. 32 and 33 the quadricovariants (the latter of them the Hessian); No. 34 is the quartinvariant or catalecticant; and No. 35 is the sextinvariant in its best form, *i. e.* a form not containing any power higher than the second of the leading coefficient *a*.

No. 30.

(a	$6b$	$15c$	$20d$	$15e$	$6f$	g	)(x, y)^6

No. 31.

$$\begin{array}{|rl|}\hline +1 & ag \\ -6 & bf \\ +15 & ce \\ -10 & d^2 \\ \hline\end{array}$$

No. 32.

$$\left(\begin{array}{|rl|rl|rl|rl|rl|}\hline +1 & ae & +2 & af & +1 & ag & +2 & bg & +1 & cg \\ -4 & bd & -6 & be & -9 & ce & -6 & cf & -4 & df \\ +3 & c^2 & +4 & cd & +8 & d^2 & +4 & de & +3 & e^2 \\ \hline\end{array}\right)\!\!\between(x, y)^4$$

No. 33.

$$\left(\begin{array}{|rl|rl|rl|rl|rl|rl|rl|rl|rl|}\hline +1 & ac & +4 & ad & +6 & ae & +4 & af & +1 & ag & +4 & bg & +6 & cg & +4 & dg & +1 & eg \\ -1 & b^2 & -4 & bc & +4 & bd & +16 & be & +14 & bf & +16 & cf & +4 & df & -4 & ef & -1 & f^2 \\ & & & & -10 & c^2 & -20 & cd & +5 & ce & -20 & de & -10 & e^2 & & & & \\ & & & & & & & & -20 & d^2 & & & & & & & & \\ \hline\end{array}\right)\!\!\between(x, y)^8$$

No. 34.

+1	$aceg$
−1	acf^2
−1	ad^2g
+2	$adef$
−1	ae^3
−1	b^2eg
+1	b^2f^2
+2	$bcdg$
−2	$bcef$
−2	bd^2f
+2	bde^2
−1	c^3g
+2	c^2df
+1	c^2e^2
−3	cd^2e
+1	d^4

No. 35.

+ 1	$a^2d^2g^2$	−42	$acde^2f$	+60	bc^2df^2
− 6	a^2defg	+12	ace^4	−30	bc^2e^2f
+ 4	a^2df^3	−20	ad^4g	+24	bcd^3g
+ 4	a^2e^3g	+24	ad^3ef	−84	bcd^2ef
− 3	$a^2e^2f^2$	− 8	ad^2e^3	+66	$bcde^3$
− 6	$abcdg^2$	+ 4	b^3dg^2	+24	bd^4f
+18	$abcefg$	−12	b^3efg	−24	bd^3e^2
−12	$abcf^3$	+ 8	b^3f^3	+12	c^4eg
+12	abd^2fg	− 3	$b^2c^2g^2$	−27	c^4f^2
−18	$abde^2g$	+30	b^2ce^2g	− 8	c^3d^2g
+ 6	abe^3f	−24	b^2cef^2	+66	c^3def
+ 4	ac^3g^2	−12	b^2d^2eg	− 8	c^3e^3
−24	ac^2e^2g	−24	$b^2d^2f^2$	−24	c^2d^3f
−18	ac^2dfg	+60	b^2de^2f	−39	$c^2d^2e^2$
+30	ac^2ef^2	−27	b^2e^4	+36	cd^4e
+54	acd^2eg	+ 6	bc^3fg	− 8	d^6
−12	acd^2f^2	−42	bc^2deg		

The sextinvariant may be thus represented by means of a determinant of the sixth order and of the quadrinvariant and quartinvariant.

$$5\times\text{No. }35=\begin{vmatrix} & a, & 2b, & 3c, & 4d, & e \\ & b, & 2c, & 3d, & 4e, & f \\ & c, & 2d, & 3e, & 4f, & g \\ a, & 4b, & 3c, & 2d, & e & \\ b, & 4c, & 3d, & 2e, & f & \\ c, & 4d, & 3e, & 2f, & g & \end{vmatrix}$$

$$+4(ag-6bf+15ce-10d^2)\begin{vmatrix} a, & b, & c, & d \\ b, & c, & d, & e \\ c, & d, & e, & f \\ d, & e, & f, & g \end{vmatrix}$$

The tables Nos. 36 and 37 relate to a septic. No. 36 is the septic itself; No. 37 the quartinvariant.

No. 36.

(a	$7\,b$	$21\,c$	$35\,d$	$35\,e$	$21\,f$	$7\,g$	h	$)(x, y)^7$

No. 37.

$-\ \ 1\ a^2h^2$	$-\ \ 40\ bd^2h$
$+\ 14\ abgh$	$-\ \ 50\ bdeg$
$-\ 18\ acfh$	$-360\ bdf^2$
$-\ 24\ acg^2$	$+240\ be^2f$
$+\ 10\ adeh$	$-360\ c^2eg$
$+\ 60\ adfg$	$-\ \ 81\ c^2f^2$
$-\ 40\ ae^2g$	$+240\ cd^2g$
$-\ 24\ b^2fh$	$+990\ cdef$
$-\ 25\ b^2g^2$	$-600\ ce^3$
$+234\ bcfg$	$-600\ d^3f$
$+\ 60\ bceh$	$+375\ d^2e^2$

The tables Nos. 38 to 45 relate to the octavic. No. 38 is the octavic itself; No. 39 the quadrinvariant; Nos. 40, 41 and 42 are the quadricovariants, the last of them being the Hessian; No. 43 is the cubinvariant; No. 44 the quartinvariant, and No. 45 the quintinvariant, which is also the catalecticant.

No. 38.

(a	$8\,b$	$28\,c$	$56\,d$	$70\,e$	$56\,f$	$28\,g$	$8\,h$	i	$)(x, y)^8$

No. 39.

$+\ \ 1\ ai$
$-\ \ 8\ bh$
$+28\ cg$
$-56\ df$
$+35\ e^2$

No. 40.

($+\ \ 1\ ag$ $-\ \ 6\ bf$ $+15\ ce$ $-10\ d^2$	$+\ \ 2\ ah$ $-10\ bg$ $+18\ cf$ $-10\ de$	$+\ \ 1\ ai$ $-\ \ 2\ bh$ $-\ \ 8\ cg$ $+34\ df$ $-25\ e^2$	$+\ \ 2\ bi$ $-10\ ch$ $+18\ dg$ $-10\ ef$	$+\ \ 1\ ci$ $-\ \ 6\ dh$ $+15\ eg$ $-10\ f^2$	$)(x, y)^4$

No. 41.

($+1\ ae$ $-4\ bd$ $+3\ c^2$	$+\ \ 4\ af$ $-12\ be$ $+\ \ 8\ cd$	$+\ \ 6\ ag$ $-\ \ 8\ bf$ $-22\ ce$ $+24\ d^2$	$+\ \ 4\ ah$ $+\ \ 8\ bg$ $-48\ cf$ $+36\ de$	$+\ \ 1\ ai$ $+12\ bh$ $-22\ cg$ $-36\ df$ $+45\ e^2$	$+\ \ 4\ bi$ $+\ \ 8\ ch$ $-48\ dg$ $+36\ ef$	$+\ \ 6\ ci$ $-\ \ 8\ dh$ $-22\ eg$ $+24\ f^2$	$+\ \ 4\ di$ $-12\ eh$ $+\ \ 8\ fg$	$+1\ ei$ $-4\ fh$ $+3\ g^2$	$)(x, y)^8$

No. 42.

$+1\ ac$	$+6\ ad$	$+15\ ae$	$+20\ af$	$+15\ ag$	$+6\ ah$	$+1\ ai$	$+6\ bi$	$+15\ ci$	$+20\ di$	$+15\ ei$	$+6\ fi$	$+1\ gi$
$-1\ b^2$	$-6\ bc$	$+6\ bd$	$+50\ be$	$+90\ bf$	$+78\ bg$	$+34\ bh$	$+78\ ch$	$+90\ dh$	$+50\ eh$	$+6\ fh$	$-6\ gh$	$-1\ h^2$
		$-21\ c^2$	$-70\ cd$	$-105\ d^2$	$+126\ cf$	$+154\ cg$	$+126\ dg$	$-105\ f^2$	$-70\ fg$	$-21\ g^2$		
					$-210\ de$	$-14\ df$	$-210\ ef$					
						$-175\ e^2$						

$$(\ \ldots\ )(x, y)^{12}$$

No. 43.

$+1\ aei$
$-4\ afh$
$+3\ ag^2$
$-4\ bdi$
$+12\ beh$
$-8\ bfg$
$+3\ c^2i$
$-8\ cdh$
$-22\ ceg$
$+24\ cf^2$
$+24\ d^2g$
$-36\ def$
$+15\ e^3$

No. 44.

$-1\ acgi$	$+3\ bcgh$	$-2\ cd^2i$
$+1\ ach^2$	$+1\ bdei$	$-23\ cdeh$
$+3\ adfi$	$-10\ bdfh$	$+27\ cdfg$
$-3\ adgh$	$+9\ bdg^2$	$+19\ ce^2g$
$-2\ ae^2i$	$+11\ be^2h$	$-21\ cef^2$
$+1\ aefh$	$-23\ befg$	$+12\ d^3h$
$+3\ aeg^2$	$+12\ bf^3$	$-21\ d^2eg$
$-2\ af^2g$	$+3\ c^2ei$	$-13\ d^2f^2$
$+1\ b^2gi$	$+9\ c^2fh$	$-32\ de^2f$
$-1\ b^2h^2$	$-12\ c^2g^2$	$-10\ e^4$
$-3\ bcfi$		

No. 45.

$+1\ acegi$	$+1\ af^4$	$-4\ bdeg^2$	$+1\ cd^2g^2$
$-1\ aceh^2$	$-1\ b^2egi$	$+2\ bdf^2g$	$-2\ cdefg$
$-1\ acf^2i$	$+1\ b^2eh^2$	$-2\ be^3h$	$-2\ cdf^3$
$+2\ acfgh$	$-2\ b^2fgh$	$+4\ be^2fg$	$-3\ ce^3g$
$-1\ acg^3$	$+1\ b^2fi^2$	$-2\ bef^3$	$+4\ ce^2dh$
$-1\ ad^2gi$	$+1\ b^2g^3$	$-1\ c^3gi$	$+3\ ce^2f^2$
$+1\ ad^2h^2$	$+2\ bcdgi$	$+1\ c^3h^2$	$+1\ d^4i$
$+2\ adefi$	$-2\ bcdh^2$	$+2\ c^2dfi$	$-2\ d^3eh$
$-2\ adegh$	$-2\ bcefi$	$-2\ c^2dgh$	$-2\ d^3fg$
$-2\ adf^2h$	$+2\ bcegh$	$+1\ c^2e^2i$	$+3\ d^2e^2g$
$+2\ adfg^2$	$+2\ bcf^2h$	$-4\ c^2efh$	$+3\ d^2ef^2$
$-1\ ae^3i$	$-2\ bcfg^2$	$+2\ c^2eg^2$	$-4\ de^3f$
$+2\ ae^2fh$	$-2\ bd^2fi$	$+1\ c^2f^2g$	$+1\ e^5$
$+1\ ae^2g^2$	$+2\ bd^2gh$	$-3\ cd^2ei$	
$-3\ aef^2g$	$+2\ bde^2i$	$+2\ cd^2fh$	

If we write

$$\begin{aligned}\text{No. } 39 &= \text{I}\\ \text{No. } 43 &= \text{J}\\ \text{No. } 44 &= \text{K}\\ \text{No. } 45 &= \text{L},\end{aligned}$$

then the determinant called the lambdaic, viz.

$$\begin{vmatrix} a, & b, & c, & d, & e-12\lambda \\ b, & c, & d, & e+3\lambda, & f \\ c, & d, & e-2\lambda, & f, & g \\ d, & e+3\lambda, & f, & g, & h \\ e-12\lambda, & f, & g, & h, & i \end{vmatrix}$$

is equal to

$$L+2\lambda K+3\lambda^2 J+18\lambda^3 I-2592\lambda^5.$$

Nos. 46 to 48 relate to the nonic. No. 46 is the nonic itself; Nos. 47 and 48 are the two quartinvariants, each of them in its best form, viz. No. 48 does not contain a^2, and No. 47 does not contain aci^2, the leading term of No. 48. The nonic is the lowest quantic with two quartinvariants.

No. 46.

$$(\,+1\ a \mid +9\ b \mid +36\ c \mid +84\ d \mid +126\ e \mid +126\ f \mid +84\ g \mid +36\ h \mid +9\ i \mid +1\ j\,)(x,y)^9$$

No. 47.

− 1	a^2j^2	∽	bf^2h
+ 18	$abij$	− 720	bfg^2
∽	aci^2	+ 432	c^2fj
− 72	$achj$	− 1728	c^2gi
+ 168	$adgj$	∽	c^2h^2
∽	$adhi$	− 720	$cdej$
− 108	$aefj$	+ 2160	$cdfi$
− 576	$aegi$	+ 4608	$cdgh$
+ 422	aeh^2	∽	ce^2i
+ 540	af^2i	− 2592	$cefh$
− 720	$afgh$	− 5760	ceg^2
+ 320	ag^3	+ 4320	cf^2g
∽	b^2hj	+ 320	d^3j
− 81	b^2i^2	− 720	d^2ei
∽	$bcgj$	− 5760	d^2fh
+ 648	$bchi$	− 1536	d^2g^2
− 576	$bdfj$	+14688	$defg$
+ 792	$bdgi$	+ 4320	de^2h
−1728	bdh^2	− 8640	df^3
+2160	$begh$	− 8640	e^3g
+ 540	be^2j	+ 5184	e^2f^2
− 972	$befi$		

No. 48.

∽	a^2j^2	+ 70	bf^2h
∽	$abij$	− 45	bfg^2
+ 2	aci^2	+ 27	c^2fj
− 2	$achj$	− 52	c^2gi
+ 7	$adgj$	+ 25	c^2h^2
− 7	$adhi$	− 45	$cdej$
− 5	$aefj$	+ 23	$cdfi$
− 22	$aegi$	+ 22	$cdgh$
+ 27	aeh^2	+ 70	ce^2i
+ 25	af^2i	−127	$cefh$
− 45	$afgh$	+ 32	ceg^2
+ 20	ag^3	+ 25	cf^2g
+ 2	b^2hj	+ 20	d^3j
− 2	b^2i^2	− 45	d^2ei
− 7	$bcgj$	+ 32	d^2fh
+ 7	$bchi$	− 47	d^2g^2
− 22	$bdfj$	+ 85	$defg$
+ 74	$bdgi$	+ 25	de^2h
− 52	bdh^2	− 50	df^3
+ 23	$begh$	− 50	e^2g
+ 25	be^2j	+ 30	e^2f^2
− 73	$befi$		

Nos. 49 and 50 relate to the dodecadic. No. 49 is the dodecadic itself; No. 50 is the cubinvariant.

No. 49.

$$(\,a \mid 12\ b \mid 66\ c \mid 220\ d \mid 495\ e \mid 792\ f \mid 924\ g \mid 792\ h \mid 495\ i \mid 220\ j \mid 66\ k \mid 12\ l \mid m\,)(x,y)^{12}$$

No. 50.

+ 3	agm	− 216	cfl	+ 540	dhi
− 12	ahl	+ 72	cgk	− 540	e^2k
+ 30	aik	+ 300	chj	+1080	efj
− 20	aj^2	− 270	ci^2	+1485	egi
− 24	bfm	− 40	d^2m	−1080	eh^2
+ 90	bgl	+ 120	del	−2160	f^2i
−108	bhk	+ 600	dfk	+2160	fgh
+ 60	bij	−1290	dgj	− 840	g^3
+ 60	cem				

Resuming now the general subject,—

54. The simplest covariant of a system of quantics of the form

$$(*\between x, y, ..)^m$$

(where the number of quantics is equal to the number of the facients of each quantic) is the functional determinant or *Jacobian*, viz. the determinant formed with the differential coefficients or derived functions of the quantics with respect to the several facients.

55. In the particular case in which the quantics are the differential coefficients or derived functions of a single quantic, we have a corresponding covariant of the single quantic, which covariant is termed the *Hessian*; in other words, the Hessian is the determinant formed with the second differential coefficients or derived functions of the quantic with respect to the several facients.

56. The expression, an *adjoint linear form*, is used to denote a linear function $\xi x+\eta y+..$, or in the notation of quantics $(\xi, \eta ...\between x, y, ..)$, having the same facients as the quantic or quantics to which it belongs, and with indeterminate coefficients $(\xi, \eta..)$. The invariants of a quantic or quantics, and of an adjoint linear form, may be considered as quantics having $(\xi, \eta ...)$ for facients, and of which the coefficients are of course functions of the coefficients of the given quantic or quantics. An invariant of the class in question is termed a contravariant of the quantic or quantics. The idea of a contravariant is due to Mr. SYLVESTER.

In the theory of binary quantics, it is hardly necessary to consider the contravariants; for any contravariant is at once turned into an invariant by writing $(y, -x)$ for (ξ, η).

57. If we imagine, as before, a system of quantics of the form

$$(*\between x, y, ..)^m,$$

where the number of quantics is equal to the number of the facients in each quantic, the function of the coefficients, which, equalled to zero, expresses the result of the elimination of the facients from the equations obtained by putting each of the quantics equal to zero, is said to be the *Resultant* of the system of quantics. The resultant is an invariant of the system of quantics.

And in the particular case in which the quantics are the differential coefficients, or derived functions of a single quantic with respect to the several facients, the resultant in question is termed the *Discriminant* of the single quantic; the discriminant is of course an invariant of the single quantic.

58. Imagine two quantics, and form the equations which express that the differential coefficients, or derived functions of the one quantic with respect to the several facients, are proportional to those of the other quantic. Join to these the equations obtained by equating each of the quantics to zero; we have a system of equations, one of which is contained in the others, and from which therefore the facients may be eliminated. The function which, equated to zero, expresses the result of the elimination is an invariant which (from its geometrical signification) might be

termed the *Tactinvariant* of the two quantics, but I do not at present propose to consider this invariant except in the particular case where the system consists of a given quantic and of an adjoint linear form. In this case the tactinvariant is a contravariant of the given quantic, viz. the contravariant termed the *Reciprocant.*

59. Consider now a quantic

$$(*\between x, y, ..)^m,$$

and let the facients $x, y, ..$ be replaced by $\lambda x+\mu X$, $\lambda y+\mu Y$, .. the resulting function may, it is clear, be considered as a quantic with the facients (λ, μ) and of the form

$$\left\{\begin{array}{l} (*\between x, y, ...)^m \\ (*\between x, y, ...)^{m-1}\,(X, Y, ...) \\ \cdot \\ \cdot \\ (*\between X, Y, ..)^m \end{array}\right\}\between \lambda, \mu)^m.$$

The coefficients of this quantic are termed *Emanants*, viz. excluding the first coefficient, which is the quantic itself (but which might be termed the 0-th emanant); the other coefficients are the first, second, and last or ultimate emanants. The ultimate emanant is, it is clear, nothing else than the quantic itself, with $(X, Y, ..)$ instead of $(x, y, ...)$ for facients: the penultimate emanant is, in like manner, obtained from the first emanant by interchanging $(x, y, ..)$ with $(X, Y, ...)$, and similarly for the other emanants. The facients $(X, Y, ..)$ may be termed the *facients of emanation,* or simply the *new facients.* The theory of emanation might be presented in a more general form by employing two or more sets of emanating facients; we might, for example, write $\lambda x+\mu X+\nu X'$, $\lambda y+\mu Y+\nu Y'$,... for $x, y, ...$, but it is not necessary to dwell upon this at present.

The invariants, in respect to the new facients of any emanant or emanants of a quantic (*i. e.* the invariants of the emanant or emanants, considered as a function or functions of the new facients), are, it is easy to see, covariants of the original quantic, and it is in many cases convenient to define a covariant in this manner; thus the Hessian is the discriminant of the second or quadric emanant of the quantic.

60. If we consider a quantic

$$(a, b, ..\between x, y, ..)^m,$$

and an adjoint linear form, the operative quantic

$$(\partial_a, \partial_b, ..\between \xi, \eta, ..)^m$$

(which is, so to speak, a contravariant operator) is termed the *Evector.* The properties of the evector have been considered in the introductory memoir, and it has been in effect shown that the evector operating upon an invariant, or more generally upon a contravariant, gives rise to a contravariant. Any such contravariant, or rather such contravariant considered as so generated, is termed an *Evectant.* In the case of a binary quantic,

$$(a, b, ..\between x, y)^m,$$

the covariant operator

$$(\partial_a, \partial_b, ..)(y, -x)^m$$

may, if not with perfect accuracy, yet without risk of ambiguity, be termed the *Evector*, and a covariant obtained by operating with it upon an invariant or covariant, or rather such covariant considered as so generated, may in like manner be termed an *Evectant*.

61. Imagine two or more quantics of the same order,

$$(a, b, ..)(x, y)^m$$
$$(\alpha, \beta, ..)(x, y)^m$$

we may have covariants such that for the coefficients of each pair of quantics the covariant is reduced to zero by the operators

$$a\partial_\alpha + b\partial_\beta + ..$$
$$\alpha\partial_a + \beta\partial_b + ..$$

Such covariants are called *Combinants*, and they possess the property of being invariantive, quoad the system, *i. e.* the covariant remains unaltered to a factor *près*, when each quantic is replaced by a linear function of all the quantics. This extremely important theory is due to Mr. SYLVESTER.

Proceeding now to the theory of ternary quadrics and cubics,—

First for a ternary quadric, we have the following tables :—

Covariant and other Tables, Nos. 51 to 56 (a ternary quadric).

No. 51.

The quadric is represented by

$$(a, b, c, f, g, h)(x, y, z)^2,$$

which means—

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy.$$

No. 52.

The first derived functions (omitting the factor 2) are—

$$(a, h, g)(x, y, z)$$
$$(h, b, f)(x, y, z)$$
$$(g, f, c)(x, y, z).$$

No. 53.

The operators which reduce a covariant to zero are—

$$\begin{array}{l}
(h,\ \ b,\ \ 2f \between \partial_g,\ \ \partial_f,\ \ \partial_c) - z\partial_y \\
(2g,\ \ f,\ \ c \between \partial_a,\ \ \partial_h,\ \ \partial_g) - x\partial_z \\
(a,\ \ 2h,\ \ g \between \partial_h,\ \ \partial_b,\ \ \partial_f) - y\partial_x \\
(g,\ \ 2f,\ \ c \between \partial_h,\ \ \partial_b,\ \ \partial_f) - y\partial_z \\
(a,\ \ h,\ \ 2g \between \partial_g,\ \ \partial_f,\ \ \partial_c) - z\partial_x \\
(2h,\ \ b,\ \ f \between \partial_a,\ \ \partial_h,\ \ \partial_g) - x\partial_y .
\end{array}$$

No. 54.

The evector is

$$(\partial_a,\ \partial_b,\ \partial_c,\ \partial_f,\ \partial_g,\ \partial_h \between \xi,\ \eta,\ \zeta)^2.$$

No. 55.

The discriminant is

$$\begin{vmatrix} a, & h, & g \\ h, & b, & f \\ g, & f, & c \end{vmatrix}$$

which is equal to

$$abc - af^2 - bg^2 - ch^2 + 2fgh.$$

No. 56.

The reciprocant is

$$-\begin{vmatrix} & \xi, & \eta, & \zeta \\ \xi, & a, & h, & g \\ \eta, & h, & b, & f \\ \zeta, & g, & f, & c \end{vmatrix}$$

which is equal to

$$(bc-f^2,\ ca-g^2,\ ab-h^2,\ gh-af,\ hf-bg,\ fg-ch \between \xi,\ \eta,\ \zeta)^2.$$

The discriminant is, it will be noticed, the same function as the Hessian. The reciprocant is the evectant of the discriminant. The covariants are the quadric itself and the discriminant; the reciprocant is the only contravariant.

Next, for a ternary cubic, we have the following Tables:—

Covariant and other Tables, Nos. 57 to 70 (a ternary cubic).

No. 57.

The cubic is U=

$$(a,\ b,\ c,\ f,\ g,\ h,\ i,\ j,\ k,\ l \between x,\ y,\ z)^3,$$

which means—

$$ax^3 + by^3 + cz^3 + 3fy^2z + 3gz^2x + 3hx^2y + 3iyz^2 + 3jzx^2 + 3kxy^2 + 6lxyz.$$

No. 58.

The first derived functions (omitting the factor 3) are—

$$
\begin{array}{l}
(a,\ k,\ g,\ l,\ j,\ h \between x,\ y,\ z)^2 \\
(h,\ b,\ i,\ f,\ l,\ k \between x,\ y,\ z)^2 \\
(j,\ f,\ c,\ i,\ g,\ l \between x,\ y,\ z)^2.
\end{array}
$$

The second derived functions (omitting the factor 6) are—

$$
\begin{array}{l}
(a,\ h,\ j \between x,\ y,\ z) \\
(k,\ b,\ f \between x,\ y,\ z) \\
(g,\ i,\ c \between x,\ y,\ z) \\
(l,\ f,\ i \between x,\ y,\ z) \\
(j,\ l,\ g \between x,\ y,\ z) \\
(h,\ k,\ l \between x,\ y,\ z).
\end{array}
$$

No. 59.

The operators which reduce a covariant to zero are—

$$
\begin{array}{l}
(\ j,\ 3f,\ c,\ 2i,\ g,\ 2l \between \partial_h,\ \partial_b,\ \partial_i,\ \partial_f,\ \partial_l,\ \partial_k) - y\partial_z \\
(\ a,\ k,\ 3g,\ 2l,\ 2j,\ h \between \partial_j,\ \partial_f,\ \partial_c,\ \partial_i,\ \partial_g,\ \partial_l) - z\partial_z \\
(3h,\ b,\ i,\ f,\ 2l,\ 2k \between \partial_a,\ \partial_k,\ \partial_g,\ \partial_l,\ \partial_j,\ \partial_h) - x\partial_y \\
(\ h,\ b,\ 3i,\ 2f,\ 2l,\ k \between \partial_j,\ \partial_f,\ \partial_c,\ \partial_i,\ \partial_g,\ \partial_l) - z\partial_y \\
(3j,\ f,\ c,\ i,\ 2g,\ 2l \between \partial_a,\ \partial_k,\ \partial_g,\ \partial_l,\ \partial_j,\ \partial_h) - x\partial_z \\
(\ a,\ 3k,\ g,\ 2l,\ j,\ 2h \between \partial_h,\ \partial_b,\ \partial_i,\ \partial_f,\ \partial_l,\ \partial_k) - y\partial_x.
\end{array}
$$

No. 60.

The evector is

$$
(\partial_a,\ \partial_b,\ \partial_c,\ \partial_f,\ \partial_g,\ \partial_h,\ \partial_i,\ \partial_j,\ \partial_k,\ \partial_l \between \xi,\ \eta,\ \zeta)^3.
$$

No. 61.

The Hessian is HU=

$$
-\left|\begin{array}{lll}
(a,\ h,\ j \between x,\ y,\ z), & (h,\ k,\ l \between x,\ y,\ z), & (j,\ l,\ g \between x,\ y,\ z) \\
(h,\ k,\ l \between x,\ y,\ z), & (k,\ b,\ f \between x,\ y,\ z), & (l,\ f,\ i \between x,\ y,\ z) \\
(j,\ l,\ g \between x,\ y,\ z), & (l,\ f,\ i \between x,\ y,\ z), & (g,\ i,\ c \between x,\ y,\ z)
\end{array}\right|
$$

which is equal to—

$$\left\{\begin{array}{c|c|c|c|c|c|c|c|c|c}
-1\ agk & -1\ bhi & -1\ cij & -1\ bch & -1\ acf & -1\ abg & -1\ bcj & -1\ ach & -1\ abi & -1\ abc \\
+1\ al^2 & +1\ bl^2 & +1\ cl^2 & +2\ bgl & +1\ ai^2 & +2\ afl & +1\ bg^2 & -1\ afg & +1\ af^2 & +1\ afi \\
+1\ gh^2 & +1\ f^2h & +1\ fg^2 & -1\ bij & +2\ chl & -1\ aik & -1\ cfh & +2\ ail & -1\ bgh & +1\ bgj \\
-2\ hjl & -2\ fkl & -2\ fkl & +1\ ck^2 & -1\ cjk & +1\ bj^2 & +2\ ckl & +1\ ch^2 & +2\ bjl & +1\ chk \\
+1\ j^2k & +1\ ik^2 & +1\ i^2j & +1\ f^2j & +1\ fgj & -2\ fhj & +1\ fij & +1\ fj^2 & -2\ fjk & -3\ fgh \\
 & & & -2\ fgk & +1\ g^2k & +1\ ghk & -2\ gik & +1\ gjk & +1\ gk^2 & +2\ fjl \\
 & & & +1\ fhi & -2\ ghi & +1\ h^2i & +1\ hi^2 & -2\ hij & +1\ hki & +2\ gkl \\
 & & & +1\ fl^2 & -1\ gl^2 & -1\ hl^2 & -1\ il^2 & -1\ jl^2 & -1\ kl^2 & +2\ hil \\
 & & & & & & & & & -3\ ijk \\
 & & & & & & & & & -2\ l^3
\end{array}\right\}(x, y, z)^3$$

No. 62.

The quartinvariant is S=

−1 $abcl$	+1 f^2j^2
+1 $abgi$	+3 $fghl$
+1 $acfk$	−1 $fgjk$
−1 af^2g	−1 $fhij$
+1 $afil$	−2 fjl^2
−1 ai^2k	+1 g^2k^2
+1 $bchj$	−1 $ghki$
−1 bg^2h	−2 gkl^2
+1 $bgjl$	+1 h^2i^2
−1 bij^2	−2 hil^2
−1 cf^2h	+3 $ijkl$
+1 $chkl$	+1 l^4
−1 cjk^2	

No. 63.

The sextinvariant is T=

+ 1 $a^2b^2c^2$	−24 acf^2hl	−12 $bcfhj^2$	−12 cfh^3i	−12 $fgjkl^2$
− 6 a^2bcfi	−12 acf^2jk	−24 $bcgh^2l$	+12 cfh^2l^2	−12 fh^2i^2j
+ 4 a^2bi^3	−12 $acfgk^2$	+18 $bcghjk$	−60 $cfhjkl$	−12 $fhijl^2$
+ 4 a^2cf^3	+18 $acfhki$	−12 bch^2ij	+24 cfj^2k^2	+36 fij^2kl
− 3 $a^2f^2i^2$	+36 $acfkl^2$	+36 $bchjl^2$	+12 $cghk^2l$	+24 fjl^4
− 6 ab^2cgj	−24 $acik^2l$	−24 bcj^2kl	−12 $cgjk^3$	+ 8 g^3k^3
+ 4 ab^2g^3	−12 af^3gj	+ 6 bfg^2hj	+12 ch^2ikl	−12 g^2hk^2i
− 6 abc^2hk	+24 af^2g^2k	+12 $bfgj^2l$	+ 6 $chijk^2$	−24 $g^2k^2l^2$
+ 6 $abcfgh$	+ 6 af^2ghi	−12 bfj^4	−12 $chkl^3$	−12 gh^2ki^2
+12 $abcfjl$	+12 af^2gl^2	−12 bg^3hk	+12 cjk^2l^2	−12 $ghkil^2$
+12 $abcgkl$	+12 af^2ijl	+24 bg^2h^2i	+ 8 f^3j^3	+36 $gijk^2l$
+12 $abchil$	−60 $afgkil$	+12 bg^2hl^2	−27 $f^2g^2h^2$	+24 gkl^4
+ 6 $abcijk$	+12 $afhi^2l$	+12 bg^2jkl	−12 f^2gj^2k	+ 8 h^3i^3
−20 $abcl^3$	+ 6 afi^2jk	−60 $bghijl$	+36 f^2ghjl	−24 $h^2i^2l^2$
+18 $abfgij$	−12 $afil^3$	+ 6 $bgij^2k$	−12 f^2hij^2	+24 hil^4
−24 $abfg^2l$	+24 agk^2i^2	−12 $bgjl^3$	−24 $f^2j^2l^2$	+36 hi^2jkl
−12 abg^2ki	−12 ahi^3k	+24 bhi^2j^2	+36 fg^2hkl	−27 $i^2j^2k^2$
−12 $abghi^2$	+12 ai^2kl^2	+12 bij^2l^2	−12 fg^2jk^2	−36 $ijkl^3$
+36 $abgil^2$	+ 4 b^2cj^3	− 3 $c^2h^2k^2$	+36 fgh^2il	− 8 l^6
−24 abi^2jl	− 3 $b^2g^2j^2$	+24 cf^2h^2j	− 6 $fghijk$	
+ 4 ac^2k^3	+ 4 bc^2h^3	+ 6 $cfgh^2k$	−36 $fghl^3$	

The discovery of the invariants S and T is due to Aronhold, the developed expressions were first obtained by Mr. Salmon.

No. 64.

There is an octicovariant for which we may take

$$\Theta\mathrm{U}=\begin{vmatrix} & \partial_x\mathrm{HU}, & \partial_y\mathrm{HU}, & \partial_z\mathrm{HU} \\ \partial_x\mathrm{HU}, & \tfrac{1}{6}\partial_x^2\ \mathrm{U}, & \tfrac{1}{6}\partial_x\partial_y\mathrm{U}, & \tfrac{1}{6}\partial_x\partial_z\mathrm{U} \\ \partial_y\mathrm{HU}, & \tfrac{1}{6}\partial_y\partial_x\mathrm{U}, & \tfrac{1}{6}\partial_y^2\ \mathrm{U}, & \tfrac{1}{6}\partial_y\partial_z\mathrm{U} \\ \partial\ \mathrm{HU}, & \tfrac{1}{6}\partial_z\partial_x\mathrm{U}, & \tfrac{1}{6}\partial_y\partial_x\mathrm{U}, & \tfrac{1}{6}\partial_z^2\ \mathrm{U} \end{vmatrix}$$

or else

$$\Theta_{\prime}\mathrm{U}=\begin{vmatrix} & \tfrac{1}{3}\partial_x\ \mathrm{U}, & \tfrac{1}{3}\partial_y\ \mathrm{U}, & \tfrac{1}{3}\partial_z\ \mathrm{U}, \\ \tfrac{1}{3}\partial_x\mathrm{U}, & \partial_x^2\ \mathrm{HU}, & \partial_x\partial_y\mathrm{HU}, & \partial_x\partial_z\mathrm{HU}, \\ \tfrac{1}{3}\partial_y\mathrm{U}, & \partial_y\partial_x\mathrm{HU}, & \partial_y^2\ \mathrm{HU}, & \partial_y\partial_z\mathrm{HU}, \\ \tfrac{1}{3}\partial_z\mathrm{U}, & \partial_z\partial_x\mathrm{HU}, & \partial_z\partial_y\mathrm{HU}, & \partial_z^2\mathrm{HU}, \end{vmatrix}$$

or else, what I believe is more simple, a function $\Theta_{\prime\prime}\mathrm{U}$, which is a linear function of the last-mentioned two functions.

The relations between $\Theta\mathrm{U}$, $\Theta_{\prime}\mathrm{U}$, $\Theta_{\prime\prime}\mathrm{U}$ are—

$$-\Theta_{\prime}\mathrm{U}+4\Theta\mathrm{U}=\mathrm{T}.\mathrm{U}^2-24\mathrm{S}.\mathrm{U}.\mathrm{HU}$$

$$\Theta_{\prime\prime}\mathrm{U}+2\Theta\mathrm{U}=\mathrm{T}.\mathrm{U}^2-10\mathrm{S}.\mathrm{U}.\mathrm{HU}.$$

I have not worked out the developed expressions.

No. 65.

The cubicontravariant is PU=

$-1\ bcl$	$-1\ acl$	$-1\ abl$	$+1\ ack$	$+1\ abi$	$+1\ bcj$	$+1\ abg$	$+1\ bch$	$+1\ acf$	$-1\ abc$
$+1\ bgi$	$+1\ agi$	$+1\ afk$	$-2\ afg$	$-1\ af^2$	$-1\ bg^2$	$+1\ afl$	$+1\ bgl$	$-1\ ai^2$	$+1\ afi$
$+1\ cfk$	$+1\ chj$	$+1\ bhj$	$+1\ ail$	$-2\ bgh$	$-2\ cfh$	$-2\ aik$	$-2\ bij$	$+1\ chl$	$+1\ bgj$
$-1\ f^2g$	$-1\ g^2h$	$-1\ fh^2$	$-1\ ch^2$	$+1\ bjl$	$+1\ ckl$	$-1\ bj^2$	$-1\ ck^2$	$-2\ cjk$	$+1\ chk$
$+1\ fil$	$+1\ gjl$	$+1\ ckl$	$+2\ fj^2$	$+3\ fhl$	$+3\ fgl$	$-1\ fhj$	$+2\ f^2j$	$-1\ fgj$	$+3\ fgh$
$-1\ i^2k$	$-1\ ij^2$	$-1\ jk^2$	$+3\ fhl$	$-1\ fjk$	$-1\ fij$	$-1\ ghk$	$-1\ fgk$	$+2\ g^2k$	$-4\ fjl$
			$-1\ gjk$	$+2\ gk^2$	$-1\ gki$	$+2\ h^2i$	$-1\ fhi$	$-1\ ghi$	$-4\ gkl$
			$-1\ hij$	$-1\ hki$	$+2\ hi^2$	$-2\ hl^2$	$-2\ fl^2$	$-2\ gl^2$	$-4\ hil$
			$-2\ jl^2$	$-2\ kl^2$	$-2\ il^2$	$+3\ jkl$	$+3\ ikl$	$+3\ ijl$	$+3\ ijk$
									$+4\ l^3$

$(\xi, \eta, \zeta)^3$

The quintic contravariant is QU =

+ 1 ab^2c^2	+ 1 a^2bc^2	+ 1 a^2b^2c	− 3 a^2bci	− 3 ab^2cj	− 3 abc^2k	− 3 a^2bcf	− 3 ab^2cg	− 3 abc^2h	+ 6 $abcfj$
− 6 $abcfi$	− 3 a^2cfi	− 3 a^2bfi	+ 6 a^2cf^2	+ 6 ab^2g^2	+ 3 $abcfg$	+ 6 a^2bi^2	+ 6 $abcfl$	+ 6 $abcgl$	+ 6 $abcgk$
+ 4 abi^3	+ 2 a^2i^3	+ 2 a^2f^3	− 3 a^2fi^2	+ 3 $abcfh$	+ 6 $abcil$	− 3 a^2f^2i	+ 3 $abcik$	+ 3 $abcij$	+ 6 $abchi$
+ 4 acf^3	− 6 $abcgj$	− 3 ab^2gj	+ 3 $abcgh$	+ 6 $abckl$	− 6 $abgi^2$	+ 6 $abchl$	+ 9 $abfgi$	− 6 abg^2i	−30 $abcl^2$
− 3 af^2i^2	+ 4 abg^3	− 6 $abchk$	+ 6 $abcjl$	+ 9 $abfij$	−12 acf^2l	+ 3 $abcjk$	−12 abi^2l	+ 6 ac^2k^2	−12 $abfg^2$
− 3 b^2cgj	− 3 ac^2hk	+ 3 $abfgh$	+ 9 $abgij$	−24 $abfgl$	+ 9 $acfki$	+ 9 $abfgj$	− 6 acf^2k	− 6 acf^2j	+36 $abgil$
+ 2 b^2g^3	+ 3 $acfgh$	+ 6 $abfjl$	−12 abg^2l	−12 $abgki$	+ 3 af^2gi	− 6 abg^2k	− 6 af^3g	−12 $acfgk$	−12 abi^2j
− 3 bc^2hk	+ 6 $acfjl$	+ 6 $abgkl$	−24 $acfhl$	− 6 $abhi^2$	+ 6 afi^2l	−12 $abghi$	+ 6 af^2il	+ 9 $acfhi$	−12 acf^2h
+ 3 $bcfgh$	+ 6 $acgkl$	+ 6 $abhil$	−12 $acfjk$	+18 $abil^2$	− 6 ai^3k	+18 $abgl^2$	+ 3 afi^2k	+18 $acfl^2$	+36 $acfkl$
+ 6 $bcfjl$	+ 6 $achil$	+ 3 $abijk$	− 6 $acgk^2$	− 6 $acfk^2$	+ 6 bc^2h^2	−24 $abijl$	+ 6 b^2cj^2	−24 $acikl$	−12 $acik^2$
+ 6 $bcgkl$	+ 3 $acijk$	−10 abl^3	+ 9 $achki$	− 6 af^3j	− 6 $bcfj^2$	+ 9 $acfhk$	− 3 b^2g^2j	+12 af^2g^2	+12 af^2gl
+ 6 $bchil$	−10 acl^3	+ 4 ack^3	+18 $ackl^2$	+24 af^2gk	−24 $bcghl$	−12 ack^2l	−12 $bcfhj$	−30 $afgil$	+ 6 af^2ij
+ 3 $bcijk$	+ 9 $afgij$	−12 af^2hl	−18 af^2gj	+ 3 af^2hi	+ 9 $bcgjk$	+ 3 af^2gh	+ 9 $bcghk$	+ 3 afi^2j	−30 $afgki$
−10 bcl^3	−12 afg^2l	− 6 af^2jk	+24 afg^2k	+ 6 af^2l^2	−12 $bchij$	+ 6 af^2jl	− 6 bch^2i	+24 $agki^2$	+ 6 $afhi^2$
+ 9 $bfgij$	− 6 ag^2ki	− 6 $afgk^2$	+ 6 $afghi$	−30 $afkil$	+18 $bcjl^2$	−30 $afgkl$	+18 $bchl^2$	− 6 ahi^3	−18 $afil^2$
−12 bfg^2l	− 6 $aghi^2$	+ 9 $afhki$	+12 $afgl^2$	+12 ak^2i^2	+ 3 bfg^2j	+12 $afhil$	−24 $bcjkl$	+ 6 ai^2l^2	+12 ai^2kl
− 6 bg^2ki	+18 $agil^2$	+18 $afkl^2$	+12 $afijl$	− 3 b^2gj^2	− 6 bg^3k	+ 6 $afijk$	+ 3 bfg^2h	+ 9 $bcghj$	−12 $bcgh^2$
− 6 $bghi^2$	−12 ai^2jl	−12 aik^2l	−30 $agkil$	−12 bch^2l	+24 bg^2hi	− 6 afl^3	+12 $bfgjl$	−12 bcj^2l	+36 $bchjl$
+18 $bgil^2$	+ 4 bcj^3	+ 2 b^2j^3	+ 6 ahi^2l	+ 9 $bchjk$	+ 6 bg^2l^2	+24 agk^2i	−18 $bfij^2$	− 6 bg^3h	−12 bcj^2k
−12 bi^2jl	− 3 bg^2j^2	+ 4 bch^3	+ 3 ai^2jk	+ 6 $bfghj$	−30 $bgijl$	−18 ahi^2k	+ 6 bg^2kl	+ 6 bg^2jl	+ 6 $bfgj^2$
+ 2 c^2k^3	+ 2 c^2h^3	− 6 $bfhj^2$	− 6 ail^3	+ 6 bfj^2l	+12 bi^2j^2	+12 $aikl^2$	−30 $bghil$	+ 3 $bgij^2$	+12 bg^2hl
−12 cf^2hl	− 6 $cfhj^2$	−12 bgh^2l	− 6 $bchj^2$	−18 bg^2hk	− 3 c^2hk^2	− 6 bch^2j	+ 6 $bgijk$	− 3 c^2h^2k	+ 6 bg^2jk
− 6 cf^2jk	−12 cgh^2l	+ 9 $bghjk$	+ 3 bg^2hj	+24 bgh^2i	+24 cf^2hj	− 6 bfj^3	− 6 bgl^3	+ 3 $cfgh^2$	−30 $bghij$
− 6 $cfgk^2$	+ 9 $cghjk$	− 6 bh^2ij	+ 6 bgj^2l	+12 $bghl^2$	+ 6 $cfghk$	+12 bg^2h^2	+24 bhi^2j	−30 $cfhjl$	−18 $bgjl^2$
+ 9 $cfhki$	− 6 ch^2ij	+18 $bhjl^2$	− 6 bij^3	+12 $bgjkl$	−18 cfh^2i	−30 $bghjl$	+12 $bijl^2$	+24 cfj^2k	+12 bi^2jl
+18 $cfkl^2$	+18 $chjl^2$	−12 bj^2kl	+24 cfh^2j	−30 $bhijl$	+12 $cfhl^2$	+ 3 bgj^2k	+12 cf^2h^2	+12 $cghkl$	+12 cfh^2l
−12 cik^2l	−12 cj^2kl	− 3 ch^2k^2	+ 3 cgh^2k	+ 3 bij^2k	−30 $cfjkl$	+24 $bhij^2$	−30 $cfhkl$	−18 $cgjk^2$	−30 $cfhjk$
− 6 f^3gj	+ 3 fg^2hj	+12 f^2h^2j	− 6 ch^3i	− 6 bjl^3	+ 6 cgk^2l	+ 6 bj^2l^2	+24 $cfjk^2$	+ 6 ch^2il	+ 6 $cghk^2$
+12 f^2g^2k	+ 6 fgj^2l	+ 3 fgh^2k	+ 6 ch^2l^2	+ 3 cfh^2k	+12 $chikl$	− 6 cfh^3	− 6 cgk^3	+ 6 $chijk$	+ 6 ch^2ik
+ 3 f^2ghi	− 6 fij^3	− 6 fh^3i	−30 $chjkl$	+ 6 chk^2l	+ 3 $cijk^2$	+ 6 ch^2kl	+ 3 $chik^2$	− 6 chl^3	−18 $chkl^2$
+ 6 f^2gl^2	− 6 g^3hk	+ 6 fh^2l^2	+12 cj^2k^2	− 6 cjk^3	− 6 ckl^3	+ 3 $chjk^2$	+ 6 ck^2l^2	+12 $cjkl^2$	+12 cjk^2l
+ 6 f^2ijl	+12 g^2h^2i	−30 $fhjkl$	+12 f^2j^3	−27 f^2gh^2	−27 f^2g^2h	− 6 f^2hj^2	+12 f^3j^2	− 6 f^2gj^2	+18 f^2ghj
−30 $fgkil$	+ 6 g^2hl^2	+12 fj^2k^2	−27 fg^2h^2	− 6 f^2j^2k	+18 f^2gjl	+18 fgh^2l	−12 f^2gjk	+18 fg^2hl	−24 f^2j^2l
+ 6 fhi^2l	+ 6 g^2jkl	+ 6 ghk^2l	−12 fgj^2k	+18 f^2hjl	− 6 f^2ij^2	− 3 $fghjk$	+18 f^2ghl	−12 fg^2jk	+18 fg^2hk
+ 3 fi^2jk	−30 $ghijl$	− 6 gjk^3	+36 $fghjl$	+36 $fghkl$	+18 fg^2kl	−12 fh^2ij	−12 f^2hij	− 3 $fghij$	+18 fgh^2i
− 6 fil^3	+ 3 gij^2k	+ 6 h^2ikl	−12 $fhij^2$	−12 $fgjk^2$	+36 $fghil$	− 6 $fhjl^2$	−24 f^2jl^2	− 6 $fgjl^2$	−54 $fghl^2$
+12 gk^2i^2	− 6 gjl^3	+ 3 $hijk^2$	−24 fj^2l^2	+18 fh^2il	− 3 $fgijk$	+18 fj^2kl	− 6 fg^2k^2	+18 fij^2l	−12 $fgjkl$
− 6 hi^3k	+12 hi^2j^2	− 6 hkl^3	+18 g^2hkl	− 3 $fhijk$	−18 fgl^3	− 6 g^2hk^2	− 3 $fghik$	+12 g^3k^2	−12 $fhijl$
+ 6 i^2kl^2	+ 6 ij^2l^2	+ 6 jk^2l^2	− 6 g^2jk^2	−18 fhl^3	−12 fhi^2j	−12 gh^2ki	− 6 $fgkl^2$	−12 g^2hki	+18 fij^2k
			+18 gh^2il	− 6 $fjkl^2$	− 6 $fijl^2$	− 6 $ghkl^2$	− 6 fh^2i^2	−24 g^2kl^2	+48 fjl^3
			− 3 $ghijk$	+12 g^2k^3	− 6 g^2k^2i	+18 gjk^2l	− 6 $fhil^2$	− 6 gh^2i^2	−24 g^2k^2l
			−18 ghl^3	−12 ghk^2i	−12 $ghki^2$	+12 h^3i^2	+36 $fijkl$	− 6 $ghil^2$	−12 $ghkil$
			− 6 $gjkl^2$	−24 gk^2l^2	− 6 $gkil^2$	−24 h^2il^2	+12 fl^4	+36 $gijkl$	+18 $gijk^2$
			− 6 h^2i^2j	− 6 h^2ki^2	+12 h^2i^3	+12 hl^4	+18 gik^2l	+12 gl^4	+48 gkl^3
			− 6 $hijl^2$	− 6 $hkil^2$	−24 hi^2l^2	+36 $hijkl$	+18 hi^2kl	+18 hi^2jl	−24 h^2i^2l
			+18 ij^2kl	+18 ijk^2l	+18 i^2jkl	−27 ij^2k^2	−27 i^2jk^2	−27 i^2j^2k	+48 hil^3
			+12 jl^4	+12 kl^4	+12 il^4	−18 jkl^3	−18 ikl^3	−18 jkl^3	+18 hi^2jk
									−54 $ijkl^2$
									−24 l^5

$(\xi, \eta, \zeta)^3$

The reciprocant is FU=

No. 67.

ξ^6.	η^6.	ζ^6.	$\eta^5\zeta$.	$\zeta^5\xi$.	$\xi^5\eta$.	$\eta\zeta^5$.	$\zeta\xi^5$.	$\xi\eta^5$.	$\eta^4\zeta^2$.	$\zeta^4\xi^2$.	$\xi^4\eta^2$.	$\eta^2\zeta^4$.	$\zeta^2\xi^4$.
$+1\ b^2c^2$	$+1\ a^2c^2$	$+1\ a^2b^2$	$-6\ a^2ci$	$-6\ ab^2j$	$-6\ bc^2k$	$-6\ a^2bf$	$-6\ b^2cg$	$-6\ ac^2h$	$+6\ a^2cf$	$+6\ ab^2g$	$+6\ bc^2h$	$+6\ a^2bi$	$+6\ b^2cj$
$-6\ bcfi$	$-6\ acgj$	$-6\ abhk$	$+6\ acgh$	$+6\ abfh$	$+6\ bcfg$	$+12\ abhl$	$+12\ bcfl$	$+12\ acgl$	$+9\ a^2i^2$	$-12\ abfl$	$-12\ bcgl$	$+9\ a^2f^2$	$+9\ b^2g^2$
$+4\ bi^3$	$+4\ ag^3$	$+4\ ak^3$	$+12\ acjl$	$+12\ abkl$	$+12\ bcil$	$+6\ abjk$	$+6\ bcik$	$+6\ acij$	$-12\ achl$	$-6\ abik$	$-6\ bcij$	$-6\ abgh$	$-6\ bcfh$
$+4\ cf^3$	$+4\ cj^3$	$+4\ bh^3$	$+18\ agij$	$-12\ afk^2$	$-12\ bgi^2$	$+18\ afhk$	$+18\ bfgi$	$-12\ ag^2i$	$-6\ acjk$	$+12\ af^2k$	$+12\ bg^2i$	$-12\ abjl$	$-12\ bckl$
$-3\ f^2i^2$	$-3\ g^2j^2$	$-3h^2k^2$	$-24\ ag^2l$	$-24\ bh^2l$	$-24\ cf^2l$	$-24\ ak^2l$	$-24\ bi^2l$	$+18\ cghj$	$-18\ afgj$	$+9\ b^2j^2$	$+9\ c^2k^2$	$-36\ afhl$	$-36\ bfgl$
			$-12\ chj^2$	$+18\ bhjk$	$+18\ cfki$	$-12\ bh^2j$	$-12\ cf^2k$	$-24\ cj^2l$	$+12\ ag^2k$	$-18\ bfhj$	$+12\ cf^2j$	$-18\ afjk$	$-18\ bfij$
			$+6\ g^2hj$	$+6\ fh^2k$	$+6\ f^2gi$	$-12\ fh^3$	$-12\ f^3g$	$-12\ g^3h$	$-18\ aghi$	$-18\ bghk$	$-18\ cfgk$	$+12\ agk^2$	$-18\ bgik$
			$+12\ gj^2l$	$+12\ hk^2l$	$+12\ fi^2l$	$+12\ h^2kl$	$+12\ f^2il$	$+12\ g^2jl$	$+48\ agl^2$	$+12\ bh^2i$	$-18\ cfhi$	$-18\ aihk$	$+12\ bhi^2$
			$-12\ ij^3$	$-12\ jk^3$	$-12\ i^3k$	$+6\ hjk^2$	$+6\ fi^2k$	$+6\ gij^2$	$-36\ aijl$	$+48\ bhl^2$	$+48\ cfl^2$	$+48\ akl^2$	$+48\ bil^2$
									$+12\ ch^2j$	$-36\ bjkl$	$-36\ cikl$	$+12\ bhj^2$	$+12\ cfk^2$
									$+12\ fj^3$	$-3\ f^2h^2$	$-3\ f^2g^2$	$-6\ gh^2k$	$+12\ f^3j$
									$-3\ g^2h^2$	$-24\ fhkl$	$-24\ fgil$	$+36\ fh^2j$	$+36\ f^2gk$
									$-24\ ghjl$	$+36\ fjk^2$	$-6\ fi^2j$	$+12\ h^3i$	$-6\ f^2hi$
									$-6\ gj^2k$	$+12\ gk^3$	$+36\ gi^2k$	$-12\ h^2l^2$	$-12\ f^2l^2$
									$+36\ hij^2$	$-6\ hik^2$	$+12\ hi^3$	$-24\ hjkl$	$-24\ fikl$
									$-12\ j^2l^2$	$-12\ k^2l^2$	$-12\ i^2l^2$	$-3\ j^2k^2$	$-3\ i^2k^2$

$\xi^2\eta^4$.	$\eta^3\zeta^3$.	$\zeta^3\xi^3$.	$\xi^3\eta^3$.	$\xi^4\eta\zeta$.	$\eta^4\zeta\xi$.	$\zeta^4\xi\eta$.	$\xi^3\eta^2\zeta$.	$\eta^3\zeta^2\xi$.	$\zeta^3\xi\eta^2$.	$\xi^3\eta\zeta^2$.	$\eta^3\zeta\xi^2$.	$\zeta^3\xi^2\eta$.	$\xi^2\eta^2\zeta^2$.
$+6\ ac^2k$	$-2\ a^2bc$	$-2\ ab^2c$	$-2\ abc^2$	$-12\ bcfj$	$-12\ acfj$	$-12\ abgk$	$+6\ abci$	$+6\ abcj$	$+6\ abck$	$-6\ abcf$	$+6\ abcg$	$+6\ abch$	$-24\ abcl$
$-6\ acfg$	$-18\ a^2fi$	$+6\ abfi$	$+6\ acfi$	$+30\ bcgk$	$-12\ acgk$	$-12\ abhi$	$-12\ acf^2$	$-12\ abg^2$	$-24\ abfg$	$-12\ abi^2$	$+36\ acfl$	$+36\ abgl$	$+6\ abgi$
$-12\ acil$	$+6\ abgj$	$-4\ af^3$	$-4\ ai^3$	$-12\ bchi$	$+30\ achi$	$+30\ abij$	$+6\ afi^2$	$-24\ acfh$	$+36\ abil$	$+6\ af^2i$	$-24\ acik$	$-24\ abij$	$+6\ acfk$
$+12\ agi^2$	$+6\ achk$	$-18\ b^2gj$	$+6\ bcgj$	$-24\ bcl^2$	$-24\ acl^2$	$-24\ abl^2$	$-24\ bcgh$	$+36\ ackl$	$+12\ af^2l$	$+36\ bchl$	$-30\ afgi$	$-12\ ack^2$	$+30\ af^2g$
$+9\ c^2h^2$	$+18\ afgh$	$+6\ bchk$	$-4\ bg^3$	$-18\ bfg^2$	$+24\ afg^2$	$-18\ af^2h$	$+36\ bcjl$	$-60\ afgl$	$-30\ afik$	$-24\ bcjk$	$+12\ ai^2l$	$-36\ af^2j$	$-48\ afil$
$+12\ cfj^2$	$+36\ afjl$	$+18\ bfgh$	$-18\ c^2hk$	$+12\ bgil$	$+12\ agil$	$+12\ afkl$	$+12\ bg^2l$	$+54\ afij$	$-12\ bch^2$	$+54\ bfgj$	$-12\ bcj^2$	$+12\ afgk$	$+30\ ai^2k$
$-36\ cghl$	$-48\ agkl$	$+36\ bfjl$	$+18\ cfgh$	$+24\ bi^2j$	$-18\ ai^2j$	$+24\ aik^2$	$-30\ bgij$	$-36\ ahi^2$	$-36\ bfj^2$	$-36\ bg^2k$	$+6\ bg^2j$	$+54\ afhi$	$+6\ bchj$
$-18\ cgjk$	$+36\ ahil$	$+36\ bgkl$	$-48\ cfjl$	$+24\ cf^2h$	$-18\ cgh^2$	$+24\ bgh^2$	$-60\ cfhl$	$+12\ agik$	$-60\ bghl$	$+12\ bghi$	$+12\ cfhj$	$+24\ afl^2$	$+30\ bg^2h$
$-18\ chij$	$+18\ aijk$	$-48\ bhil$	$+36\ cgkl$	$+12\ cfkl$	$+12\ chjl$	$+12\ bhjl$	$+12\ cfjk$	$+24\ ail^2$	$+54\ bgjk$	$+24\ bgl^2$	$+54\ cghk$	$-60\ aikl$	$-48\ bgjl$
$+48\ cjl^2$	$-32\ al^3$	$+18\ bijk$	$+18\ cijk$	$-18\ cik^2$	$+24\ cj^2k$	$-18\ bj^2k$	$-36\ cgk^2$	$+6\ bgj^2$	$+12\ bhij$	$-60\ bijl$	$-36\ ch^2i$	$-30\ bghj$	$+30\ bij^2$
$-6\ fg^2j$	$-4\ bj^3$	$-32\ bl^3$	$+36\ chil$	$+60\ f^2gl$	$-12\ fgj^2$	$+60\ fh^2l$	$+54\ chik$	$+12\ ch^2l$	$+24\ bjl^2$	$-30\ cfhk$	$+24\ chl^2$	$+12\ bj^2l$	$+30\ cfh^2$
$+12\ g^3k$	$-4\ ch^3$	$-4\ ck^3$	$-32\ cl^3$	$-12\ f^2ij$	$+60\ g^2hl$	$-66\ fhjk$	$+24\ ckl^2$	$-30\ chjk$	$+6\ chk^2$	$+12\ ck^2l$	$-60\ cjkl$	$+6\ ch^2k$	$-48\ chkl$
$+36\ g^2hi$	$-36\ fhj^2$	$+12\ f^2hl$	$+12\ fg^2l$	$-66\ fgik$	$-12\ g^2jk$	$-12\ ghk^2$	$-24\ f^2gj$	$+78\ fghj$	$+60\ f^2hj$	$-66\ f^2gh$	$-66\ fg^2h$	$-66\ fgh^2$	$+30\ cjk^2$
$-12\ g^2l^2$	$+12\ gh^2l$	$-36\ f^2jk$	$+12\ fgij$	$-12\ fhi^2$	$-66\ ghij$	$-12\ h^2ki$	$+60\ fg^2k$	$-48\ fj^2l$	$+78\ fghk$	$-48\ f^2jl$	$+72\ fgjl$	$-12\ fhjl$	$+24\ f^2j^2$
$-24\ gijl$	$+12\ ghjk$	$-36\ fgk^2$	$-36\ g^2ki$	$-48\ fil^2$	$-48\ gjl^2$	$-48\ hkl^2$	$+78\ fghi$	$-24\ g^2hk$	$-24\ fh^2i$	$-12\ fgkl$	$-24\ fij^2$	$+60\ fj^2k$	$+108\ fghl$
$-3\ i^2j^2$	$-36\ h^2ij$	$+12\ fhki$	$-36\ ghi^2$	$+60\ i^2kl$	$+60\ j^2il$	$+60\ jk^2l$	$-96\ fgl^2$	$+60\ gh^2i$	$-96\ fhl^2$	$+72\ fhil$	$-48\ g^2kl$	$+72\ ghkl$	$-114\ fgjk$
	$+24\ hjl^2$	$+24\ fkl^2$	$+24\ gil^2$				$+72\ fijl$	$-96\ ghl^2$	$-12\ fjkl$	$+78\ fijk$	$-12\ ghil$	$-24\ gjk^2$	$-114\ fhij$
	$+12\ j^2kl$	$+12\ ik^2l$	$+12\ i^2jl$				$-12\ gikl$	$+72\ gjkl$	$-48\ gk^2l$	$+48\ fl^3$	$+78\ gijk$	$-48\ h^2il$	$+24\ fjl^2$
							$-48\ hi^2l$	$-12\ hijl$	$+72\ hikl$	$+60\ gik^2$	$+48\ gl^3$	$+78\ hijk$	$+24\ g^2k^2$
							$-66\ i^2jk$	$-66\ ij^2k$	$-66\ ijk^2$	$-24\ hi^2k$	$+60\ hi^2j$	$+48\ hl^3$	$-114\ ghki$
							$+48\ il^3$	$+48\ jl^3$	$+48\ kl^3$	$-96\ ikl^2$	$-96\ ijl^2$	$-96\ jkl^2$	$+24\ gkl^2$
													$+24\ h^2i^2$
													$+24\ hil^2$
													$+108\ ijkl$
													$-48\ l^4$

$(\xi,$

The preceding Tables contain the complete system of the covariants and contravariants of the ternary cubic, *i. e.* the covariants are the cubic itself U, the quartinvariant S, the sextinvariant T, the Hessian HU, and an octicovariant, say ΘU ; the contravariants are the cubicontravariant PU, the quinticontravariant QU, and the reciprocant FU.

The contravariants are all of them evectants, viz. PU is the evectant of S, QU is the evectant of T, and the reciprocant FU is the evectant of QU, or what is the same thing, the second evectant of T.

The discriminant is a rational and integral function of the two invariants; representing it by R, we have $R=64\,S^3-T^2$.

If we combine U and HU by arbitrary multipliers, say α and 6β, so as to form the sum $\alpha U+6\beta HU$, this is a cubic, and the question arises, to find the covariants and contravariants of this cubic: the results are given in the following Table:—

No. 68.

$$
\begin{array}{ll}
\alpha U+6\beta HU & =\alpha U+6\beta HU. \\
H(\alpha U+6\beta HU)= & (0,\ 2S,\quad T,\quad 8S^2 \between \alpha,\beta)^3 U \\
 & +(1,\ 0,\ -12S,\ -2T \between \alpha,\beta)^3 HU. \\
P(\alpha U+6\beta HU)= & (1,\ 0,\ 12S,\quad 4T \between \alpha,\beta)^3 PU \\
 & +(0,\ 1,\quad 0,\ -4S \between \alpha,\beta)^3 QU. \\
Q(\alpha U+6\beta HU)= & (0,\ 60S,\ 30T,\quad 0,\ -120TS,\ -24T^2+576S^3 \between \alpha,\beta)^5 PU \\
 & +(1,\quad 0,\quad 0,\ 10T,\quad 240S^2,\qquad 24TS \between \alpha,\beta)^5 QU. \\
S(\alpha U+6\beta HU)= & (S,\ T,\ 24S^2,\ 4TS,\ T^2-48S^3 \between \alpha,\beta)^4. \\
T(\alpha U+6\beta HU)= & (T,\ 96S^2,\ 60TS,\ 20T^2,\ 240TS^2,\ -48T^2S+4608S^4,\ -8T^3+576TS^3 \between \alpha,\beta)^6. \\
R(\alpha U+6\beta HU)= & [(1,\ 0,\ -24S,\ -8T,\ -48S^2 \between \alpha,\beta)^4]^3 R. \\
F(\alpha U+6\beta HU)= & (1,\quad 0,\ -24S,\ -8T,\ -48S^2 \between \alpha,\beta)^4 FU \\
 & +(0,\ 24,\quad 0,\quad 0,\ -48T \between \alpha,\beta)^4 (PU)^2 \\
 & +(0,\quad 0,\ 24,\quad 0,\quad 96S \between \alpha,\beta)^4 PU.QU \\
 & +(0,\quad 0,\quad 0,\quad 8,\quad 0 \between \alpha,\beta)^4.(QU)^2.
\end{array}
$$

We have, in like manner, for the covariants and contravariants of the cubic $6\alpha PU+\beta QU$, the following Table:—

No. 69.

$$
\begin{array}{ll}
6\alpha PU+\beta QU & =6\alpha PU+\beta QU. \\
H(6\alpha PU+\beta QU)= & (-2T,\ 48S^2,\ 18TS,\ T^2+16S^3 \between \alpha,\beta)^3 PU \\
 & +(8S,\quad T,\ -8S^2,\quad -TS \between \alpha,\beta)^3 QU. \\
P(6\alpha PU+\beta QU)= & (32S^2,\ 12TS,\ T^2+32S^3,\quad 4TS^2 \between \alpha,\beta)^3 U \\
 & +(4T,\ 96S^2,\quad 12TS,\ T^2-32S^3 \between \alpha,\beta)^3 HU.
\end{array}
$$

$$
\mathrm{Q}(6\alpha\mathrm{PU}+\beta\mathrm{QU})=\left[\begin{array}{l}
+384\mathrm{T}\,\mathrm{S}^2,\\
+120\mathrm{T}^2\mathrm{S}+7680\ \mathrm{S}^4,\\
+\ 10\mathrm{T}^3\ +3200\mathrm{T}\,\mathrm{S}^3,\\
+480\mathrm{T}^2\mathrm{S}^2,\\
+\ 30\mathrm{T}^3\mathrm{S},\\
+\ 1\mathrm{T}^4\ -\ 24\mathrm{T}^2\mathrm{S}^3+512\mathrm{S}^6
\end{array}\right]^{\nwarrow}(\alpha,\beta)^5\mathrm{U}
$$

$$
+\left[\begin{array}{l}
-\ 24\mathrm{T}^2\ +\ 4608\mathrm{S}^3,\\
+1920\mathrm{TS}^2,\\
+\ 480\mathrm{T}^2\mathrm{S},\\
+\ 30\mathrm{T}^3\ +1920\mathrm{TS}^3,\\
+\ 120\mathrm{T}^2\mathrm{S}^2+7680\ \mathrm{S}^5,\\
-\ 6\mathrm{T}^3\mathrm{S}+\ 768\mathrm{TS}^4
\end{array}\right]^{\nwarrow}(\alpha,\beta)^5\mathrm{HU}
$$

$$
\mathrm{S}(6\alpha\mathrm{PU}+\beta\mathrm{QU})=\left[\begin{array}{l}
+\ 1\mathrm{T}^2\ +192\ \mathrm{S}^3,\\
+128\mathrm{T}\,\mathrm{S}^2,\\
+\ 18\mathrm{T}^2\mathrm{S}+384\ \mathrm{S}^4,\\
+\ 1\mathrm{T}^3\ +\ 64\mathrm{TS}^3,\\
+\ 5\mathrm{T}^2\mathrm{S}^2-\ 64\ \mathrm{S}^5
\end{array}\right]^{\nwarrow}(\alpha,\beta)^4
$$

$$
\mathrm{T}(6\alpha\mathrm{PU}+\beta\mathrm{QU})=\left[\begin{array}{l}
-\ 8\mathrm{T}^3\ +\ 4608\mathrm{TS}^3,\\
+1920\mathrm{T}^2\mathrm{S}^2+73728\ \mathrm{S}^5,\\
+\ 360\mathrm{T}^3\mathrm{S}+38400\mathrm{TS}^4,\\
+\ 20\mathrm{T}^4\ +\ 8960\mathrm{T}^2\mathrm{S}^3,\\
+\ 840\mathrm{T}^3\mathrm{S}^2+\ 7680\mathrm{TS}^5,\\
+\ 36\mathrm{T}^4\mathrm{S}+\ 384\mathrm{T}^2\mathrm{S}^4+24576\ \mathrm{S}^7,\\
+\ 1\mathrm{T}^5\ -\ 40\mathrm{T}^3\mathrm{S}^2+\ 2560\mathrm{TS}^6
\end{array}\right]^{\nwarrow}(\alpha,\beta)^6
$$

$$
\mathrm{R}(6\alpha\mathrm{PU}+\beta\mathrm{QU})=[(48\mathrm{S},\ 8\mathrm{T},\ -96\mathrm{S}^2,\ -24\mathrm{TS},\ -\mathrm{T}^2-16\mathrm{S}^3)(\alpha,\beta)^4]^3\mathrm{R}^2.
$$

$$
\begin{array}{l}
\mathrm{F}(6\alpha\mathrm{PU}+\beta\mathrm{QU}=(192\,\mathrm{S},\ 32\mathrm{T},\ -384\ \mathrm{S}^2,\ -96\mathrm{T}\,\mathrm{S},\ -4\mathrm{T}^2-64\mathrm{S}^3)(\alpha,\beta)^4\Theta\mathrm{U}\\
\qquad+(0,\ 512\ \mathrm{S}^3,\ 192\mathrm{T}\,\mathrm{S}^2,\ 24\mathrm{T}^2\mathrm{S},\ \mathrm{T}^3)(\alpha,\beta)^4.\mathrm{U}^2\\
\qquad+(1344\mathrm{S}^2,\ 352\mathrm{TS},\ 24\mathrm{T}^2-1152\mathrm{S}^3,\ -288\mathrm{T}\,\mathrm{S}^2,\ -20\mathrm{T}^2\mathrm{S}+64\mathrm{S}^4)(\alpha,\beta)^4\mathrm{U}.\mathrm{HU}\\
\qquad+(48\mathrm{T},\ 0,\ 288\mathrm{T}\,\mathrm{S},\ 24\mathrm{T}^2+1536\mathrm{S}^3,\ 144\mathrm{T}\,\mathrm{S}^2)(\alpha,\beta)^4(\mathrm{HU})^2.
\end{array}
$$

The tables for the ternary cubic become much more simple if we suppose that the cubic is expressed in HESSE's canonical form; we have then the following table:—

No. 70.

$$\mathrm{U}=x^3+y^3+z^3+6lxyz.$$

$$\mathrm{S}=-l+l^4.$$

$$\begin{aligned}
\mathrm{T}\ &=1-20l^3-8l^6.\\
\mathrm{R}\ &=-(1+8l^3)^3.\\
\mathrm{HU}\ &=l^2(x^3+y^3+z^3)-(1+2l^3)xyz.\\
\Theta\mathrm{U}\ &=(1+8l^3)^2(y^3z^3+z^3x^3+x^3y^3)\\
&\quad+(-9l^6)(x^3+y^3+z^3)^2\\
&\quad+(-2l-5l^4-20l^7)(x^3+y^3+z^3)xyz\\
&\quad+(-15l^2-78l^5+12l^8)x^2y^2z^2.\\
\Theta_{\prime}\mathrm{U}\ &=4(1+8l^3)^2(y^3z^3+z^3x^3+x^3y^3)\\
&\quad+(-1-4l^3-4l^6)(x^3+y^3+z^3)^2\\
&\quad+(4l+100l^4+112l^7)(x^3+y^3+z^3)xyz\\
&\quad+(48l^2+552l^5+48l^8)x^2y^2z^2.\\
\Theta_{\prime\prime}\mathrm{U}&=-2(1+8l^3)^2(y^3z^3+z^3x^3+x^3y^3)\\
&\quad+(1-10l^3)(x^3+y^3+z^3)^2\\
&\quad+(6l-180l^4-96l^7)(x^3+y^3+z^3)xyz\\
&\quad+(6l^2-624l^5-192l^8)x^2y^2z^2.\\
\mathrm{PU}\ &=-l(\xi^3+\eta^3+\zeta^3)+(-1+4l^3)\xi\eta\zeta.\\
\mathrm{QU}\ &=(1-10l^3)(\xi^3+\eta^3+\zeta^3)-6l^2(5+4l^3)\xi\eta\zeta.\\
\mathrm{FU}\ &=-4(1+8l^3)(\eta^3\zeta^3+\zeta^3\xi^3+\xi^3\eta^3)\\
&\quad+(\xi^3+\eta^3+\zeta^3)^2\\
&\quad-24l^2(\xi^3+\eta^3+\zeta^3)\xi\eta\zeta\\
&\quad-24l(1+2l^3)\xi^2\eta^2\zeta^2,
\end{aligned}$$

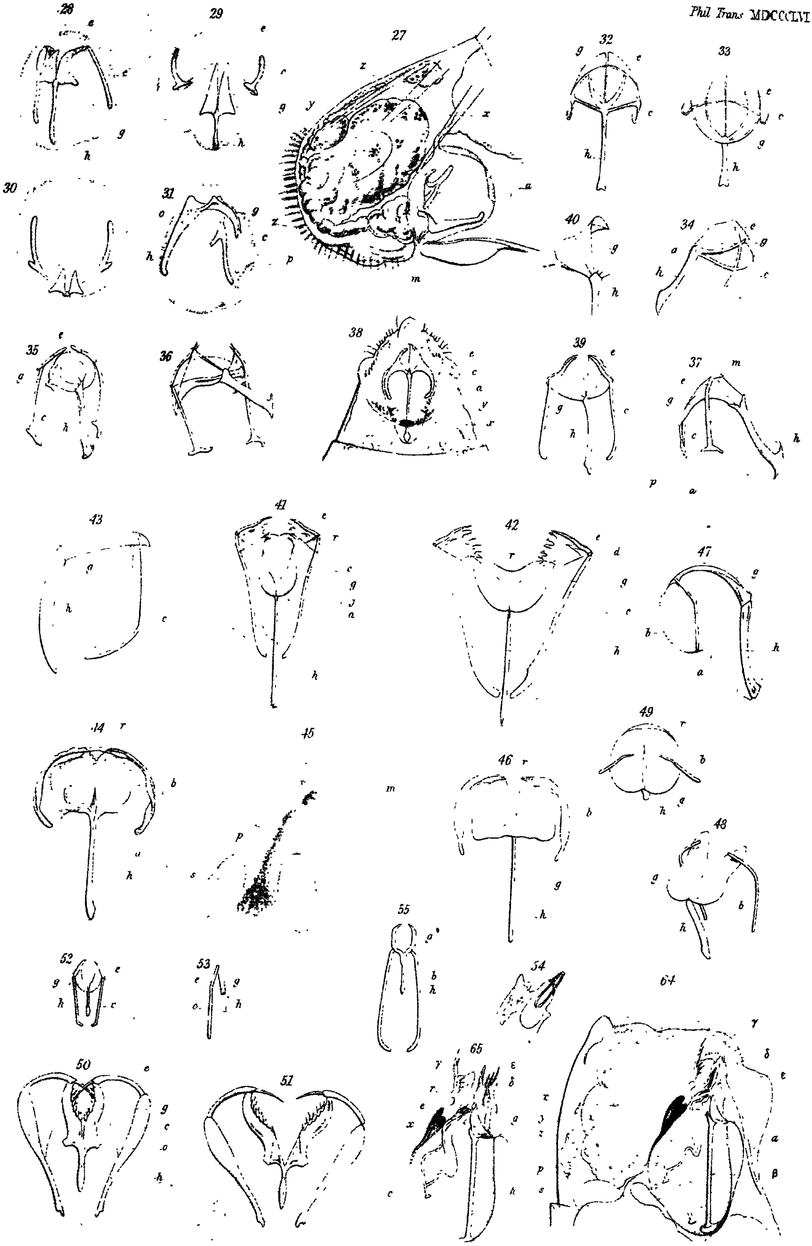
to which it is proper to join the following transformed expressions for $\Theta\mathrm{U}$, $\Theta_{\prime}\mathrm{U}$, $\Theta_{\prime\prime}\mathrm{U}$, viz.

$$\begin{aligned}
\Theta\mathrm{U}\ &=(1+8l^3)^2(y^3z^3+z^3x^3+x^3y^3)\\
&\quad+(-2l^3-l^6)\mathrm{U}^2\\
&\quad+(2l-5l^4)\mathrm{U.HU}\\
&\quad+(-3l^2\quad)(\mathrm{HU})^2.\\
\Theta_{\prime}\mathrm{U}\ &=4(1+8l^3)^2(y^3z^3+z^3x^3+x^3y^3)\\
&\quad+\ (-1+12l^3+4l^6)\mathrm{U}^2\\
&\quad+\ (-16l+4l^4\qquad)\mathrm{U.HU}\\
&\quad+\ (-12l^2\qquad\qquad)(\mathrm{HU})^2.\\
\Theta_{\prime\prime}\mathrm{U}&=-2(1+8l^3)^2(y^3z^3+z^3x^3+x^3y^3)\\
&\quad+\ (1-16l^3-6l^6)\mathrm{U}^2\\
&\quad+\ (6l\qquad\qquad\quad)\mathrm{U.HU}\\
&\quad+\ (6l^2\qquad\qquad\quad)(\mathrm{HU})^2.
\end{aligned}$$

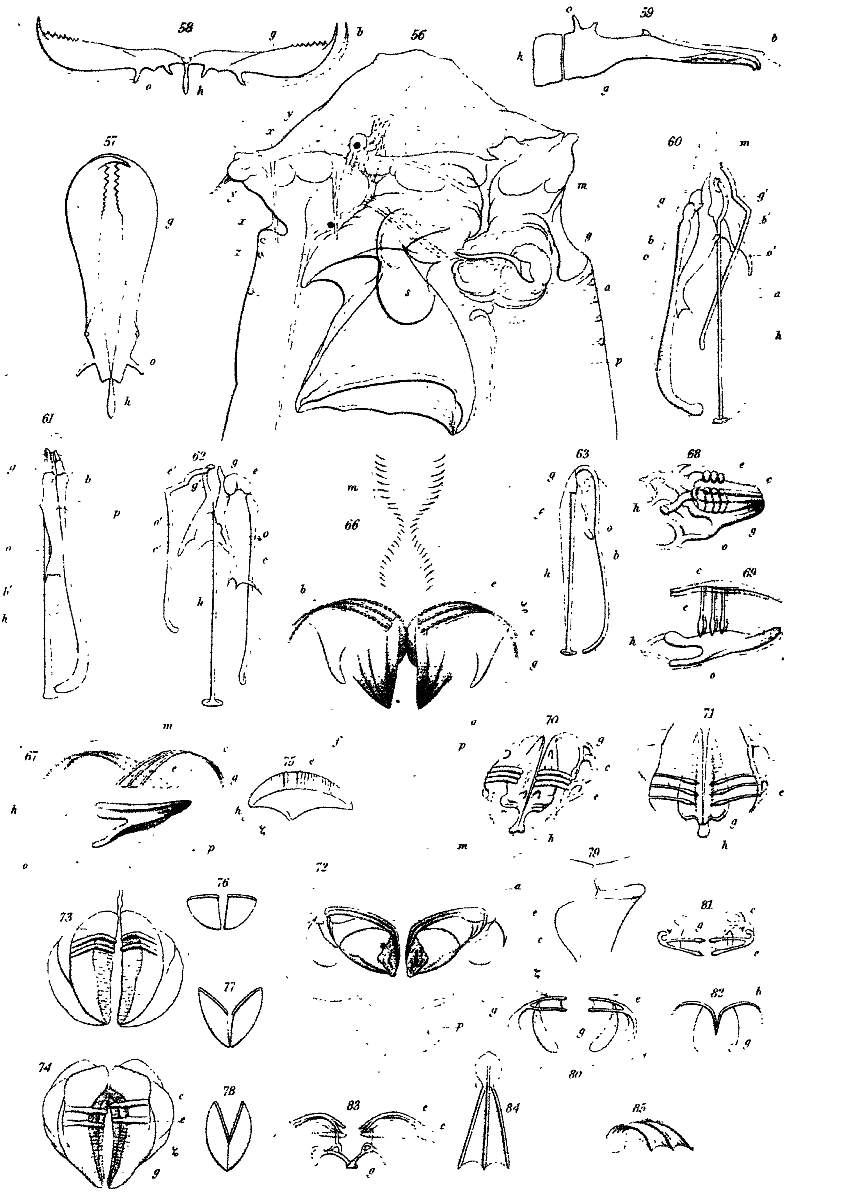
The last preceding table affords a complete solution of the problem to reduce a ternary cubic to its canonical form.

P.H. Gosse. del. ad. viv.

J. Basire.



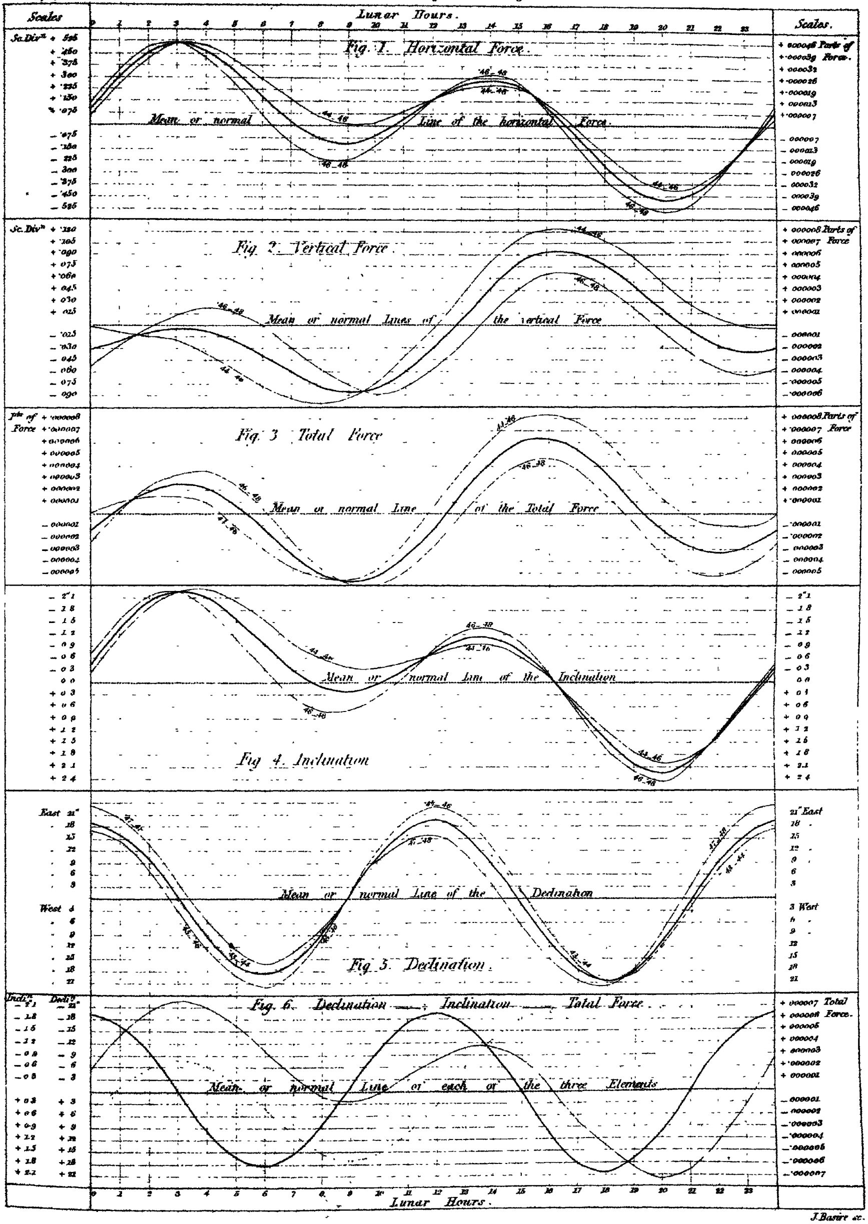
Gosse ad viv. del.



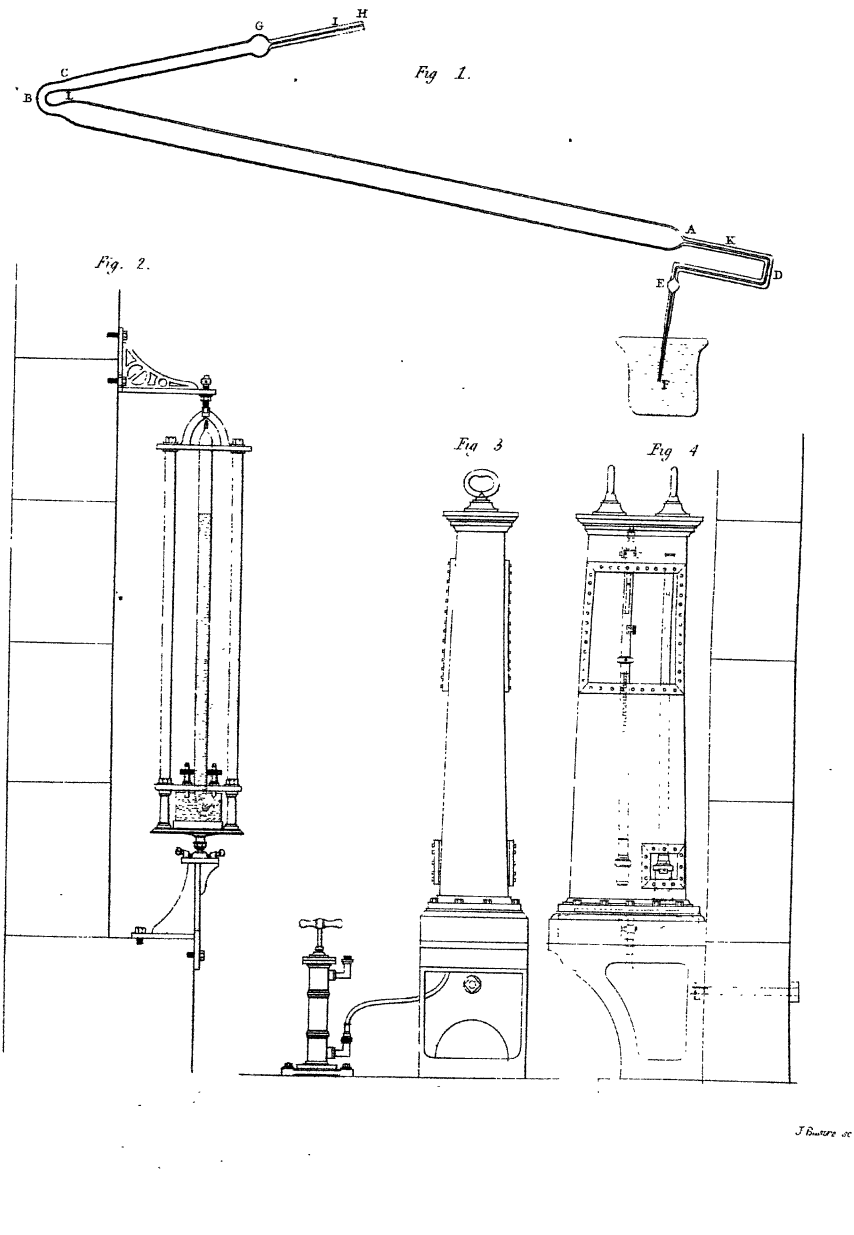
H. Gosse, del. ad viv.

J. Basire, sc.

Lunar-diurnal Variations of the Magnetic Elements.



J. Basire sc.



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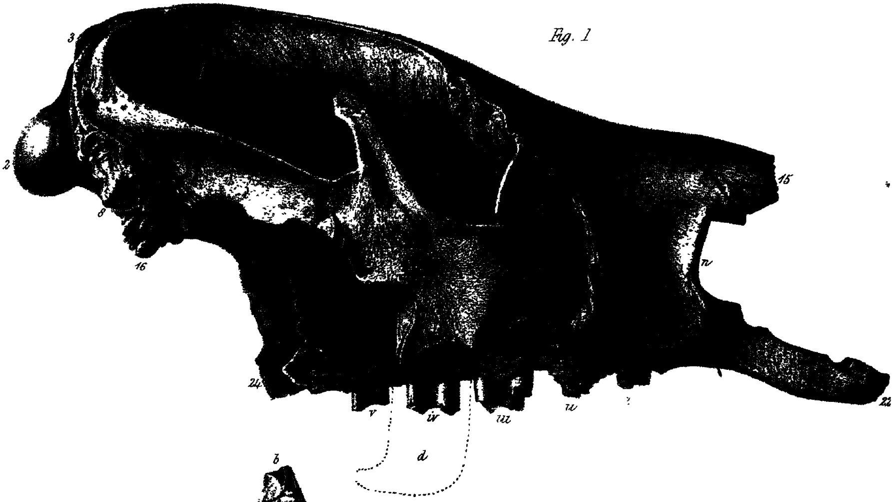


Fig. 1

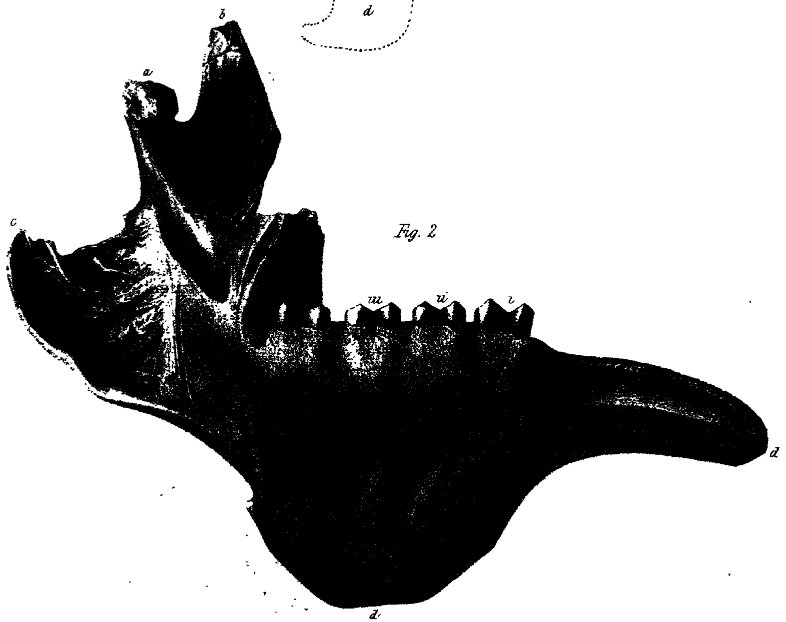


Fig. 2

Jos. Dinkel, del.

Printed by J. Basire

Fig 1

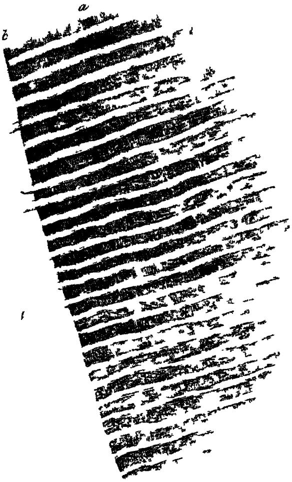
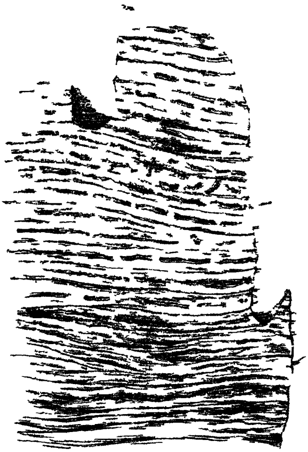
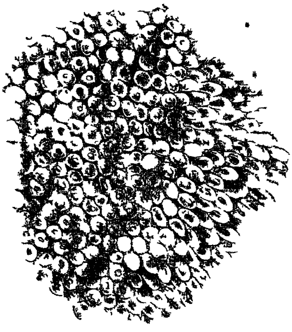


Fig 2





E Morrett Williams del et sc

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Fig 1.

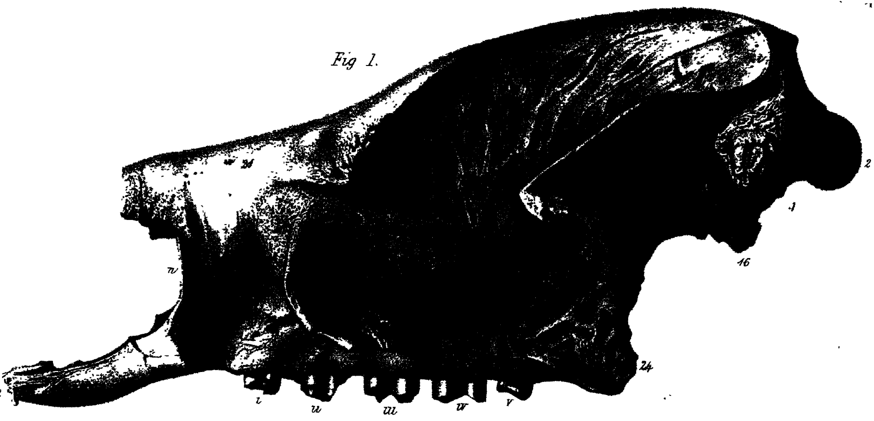


Fig 2.

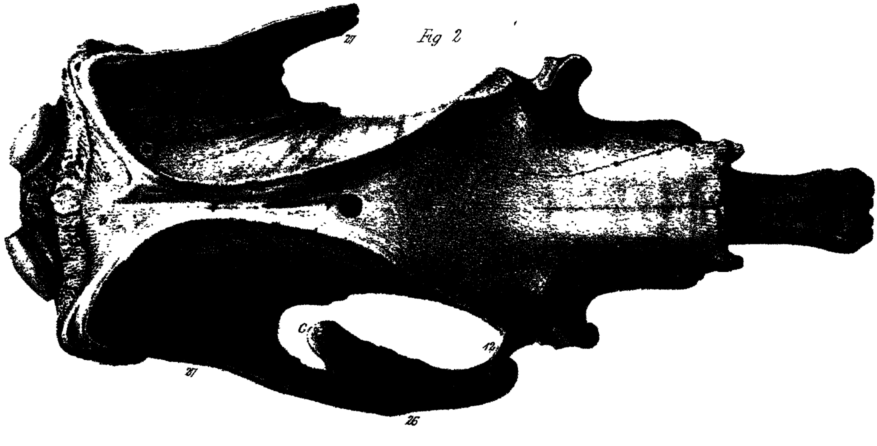
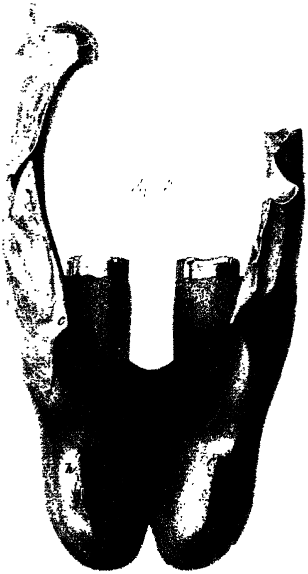
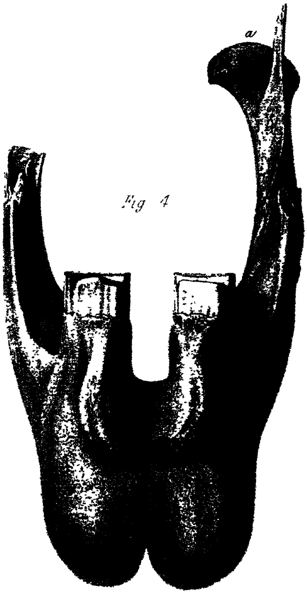


Fig 1.

Fig 2.





Jas.th Dinkel, del.
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Fig. 1

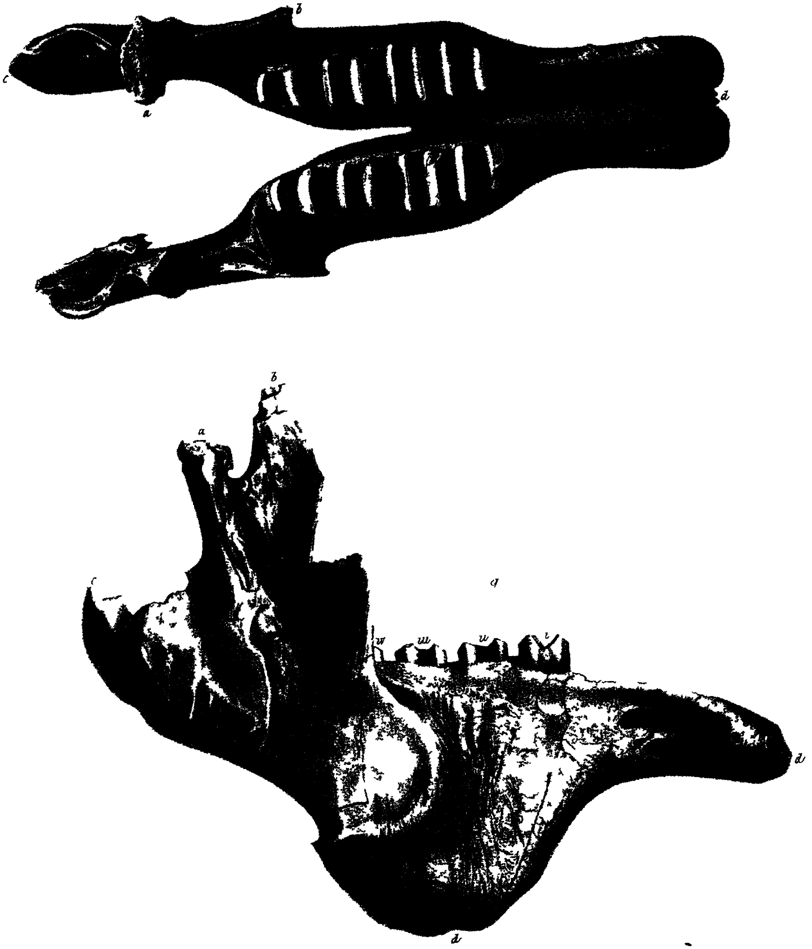


Fig. 1.

Fig. 2.

¾ Nat. size

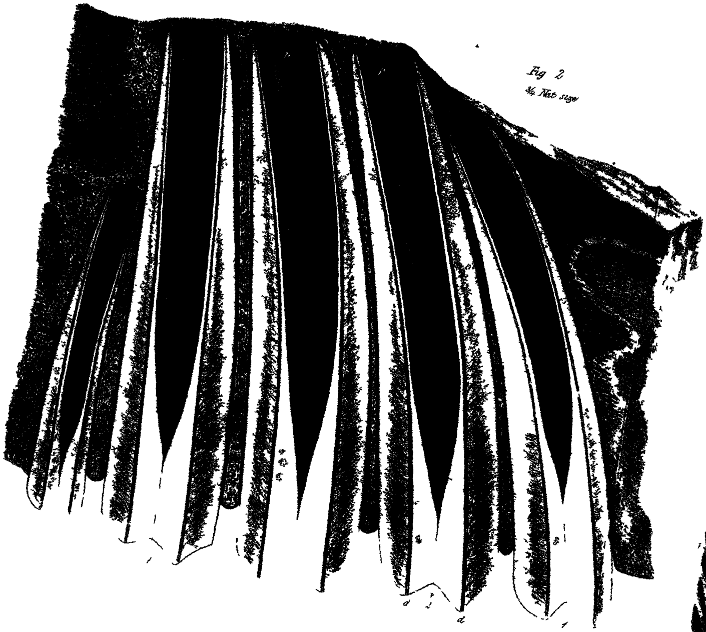
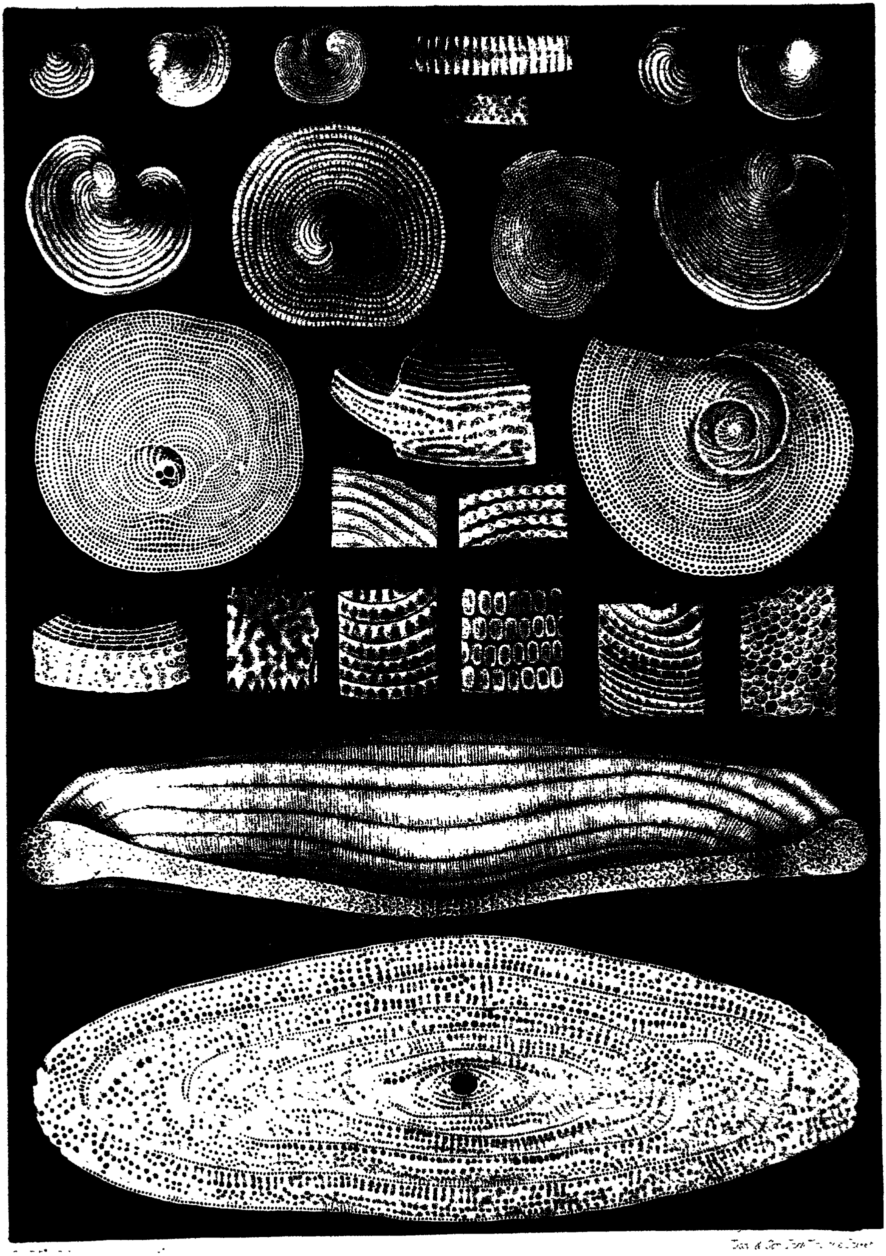
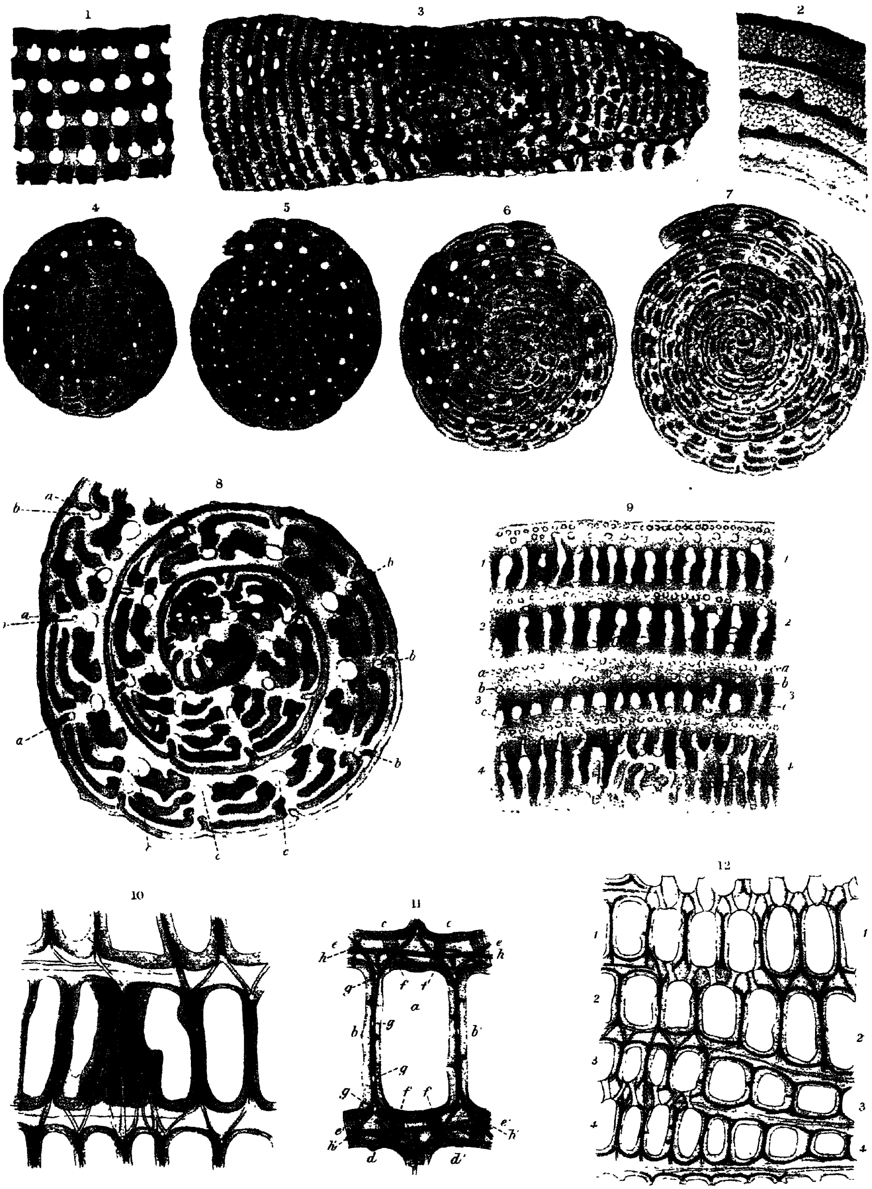


Fig. 3.

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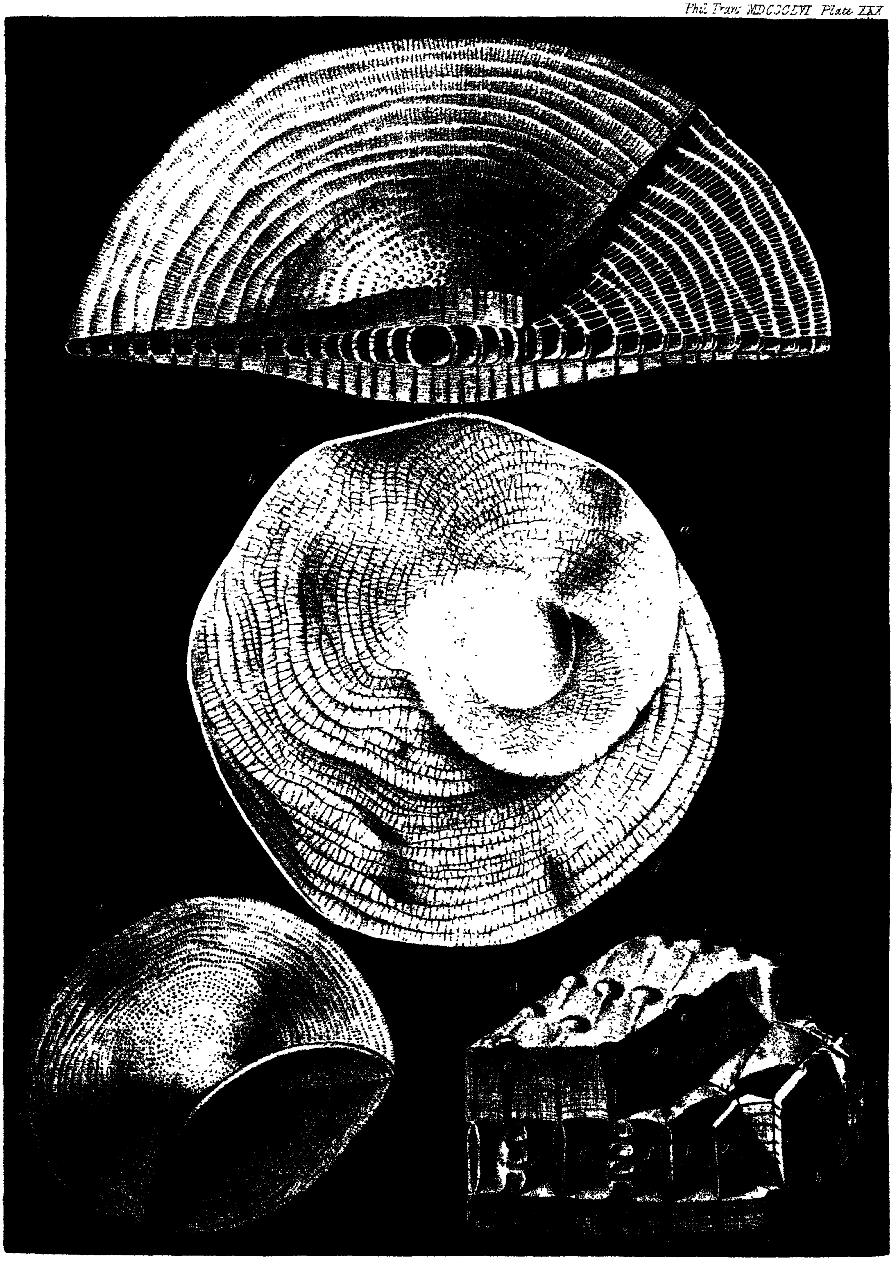
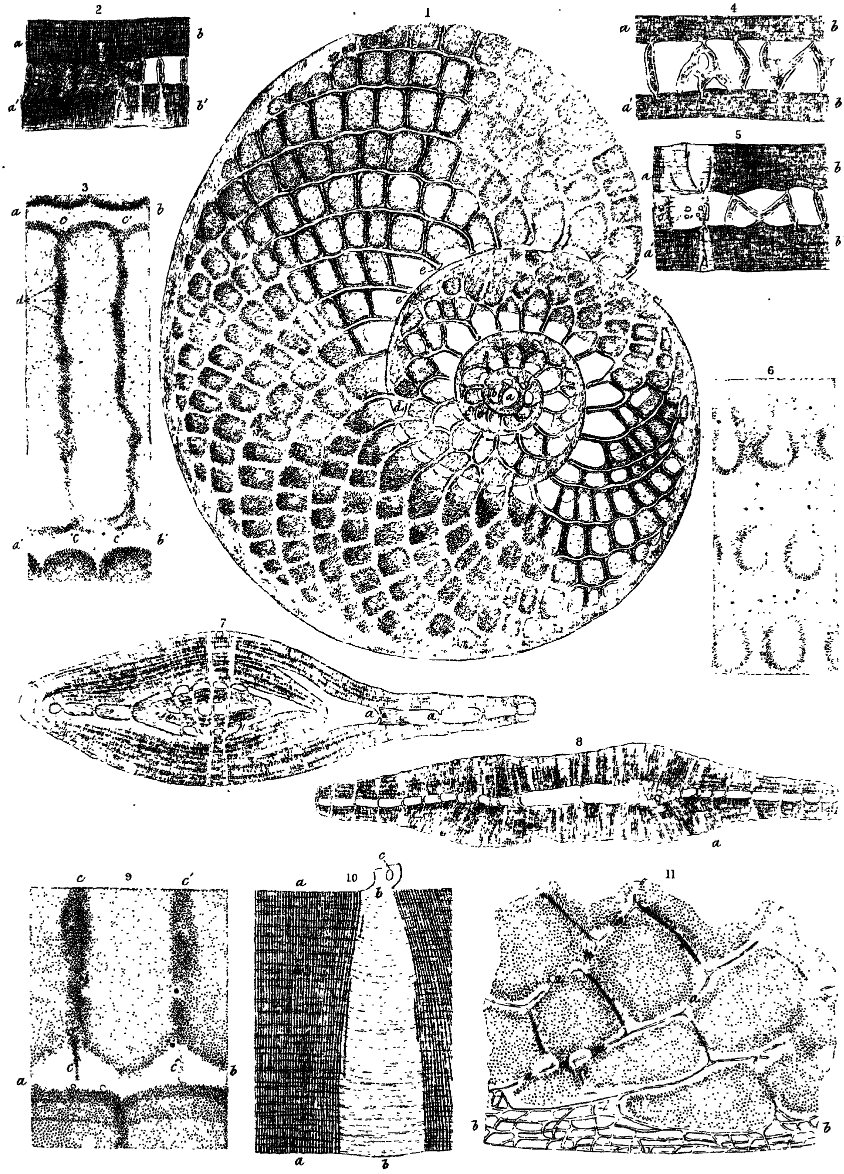


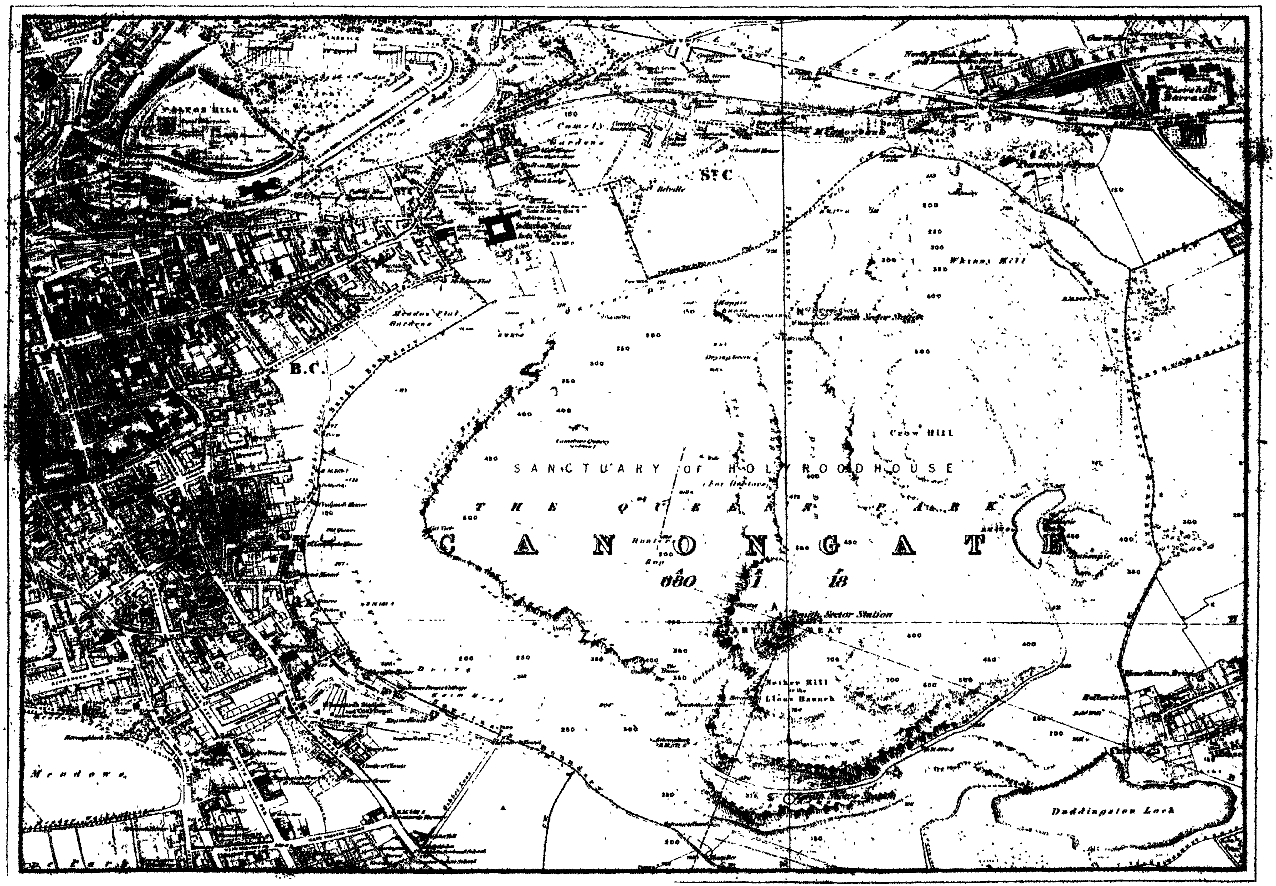
Joo West del. et lith.

Jas. & Son Lith. to The Queen



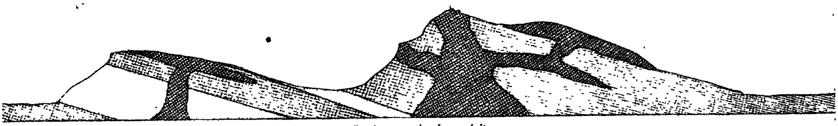
Geo West del et lith.

Day & Son Lithrs to the Queen.

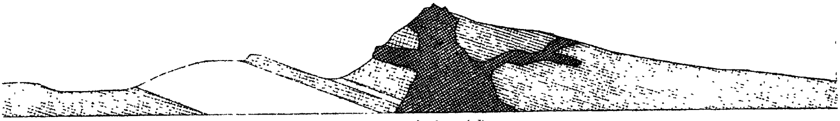

SANCTUARY OF HOLYROODHOUSE
THE QUEEN'S PARK
CANONGATE
Whinny Hill
Crow Hill
Duddingston Loch
Meadows
Nether Hill
Lions Haunch
Zenith Sector Station
St C
B.C.
Camely Gardens
Meadow Flat Gardens
Parliamentary Boundary

SECTIONS THROUGH ARTHUR'S SEAT

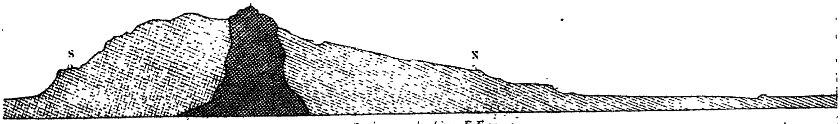
Trap Rock *Sandstone*



Section on the Line A B



Section on the Line C D



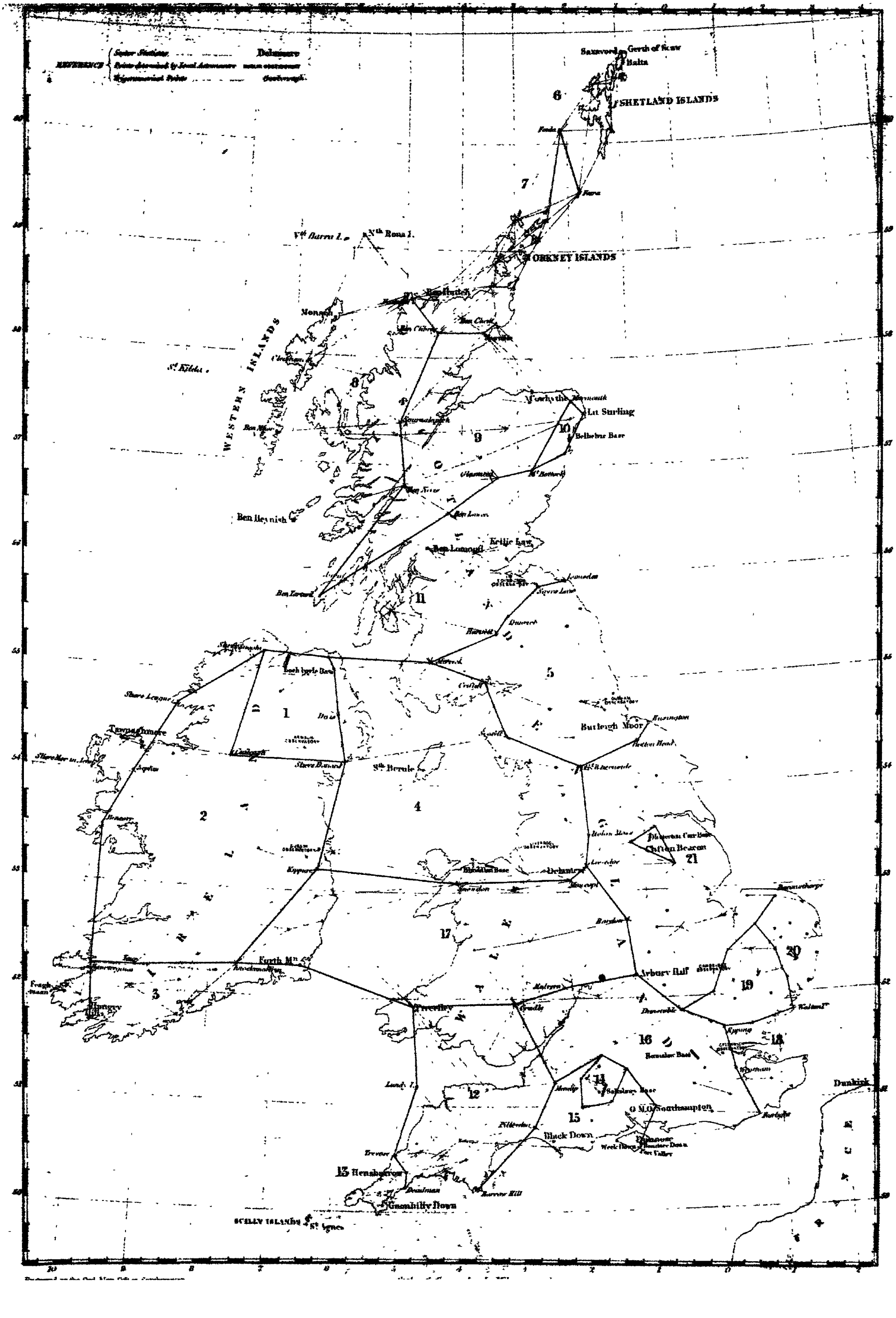
Section on the Line E F.

Table of the Specific Gravities of the Rocks in Arthur's Seat

Nº 1 Trap tuff	2.517	Nº 6 Porphyry	2.679
2 Trap conglomerate	3.000	7 Greenstone (Salisbury Crags)	2.689
3 Amygdaloidal trap	2.663	8 Dº very compact	2.856
4 Basalt St Anthony's	2.850	9 Sandstone	2.676
5 Basalt under apex	2.692	10 Dº	2.451
		Mean	2.710

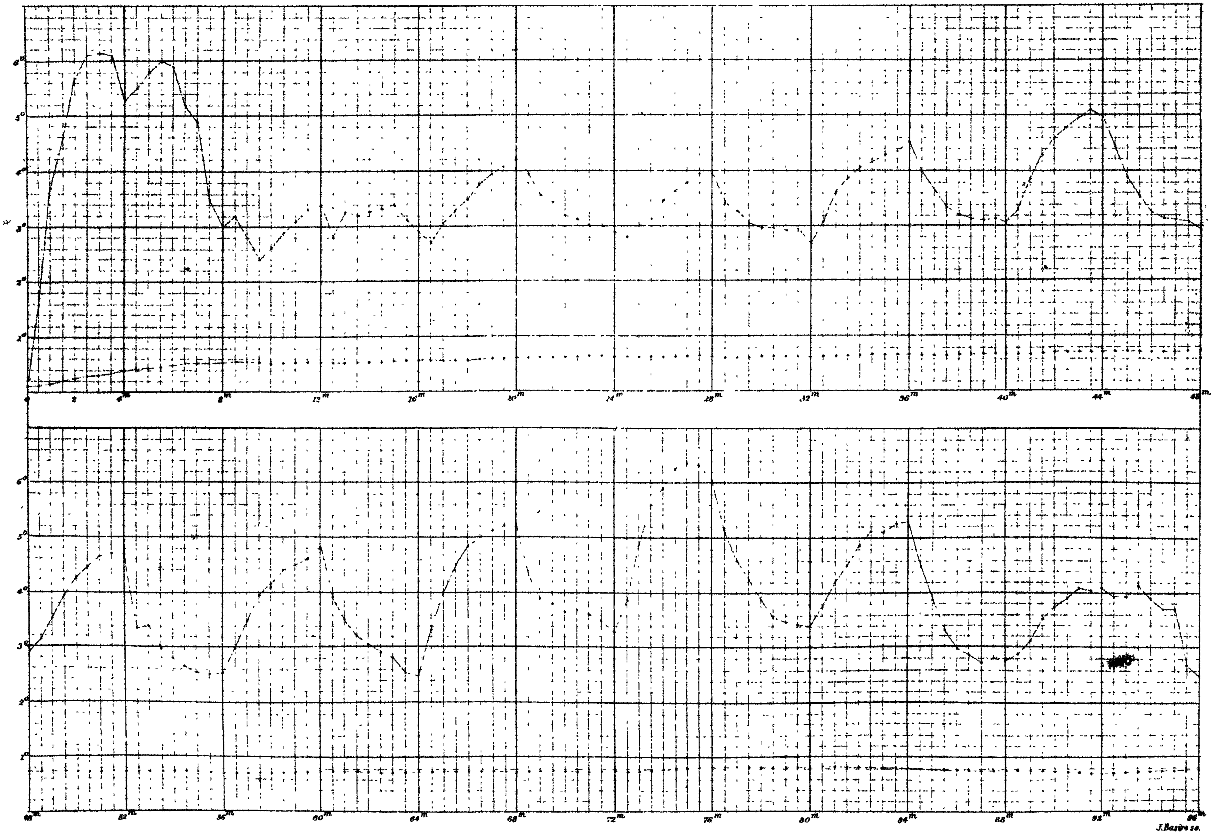
2.75 to be used as the Mean in the Computations

Engraved at the Ord. Map Office Southampton



Saxavord
Gerth of Scaw
Balta
SHETLAND ISLANDS
ORKNEY ISLANDS
WESTERN ISLANDS
St Kilda
Monach
Ben Heynish
Ben Lomond
Kellie Law
Belhelvie Base
Burleigh Moor
Clifton Beacon
Delamere
Forth Mn
Precelly
Black Down
Southampton
Dunkirk
SCILLY ISLANDS
St Agnes

The current flowing twelve times four minutes in each direction temperatures observed every half minute The ordinates of the upper curve show the differences of temperature at the times corresponding to the abscissas and those of the lower curve show one one-hundredth of half the sums of the same temperatures



J. Basire sc.

CONTENTS.

PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCLVI.

VOL. 146.

LONDON:

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MDCCCLVI.

CONTENTS

OF VOL. 146.

APPENDIX.

ERRATA.

Page 618, line 19, *for* Makerstown *read* Brisbane.

—— 644, Table 67. The bracket should have the arrowhead (↖).

—— 775, line 18, *for* inch *read* inches.

—— 805, after line 15, *insert* G lb. No. 9. (M) 7·376.

—— 805, line 4 from below, *for* 8·512 *read* 8·515.

—— 861, line 3 from below, *for* 0·38035 *read* 0·39644.

—— 861, line 3 from below, *for* 0·38019 *read* 0·39628.

—— 862, line 5, *for* 0·01303 *read* 0·01304.

—— 886, line 17 from below, *for* 30 *read* 34.

—— 888, line 16 from below, *for* 28 *read* 22.

—— 903, line 7 from below, *for* comparisons *read* weighings.

XXXI. The Bakerian Lecture.—*On the Electro-dynamic Qualities of Metals**.

By Professor William Thomson, *M.A.*, *F.R.S.*

Received February 28,—Read February 28, 1856.

1. An electrified body may be regarded as a reservoir of potential energy, and any material combination in virtue of which bodies can receive charges of electricity is a ource of motive power. The development of mechanical effect from the potential energy of electricity, or through electric means from any source of motive power, may take place in a great variety of ways. For instance, electro-statical attractions and repulsions may become direct moving forces (as in "Franklin's Spider"), to do work in the discharge of an electrified conductor or in the continued use of a continuous supply of electricity; or the forces of current electricity may, as in any kind of electro-magnetic engine, become working forces on bodies in motion; or the whole energy of the discharge may, as discovered by Joule, be converted into heat, which again may be transformed into other kinds of energy; or the heat evolved and absorbed by electricity, in a circuit of two different metals, at the places where it crosses the junctions from one metal to the other, being a thermal result of dynamic moment† when the junction at which heat is evolved is at a higher temperature than the junction at which heat is absorbed, may be used in a thermo-dynamic engine. Again, a thermo-electric current is a dynamic result derived from a definite absorption of heat in one locality and a definite evolution of heat in a locality of lower temperature.

2. Of these various kinds of action, all except the first mentioned, depend essen-

* The author has to acknowledge much valuable assistance in the various experimental investigations described in this paper, from his assistant Mr. McFarlane, and from M. C. A. Smith, Mr. R. Davidson, Mr. F. Maclean, Mr. John Murray, and other pupils in his laboratory.

† Either an evolution of heat at a temperature higher, or an absorption of heat at a temperature lower, than that of the atmosphere, may be taken advantage of to work an engine giving mechanical effect from heat; by using the atmosphere in one case as a recipient for discharged heat, in the other as the source of the heat taken in. Or an evolution of heat at any temperature and an absorption of heat at any lower temperature, may be taken advantage of for the same purpose, in a limited material system, neither taking heat from nor parting with heat to any external matter. Hence such a double thermal effect may be said to possess "dynamic moment." See the author's "Account of Carnot's Theory of the Motive Power of Heat," §§ 4 to 11, Trans. Roy. Soc. Edinb. Jan. 2, 1849; also his "Dynamical Theory of Heat," §§ 8, 13, 23 to 30, Trans. Roy. Soc. Edinb., March 17, 1851, and "Dynamical Theory of Heat, Part VI. Thermo-electric Currents," § 102, Trans. Roy. Soc. Edinb., May 1, 1854. The series of articles under the general title "Dynamical Theory of Heat," have been republished in a succession of Numbers of the Philosophical Magazine, viz. §§ 1 to 80, Vol. July to Dec. 1852: §§ 81 to 96, Vol. Jan. to June, 1855; §§ 97 to 181, Vol. Jan. to June, 1856.

tially on certain definite properties of matter in regard to which different metals have remarkably different qualities. Thus in electro-magnetic engines the electric conductivity of the coils through which the current passes, and the magnetic inductive capacity and retentiveness of the iron cores of the electro-magnets, are essentially involved; and as essentially, when permanent magnets are used, the magnetic properties of steel, loadstone, or other bodies possessing strong retentiveness for magnetism. In the simple conversion of any kind of energy into heat by means of electric currents in metals, their electric conductivities are essentially and solely concerned.

The inverse thermo-electric transformation of energy into an evolution and absorption of heat, at localities of different temperature, in quantities differing from one another *by the thermal equivalent* of the work spent in maintaining the current*, depends essentially on certain distinct properties of metals in regard to which their various qualities are shown by the differences of their positions in the thermo-electric series at different temperatures; and the accessory circumstances of such operations are influenced by the electric and thermal conductivities of the metals used. The same properties are involved in the direct thermo-electric transformation of energy in which electric currents, sustained by the communication of heat in a hot locality and the abstraction of a less quantity of heat in a locality lower in temperature, either produce any mechanical action, or are allowed to waste all their motive power in the frictional generation of heat†.

* See "Dynamical Theory of Heat, Part VI. Thermo-electric Currents," §§ 105, 110.

† ". . . . a current cannot pass through a homogeneous conductor without generating heat in overcoming resistance. This effect, which we shall call the *frictional generation of heat*, has been discovered by Joule to be produced at a rate proportional to the square of the strength of the current; and, taking place equally with the current in one direction or the contrary, is obviously of an irreversible kind" (Dyn. Th. Heat, § 104). This definition was given merely to render circumlocution unnecessary in frequently referring to a mode of electric action which bore an obvious analogy to the action of a common fluid generating heat by friction among its particles as a dynamical equivalent to work spent upon it from without in forcing it to circulate in a tube, or otherwise keeping it in motion. It appears to me highly probable, however, that what I have, with reference only to recognized electric currents, defined as the frictional generation of heat, is precisely the mode of action by which all the heat is generated in every case when two solids are rubbed together. Certainly when two bad conductors of electricity are rubbed together, a portion of the heat of friction is generated in visible electric flashes; and a charged Leyden battery contains, in potential energy, a dynamic equivalent for a portion of the heat of friction between rubber and glass never made till the battery is discharged. As certainly a portion of the heat of friction between a metal and a bad conducter of electricity is *invisibly* generated by electric currents through a very minute depth of the metallic substance beside its rubbed surface. The first effect of chemical forces of affinity, as Joule has so powerfully demonstrated in a variety of cases, is to press electricity into motion; which motion may either subside into heat close to the locality of the combination (as when rough zinc is dropped into dilute sulphuric acid), or, reactively resisting the chemical combination, may transmit the work to a locality distant from the source, and may there either generate heat in a permanent metallic or other undecomposable conductor, or may, without any generation of heat at all, be wholly spent in effecting decompositions against chemical affinities infinitely little less powerful than those from which it proceeds, or in raising weights. So it appears highly probable that the first effect of the force by which one solid is made to slide upon

3. All properties, then, of electric and thermal conductivity, of magnetic inductive capacity and retentiveness, and of thermo-electric rank and its variations from one temperature to another, may be characterized as electro-dynamic; and the degrees to which these properties are possessed by different substances may be called their electro-dynamic qualities. Again, the variation which absolute magnetic inductive capacity, and magnecrystallic axial differences, experience with change of temperature may obviously be made the means of a transformation of heat into common mechanical energy, and we have thus a set of magneto-dynamic properties of matter which may almost in the present state of science be regarded as intrinsically electric, but which at all events (when we consider that the motions contemplated, taking place as they do under magnetic force, cannot but be accompanied by electric currents) may be fairly classed under the general designation of electro-dynamic. The variations of intrinsic magnetism, of magnetic inductive capacity, and of magnecrystallic properties, produced by variations of temperature, are therefore included among the electro-dynamic qualities of metals which I propose to investigate, although I have as yet made no progress in this branch of the subject.

PART I. ON THE ELECTRIC CONVECTION OF HEAT, §§ 4 to 77.

§§ 4 to 18. THEORETICAL INDICATIONS.

§§ 4 to 9. *Origin of the Investigation.*

4. In first attempting an application of the principles of the Dynamical Theory of Heat to show the mechanical relation between cause and effect in thermo-electric currents, I supposed the effects thermal and mechanical that can be produced by a thermo-electric current in any part of its circuit to be, as first suggested by JOULE, due to the heat absorbed, according to PELTIER's discovery, at the hot junction in virtue of the current crossing it, and I pointed out that the current crossing the cold junction must evolve a quantity of heat which, were this supposition true, would be less than that absorbed at the hot junction, by an amount precisely equivalent to all the effects, produced by the current in the rest of the circuit *.

5. Introducing CARNOT's principle, as modified in the Dynamical Theory of

another is electricity set into a state of motion; that this electric motion subsides wholly into heat in most cases, either close to its origin and instantaneously, as when the solids are both of metal, or at sensible distances from the actual locality of friction and during appreciable intervals of time, as when the substance of one or both the bodies is of low conducting power for electricity; and that it only fails to produce the full equivalent in heat for the work spent in overcoming the friction, when the electric currents are partially diverted from closed circuits in the two bodies and in the space between them, and are conducted away to produce other effects in other localities. Still, no hypothesis need be implied by using the expression "the frictional generation of heat by an electric current," as defined in the passage quoted, and it is introduced into the present paper with no other justification than its convenience.

* Dynamical Theory of Heat, March 17, 1851, § 17.

Heat*, I found a relation between the quantities of heat absorbed or evolved by currents crossing metallic junctions at different temperatures; which led immediately to a general expression for the electrical condition of a circuit of two metals with their junctions kept at any stated temperatures.

6. From this it appeared that the electro-motive force should follow the same law of variation in every case, being expressed by a constant, (representing the thermo-electric difference between the two metals,) multiplied into an absolute function of the temperatures of their junctions, namely, the difference of their temperature on the absolute thermometric scale since proposed by Mr. JOULE and myself, and demonstrated by our experiments† to agree very approximately with their difference of temperature as indicated by an air-thermometer. Finding this conclusion contradictory to the statements made by experimenters, that the electro-motive force does not vary with the temperature of the junctions according to the same law in circuits composed of different metals, I perceived that PELTIER's discovery did not afford a sufficient explanation of the source whence a thermo-electric current derives its energy, but that electric currents must possess the previously undiscovered property of producing different thermal effects in passing from cold to hot and from hot to cold in the same metal, and must possess this property to different amounts in different metals.

7. Taking this new property of electric currents into account along with that discovered by PELTIER, and introducing an application of CARNOT's principle, I arrived at expressions for the relations between the heat absorbed and evolved in various parts of a circuit of any different metals, and between the electro-motive force and the temperatures of the junctions, which appear to be in complete accordance with the facts. These investigations were communicated in December 1851, to the Royal Society of Edinburgh‡.

8. Still simpler theoretical considerations (§§ 10 to 18 below) regarding the source of energy drawn upon in a thermo-electric current, make it certain that the phenomena of inversion discovered by CUMMING could not exist, unless the metals presenting them had the property of experiencing, when unequally heated, unequal thermal effects from electric currents passing through them from hot to cold, and from cold to hot. Having satisfied myself, both by an examination of the evidence afforded by BECQUEREL's experiments (the original investigation on the subject by CUMMING being at that time unknown to me), and by actual observation, in experiments of my own §,

* This, the true form of CARNOT's principle, was first published by CLAUSIUS in May 1850 (POGGENDORFF's 'Annalen'). It had occurred to myself, and I had used it in discovering the true expression for the duty of a perfect thermo-dynamic engine shortly before that time. It was not, however, until the beginning of the year 1851 that I thought on a demonstration which would probably be admitted as conclusive in establishing the principle, and my investigation on the subject was only communicated in March 1851 to the Royal Society of Edinburgh. See Trans. Roy. Soc. Ed. of that date, "Dynamical Theory of Heat," § 14.

† "On the Thermal Effects of Fluids in Motion," Transactions, June 1854.

‡ See Proceedings of that date, and Philosophical Magazine, June 1852.

§ See below, Part II. §§ 79, 80, 81, 83, 84, &c.

that the doubts which various writers had thrown on the existence of thermo-electric inversions were groundless, I concluded with certainty that the newly conceived thermal effect of electricity in unequally heated metals really exists. But the theory left it undecided what the absolute nature and amount of this effect may be, and only showed how, by observations on thermo-electric currents, its difference in different metals may be determined.

9. I therefore had recourse to direct experiment on the thermal effects of electric currents in unequally heated conductors, not to demonstrate the existence of the peculiar effect anticipated, but to ascertain its nature, with moreover a view of ultimately determining its absolute amount, in some particular metal or metals. Before proceeding to describe experiments, by which I have now discovered the quality of the new effect in several cases, I shall, without entering on the mathematical details of the theory, or the full application of CARNOT's principle, repeat in a few words so much of my first communication on the subject to the Royal Society of Edinburgh, as to show the reasoning, founded on incontrovertible mechanical principles, which made me commence the experimental research with the certainty that the property looked for existed, whether I could find it or not.

§§ 10 to 15. *General inferences regarding the Electric Convection of Heat from Dynamical Principles.*

10. CUMMING has discovered that in many cases when one of the junctions of a thermo-electric circuit of two metals is kept at a fixed temperature, if that of the other be elevated gradually from equality, an electro-motive force is produced, which first increases to a maximum, then diminishes, vanishes for a certain temperature of the junction, and acts in the contrary direction with gradually increasing strength as the temperature is further raised. It is clear that, at exactly that temperature of the hot junction for which in any such case the electro-motive force is a maximum, the two metals must be thermo-electrically neutral to one another, and must present reverse thermo-electric relations for temperatures below and above this point. Hence the *thermal effect depending on the direction of a current crossing the junction of two such metals* must be for temperatures above, the reverse of what it is for temperatures below, the neutral point, and must vanish when the metals are exactly at this temperature.

11. For although PELTIER himself supposed the effect he had discovered to depend on the conducting powers of the two metals for heat, and remarked as an anomaly the case of bismuth and copper, for which his supposition was violated, his own experiments show the truth to be, that in a circuit of two metals an absorption of heat at the junction where the temperature is higher, and an evolution of heat at the other, must be produced by the thermo-electric current which is caused by the maintaining of the difference of temperature between the junctions. That this is universally true when the temperatures of the two junctions are on the same side of

the neutral point, cannot, in the present state of science as regards the theory of heat, be reasonably doubted.

12. If, therefore, a circuit of two metals have one junction kept at the neutral point, and the other at some lower temperature, the current excited will cause the evolution of heat at the cold junction, but neither absorption nor evolution of heat at the hot junction; and in the rest of the circuit there will be effects either purely thermal, or thermal and mechanical or chemical, according to the nature of the resistance against which the electro-motive force is allowed to work. The source from which the electro-motive force derives its energy to produce these effects cannot be at the hot junction (§ 10), where heat is neither absorbed nor evolved, nor at the cold junction (§ 11), where heat is evolved, nor of course in any uniformly heated part in either metal, through all of which, provided the metal has no thermo-electric crystalline characteristic, there can be nothing but a frictional evolution of heat; that is, it is nowhere but in those portions of the circuit where the temperature varies between that of the cold and that of the hot junction. In those portions, therefore, there must be as much heat absorbed, in virtue of the current, as is equivalent to the aggregate mechanical value of the heat evolved at the cold junction, and all the effects, thermal, mechanical, and chemical, produced in the rest of the circuit.

13. If, for example, an electro-magnetic engine be introduced into the circuit, and be allowed to work at such a rate as to reduce, by its inductive reaction, the strength of the thermo-electric current to an infinitely small fraction of what it is when the engine is at rest, the heat absorbed in virtue of the current in the unequally heated parts of the two metals will be equal to the heat evolved at the cold junction, together with the thermal equivalent of the work done by the engine, and will be simply proportional to the strength of the current. On the other hand, if the engine be forced to work a little faster, so as to overbalance by an infinitely small amount the thermal electro-motive force, and cause a reverse current in the circuit, there must be heat evolved in virtue of this current in the unequally heated parts of the two metals to an amount equal to the heat absorbed at the cold junction, together with the thermal equivalent to the work done against electro-magnetic forces in the engine. It follows that in the unequally heated portions of the two metals, the current passing from cold to hot in one, and from hot to cold in the other, must produce a thermal effect, in simple proportion to its own strength, constituting on the whole an absorption of heat when the thermal electro-motive force is allowed to produce a current, and an evolution of heat when a current is forced by other means in the contrary direction.

14. Hence, for any two metals which are thermo-electrically neutral to one another at a certain temperature, and which possess reverse thermo-electric properties for temperatures above and below the neutral point, we conclude the following propositions:—

(1) In one or other of the metals (and most probably in both) there must be a thermal effect due to the passage of electricity through a non-uniformly heated portion of it, which must be an absorption of heat or an evolution of heat, according to the direction of the current between the hot and cold parts, and proportional in amount to the whole quantity of electricity that passes in a stated time.

(2) The amount of this effect, with the same strength of current and the same difference of temperatures, must differ in the two metals to such an extent, that the effect of a current in passing from cold to hot in one metal, together with the effect of an equal current passing from a place equally hot to a place equally cold in the other, may amount to the absorption or evolution, the existence of which has been demonstrated.

15. The *reversible thermal effect* * of electric currents in single metals of non-uniform temperature, which has been thus established, may obviously be called a Convection of Heat by electricity in motion. To avoid circumlocution, I shall express it that *the Vitreous Electricity carries heat with it*, when this convection is in the "nominal direction of the current." On the other hand, when the convection is against the "nominal direction of the current," it will be said that *the Resinous Electricity carries heat with it.*

§§ 16 to 18. *Dynamical Theory applied to draw, from thermo-electric data, inferences regarding the Electric Convection of Heat in Copper and in Iron.*

16. The application of the preceding theorem to the particular case of copper and iron is a consequence of CUMMING's discovery, that, if one junction in a circuit of two arcs of those two metals be kept cold, and the other be heated gradually, a current at first sets from copper to iron through the hot junction with increasing strength; but begins to diminish after a certain temperature, which BECQUEREL found to be about 300° Cent., is exceeded; falls away to nothing when a red heat is attained; and sets in the reverse † direction when the elevation of temperature is pushed higher.

* See an article by the author, entitled "On a Universal Tendency in Nature to the Dissipation of Mechanical Energy" (Proceedings Roy. Soc. Edinb., Feb. 16, 1852, and Phil. Mag., Oct. 1852), where all natural operations are divided into two great classes, "reversible" and "irreversible." See foot-note on § 2 above, for an example of the second class.

† Having myself experienced some difficulty in obtaining the reverse current in the manner described by M. BECQUEREL, in which one junction was heated in the flame of a spirit-lamp, while the other was kept at the atmospheric temperature, I found that it could be obtained so as to be observed with the greatest ease by means of a very ordinary galvanometer and an iron wire with copper wires twisted round its ends, by keeping the lower junction at a temperature considerably above that of the atmosphere, at 100° Cent. for instance; and I ascertained that when both junctions were kept at a very high temperature, in the flame of a spirit-lamp for instance, and one of them cooled a little below the temperature of the other, the current produced was the reverse of that which the same difference occasioned when both junctions were at ordinary temperatures. See Part II. below for further developments on this subject.

Some observations of REGNAULT's having appeared to indicate 240° Cent. as, more nearly than 300°, the temperature of the hot junction which gives the current its maximum strength, I concluded the following proposition:—

17. "When a thermo-electric current passes through a piece of iron from one end kept at about 240° Cent.*, to the other end kept cold, in a circuit of which the remainder is copper, including a long resistance wire of uniform temperature throughout, or an electro-magnetic engine raising weights, there is heat evolved at the cold junction of the copper and iron, and (no heat being either absorbed or evolved at the hot junction) there must be a quantity of heat absorbed on the whole in the rest of the circuit. When there is no engine raising weights, in the circuit, the sum of the quantities evolved, at the cold junction, and generated in the 'resistance wire,' is equal to the quantity absorbed on the whole in the other parts of the circuit. When there is an engine in the circuit, the sum of the heat evolved at the cold junction and the thermal equivalent of the weights raised, is equal to the quantity of heat absorbed on the whole in all the circuit, except the cold junction†."

18. Hence, if the reversible part of the effect of a current from hot to cold in iron is an evolution of heat, the corresponding effect in copper must be a greater evolution of heat. But if, on the other hand, a cooling effect be produced by a current from hot to cold in iron, there must be either a less effect of the same kind, or a reverse effect, in copper. It is left to experiment to determine which of the two hypotheses is true regarding iron; and should it turn out to be the latter, to ascertain which of the two remaining alternatives regarding copper must be concluded. With this object I commenced the experimental researches which I now proceed to describe.

§§ 19 to 77. *Experimental Investigation of the Electric Convection of Heat in Copper, in Iron, and in some other Metals.*

§§ 19, 20. *Unsuccessful attempts, and first result.*

19. I began, more than four years ago, by observing carefully the ignition produced in short wires of copper, iron, and platinum by electric currents alternately in the two directions, thinking that some of the effects described by various experimenters, as showing a superior heating power in the positive electrode, might possibly be dependent on the convective agency which I was endeavouring to discover. But I never observed the slightest variation in the position of the incandescent part of the

* I have since ascertained (see Part II. below), by keeping the ends of an iron wire, with copper wires from the galvanometer soldered to them, in separate vessels of hot oil, and determining different temperatures of the two which give no current, that the neutral point for the particular specimens of iron and copper which I used must be about 284° Cent. I should therefore, at present, substitute 284° for 240° in the proposition quoted in the text; without further research, however, it is impossible to pronounce upon the limits between which the neutral point of various specimens of copper and iron wires may be found to lie.

† Proceedings of the Royal Society of Edinburgh, Dec. 15, 1851, republished in Phil. Mag., June 1852.

wire, with a sudden reversal of the current. Sometimes the incandescence was assisted by a spirit-lamp flame applied to the middle part of the wire, and the ends were kept cool by wet threads. Sometimes in a long wire with a current through it not quite strong enough to keep it at a red heat, a small part was made incandescent by a slight application of heat as nearly as possible at one point, by a spirit-lamp flame. Still there was never observed the slightest motion of the incandescent part, when the current was suddenly reversed, and I concluded that whatever had been observed in the way of different heating effects of the positive and negative electrodes, must have been owing to peculiar agencies of the current in passing between metal and rarefied air, or to some other cause than thermal convection in metals; and I saw that more powerful tests would be required to bring out the result I looked for.

20. I next made experiments on a conductor of bar iron bent into two equal upward vertical branches on each side of the horizontal part, which was kept immersed in a vessel of hot oil, while the upper ends of the vertical branches were kept cool by streams of cold water. Vessels of water were applied round the two vertical branches, as calorimetric arrangements to test heat evolved or absorbed in them by the agency of a current sent down one and up the other from a nitric acid battery of sixteen small iron cells, arranged as a single element.

The current was sent first for half an hour in one direction, then half an hour in the contrary direction; and so on, with a reversal every half-hour. The water round the two vertical branches was kept constantly stirred, and thermometers in fixed positions in them were observed at frequent intervals during the experiments, which were each continued for about two hours. A comparison of all the readings taken showed a rather higher mean temperature in the branch down which the current was passing than in the other; indicating, differentially, a *cooling* effect in the branch through which the current passes from the hot middle, and a heating effect in the other. This experiment appeared to show that "the resinous electricity" carries heat with it in an iron conductor; but the irregular variations of temperature in each thermometer were so much greater than the differential effect deduced, that I could not consider the conclusion satisfactorily established.

§§ 21 to 29. *Unsuccessful attempts with large bar conductors.*

21. There were difficulties connected with the arrangements of the calorimetric vessel, which made me judge that it would be better, instead of testing the average temperature of two portions of the conductor, each extending the whole way from the hot middle to the cold ends, to simply test the temperature of as nearly as possible one point midway between the hot and cold on each side; and it appeared that the heating could be more easily applied and better regulated by a source of heat at the middle of a straight horizontal conductor, than by the plan I had followed in the arrangement just described. I therefore got bars of copper and iron, with holes to admit the bulbs of sensitive thermometers, made to the following dimensions:—

	Copper conductor. inches.	Iron conductor. inches.
Whole length . . .	16	24
Breadth	1	2
Depth	$2\frac{1}{2}$	3
Depth of hollows . .	$2\frac{1}{10}$	$2\frac{1}{2}$
Diameter of hollows .	$\frac{1}{3}$	$\frac{1}{5}$

These relative dimensions were chosen so that the conducting powers of the two bars for electric currents, and consequently for heat also, might be not very unequal. A vessel of tin-plate, perforated to admit the bar through its sides, was soldered round the middle of each conductor, and two others so as to leave about 2 inches at the ends of the conductor projecting beyond them. The parts of the conductors within these vessels were about 3 inches long in the copper and $4\frac{1}{2}$ inches in the iron, and the parts between the middle vessel and the vessels at the two sides were $2\frac{1}{2}$ and 2 inches respectively. The bores for the thermometer bulbs were exactly in the middle of the last-mentioned parts. In experimenting on either conductor, the central vessel was generally filled with oil or water, and kept hot by a gas-lamp below it. Streams of cold water from the town supply-pipes were kept flowing through the two lateral vessels.

22. To make these streams constant, whatever variations of pressure might occur in the supply-pipes, a cistern in a fixed position above the conductor was kept full (overflowing), and the coolers were supplied by pipes from this cistern. The supply often failed for several minutes, and sometimes for much longer; and after an experiment (Nov. 19, 1853) was nearly lost from this cause, a plan was arranged to lift water up from a larger cistern (into which the exit-streams from the coolers were discharged), and to pour it into the smaller cistern above, so as to keep the stream constant in quantity (although not quite invariable in temperature) even when the proper supply failed.

23. A galvanic battery for exciting a current through these conductors was prepared, consisting at first of four, and ultimately of eight, large iron cells, each measuring internally 12 inches deep, $10\frac{1}{2}$ inches broad, and $2\frac{1}{2}$ inches from side to side; eight porous cells, each 12 inches deep, 10 inches broad, and 2 inches from side to side; and eight zinc plates, each $9\frac{1}{2}$ inches by 10 inches. The iron cells were charged with a mixture of nitric acid two parts (bulk), sulphuric acid three parts, and water two. The porous cells were charged with dilute sulphuric acid. In each of the cells there were $1\frac{1}{3}$ square feet of zinc surface exposed to $2\frac{1}{2}$ square feet of iron, and the electro-motive force was not far from double that of a single cell of DANIELL's.

24. After preliminary experiments in which, with oil in the central vessel kept hot by a gas-lamp, the temperatures were too unsteady to allow any results of value to be obtained, water was substituted for oil in the central vessel, and was kept boiling briskly by the gas, the place of the water evaporated being frequently supplied by small quantities of boiling water poured in, so that ebullition never ceased. The

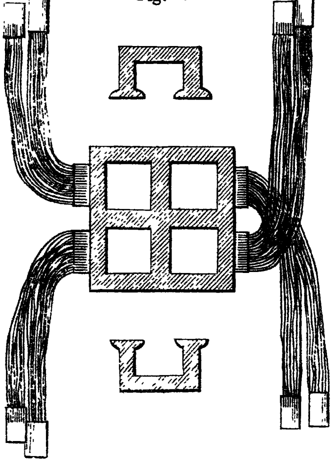
irregularities having been found to be much diminished, experiments were made in the following manner.

25. Four of the large iron cells, arranged as a single galvanic element, were used to excite the current. The experiment lasted about two hours, during which the current was sent through the conductor for twenty minutes at a time, alternately in the two directions, twice in each direction. Several minutes were spent in changing the direction of the current; the stiffness of the electrodes, and the clamps used for the connexions which had to be changed, rendering the process very troublesome. Readings of the thermometers were taken at intervals of five minutes during the flow of the current in both directions, as well as for some time before the current commenced and after it ceased.

26. The results of this experiment manifested, among great irregularities in the indications of the thermometers, a very decided differential variation between the two every time the direction of the current was changed; and appeared so promising, that a series of further experiments on the same copper conductor, and on the iron conductor similarly arranged, were immediately commenced, for the purpose of testing decisively the conclusion which had been indicated, and for discovering the corresponding effect in iron. To avoid the loss of time and the derangement in the position of the conductor by the shifting of the heavy clamps and stiff electrodes between its ends, in changing the direction of the current, a commutator, by which the change could be effected nearly instantaneously, was constructed on the following plan.

27. Four square holes, each of 1 inch, in a square block of mahogany, were fitted

Fig. 1.



with bottoms of thick copper slabs, passing through the mahogany, and cemented

with red lead so as to hold mercury, which was poured into each hole. The copper slabs projected outside to distances of about an inch, and each bore a bundle of 100 No. 18 copper wires soldered to it, two of which, connected with diagonally opposite copper slabs, served as battery electrodes, while the other two were clamped to the ends of the conductor to be tested. The four slabs have only to be connected by two conducting arcs parallel to one pair or to the other of the sides of the square, to send the current one way or the other through the conductor; which was done by means of two heavy brass castings, as shown in the diagram (fig. 1). This commutator has been used in a considerable variety of experiments, and has been found very convenient. It gives the means of reversing almost instantaneously a very powerful current, without the necessity of bending any of the electrodes or deranging any part of the apparatus, and the conductors involved in it are so strong that it occasions very little resistance.

28. As the supposed differential effect had appeared not to be increased after the first five minutes of the flow of the current in either direction, shorter periods of various lengths were tried, and more frequent observations of the thermometers were made, for the purpose of discovering the gradual variation of the temperature in the conductor, towards its final distribution as affected by the current. Four more large iron cells were added to the battery, which made it consist in all of eight cells, arranged as a single galvanic element, exposing 20 square feet of iron to $10\frac{1}{2}$ square feet of zinc surface. As the strength of current thus produced would be nearly double of that given in the previous experiment, any true effect of the kind sought would be augmented in the same ratio; and might be expected, both on this account and because of the improved system of observation, to become much more decided. These expectations, however, were not borne out by the results. The irregularities certainly became much diminished, but with these the differential effect on the thermometers, following the reversals of the current, either quite disappeared, or became very much less considerable than that which had been observed in the first experiment, and which I afterwards was led to attribute to some derangement in the position of the conductor occasioned by shifting the heavy clamps and stiff electrodes from between its two ends, causing the thermometer bulbs to alter a little in their positions in the hollows.

29. Many experiments, both on the copper and iron conductor, were made, from October 1852 to March 1853, and the results of the observations (on each of the two principal thermometers either every half-minute, or every quarter-minute, during an experiment of about two hours) carefully reduced; with much labour at first when arbitrary scale thermometers were employed, but afterwards with far greater ease when centigrade thermometers, constructed for this investigation at the Kew Establishment, were received and brought into use.

In the months of September and October 1853 the investigation was taken up again. The thermometric observations which had been made in the previously com-

pleted experiments, were all reduced, on the plan of the Tables given (§§ 47 and 56) below for subsequent experiments, and, when thus tested, they appeared to contain some indications of the effect looked for. Several more sets of observations of the same kind were therefore executed, but with various modifications of details. Still no decided result could be obtained, and I concluded from all the experiments which had been made, that the anticipated effect must be too small to be discovered without either increasing the sensibility of the test or diminishing the irregularities. I therefore prepared new apparatus, by which the former, and as much as possible of the latter object, would be attained.

§§ 30 to 34. *Improvements and Modifications of Apparatus.*

30. Instead of increasing the power of the battery, which I reserved as a later resource, if necessary, or of increasing the length of the conductor between the heater and the coolers on each side, which, while it would increase in the same ratio the amount of the effect looked for, would increase in a duplicate ratio the time that would have to be given to allow it to reach a stated proportion of its limiting value, I had conductors made of about the same length as the others, but of considerably less section.

31. With a view to perfecting and testing the action of the heater and coolers, each conductor was made up of a number of slips of flat sheet metal, bent and placed together, as shown in the accompanying diagram (fig. 2). The slips were held firmly

Fig. 2.

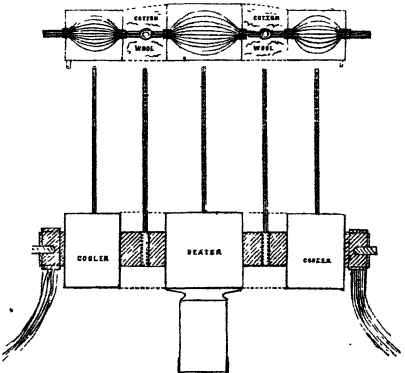


Fig. 3.

together by a vice, while collars of sheet copper, separated from them by vulcanized india-rubber, were soldered round them in the places for the sides of the heater and coolers. Tin-plate vessels, as shown in the diagram, were then put together, and soldered to these collars. The interstices between the slips and the india-rubber, and

the metal collars round the india-rubber, were stopped with red lead, and after some trouble were made water-tight. Thus the heater and coolers, without any metallic communication with the conductor, served the purpose of keeping the required supplies of hot and cold water round it in the proper places. The spaces for the thermometers were firmly stopped below with corks fitted to support the lower ends of the bulbs in perfectly fixed positions (fig. 3). Little collars of cork were put round the tubes just above the bulbs, and pushed down into the upper end of the hollow so as to hold the thermometers firmly and prevent all motion of their bulbs.

32. Various methods of heating the central part of the conductor were tried. First, as in previous experiments, water in the central vessel was kept boiling either by a gas-lamp under it, or by steam blown into it from a separate boiler; then a complex system with a boiling fountain, by which I attempted to get a perfectly uniform stream of water at a constant temperature, as little short of boiling as possible, to flow through the open spaces between the different slips within the central vessel, was used during several experiments. Lastly, water filling the central vessel was kept at a very constant temperature, near the boiling-point, by a gas-lamp below it, regulated by a person watching the indications of a thermometer with its bulb fixed in the middle space between the slips, as nearly as possible in the centre of the compound conductor. This last I found to be by far the best plan, and I used it in all subsequent experiments in which any external application of heat to the conductor was required.

33. Each cooler was divided into four compartments by partitions of tin-plate, stopping all communication from one to another, except through the spaces between the different slips composing the conductor. A constant stream of cold water (§ 22), introduced by the compartment nearest to the middle of the conductor, and drawn off by an overflow pipe, from a compartment next the end, was thus forced to flow all through among the different slips, and, as I found by placing thermometers in various positions in each compartment, gave a very satisfactory effect in fixing the temperature of the whole section of the conductor.

34. The experiments were made in other respects exactly as described above (§§ 28 and 29); the electric current, however, not being often again kept up for a longer time than ninety-six minutes, since the fumes, which always began to rise from the battery after the current had been flowing for about an hour, began after half an hour more to occasion great irregularities and inconvenience by causing the liquid (which sometimes became very hot,) to foam and overflow in some of the iron cells. The atmosphere had been in previous experiments sometimes rendered intolerable for the observers, by the acid vapour; but this evil was done away by covering the battery with cloths kept moist with ammonia and water, and by moistening other surfaces in the neighbourhood in the same way, so that the fumes never got far without meeting vapour of ammonia and combining into white clouds, which were perfectly innocuous.

§§ 35 to 38. *First Experiments with Multiple Sheet Copper Conductor.*

35. The copper conductor on the new plan was first used in an experiment on the 28th of October, 1853; with the central vessel heated by steam, and a current from the eight large iron cells kept flowing for seventy-two minutes, alternately in contrary directions, six times six minutes each way. The thermometers were noted every half-minute. The observations thus recorded, when thoroughly examined, indicated a slight differential cooling effect in the part of the conductor in which the nominal current was from cold to hot, and a heating effect where it passed from hot to cold; that is to say, a convection of heat in the nominal direction of the current, or as I shall call it to avoid circumlocution, *a convection of heat by vitreous electricity.*

36. A second experiment with the same conductor was made on the 2nd of November, 1853, in which the current was kept flowing for ninety-six minutes, eight times six minutes each way, and the thermometers were noted every quarter-minute. An examination of the recorded results indicated still the same kind of effect, but to a much smaller extent. Thus the final average, for the alteration of difference, between the temperatures at A and B due to the flow of the current for six minutes in one direction, after it had been flowing for six minutes in the contrary direction, amounted to ·039° Cent. in the first experiment, and to only ·0143° in the second experiment. A full analysis of the progress of the differential variation of temperature during the flow of the current is given in Tables I. and II., § 56 below, and shows through what fluctuations the final alterations are reached. The temperatures, at the ends of the successive times of flow in one direction or the other, and the evaluation of the mean final effect, are shown, for each experiment, in the following abridged Tables.

37. The observations made during the first period (that is the time from starting till the second reversal of the current) are rejected from the average in every case of experiments on the new conductors, because they were found to show so great absolute elevations of temperature (due to the frictional generation of heat by the current) that no alteration of difference between the thermometer observed during them could be relied on as an effect depending on the direction of the current.

38. Conductor composed of thirteen slips of sheet copper.

Experiment I. October 28th, 1853.

Periods.	(Current six times six minutes in each direction.) Differences of temperatures after six minutes of current entering. By end next A. $T_A-T_B=D$.	By end next B. $T_A-T_B=D'$.	Augmentations of differences from middles to ends of periods. $D'-D$.
I.	$2^{\circ}\cdot18$	$2^{\circ}\cdot09$	$-{}^{\circ}\cdot09$
II.	2·07	2·10	·03
III.	2·15	2·11	—·04
IV.	2·16	2·13	—·03
V.	2·01	1·99	—·02
VI.	1·80	2·02	·22
Means for five periods..	2·038	2·070	·032

Augmentation of difference during periods included .... } —·07
Deduct average augmentation per half-period .... } —·007
Effect due to reversal of current .... $0^{\circ}\cdot039$,
in favour of *Vitreous Electricity*.

Experiment II. November 2nd, 1853.

Periods.	By end next A. T_A.	T_B.	$T_A-T_B=D$.	By end next B. T_A.	T_B.	$T_A-T_B=D'$.	Augmentations of differences from middles to ends of periods. $D'-D$.
	(Current eight times six minutes in each direction.) Temperatures and differences of temperature after six minutes of current entering						
I.	$53^{\circ}\cdot79$	$52^{\circ}\cdot43$	$1^{\circ}\cdot36$	$54^{\circ}\cdot00$	$52^{\circ}\cdot70$	$1^{\circ}\cdot30$	$-{}^{\circ}\cdot06$
II.	53·68	52·42	1·26	53·51	52·23	1·28	·02
III.	53·86	52·57	1·29	53·79	52·63	1·16	—·13
IV.	53·90	52·60	1·30	53·99	52·63	1·36	·06
V.	54·09	52·72	1·37	54·05	52·68	1·37	·00
VI.	55·10	54·00	1·10	54·15	52·86	1·29	·19
VII.	53·99	52·80	1·19	54·10	52·80	1·30	·11
VIII.	54·00	52·50	1·50	54·00	52·60	1·40	—·10
Means for seven periods	54·0886	52·8014	1·2871	53·9414	52·6329	1·3086	·02143

Augmentation of difference during periods included ·10
Deduct average augmentation per half-period .... ·00714
Effect due to reversal of current .... $0^{\circ}\cdot01429$,
in favour of *Vitreous Electricity*.

§§ 39 to 43. *Decisive Experiments with Multiple Sheet Iron Conductor.*

39. These experiments seemed therefore on the whole to establish a probability in favour of the convection of heat by the so-called positive electricity, when a current is kept up through an unequally heated conductor. The convective effect, if of this kind, ought (§ 18) to be less in iron than in copper, I therefore had little expectation of finding an indication of it in the iron conductor which (§ 31) had been in the course of preparation; but as soon as it was ready for use I made the following experiments, and was much surprised by the result, which became manifest before the first of them was finished.

40. Conductor composed of thirty slips of sheet iron.

Experiment III. November 12th, 1853.

	(Current six times eight minutes in each direction.) Temperatures and differences of temperature after eight minutes of current entering						Augmentations of differences from middles to ends of periods.
	By end next A.			By end next B.			
Periods.	T_A.	T_B.	$T_B - T_A = D$.	T_A.	T_B.	$T_B - T_A = D'$.	$D' - D$.
I.	51°·43	53°·56	2°·13	51°·48	53°·49	2°·01	−°·12
II.	51·62	53·30	1·68	51·41	53·21	1·80	·12
III.	51·73	53·26	1·53	52·03	53·87	1·84	·31
IV.	52·01	53·80	1·79	51·32	53·42	2·10	·31
V.	51·30	53·00	1·70	51·00	52·95	1·95	·25
VI.	51·14	52·98	1·84	50·69	52·80	2·11	·27
Means for five periods..	51·56	53·268	1·708	51·29	53·25	1·96	·252

Augmentation of difference during periods included ... ·10

Deduct average augmentation per half-period .... ·010

Effect due to reversal of current .... 0°·242,

in favour of *Resinous Electricity*.

Experiment IV. November 19th, 1853.

	(Current seven times eight minutes in each direction.) Temperatures and differences of temperature after eight minutes of current entering						Augmentations of differences from middles to ends of periods.
	By end next A.			By end next B.			
Periods.	T_A.	T_B.	$T_B-T_A=D$.	T_A.	T_B.	T_B-T_A.	$D'-D$.
I.	57°·50	59°·30	1°·80	58°·02	59°·84	1°·82	°·02
II.*	48·20	51·15	2·95	46·82	49·79	2·97	·02
III.	46·49	49·13	2·64	47·01	49·95	2·94	·30
IV.	48·41	51·69	3·28	48·31	51·99	3·68	·40
V.	48·36	51·74	3·38	48·18	51·80	3·62	·24
VI.	48·00	51·20	3·20	48·00	51·49	3·49	·29
VII.	48·31	51·51	3·20	48·06	51·60	3·54	·34
Means for five periods..	49·3243	52·2457	3·14	49·20000	52·35143	3·454	·314

Augmentation of difference during periods included... 0·57
Deduct average augmentation per half-period.......................... ·057
Effect due to reversal of current .... 0°·257,
in favour of *Resinous Electricity*.

41. A full analysis of the differential variations throughout each of these experiments, derived from observations of the thermometer taken every quarter of a minute, was made in each case immediately after the conclusion of the experiment (see Tables I. and II. § 47 below), and was sufficient to convince me that the true effect in the iron conductor is of the kind indicated by the preceding summary of the effects apparent at the ends of the periods.

42. To try whether or not the very considerable effect thus discovered depended on some inequality in the conductor itself, I made an experiment on the 25th of November 1853 exactly like the two preceding, with the exception that the middle vessel previously used as a heater was filled with cold water at the commencement. The current was sent six times eight minutes in each direction; the thermometers were noted every quarter of a minute; and the observations were reduced and compared in the usual way. The result gave no effect of the kind observed in the preceding experiments, but (probably because of a temporary failure in the water-supply for the coolers) showed, on the contrary, a deviation in the mean difference of temperature amounting to °·029 Cent., being about a tenth part of the amount of that effect, but in the opposite way according to the direction of the current through the conductor. Before the experiment was concluded boiling water was poured through the central vessel and left filling it, but with no lamp below. The two thermometers (A and B) being thus raised to about 27° Cent., the current was again started and was sent through the conductor for three times four minutes in each direction. The thermometers rose each nearly 2°, but fell again by nearly $5\frac{1}{2}°$ before the conclusion. The mean differential result, whether from these three periods (amounting to °·05 Cent.),

* Rejected because of a failure in the water-supply through the coolers during the whole of Period I.

or from the last two of them without the first (°·025 Cent.), was of the same kind as in the first two experiments. This experiment then conclusively demonstrated that the effect previously discovered was really owing to the heat in the central part of the conductor, and not to any inequality in the metal of the conductor itself, nor to any accidental disturbing agency.

43. It was thus established, that the *Resinous Electricity carries heat with it in an iron conductor.*

§§ 44 and 45. *Experiment with Copper Conductor, repeated.*

44. The very small effect I had discovered of the opposite kind in the copper conductor required confirmation; and indeed the analysis of the progress of the variation (see Tables I. and II. § 56 below) was so unsatisfactory, that I felt it quite an open question, whether it was the true effect, or merely an accidental coincidence of irregularities; and I thought it improbable that contrary effects should really exist in copper and in iron. I made on the 26th of November another experiment on the copper conductor, with the current flowing six times eight minutes each way (instead of eight times six minutes, as before, because the analysis seemed to show that the effects which had chanced to appear in the average results of the six minutes might disappear with longer periods*); but I got still a very small result of the same kind. The full analysis (Table III.) was equally unsatisfactory with those of the two preceding experiments on the same conductor. The following numbers show the temperatures at the reversals, and the final result, as in the previous abridged tables.

45. Conductor composed of thirteen slips of sheet copper.

Experiment V. November 26, 1853.

Periods.	(Current six times eight minutes in each direction.) Temperatures and differences of temperature after eight minutes of current entering By end next A.			By end next B.			Diminutions of differences from middles to ends of periods.
	T_A.	T_B.	$T_B-T_A=D$.	T_A.	T_B.	$T_B-T_A=D'$.	$D'-D$.
I.	50°·88	52°·78	1°·90	50°·88	52°·72	1°·92	−°·02
II.	50·64	52·49	1·85	50·53	52·32	1·79	+·06
III.	50·38	52·29	1·91	50·01	52·00	1·99	−·08
IV.	50·14	52·08	1·94	49·90	51·83	1·93	+·01
V.	49·60	51·61	2·01	49·48	51·52	2·04	−·03
VI.	49·11	51·22	2·11	48·80	50·92	2·12	−·01
Means for five periods	49·974	51·938	1·964	49·744	51·718	1·974	−·01

Augmentation of difference during periods included... ·20

Add average augmentation per half-period .... +0·02

Effect due to reversal of current .... ·01,

in favour of *Vitreous Electricity.*

* I now believe that a true effect, amounting to from °·01 to °·02, was really reached in three or four minutes, and that in the latter parts of the half-periods there was no sensible augmentation of this effect, but

46. Another experiment was also made on the new iron conductor, and results as decisive as those in the first two experiments were obtained. The following abridged Table shows sufficiently the character of the effect demonstrated; and the analysis of the progress of variation is given in the full table (Table III. § 47) below.

Experiment VI. December 2, 1853.

Conductor composed of thirty slips of sheet iron.

Periods.	(Current six times eight minutes in each direction.) Temperatures and differences of temperature after eight minutes of current entering. By end next A.			By end next B.			Augmentations of differences from middles to ends of periods.
	T_A.	T_B.	$T_B-T_A=D$.	T_A.	T_B.	$T_B-T_A=D'$.	$D'-D$.
I.	54°·76	56°·33	1°·57	54°·80	56°·66	1°·86	°·29
II.	54·97	56·68	1·71	54·89	56·80	1·91	·20
III.	55·01	56·70	1·69	54·93	56·86	1·93	·24
IV.	55·22	56·90	1·68	55·08	57·10	2·02	·34
V.	55·31	57·08	1·77	55·07	57·12	2·05	·28
VI.	55·12	57·00	1·88	54·84	57·03	2·19	·31
Means for five periods	55·126	56·872	1·746	54·962	56·982	2·020	·274

Augmentation of difference T_B-T_A during included periods ·33
Deduct average augmentation per half-period .... ·033
Effect due to reversal of current .... 0°·241,
in favour of *Resinous Electricity*.

47. The following Tables show the progress of variation of the difference between the temperatures of the two tested localities (A, B) of the iron conductor, during each of the three regular experiments referred to above, as derived directly from the quarter-minute or half-minute observations actually made in the course of each experiment.

only irregular fluctuations, sometimes counteracting and reversing the true effect, but generally only diminishing it and increasing it alternately, and always maintaining, during the whole latter half of the aggregate of the half-periods, an average deviation of the kind noted as the final result. A careful consideration of the Tables I., II. and III. given below, § 56, for the copper conductor and of their graphical representation (see Diagram, § 57), is, I think, sufficient to establish this view. [April 9, 1856.]

TABLE I. November 12th, 1853. Conductor composed of thirty slips of sheet iron.

Middle of conductor in water kept hot by steam blown into it.

Streams of water at temperature 6° running through the coolers.

Temperatures at middle points A, B of the parts between heater and coolers:—Initial $T_A=51°{\cdot}23$, $T_B=53°{\cdot}30$; Final $T_A=50°{\cdot}69$, $T_B=52°{\cdot}80$.

Augmentations of difference T_B-T_A (in hundredths of a degree Cent.), during Periods	1.		2.		3.		4.		5.		6.		7.		8.	
	First half-minute of current entering by end next		Second half-minute of current entering by end next		Third half-minute of current entering by end next		Fourth half-minute of current entering by end next		Fifth half-minute of current entering by end next		Sixth half-minute of current entering by end next		Seventh half-minute of current entering by end next		Eighth half-minute of current entering by end next	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
I.	0	−2	−1	1	2	0	0	0	2	0	0	1	0	3	−3	−1
II.	−4	1	−4	−2	−2	7	−6	4	−1	1	−1	1	−2	5	−1	2
III.	0	0	−1	4	−6	2	−2	4	−2	3	0	−1	−6	3	0	2
IV.	2	−2	−1	2	−2	3	−3	5	−3	4	−1	5	−1	4	0	5
V.	1	5	−3	3	−5	0	−9	3	−6	5	−5	0	−2	3	−1	0
VI.	−2	2	−3	0	1	3	−3	3	−4	1	−1	6	0	0	−1	4
Augmentations during half-minutes, summed for five periods	−3	6	−12	7	−14	15	−23	19	−16	14	−8	+11	−11	15	−3	13
Differences of augmentation during corresponding half-minutes of first and second halves of a period, summed for five periods...............	9		19		29		42		30		19		26		16	
Differences of augmentation in equal intervals from beginning and middle of a period, summed for five periods....................................	9		28		57		99		129		148		174		190	
Mean augmentation of difference in favour of thermometer next entering current	°·009		°·028		°·057		°·099		°·129		°·148		°·171		°·190	
.... after reversal and flow for	½ min.		1 min.		1½ min.		2 min.		2½ min.		3 min.		3½ min.		4 min.	

Augmentations of difference T_B-T_A (in hundredths of a degree Cent.), during Periods	9.		10.		11.		12.		13.		14.		15.		16.	
	Ninth half-minute of current entering by end next		Tenth half-minute of current entering by end next		Eleventh half-minute of current entering by end next		Twelfth half-minute of current entering by end next		Thirteenth half-minute of current entering by end next		Fourteenth half-minute of current entering by end next		Fifteenth half-minute of current entering by end next		Sixteenth half-minute of current entering by end next	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
I.	5	−1	5	−1	1	−1	1	−3	0	−2	−1	−5	−1	0	−4	1
II.	−2	−1	1	−5	−10	2	−2	−2	−1	0	−5	−2	−1	1	8	0
III.	−3	0	−2	4	−3	2	−1	4	5	0	−3	−1	−3	1	0	4
IV.	2	−2	1	2	0	3	−2	−6	−1	2	1	−3	2	10	1	−1
V.	−3	0	−1	1	−3	0	1	3	−2	−3	0	1	−2	−1	0	5
VI.	0	−1	−2	0	0	4	0	4	1	0	0	1	2	0	1	0
Augmentations during half-minutes, summed for five periods	−6	−4	−3	2	−16	11	−4	3	2	−1	−7	−4	−2	11	10	8
Differences of augmentation during corresponding half-minutes of first and second halves of a period, summed for five periods...............	2		5		27		7		−3		3		13		−2	
Differences of augmentation in equal intervals from beginning and middle of a period, summed for five periods....................................	192		197		224		231		228		231		244		242	
Mean augmentation of difference in favour of thermometer next entering current	°·192		°·197		°·224		°·231		°·228		°·231		°·231		°·242	
.... after reversal and flow for	4½ min.		5 min.		5½ min.		6 min.		6½ min.		7 min.		7½ min.		8 min.	

Table II. November 19th, 1853. Con-

Middle of conductor in water kept

Streams of water at temperature

Temperatures at middle points A, B of the parts between heater and

Augmentations of difference $T_B - T_A$ (in hundredths of a degree Cent.), during Periods	1. First quarter-minute of current entering by end next		2. Second quarter-minute of current entering by end next		3. Third quarter-minute of current entering by end next		4. Fourth quarter-minute of current entering by end next		5. Fifth quarter-minute of current entering by end next		6. Sixth quarter-minute of current entering by end next		7. Seventh quarter-minute of current entering by end next		8. Eighth quarter-minute of current entering by end next		9. Ninth quarter-minute of current entering by end next		10. Tenth quarter-minute of current entering by end next		11. Eleventh quarter-minute of current entering by end next		12. Twelfth quarter-minute of current entering by end next		13. Thirteenth quarter-minute of current entering by end next	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
I.	−3	−2	−1	−1	−4	−2	−4	−1	4	−2	−4	4	3	1	−1	2	2	−2	1	1	−1	2	1	−1	−3	1
II*.	3	5	−1	1	0	−5	6	4	5	−2	15	1	7	0	10	−2	38	2	−6	9	8	−5	0	0	13	−1
III.	0	1	−2	3	−3	−3	−1	1	−1	1	0	−3	−1	5	−3	2	−4	−2	−2	3	1	2	−2	−1	0	0
IV.	0	−3	0	1	3	3	−3	2	5	2	−1	3	4	4	5	1	−4	0	6	1	−3	2	4	4	4	2
V.	1	0	−1	0	−4	3	−3	2	−2	1	0	3	−1	1	−1	1	−1	1	0	1	−1	1	−2	0	−3	1
VI.	0	2	−3	2	−5	3	−2	3	0	1	−3	1	−2	3	−6	3	−1	0	−2	1	−1	1	−2	−1	−1	2
VII.	0	1	1	0	−3	1	−2	0	−2	2	−2	3	−1	3	0	2	−1	2	−1	4	−2	3	−1	2	−2	1
ugmentations during quarter-minutes, summed for five periods	1	1	−5	6	−12	7	−11	8	0	7	−6	7	−1	16	−5	9	−11	1	1	10	−6	9	−3	4	−2	6
ifferences of augmentation during corresponding quarter-minutes of first and second halves of a period, summed for five periods	0		11		19		19		7		13		17		14		12		9		15		7		8	
ifferences of augmentation in equal intervals from beginning and middle of a period, summed for five periods	0		11		30		49		56		69		86		100		112		121		136		143		151	
lean augmentation of difference in favour of thermometer next entering current	°·000		°·011		°·030		°·049		°·056		°·069		°·086		°·100		°·112		°·121		°·136		°·143		°·151	
after reversal and flow for	¼ min.		½ min.		¾ min.		1 min.		1¼ min.		1½ min.		1¾ min.		2 min.		2¼ min.		2½ min.		2¾ min.		3 min.		3¼ min.	

After the conclusion of this experiment the current was started in one direction and broken, and fect, when the heater and coolers were kept regular, on either thermometer amounted to about 1°·4.

* Period II. rejected because the water-supply, which had failed during the whole of Period I.,

ductor composed of thirty slips of sheet iron.

boiling by steam blown into it.

6° running through the coolers.

coolers:—Initial $T_A=56°·44$, $T_B=58°·28$; Final $T_A=48°·06$, $T_B=51°·59$.

14.		15.		16.		17.		18.		19.		20.		21.		22.		23.	
Fourteenth quarter-minute of current entering by end next		Fifteenth quarter-minute of current entering by end next		Sixteenth quarter-minute of current entering by end next		Seventeenth quarter-minute of current entering by end next		Eighteenth quarter-minute of current entering by end next		Nineteenth quarter-minute of current entering by end next		Twentieth quarter-minute of current entering by end next		Twenty-first quarter-minute of current entering by end next		Twenty-second quarter-minute of current entering by end next		Twenty-third quarter-minute of current entering by end next	
A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
4	3	−4	2	1	−5	2	2	−3	2	3	−1	1	1	0	−1	−5	2	2	−2
6	−3	4	2	2	−1	10	−1	−2	−1	−3	2	−1	−2	1	1	5	−1	1	0
0	0	−3	0	−3	0	2	10	0	−7	0	5	−3	0	−1	1	1	−7	2	8
4	0	−3	0	2	2	−1	1	5	3	0	2	2	1	−2	1	4	1	−1	0
−2	2	−1	3	−2	1	−3	0	0	1	0	1	0	0	−1	−1	1	0	0	0
1	0	−2	0	0	1	0	1	−2	1	0	3	−1	2	−1	−1	−2	0	−1	0
0	2	−1	−2	−1	0	−2	3	−1	1	−1	1	0	1	0	2	−2	0	−1	−1
3	4	−10	1	−4	4	−4	15	2	−1	−1	12	−2	4	−5	2	2	−6	−1	7
1		11		8		19		−3		13		6		7		−8		8	
152		163		171		190		187		200		206		213		205		213	
°·152		°·163		°·171		°·190		°·187		°·200		°·206		°·213		°·205		°·213	
3½ min.		3¾ min.		4 min.		4¼ min.		4½ min.		4¾ min.		5 min.		5¼ min.		5½ min.		5¾ min.	

24.		25.		26.		27.		28.		29.		30.		31.		32.	
Twenty-fourth quarter-minute of current entering by end next		Twenty-fifth quarter-minute of current entering by end next		Twenty-sixth quarter-minute of current entering by end next		Twenty-seventh quarter-minute of current entering by end next		Twenty-eighth quarter-minute of current entering by end next		Twenty-ninth quarter-minute of current entering by end next		Thirtieth quarter-minute of current entering by end next		Thirty-first quarter-minute of current entering by end next		Thirty-second quarter-minute of current entering by end next	
A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
2	0	0	1	−1	−3	−1	−1	2	−1	2	1	−1	−1	2	1	0	2
−1	1	−12	0	5	−2	5	−2	−7	−1	1	0	16	2	−11	0	−4	1
2	7	−1	−3	−2	−1	−3	5	0	0	2	−2	−3	0	−2	5	−3	0
2	0	−1	0	1	3	2	2	−5	1	4	1	−1	1	−2	−1	4	0
−1	0	0	1	0	0	1	−1	−1	2	−2	0	0	0	0	0	−1	0
−1	1	0	1	3	0	0	1	−1	−2	−2	1	−2	−2	−1	1	−2	0
0	0	−2	0	0	−1	−3	1	0	1	1	0	0	0	1	1	−1	1
2	8	−4	−1	2	1	−3	8	−7	2	3	0	−6	−1	−4	6	−3	1
6		3		−1		11		9		−3		5		10		4	
219		222		221		232		241		238		243		253		257	
°·219		°·222		°·221		°·232		°·241		°·238		°·243		°·253		°·257	
6 min.		6¼ min.		6½ min.		6¾ min.		7 min.		7¼ min.		7½ min.		7¾ min.		8 min.	

started in the other direction and broken, several times; and it was found that its absolute heating

only commenced giving a stream through the coolers at the commencement of the second period.

TABLE III. December 2nd, 1853. Con-

Middle of conductor at

Streams of water at temperature

Temperatures at middle points A, B of the parts between heater and

Augmentations of difference $T_B - T_A$ (in hundredths of a degree Cent.), during Periods	1.		2.		3.		4.		5.		6.		7.		8.		9.		10.		11.		12.		13.	
	First quarter-minute of current entering by end next		Second quarter-minute of current entering by end next		Third quarter-minute of current entering by end next		Fourth quarter-minute of current entering by end next		Fifth quarter-minute of current entering by end next		Sixth quarter-minute of current entering by end next		Seventh quarter-minute of current entering by end next		Eighth quarter-minute of current entering by end next		Ninth quarter-minute of current entering by end next		Tenth quarter-minute of current entering by end next		Eleventh quarter-minute of current entering by end next		Twelfth quarter-minute of current entering by end next		Thirteenth quarter-minute of current entering by end next	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
I.	0	0	1	2	0	1	−2	2	−1	1	3	1	−1	2	0	2	2	2	0	2	−2	2	3	3	−1	1
II.	4	0	−1	0	−2	1	−1	3	−2	1	−1	1	−2	2	−3	1	0	1	−1	0	0	0	1	0	−4	2
III.	1	0	0	0	0	0	−1	1	0	1	−1	0	0	2	0	3	0	3	0	3	−1	3	−1	2	−2	1
IV.	0	1	0	1	−2	0	1	0	−2	1	−4	1	−3	3	−2	2	0	3	−2	2	−2	1	−2	2	−1	0
V.	0	1	−1	−3	0	3	−1	4	−1	0	0	−1	−1	4	−3	5	−1	1	−2	2	−1	1	−1	2	−2	−1
VI.	0	0	−1	0	0	1	−1	1	−1	0	−1	0	0	0	0	2	−1	1	−1	2	0	2	0	3	−2	0
Augmentations during quarter-minutes, summed for five periods	5	2	−3	−2	−4	5	−3	9	−6	3	−7	1	−6	11	−8	13	−2	9	−6	9	−4	7	−3	9	−11	2
Differences of augmentation during corresponding quarter-minutes of first and second halves of a period, summed for five periods............	−3		1		9		12		9		8		17		21		11		15		11		12		13	
Differences of augmentation in equal intervals from beginning and middle of a period, summed for five periods..........................	−3		−2		+7		19		28		36		53		74		85		100		111		123		136	
Mean augmentation of difference in favour of thermometer next entering current	°·003		°·002		°·007		°·019		°·028		°·036		°·053		°·074		°·085		°·100		°·111		°·123		°·136	
.... after reversal and flow for	¼ min.		½ min.		¾ min.		1 min.		1¼ min.		1½ min.		1¾ min.		2 min.		2¼ min.		2½ min.		2¾ min.		3 min.		3¼ min.	

ductor composed of thirty slips of sheet iron.

temperature 95° Cent.

7°·5 running through the coolers.

coolers:—Initial $T_A=54°·20$, $T_B=55°·70$; Final $T_A=54°·84$, $T_B=57°·03$.

14.		15.		16.		17.		18.		19.		20.		21.		22.	
Fourteenth quarter-minute of current entering by end next		Fifteenth quarter-minute of current entering by end next		Sixteenth quarter-minute of current entering by end next		Seventeenth quarter-minute of current entering by end next		Eighteenth quarter-minute of current entering by end next		Nineteenth quarter-minute of current entering by end next		Twentieth quarter-minute of current entering by end next		Twenty-first quarter-minute of current entering by end next		Twenty-second quarter-minute of current entering by end next	
A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
0	2	−2	0	3	0	2	−1	1	1	0	1	−1	1	−1	1	1	1
−1 −3 −1 −1 0	1 0 2 1 1	0 −4 −1 −1 0	0 1 0 0 1	−1 −1 0 −2 −1	0 1 1 1 1	−1 0 −1 −3 −1	1 0 1 1 2	0 −2 0 −1 −1	1 1 2 1 2	0 −1 0 0 −1	0 1 0 0 3	−1 −1 −1 −1 0	1 1 2 1 2	0 −1 0 −2 −2	0 1 0 1 1	0 0 0 −5 −1	0 1 2 1 0
−6	5	−6	2	−5	4	−6	5	−4	7	−2	4	−4	7	−5	3	−6	4
11		8		9		11		11		6		11		8		10	
147		155		164		175		186		192		203		211		221	
°·147		°·155		°·164		°·175		°·186		°·192		°·203		°·211		°·221	
3½ min.		3¾ min.		4 min.		4¼ min.		4½ min.		4¾ min.		5 min.		5¼ min.		5½ min.	

23.		24.		25.		26.		27.		28.		29.		30.		31.		32.	
Twenty-third quarter-minute of current entering by end next		Twenty-fourth quarter-minute of current entering by end next		Twenty-fifth quarter-minute of current entering by end next		Twenty-sixth quarter-minute of current entering by end next		Twenty-seventh quarter-minute of current entering by end next		Twenty-eighth quarter-minute of current entering by end next		Twenty-ninth quarter-minute of current entering by end next		Thirtieth quarter-minute of current entering by end next		Thirty-first quarter-minute of current entering by end next		Thirty-second quarter-minute of current entering by end next	
A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
3	1	3	−1	0	−1	0	−1	−1	0	1	2	−1	−1	−1	1	0	1	0	1
0 −1 −1 0 0	1 0 1 0 1	0 0 0 1 0	0 0 1 1 0	0 −1 0 1 −1	0 −2 1 1 0	0 −1 −1 1 0	1 −2 0 1 1	0 0 0 0 0	0 1 1 −1 −2	1 0 0 0 0	1 0 1 0 1	0 0 0 0 −1	0 0 0 0 1	0 0 0 1 0	0 0 1 1 0	0 0 0 0 0	1 0 0 −1 3	0 −1 0 1 0	0 1 1 1 1
−2	3	1	2	−1	0	−1	1	0	−1	1	3	−1	1	1	2	0	3	0	4
5		1		1		2		−1		2		2		1		3		4	
226		227		228		230		229		231		233		234		237		241	
°·226		°·227		°·228		°·230		°·229		°·231		°·233		°·234		°·237		°·241	
5¾ min.		6 min.		6¼ min.		6½ min.		6¾ min.		7 min.		7¼ min.		7½ min.		7¾ min.		8 min.	

48. The gradual augmentation of the difference $T_A - T_B$ from its value at a time when the current had been flowing for eight minutes entering by the end next B, consequent upon reversing the current and letting it flow continuously entering by the end next A, is shown by the numbers at the foot of each table, as a mean result derived from a single experiment. The mean of the results of the three experiments is shown by the following numbers, and is exhibited by a curve in the Diagram (fig. 4) of § 57 below.

Current entering by end next A.

Time from instant of reversal in quarter-minutes	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Mean Augmentation of difference $T_A - T_B$	·000	·001	·006	·0185	·032	·042	·054	·0695	·091	·0985	·1167	·1235	·138	·1435	·1578	·1590	·175

Time from instant of reversal in quarter-minutes	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Mean Augmentation of difference $T_A - T_B$	·1825	·1883	·196	·202	·212	·2167	·2195	·229	·225	·2263	·2305	·2343	·2355	·2403	·245	·2467

49. That Vitreous Electricity carries heat with it in copper is indicated by each of the three experiments on the thirteen slip conductor adduced above, but by so narrow an effect; amounting on an average to only $0^{\circ}{\cdot}02$ Cent., which corresponds to a reading of half that amount, or $\frac{1}{100}$th of a degree, being $\frac{1}{10}$th of a division on the scale of each thermometer; with such discrepancies among the results of the different experiments (Oct. 28th, effect ·039, Nov. 2nd, ·0143, Nov. 26th, ·01); and with so great fluctuations in the course of each experiment (see Tables I., II., III., § 56 below); that I did not venture to draw from them so seemingly improbable a conclusion, as that the convective effects in copper and in iron should be in contrary directions. The dynamic theory (§ 18) was fully satisfied by the demonstration which the experiments gave, that the convective effect is undoubtedly in iron a *conveying of heat in the direction of the Resinous Electricity*, and that it is *less in amount* in copper, whether in the same direction as in iron or in the contrary direction. But it was still an object of great interest, (in fact an object of much greater interest than any verification of conclusions from the dynamic theory, which were in reality as certain before as after the experiments directly demonstrating them,) to ascertain the actual nature of the convective effect in copper, and I therefore endeavoured to make more decisive experiments for discovering it.

50. The three experiments which had been made were quite sufficient to prove that the convective effect, whatever its true nature might be, was nearly insensible to my thermometers without either more powerful currents or a more sensitive conductor. To work with more powerful currents would have increased immensely the labour of carrying out the experiments, and would besides involve a large addition to the battery which had been used hitherto. I preferred therefore to make the conductor more sensitive, which I saw could be done by diminishing the body of metal in the tested parts, and so preventing the looked-for thermal effect from being so much conducted away from the localities of the thermometers as it had been. I accordingly had

several slips of copper cut away from each side of the conductor in the parts between the heater and the coolers, leaving the parts within these vessels unchanged.

51. Several experiments were made on the conductor thus reduced successively to smaller and smaller numbers of slips; but the results did not appear much more decided than they had been in the experiments on the unreduced conductor, until it was tried with all the slips but two cut away. Thus with four slips left, the following results were obtained:—

52. Copper conductor reduced to four slips.

Experiment VII. February 1854.

	(Current six times eight minutes in each direction.) Temperatures and differences of temperature after eight minutes of current entering. By end next A.			By end next B.			Augmentations of differences from middles to ends of periods.
Periods.	T_A.	T_B.	$T_A-T_B=D$.	T_A.	T_B.	$T_A-T_B=D'$.	$D'-D$.
I.	$46^{\circ}\!\cdot00$	$45^{\circ}\!\cdot24$	·76	$45^{\circ}\!\cdot92$	$45^{\circ}\!\cdot27$	·65	—·11
II.	45·82	45·19	·63	46·01	45·43	·58	—·05
III.	46·00	45·50	·50	46·02	45·54	·48	—·02
IV.	46·20	45·77	·43	46·21	45·76	·45	·02
V.	46·18	45·80	·38	46·09	45·71	·38	·00
VI.	46·09	45·72	·37	46·05	45·68	·37	·00
Means for five periods.	46·058	45·596	·462	46·076	45·624	·452	—·010

Diminution of difference during periods included ...... ·28
Add average diminution per half-period .... +·028
Effect due to reversal of current .... ·018,
in favour of *Vitreous Electricity*.

The effect shown here is of the same kind as had been found in all the previous experiments, but was still too small to be very satisfactory. Some unknown cause made the difference T_A-T_B to diminish so much through the whole experiment as to overpower the apparent tendency of the current from B to A to increase it, and the abridged table has on this account a very unsatisfactory appearance as regards the conclusion drawn from it after the proper correction for their diminution is applied: but the full examination of the progress of variation in the course of the experiment shown in Table IV. below is much less unsatisfactory, and shows undoubtedly the true convective effect in copper.

53. The following results, derived from the first experiment made on the conductor reduced to two slips, show a very marked increase in the effect, and make the result quite apparent even without the full analysis given below in Table V.

Experiment VIII. February 23rd, 1854.

Copper conductor reduced to two slips.

	(Current seven times six minutes in each direction.) Temperatures and differences of temperature after eight minutes of current entering. By end next A.			By end next B.			Diminutions of differences from middles to ends of periods.
Periods.	T_A.	T_B.	$T_B-T_A=D$.	T_A.	T_B.	$T_B-T_A=D'$.	$D'-D$.
I.	54°·81	56°·31	1°·50	55°·47	56°·88	1°·41	·09
II.	55·90	57·32	1·42	56·37	57·63	1·26	·14
III.	56·70	57·99	1·29	57·16	58·29	1·13	·16
IV.	57·49	58·70	1·21	57·80	58·91	1·11	·10
V.	57·80	58·98	1·18	58·18	59·20	1·02	·16
VI.	58·29	59·80	1·51	58·37	59·80	1·43	·08
VII.	58·27	59·80	1·53	58·08	59·50	1·42	·11
Means for six periods...	57·4083	58·7650	1·3567	57·663	58·888	1·2283	·1283

Augmentation of differences during periods included ·01
Add average augmentation per half-period .... ·0008
Effect due to reversal of current .... 0·°1291,
in favour of *Vitreous Electricity*.

54. The effect here shown is, as regards amount, unmistakeable, and it appeared quite to establish the conclusion I had not ventured to draw from the previous experiments on the copper conductor. As it was clear that diminution in the power of the conductor had now begun to augment its sensibility for the effect under investigation, I had it further reduced by cutting away from its breadth, above and below, between the heaters and the bends for the thermometer-bulbs on the two sides, and on the other sides of those bends as far as the coolers. An experiment was then made which led to the following results:—

Experiment IX. March 7th, 1854.

Copper conductor diminished in breadth.

	(Current seven times six minutes in each direction.) Temperatures and differences of temperature after six minutes of current entering. By end next A.			By end next B.			Diminutions of differences from middles to ends of periods.
Periods.	T_A.	T_B.	$T_B-T_A=D$.	T_A.	T_B.	$T_B-T_A=D'$.	$D'-D$.
I.	56°·61	58°·00	1°·39	57°·56	58°·98	1°·42	—·03
II.	58·19	59·69	1·50	58·74	60·01	1·27	·23
III.	59·12	60·70	1·58	59·61	61·08	1·47	·11
IV.	59·91	61·53	1·62	60·18	61·75	1·57	·05
V.	60·50	62·18	1·68	61·15	62·53	1·38	·30
VI.	61·71	63·02	1·31	61·87	62·91	1·04	·27
VII.	61·07	62·80	1·73	61·07	62·62	1·55	·18
Means for six periods...	60·083	61·653	1·57	60·4367	61·8167	1·38	·19

Augmentation of differences during periods included... ·13
Add average augmentation per half-period........................... =·01083
·20083

After this experiment I considered it quite established that the *Vitreous Electricity carries heat with it in copper.*

55. The conductor was still further diminished in breadth (so as to be only an inch broad in the parts between the heater and coolers on each side), and an experiment was made before my class on the 19th April, 1854, leading to the following results, shown as in the abridged tables of the preceding experiments.

Experiment X. April 19th, 1854.

Copper conductor of two slips, further diminished in breadth.

	(Current seven times six minutes each way.) Temperatures and differences of temperatures after six minutes of current entering. By end next A.			By end next B.			Diminutions of differences from middles to ends of periods.
Periods.	T_A.	T_B.	$T_B-T_A=D$.	T_A.	T_B.	$T_B-T_A=D'$.	$D-D'$.
I.	74°·30	76°·60	2°·30	74·81	77°·50	2°·69	—·39
II.	73·80	76·48	2·68	75·32	78·10	2·78	—·10
III.	76·25	79·42	3·17	76·17	79·18	3·01	·16
IV.	76·33	79·51	3·18	76·28	79·37	3·09	·09
V.	75·60	78·69	3·09	75·20	78·07	2·87	·22
VI.	74·80	77·70	2·90	75·00	77·75	2·75	·15
VII.	74·10	76·84	2·74	75·42	78·20	2·78	—·04
Means, Period I. off ...	75·147	78·107	2·96	75·565	78·445	2·88	·08
	Augmentation of differences during periods included... ·09						
	Add average augmentation per half-period						·0075
	Effect due to reversal of current						°·0875

Means, Periods I. and VII. off	75·356	78·36	3·004	75·594	78·494	2·90	·104
	Augmentation of differences during periods included... ·06						
	Deduct average augmentation per half-period						·006
	Effect due to reversal of current						·110

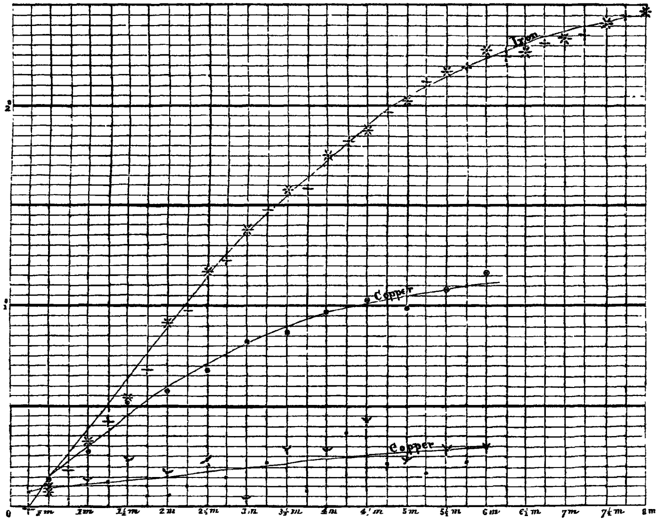
The effect here obtained, although of quite a decisive character, does not appear to show any increased sensibility resulting from the further diminution in the breadth of the conductor.

56. The following Tables [*printed after a part of* § 58] show a complete analysis of the results of the seven experiments on the copper conductor which have been adduced.

57. The average progress towards the final effect of a reversal, as indicated by the numbers at the ends of these Tables, is exhibited graphically for the copper conductor in the different states in which it was used in the experiments, in the following Diagram, along with the curve exhibiting the corresponding reverse effect in the iron conductor.

The uppermost curve represents the results of three experiments with the Iron conductor (thirty slips), the points marked * representing the mean of three days' observation, and the points marked -+- that of two days' observation. The lowest curve represents the results of three experiments with the Copper conductor (thirteen slips), the points marked Y denoting three days', and the simple dots • two days' observation. The middle curve represents three experiments with the Copper conductor (two slips), the points marked ⊙ denoting the mean of three days' observation.

Fig. 4.



58. The diminution of the conducting power in the copper conductor had so markedly augmented the looked-for indication of a convective effect, that it was to be expected a corresponding augmentation might be obtained by treating the iron conductor similarly. Instead, however, of cutting up the iron conductor, which, as it stood, possessed sensibility enough to give a very decided result, I prepared a new iron conductor on a much smaller scale. It appeared that the smaller the conducting power for the same strength of current, and the same difference of temperatures between hot and cold, the greater would be the indication of convective effect; and the greatest indication would therefore be obtained by [*Continued after* TABLE VII.]

[Tables belonging to § 56.]

TABLE I. October 28th, 1853. Conductor composed of thirteen slips of sheet copper.

Middle of conductor in water kept boiling.

Streams of water at temperature 6° running through the coolers.

Temperatures at middle points A, B of the parts between heater and coolers:—Initial $T_A=55^\circ{\cdot}20$, $T_B=52^\circ{\cdot}94$; Final $T_A=54^\circ{\cdot}70$, $T_B=52^\circ{\cdot}68$.

Augmentations of difference T_A-T_B (in hundredths of a degree Cent.), during Periods	1. First half-minute of current entering by end next		2. Second half-minute of current entering by end next		3. Third half-minute of current entering by end next		4. Fourth half-minute of current entering by end next		5. Fifth half-minute of current entering by end next		6. Sixth half-minute of current entering by end next	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
I.	−5	10	12	2	−23	17	16	−16	−17	4	22	−5
II.	−1	−6	−4	7	6	2	2	−3	−2	9	−2	−6
III.	2	5	6	−10	0	13	0	4	11	−4	−2	−15
IV.	−5	0	1	7	4	−7	6	−1	−6	3	6	−2
V.	−1	15	7	−7	−8	1	8	0	1	−1	−1	−11
VI.	−8	10	9	8	−5	12	1	−3	3	3	−10	−9
Augmentations during half-minutes, summed for five periods	−13	24	19	5	−3	21	17	−3	7	10	−9	−43
Differences of augmentation during corresponding half-minutes of first and second halves of a period, summed for five periods	37		−14		24		−20		3		−34	
Differences of augmentation in equal intervals from beginning and middle of a period, summed for five periods	37		23		47		27		30		−4	
Mean augmentation of difference in favour of thermometer remote from entering current	°·037		°·023		°·047		°·027		°·030		−°·004	
after reversal and flow for	½ min.		1 min.		1½ min.		2 min.		2½ min.		3 min.	

Augmentations of difference T_A-T_B (in hundredths of a degree Cent.), during Periods	7. Seventh half-minute of current entering by end next		8. Eighth half-minute of current entering by end next		9. Ninth half-minute of current entering by end next		10. Tenth half-minute of current entering by end next		11. Eleventh half-minute of current entering by end next		12. Twelfth half-minute of current entering by end next	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
I.	−13	2	−4	2	8	−14	−10	1	2	−4	4	−8
II.	−2	−1	−6	7	5	−1	0	3	−5	−8	7	0
III.	−12	3	9	−3	−21	1	13	1	−5	0	4	1
IV.	5	−8	−5	14	1	3	0	−11	0	4	−2	−5
V.	−7	6	−3	−2	−6	−4	3	1	3	5	−8	−5
VI.	1	5	1	2	0	3	−7	−2	2	−6	−6	−1
Augmentations during half-minutes, summed for five periods	−15	5	−4	18	−21	2	9	−8	−5	−5	−5	−10
Differences of augmentation during corresponding half-minutes of first and second halves of a period, summed for five periods	20		22		23		−17		0		−5	
Differences of augmentation in equal intervals from beginning and middle of a period, summed for five periods	16		38		61		44		44		39	
Mean augmentation of difference in favour of thermometer remote from entering current	°·016		°·038		°·061		°·044		°·044		°·039	
after reversal and flow for	3½ min.		4 min.		4½ min.		5 min.		5½ min.		6 min.	

TABLE II. November 2nd, 1853. Con-

Middle of conductor in water kept

Streams of water at temperature

Temperatures of middle points A, B of the parts between heater and

Augmentations of difference $T_A - T_B$ (in hundredths of a degree Cent.), during Periods	1. First quarter-minute of current entering by end next		2. Second quarter-minute of current entering by end next		3. Third quarter-minute of current entering by end next		4. Fourth quarter-minute of current entering by end next		5. Fifth quarter-minute of current entering by end next		6. Sixth quarter-minute of current entering by end next		7. Seventh quarter-minute of current entering by end next		8. Eighth quarter-minute of current entering by end next		9. Ninth quarter-minute of current entering by end next		10. Tenth quarter-minute of current entering by end next	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
I.	−6	− 7	5	1	0	− 4	−3	−2	1	−5	−5	− 9	0	22	− 1	−5	−1	0	2	−11
II.	0	13	0	− 1	− 8	− 3	4	−6	10	−3	−4	− 4	− 1	2	− 4	7	2	9	− 5	−10
III.	7	− 1	6	0	− 5	3	−6	7	− 2	0	−7	− 8	7	−1	11	− 9	− 3	7	− 7	11
IV.	1	− 1	3	4	0	− 1	1	−2	− 1	−1	−6	2	− 2	9	7	− 3	10	− 2	− 3	− 5
V.	−5	9	2	− 5	13	0	−5	7	− 2	−1	3	− 6	5	−1	− 3	8	− 4	2	− 7	− 7
VI.	4	10	1	−15	1	18	−2	2	− 2	0	−3	0	− 3	3	3	− 1	10	0	− 6	0
VII.	−5	9	−1	3	− 5	−10	4	−1	7	5	4	− 9	5	4	− 8	15	− 4	1	− 6	− 7
VIII.	7	− 9	−2	− 1	− 5	0	−1	−2	− 8	4	6	5	12	−1	−10	− 5	2	− 6	− 1	0
Augmentations during quarter-minutes, summed for seven periods.................................	9	30	9	−15	− 9	7	−5	5	2	4	−7	−20	23	15	− 4	12	13	11	−35	−18
Differences of augmentation during corresponding quarter-minutes of first and second halves of a period, summed for seven periods	21		−24		16		10		2		−13		−8		16		−2		17	
Differences of augmentation in equal intervals from beginning and middle of a period, summed for seven periods.................................	21		−3		13		23		25		12		4		20		18		35	
Mean augmentation of difference in favour of thermometer remote from entering current...	°·015		−°·002		°·009		°·016		°·018		°·009		°·003		°·014		°·013		°·025	
.... after reversal and flow for	¼ min.		½ min.		¾ min.		1 min.		1¼ min.		1½ min.		1¾ min.		2 min.		2¼ min.		2½ min.	

ductor composed of thirteen slips of sheet copper.

boiling-hot by steam blown into it.

$10^{\circ}\cdot4$ running through the coolers.

coolers:—Initial $T_A=54^{\circ}\cdot10$, $T_B=52^{\circ}\cdot80$; Final $T_A=54^{\circ}\cdot68$, $T_B=53^{\circ}\cdot38$.

11.		12.		13.		14.		15.		16.		17.	
Eleventh quarter-minute of current entering by end next		Twelfth quarter-minute of current entering by end next		Thirteenth quarter-minute of current entering by end next		Fourteenth quarter-minute of current entering by end next		Fifteenth quarter-minute of current entering by end next		Sixteenth quarter-minute of current entering by end next		Seventeenth quarter-minute of current entering by end next	
A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
5	3	−6	10	−1	9	−7	−6	18	−11	−3	4	−5	−4
4	−1	0	−4	4	−2	−2	5	0	−2	−5	6	−4	9
3	−3	−6	−11	−2	2	6	7	14	0	−10	−10	−4	−3
−7	6	6	0	−5	0	−4	1	4	−5	5	8	4	3
0	−5	3	−8	−2	8	0	−4	8	8	−4	7	−7	3
10	−3	−10	1	−46	7	−24	5	57	−7	−12	−1	−9	−4
2	−2	4	4	5	−8	−8	0	−1	3	−1	6	7	−1
0	−5	−1	1	0	−2	3	1	3	2	3	−1	4	6
12	−13	−4	−17	−46	5	−29	15	85	−1	−24	15	−9	13
−25		−13		51		44		−86		39		22	
10		−3		48		92		6		45		67	
°·007		−°·002		°·034		°·066		°·004		°·032		°·048	
2¾ min.		3 min.		3¼ min.		3½ min.		3¾ min.		4 min.		4¼ min.	

18.		19.		20.		21.		22.		23.		24.	
Eighteenth quarter-minute of current entering by end next		Nineteenth quarter-minute of current entering by end next		Twentieth quarter-minute of current entering by end next		Twenty-first quarter-minute of current entering by end next		Twenty-second quarter-minute of current entering by end next		Twenty-third quarter minute of current entering by end next		Twenty-fourth quarter-minute of current entering by end next	
A	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
−2	3	10	6	−4	−5	4	9	−6	−13	7	−2	4	11
10	0	6	−2	3	−2	−7	−5	−7	1	−4	−6	4	1
−4	−9	−2	7	5	3	10	−1	−2	−4	−12	4	4	−4
−4	−4	1	−8	−4	1	−2	13	3	−1	10	−5	−3	−3
1	−8	13	−2	−4	−12	−4	8	−6	0	0	−2	6	1
−17	0	24	10	−3	3	4	−6	3	−3	−2	4	−5	−4
5	−1	−3	−3	8	−1	−9	13	−3	−3	−7	2	0	−8
−1	2	−3	2	2	4	13	0	−9	0	6	−4	0	−1
−10	−20	36	4	7	−4	5	22	−21	−10	−9	−7	6	−18
−10		−32		−11		17		11		2		−24	
57		25		14		31		42		44		20	
°·041		°·018		°·010		°·022		°·030		°·031		°·014	
4½ min.		4¾ min.		5 min.		5¼ min.		5½ min.		5¾ min.		6 min.	

TABLE III. November 26th, 1853. Con-

Middle of con-

Temperatures at the middle points A, B of the parts between heater and

Diminutions of difference $T_B - T_A$ (in hundredths of a degree Cent.), during Periods	1. First quarter-minute of current entering by end next A.	B.	2. Second quarter-minute of current entering by end next A.	B.	3. Third quarter-minute of current entering by end next A.	B.	4. Fourth quarter-minute of current entering by end next A.	B.	5. Fifth quarter-minute of current entering by end next A.	B.	6. Sixth quarter-minute of current entering by end next A.	B.	7. Seventh quarter-minute of current entering by end next A.	B.	8. Eighth quarter-minute of current entering by end next A.	B.	9. Ninth quarter-minute of current entering by end next A.	B.	10. Tenth quarter-minute of current entering by end next A.	B.	11. Eleventh quarter-minute of current entering by end next A.	B.	12. Twelfth quarter-minute of current entering by end next A.	B.	13. Thirteenth quarter-minute of current entering by end next A.	B.
I.	−1	−3	0	2	3	0	0	1	1	2	1	1	−1	1	−1	1	1	−2	1	0	−1	1	0	0	0	1
II.	0	0	0	0	0	1	0	1	0	0	1	1	0	0	0	1	0	1	0	0	−1	−1	1	1	1	0
III.	1	−1	2	−2	0	1	−1	0	−1	0	−1	0	1	0	0	−2	0	−2	0	0	−2	2	1	1	0	0
IV.	0	0	0	−1	−1	0	0	1	0	0	0	0	1	−2	0	1	0	1	0	1	1	2	1	1	2	0
V.	1	0	0	0	−1	1	0	−1	−2	0	−2	−1	−2	0	−1	0	−3	−1	−2	1	1	1	2	−1	0	−2
VI.	0	0	0	2	−1	−1	0	−1	−2	1	−1	0	−1	−1	0	−1	0	−2	−3	0	0	4	1	−1	1	−2
Diminutions during quarter-minutes, summed for five periods	2	−1	2	−1	−3	2	−1	0	−5	1	−3	0	−1	−3	−1	−1	−3	−3	−5	2	−1	8	6	1	4	−4
Differences of diminution during corresponding quarter-minutes of first and second halves of a period, summed for five periods	−3		−3		5		1		6		3		−2		0		0		7		9		−5		−8	
Differences of diminution in equal intervals from beginning and middle of a period, summed for five periods	−3		−6		−1		0		6		9		7		7		7		14		23		18		10	
Mean augmentation of difference in favour of thermometer remote from entering current ...	−°·003		−°·006		−°·001		°·000		°·006		°·009		°·007		°·007		°·007		°·014		°·023		°·018		°·010	
.... after reversal and flow for	¼ min.		½ min.		¾ min.		1 min.		1¼ min.		1½ min.		1¾ min		2 min.		2¼ min.		2½ min.		2¾ min.		3 min.		3¼ min.	

ductor composed of thirteen slips of sheet copper.

ductor at 99° Cent.

coolers:—Initial $T_A=50°{\cdot}96$, $T_B=52°{\cdot}89$; Final $T_A=48°{\cdot}80$, $T_B=50°{\cdot}92$.

14.		15.		16.		17.		18.		19.		20.		21.		22.		23.		24.		25.		26.		27.		28.		29.		30.		31.		32.	
Fourteenth quarter-minute of current entering by end next		Fifteenth quarter-minute of current entering by end next		Sixteenth quarter-minute of current entering by end next		Seventeenth quarter-minute of current entering by end next		Eighteenth quarter-minute of current entering by end next		Nineteenth quarter-minute of current entering by end next		Twentieth quarter-minute of current entering by end next		Twenty-first quarter-minute of current entering by end next		Twenty-second quarter-minute of current entering by end next		Twenty-third quarter-minute of current entering by end next		Twenty-fourth quarter-minute of current entering by end next		Twenty-fifth quarter-minute of current entering by end next		Twenty-sixth quarter-minute of current entering by end next		Twenty-seventh quarter-minute of current entering by end next		Twenty-eighth quarter-minute of current entering by end next		Twenty-ninth quarter-minute of current entering by end next		Thirtieth quarter-minute of current entering by end next		Thirty-first quarter-minute of current entering by end next		Thirty-second quarter-minute of current entering by end next	
A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
1	1	0	0	0	0	2	1	0	−2	−1	−1	1	−1	0	−1	0	−1	1	0	0	−1	0	0	0	0	0	−1	−1	−1	−1	2	−1	2	0	0	1	0
0	1	0	1	−1	−1	1	2	1	−1	0	−3	−1	−2	0	1	1	2	1	1	2	−2	1	−2	1	−1	0	1	0	0	−1	2	−1	0	1	1	0	1
1	1	−4	1	−1	0	−1	1	0	−1	−1	0	1	−3	0	−2	−1	−2	−1	0	−1	2	−2	−2	−1	0	0	−1	0	0	0	0	0	0	−1	1	0	0
3	1	1	0	−1	0	−3	0	−2	0	−1	−2	0	−1	1	0	0	2	−1	−1	1	1	0	0	−1	0	1	−1	1	−1	0	−1	1	−1	0	1	1	0
−2	−2	0	−1	1	1	0	1	1	1	0	0	1	2	−1	0	3	0	0	0	0	0	1	0	1	0	−1	−1	0	0	0	0	−2	0	0	−1	−1	0
1	−2	0	0	0	2	−4	1	−1	0	0	−2	1	0	1	0	0	1	0	−1	0	0	−1	−3	−1	0	−1	1	4	0	−1	1	0	1	0	2	1	0
3	−1	−3	1	−2	2	−7	5	−1	−1	−2	−7	2	−4	1	−1	3	3	−1	−1	2	1	−1	−7	−1	−1	−1	−1	5	−1	−2	2	−2	0	0	4	1	1
−4		4		4		12		0		−5		−6		−2		0		0		−1		−6		0		0		−6		4		2		4		0	
6		10		14		26		26		21		15		13		13		13		12		6		6		6		0		4		6		10		10	
°·006		°·010		°·014		°·024		°·026		°·026		°·015		°·013		°·013		°·013		°·012		°·006		°·006		°·006		°·000		°·004		°·006		°·010		°·010	
3½ min.		3¾ min.		4 min.		4¼ min.		4½ min.		4¾ min.		5 min.		5¼ min.		5½ min.		5¾ min.		6 min.		6¼ min.		6½ min.		6¾ min.		7 min.		7¼ min.		7½ min.		7¾ min.		8 min.	

TABLE IV. February 1854. Copper conductor reduced to four slips.

Stream of water at 60° Cent. through middle of conductor.

Streams of water at temperature 5°·9 running through coolers.

Temperatures at middle points A, B of the parts between heater and coolers:—Initial $T_A=44°·17$, $T_B=45°·36$; Final $T_A=45°·68$, $T_B=46°·05$.

Diminutions of difference T_B-T_A (in hundredths of a degree Cent.), during Periods	1. First half-minute of current entering by end next		2. Second half-minute of current entering by end next		3. Third half-minute of current entering by end next		4. Fourth half-minute of current entering by end next		5. Fifth half-minute of current entering by end next		6. Sixth half-minute of current entering by end next		7. Seventh half-minute of current entering by end next		8. Eighth half-minute of current entering by end next		9. Ninth half-minute of current entering by end next		10. Tenth half-minute of current entering by end next		11. Eleventh half-minute of current entering by end next		12. Twelfth half-minute of current entering by end next		13. Thirteenth half-minute of current entering by end next		14. Fourteenth half-minute of current entering by end next		15. Fifteenth half-minute of current entering by end next		16. Sixteenth half-minute of current entering by end next	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
I.	−1	4	−1	5	−2	7	1	4	1	6	−2	5	0	7	0	1	2	0	−1	0	2	1	0	3	2	1	4	−2	4	0	2	1
II.	4	−4	−1	−3	1	2	3	1	−3	2	6	1	0	2	−5	0	−1	1	−1	0	1	0	−1	0	−1	0	2	1	−1	0	2	−1
III.	−1	−1	3	0	−1	2	−1	1	1	1	0	0	0	2	0	2	−2	1	0	−1	1	0	1	1	−1	0	0	1	2	−1	0	0
IV.	−2	0	−1	3	3	2	−2	−2	−1	−2	2	1	−1	3	0	2	0	−1	0	−1	−1	1	1	0	0	1	−1	−1	1	0	0	−1
V.	−1	1	2	0	1	0	1	1	−1	1	0	1	0	2	−1	0	0	0	1	0	0	0	−1	0	1	0	−1	1	−1	0	0	0
VI.	−1	0	−1	2	1	−1	0	1	−1	1	0	0	0	−1	0	−1	0	0	−1	0	0	−1	0	1	0	0	3	0	0	0	0	0
Diminutions during half-minutes, summed for five periods	−1	−4	2	2	5	5	1	2	−5	3	8	3	−1	8	−6	3	−3	1	−1	−2	1	0	0	2	−1	1	3	2	1	−1	2	−2
Differences of diminution during corresponding half-minutes of first and second halves of a period, summed for five periods	−3		0		0		1		8		−5		9		9		4		−1		−1		2		2		−1		−2		−4	
Differences of diminution in equal intervals from beginning and middle of a period, summed for five periods	−3		−3		−3		−2		6		1		10		19		23		22		21		23		25		24		22		18	
Mean augmentation of difference in favour of thermometer remote from entering current	−°·003		−°·003		−°·003		−°·002		°·006		°·001		°·010		°·019		°·023		°·022		°·021		°·023		°·025		°·024		°·022		°·018	
after reversal and flow for	½ min.		1 min.		1½ min.		2 min.		2½ min.		3 min.		3½ min.		4 min.		4½ min.		5 min.		5½ min.		6 min.		6½ min.		7 min.		7½ min.		8 min.	

TABLE V. February 23rd, 1854. Copper conductor reduced to two slips.

Water kept at temperature 99°·5 Cent. in heater.

Streams of water at temperature 5°·6 through coolers.

Temperatures at middle points A, B of the parts of conductor between heater and coolers:—
Initial $T_A=51°·23$, $T_B=52°·48$; Final $T_A=58°·10$, $T_B=59°·61$.

Diminutions of difference T_B-T_A (in hundredths of a degree Cent.), during Periods	1. First half-minute of current entering by end next A.	1. B.	2. Second half-minute of current entering by end next A.	2. B.	3. Third half-minute of current entering by end next A.	3. B.	4. Fourth half-minute of current entering by end next A.	4. B.	5. Fifth half-minute of current entering by end next A.	5. B.	6. Sixth half-minute of current entering by end next A.	6. B.	7. Seventh half-minute of current entering by end next A.	7. B.	8. Eighth half-minute of current entering by end next A.	8. B.	9. Ninth half-minute of current entering by end next A.	9. B.	10. Tenth half-minute of current entering by end next A.	10. B.	11. Eleventh half-minute of current entering by end next A.	11. B.	12. Twelfth half-minute of current entering by end next A.	12. B.
I.	−5	−7	−14	5	−14	4	1	6	2	0	3	−1	−7	−1	−1	2	−4	−3	5	4	−6	−2	5	2
II.	−5	−1	4	5	−5	4	2	0	−4	2	4	2	3	0	3	−1	−2	3	1	−2	−2	−1	0	5
III.	−1	3	1	5	−3	5	2	−1	0	3	−3	−1	3	0	−1	0	−2	−1	3	3	0	1	−2	−1
IV.	−3	2	0	−1	0	5	−5	1	2	0	1	−1	0	4	1	0	−3	−1	2	−3	−2	1	−1	3
V.	1	0	−1	3	0	5	0	2	−6	1	−3	2	−1	1	2	−1	−1	−1	1	3	0	0	1	1
VI.	−3	−1	−3	6	−9	0	−5	2	−6	−2	−12	0	−1	2	−4	2	−1	1	−5	−2	1	2	−1	−2
VII.	−1	3	3	2	−2	1	−1	−3	−6	−6	0	−1	0	2	0	1	0	0	0	1	0	2	−3	9
Diminutions during half-minutes, summed for six periods	−12	6	4	20	−19	20	−7	1	−20	−2	−13	1	4	9	1	1	−9	1	2	0	−3	5	−6	15
Differences of diminution during corresponding half-minutes of first and second halves of a period, summed for six periods	18		16		39		8		18		14		5		0		10		−2		8		21	
Differences of diminution in equal intervals from beginning and middle of a period, summed for six periods	18		34		73		81		99		113		118		118		128		126		134		155	
Mean augmentation of difference in favour of thermometer remote from entering current ...	°·015		°·0283		°·06083		°·0675		°·0825		°·09416		°·0983		°·0983		°·107		°·105		°·11175		°·1291	
... after reversal and flow for	½ min.		1 min.		1½ min.		2 min.		2½ min.		3 min.		3½ min.		4 min.		4½ min.		5 min.		5½ min.		6 min.	

TABLE VI. March 7th, 1854. Copper conductor to two slips, and diminished in breadth to $1\frac{1}{2}$ inch.

Water kept at temperature 99° Cent. in heater. Streams of water about temperature 6° through coolers.

Temperatures at middle points A, B of the parts of the conductor between heater and coolers:—
Initial $T_A=50^\circ{\cdot}33$, $T_B=50^\circ{\cdot}70$; Final $T_A=61^\circ{\cdot}07$, $T_B=62^\circ{\cdot}62$.

Diminutions of difference T_B-T_A (in hundredths of a degree Cent.), during Periods	1. First half-minute of current entering by end next		2. Second half-minute of current entering by end next		3. Third half-minute of current entering by end next		4. Fourth half-minute of current entering by end next		5. Fifth half-minute of current entering by end next		6. Sixth half-minute of current entering by end next		7. Seventh half-minute of current entering by end next		8. Eighth half-minute of current entering by end next		9. Ninth half-minute of current entering by end next		10. Tenth half-minute of current entering by end next		11. Eleventh half-minute of current entering by end next		12. Twelfth half-minute of current entering by end next	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
I.	−1	−5	−4	4	−26	2	−13	−2	−38	2	−9	−1	−7	−4	·0	3	−7	0	−3	−1	0	−1	6	0
II.	−2	2	0	7	0	4	−2	1	1	1	−1	1	−3	1	4	2	−2	3	1	−4	0	2	−4	3
III.	−3	2	−3	6	−3	1	−2	−2	−3	5	−4	−2	1	−1	−4	−3	−2	1	0	0	−7	2	−1	2
IV.	1	10	−4	1	0	7	−2	−11	−2	−2	−4	2	3	3	−3	−3	0	−1	−2	0	0	1	−2	−2
V.	1	4	−1	−1	−2	5	−3	−3	6	3	−9	9	2	1	−5	9	2	−2	5	−7	−4	8	−3	4
VI.	1	2	2	3	−8	3	1	7	7	−2	−3	3	5	2	−4	4	2	0	4	0	−1	4	1	1
VII.	−12	−1	−4	6	−16	1	−15	1	−8	5	−4	3	−4	0	0	4	−9	0	−3	0	−3	−1	9	0
Diminutions during half-minutes, summed for six periods	−14	19	−10	22	−29	21	−23	−7	1	10	−25	16	4	6	−12	13	−9	1	5	−11	−15	16	0	8
Differences of diminution during corresponding half-minutes of first and second halves of a period, summed for six periods	33		32		50		16		9		41		2		25		10		−16		31		8	
Differences of diminution in equal intervals from beginning and middle of a period, summed for six periods	33		65		115		131		140		181		183		208		218		202		233		241	
Mean augmentation of difference in favour of thermometer remote from entering current ...	°·0275		°·0542		°·0958		°·1092		°·1167		°·1508		°·1525		°·1733		°·1817		°·1683		°·1933		°·2008	
... after reversal and flow for	½ min.		1 min.		1½ min.		2 min.		2½ min.		3 min.		3½ min.		4 min.		4½ min.		5 min.		5½ min.		6 min.	

TABLE VII. April 19th, 1854. Copper conductor two slips further diminished in breadth.

Water kept at temperature 99° Cent. in heater. Streams of water at about temperature 6° through coolers.

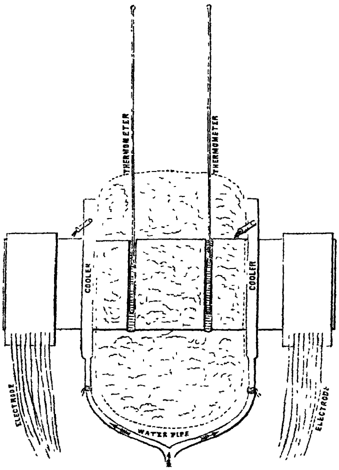
Temperatures at middle points A, B of the parts of the conductor between heater and coolers:— Initial $T_A=63°·65$, $T_B=63°·81$; Final $T_A=72°·90$, $T_B=75°·05$.

Diminutions of difference T_B-T_A (in hundredths of a degree Cent.), during Periods	1. First half-minute of current entering by end next A.	B.	2. Second half-minute of current entering by end next A.	B.	3. Third half-minute of current entering by end next A.	B.	4. Fourth half-minute of current entering by end next A.	B.	5. Fifth half-minute of current entering by end next A.	B.	6. Sixth half-minute of current entering by end next A.	B.	7. Seventh half-minute of current entering by end next A.	B.	8. Eighth half-minute of current entering by end next A.	B.	9. Ninth half-minute of current entering by end next A.	B.	10. Tenth half-minute of current entering by end next A.	B.	11. Eleventh half-minute of current entering by end next A.	B.	12. Twelfth half-minute of current entering by end next A.	B.
I.	0	−25	−11	−12	−40	−2	−29	8	−37	−2	−24	−6	−15	11	−39	1	−9	0	−11	−3	−9	−7	18	−2
II.	−1	−2	2	12	−1	1	6	−5	−2	2	−7	−3	3	−6	1	−5	−2	−2	2	−3	1	1	−1	0
III.	−2	−5	−2	1	−8	12	−5	6	−4	−1	−4	4	7	1	−14	0	−2	1	1	−1	−5	−1	−1	−1
IV.	−2	0	−2	5	1	2	−2	4	−2	−3	−6	1	−7	−1	0	−3	−1	2	0	1	3	0	1	1
V.	1	1	6	7	−1	7	−1	3	0	6	1	1	−1	−2	−4	4	−1	−2	0	4	1	−6	−1	−1
VI.	0	7	5	4	0	5	−1	1	−1	0	−2	2	1	2	−4	−1	−2	−2	−2	−3	−1	3	4	−3
VII.	1	3	−5	1	4	6	0	4	3	5	−6	−2	4	−5	−4	8	53	−16	−5	−5	3	−3	−47	0
Diminutions during half-minutes, summed for six periods	−3	4	4	30	−5	33	−3	13	−6	9	−24	3	7	−11	−25	3	45	−19	−4	−7	2	−6	−45	−4
Differences of diminution during corresponding half-minutes of first and second halves of a period, summed for six periods	7		26		38		16		15		27		−18		28		−64		−3		−8		41	
Differences of diminution in equal intervals from beginning and middle of a period, summed for six periods	7		33		71		87		102		129		111		139		75		72		64		105	
	°·0058		°·0275		°·0582		°·0725		°·0850		°·1075		°·0925		°·1158		°·0625		°·0600		°·0533		°·0875	
Periods I. and VII. off.																								
Diminutions during half-minutes, summed for five periods	−4	−1	9	29	−9	27	−3	9	−9	4	−18	5	3	−6	−21	−5	−8	−3	1	−2	−1	−3	2	−4
Differences of diminution during corresponding half-minutes of first and second halves of a period, summed for five periods	5		20		36		12		13		23		−9		16		5		−3		−2		−6	
Differences of diminution in equal intervals from beginning and middle of a period, summed for five periods	5		25		61		73		86		109		100		116		121		118		116		110	
Mean augmentation of difference in favour of thermometer remote from entering current	°·005		°·025		°·061		°·073		°·086		°·109		°·100		·116		°·121		°·118		°·116		°·110	
...after reversal and flow for	½ min.		1 min.		1½ min.		2 min.		2½ min.		3 min.		3½ min.		4 min.		4½ min.		5 min.		5½ min.		6 min.	

reducing the conductor so much that the current through it would generate heat enough to keep up the required difference of temperatures without any external heater.

59. The new conductor was therefore made of just two slips of sheet iron broad enough to admit the whole length of the thermometer-bulbs in the same manner as in the conductor previously used; these slips were bent in the places for the thermometer-bulbs, but were kept straight and bound close together elsewhere. Gutta-percha pipes were cut and cemented upon the iron slips near their ends, so as to lead streams of cold water across them. The part of the conductor between these coolers was packed round with a large mass of cotton wool, the thermometer-bulbs being

Fig. 5.



steadied in the apertures prepared for them by means of corks, as before (§ 31). The breadth of the conductor was $2\frac{1}{2}$ inches, the length between the coolers only $3\frac{1}{2}$ inches (instead of 10 inches, as in the iron conductors used previously), so that too great a time might not elapse before such a nearly permanent state of temperature as depended on the heating effect of the current would be reached.

60. On the 25th of March, 1854, an experiment was made with this conductor in the following manner:—A constant stream of cold water was maintained through each of the coolers; a current from the full nitric acid battery of eight large iron cells was sent through the conductor for twelve times four minutes in each direction, that is for ninety-six minutes in all, and the thermometers were noted every half-minute.

The actual observations of temperature are required to show the circumstances of this experiment, and I therefore give them as follows; instead of an analytical table, such as those by which the results of the preceding experiments were exhibited:—

61. Iron conductor heated by electric current. Ends kept cool by streams of water about 6° Cent. March 25, 1854.

Time in minutes and half-minutes from commencement of current.	Current.	Observed temperature at A by Kew, No. 114, T_A.	Observed temperature at B by Kew, No. 91, T_B.	Difference of observed temperatures $T_A - T_B = D$.
m 0	Started.	8·40	8·20	0·20
1	Entering by end next A.	12·70	11·00	1·70
2		18·40	14·70	3·70
3		22·60	18·00	4·60
4		28·00	22·30	5·70
5		32·30	26·20	6·10
6		35·84	29·70	6·14
7		39·60	33·50	6·10
4 0	Change.	41·30	36·00	5·30
1	Entering by end next B.	44·10	38·60	5·50
2		46·80	41·00	5·80
3		50·00	44·00	6·00
4		51·30	45·40	5·90
5		52·40	47·20	5·20
6		53·80	48·90	4·90
7		54·00	50·50	3·50
8 0	Change.	55·00	52·00	3·00
1	Entering by end next A.	55·60	52·40	3·20
2		54·70	51·90	2·80
3		53·50	51·10	2·40
4		53·42	50·80	2·62
5		53·65	50·70	2·95
6		53·84	50·75	3·09
7		54·19	50·88	3·31
12 0	Change.	54·50	51·10	3·40
1	Entering by end next B.	54·71	51·92	2·79
2		55·66	52·41	3·25
3		56·41	53·22	3·19
4		57·50	54·23	3·27
5		58·65	55·30	3·35
6		59·63	56·23	3·40
7		60·31	57·19	3·12
16 0	Change.	60·65	57·80	2·85
1	Entering by end next A.	60·40	57·70	2·70
2		60·65	57·63	3·02
3		60·92	57·63	3·29
4		61·30	57·79	3·51
5		61·87	58·09	3·78
6		62·41	58·45	3·96
7		62·82	58·76	4·06
20 0	Change.	63·39	59·18	4·21
1	Entering by end next B.	63·30	59·37	3·93
2		63·70	60·13	3·57
3		64·19	60·75	3·44
4		64·84	61·64	3·20
5		65·18	62·08	3·10
6		65·71	62·70	3·01
7		66·14	63·14	3·00

Time in minutes and half-minutes from commencement of current.	Current.	Observed temperature at A by Kew, No. 114, T_A.	Observed temperature at B by Kew, No. 91, T_B.	Difference of observed temperatures $T_A - T_B = D$.
m 24 0	Change.	66·61	63·71	2·90
1	Entering by end next A.	66·49	63·70	2·79
2		66·50	63·50	3·00
3		66·50	63·32	3·18
4		66·58	63·11	3·47
5		66·56	62·92	3·64
6		66·60	62·80	3·80
7		66·69	62·73	3·96
28 0	Change.	66·53	62·59	3·94
1	Entering by end next B.	65·72	62·31	3·41
2		65·63	62·32	3·31
3		65·54	62·50	3·04
4		65·64	62·66	2·98
5		65·91	62·93	2·98
6		66·19	63·28	2·91
7		66·64	63·71	2·93
32 0	Change.	66·82	61·15	2·67
1	Entering by end next A.	67·09	64·01	3·08
2		67·75	64·13	3·62
3		68·18	64·30	3·88
4		68·43	64·39	4·04
5		68·57	64·41	4·16
6		68·71	61·43	4·28
7		68·83	64·48	4·35
36 0	Change.	69·03	64·48	4·55
1	Entering by end next B.	68·28	64·32	3·96
2		68·00	64·38	3·62
3		67·85	64·50	3·35
4		67·91	64·72	3·19
5		68·16	65·03	3·13
6		68·59	65·50	3·09
7		69·02	65·91	3·11
40 0	Change.	69·45	66·40	3·05
1	Entering by end next A.	69·92	66·65	3·27
2		70·92	67·08	3·84
3		72·00	67·70	4·30
4		72·59	68·00	4·59
5		73·01	68·20	4·81
6		73·45	68·50	4·95
7		73·90	68·80	5·10
44 0	Change.	73·91	68·95	4·96
1	Entering by end next B.	73·14	68·72	4·42
2		72·70	68·82	3·88
3		72·61	69·08	3·53
4		72·76	69·53	3·23
5		73·28	70·15	3·13
6		73·81	70·70	3·11
7		74·18	71·10	3·08

Time in minutes and half-minutes from commencement of current.	Current.	Observed temperature at A by Kew, No. 114, T_A.	Observed temperature at B by Kew, No. 91, T_B.	Difference of observed temperatures $T_A - T_B = D$.
m 48 0	Change.	74·56	71·58	2·98
1	Entering by end next A.	74·64	71·56	3·08
2		74·93	71·40	3·53
3		75·23	71·30	3·93
4		75·21	71·00	4·21
5		75·30	70·80	4·50
6		75·32	70·71	4·61
7		75·33	70·63	4·70
52 0	Change.	75·12	70·40	4·72
1	Entering by end next B.	73·61	70·24	3·37
2		72·60	69·20	3·40
3		71·88	68·90	2·98
4		71·53	68·74	2·79
5		71·25	68·63	2·62
6		71·14	68·60	2·54
7		71·13	68·61	2·52
56 0	Change.	71·12	68·59	2·53
1	Entering by end next A.	71·48	68·56	2·92
2		72·30	68·80	3·50
3		73·19	69·22	3·97
4		73·71	69·53	4·18
5		74·32	69·92	4·40
6		74·82	70·30	4·52
7		75·20	70·60	4·60
60 0	Change.	75·56	70·75	4·81
1	Entering by end next B.	75·12	71·18	3·94
2		75·16	71·61	3·55
3		75·23	72·03	3·20
4		75·47	72·46	3·01
5		75·82	72·90	2·92
6		76·14	73·35	2·79
7		76·15	73·60	2·55
64 0	Change.	76·40	73·72	2·68
1	Entering by end next A.	76·90	73·55	3·35
2		77·60	73·60	4·00
3		78·12	73·60	4·52
4		78·40	73·57	4·83
5		78·50	73·50	5·00
6		78·54	73·40	5·14
7		78·50	73·31	5·19
68 0	Change.	78·40	73·16	5·24
1	Entering by end next B.	76·80	72·46	4·34
2		76·61	72·70	3·91
3		76·90	73·10	3·80
4		77·30	73·65	3·65
5		77·91	74·22	3·69
6		78·41	74·82	3·59
7		78·90	75·40	3·50

Time in minutes and half-minutes from commencement of current.	Current.	Observed temperature at A by Kew, No. 114, T_A.	Observed temperature at B by Kew, No. 91, T_B.	Difference of observed temperatures $T_A - T_B = D$.
m 72 0	Change.	78·70	75·40	3·30
1	Entering by end next A.	79·55	75·70	3·85
2		81·31	76·41	4·90
3		82·82	77·21	5·61
4		83·90	78·00	5·90
5		84·81	78·58	6·23
6		85·47	79·09	6·38
7		85·73	79·42	6·31
76 0	Change.	85·60	79·58	6·02
1	Entering by end next B.	84·50	79·45	5·05
2		84·20	79·61	4·59
3		84·11	79·90	4·21
4		84·20	80·30	3·90
5		84·49	80·82	3·67
6		84·68	81·20	3·48
7		84·91	81·50	3·41
80 0	Change.	85·10	81·71	3·39
1	Entering by end next A.	84·96	80·20	4·76
2		85·11	80·92	4·19
3		85·19	80·63	4·45
4		85·20	80·35	4·85
5		85·09	79·95	5·14
6		84·66	79·54	5·12
7		84·31	79·07	5·24
84 0	Change.	84·00	78·70	5·30
1	Entering by end next B.	82·48	77·93	4·55
2		81·30	77·50	3·80
3		80·50	77·18	3·32
4		79·81	76·84	2·97
5		79·42	76·58	2·84
6		79·11	76·40	2·71
7		79·02	76·22	2·80
88 0	Change.	78·80	76·05	2·75
1	Entering by end next A.	77·89	75·03	2·86
2		77·50	74·36	3·14
3		77·10	73·57	3·53
4		76·70	72·97	3·73
5		76·30	72·40	3·90
6		76·00	71·90	4·10
7		75·50	71·45	4·05
92 0	Change.	75·18	71·10	4·08
1	Entering by end next B.	75·14	71·20	3·94
2		75·91	71·95	3·96
3		76·68	72·50	4·18
4		77·43	73·55	3·88
5		78·10	74·40	3·70
6		78·70	75·00	3·70
7		78·40	75·70	2·70
96 0		78·70	76·20	

The differences of temperatures here tabulated, and the half sums of the same temperatures, are graphically represented in Plate XXXV. The result is obvious, either with or without the graphical representation, and affords a striking confirmation of the conclusion first arrived at by so different an apparatus (§ 31), that *the Resinous Electricity carries heat with it in iron.*

62. About the same time another form of the experiment was tried on a copper tube, with a vessel of oil fitted round it in the middle, and kept hot by a lamp below it, and with gutta-percha tubes fitted to conduct streams of cold water round it. A current from the battery was sent alternately in the two directions through it, as in the previous experiments, and it was attempted to observe the thermal effects by means of two open thermometer-tubes with small spherical bulbs, pushed into the copper tube from each end, and bent down at right angles outside it, with their lower ends immersed in two cups of spirits of wine. The want of any sufficient regulation of temperature to keep the liquid column of these air-thermometers within range, made it impossible to get any clear indication of a result by this experiment; but on the whole, there appeared to be an effect of the same kind as had been previously discovered in copper.

63. A few weeks ago, I began again to make direct experiments on electrical convection with a view to obtaining additional evidence in support of the conclusion which I had arrived at previously, and to investigate methods by which the nature of the quality in other metals could be discovered more readily, and the specific heat of electricity in any metal determined in absolute units. I had determined to give up the use of the nitric acid battery in consequence of the inconveniences which had been alluded to above (§ 34), and accordingly I had constructed a large DANIELL's battery: consisting altogether of eight wooden cells lined with gutta percha, and fitted with sheet copper, suitably arranged with shelves to bear crystals of sulphate of copper; sixteen porous cells, some of which had served previously in the iron battery; and sixteen zinc plates of the same dimensions as those previously used. Each wooden cell had sheet copper not only round its interior, but also a portion of the same sheet carried across it so as to divide it into two spaces, each completely surrounded by the metallic surface. A porous cell is put into each of these spaces, and a zinc plate into each porous cell; the two zincs in the porous cells contained in the same wooden cell being always united. The ordinary liquids of a DANIELL's battery, acidulated solution of sulphate of copper and dilute sulphuric acid, are used. The whole battery power thus consists of eight independent cells, which, with the connexions in ordinary use, may be arranged either in one or in two elements, but which may also, should there be occasion, be readily enough set up in four or in eight elements. Any power may of course be used down to the lowest, with only a single porous cell and a single zinc plate in one of the wooden cells. The sulphate of copper solution is kept constantly in the wooden cells, which remain in a fixed position on a shelf. Electrodes from the large commutator (§ 27), which is fixed

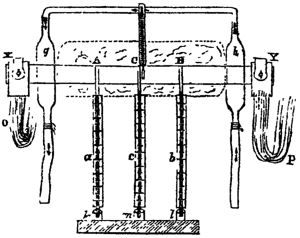
to the wall in an adjoining apartment as near as possible to the middle of the wooden cells, are brought through the partition between the two rooms, and kept always ready to be put in communication with the two poles of the battery, however arranged.

This battery, or parts of it, have been used in nearly all the experiments described below in Parts IV. and V., and it has been found very convenient. Some of the wooden cells have contained the acidulated solution of sulphate of copper now for more than a year [*for more than two years now, Nov.* 1856], and as yet their gutta-percha linings have shown no signs of injury.

64. The first of the recent experiments on electrical convection was made with an iron conductor prepared as follows :—

The conductor, XY, consists of two pieces of thin sheet iron $8\frac{1}{2}$ inches long and $\frac{3}{4}$ of an inch broad, and bent so that when put together they form three tubular spaces, A, B, C, fig. 6. The iron is cut so as to make prolongations of these tubes of about

Fig. 6.



an inch beyond one side of the conductor. The slips thus put together are soldered so as to make the tubes perfectly air-tight, one end being closed, and the other left open to receive the thermometer-tubes *a, b, c,* which were cemented air-tight with wax. In soldering, great care was taken to prevent the solder from spreading between the iron slips. Copper electrodes were now soldered to the ends of the conductor, and the junctions were enclosed within pieces of gutta-percha tube, *g, h,* through which a continuous stream of cold water was made to flow. The distance between the coolers was $7\frac{1}{2}$ inches, and they were placed so that the four spaces between them and A, B, C were all equal. Divided scales were attached to the tubes, of which the lower ends were immersed in small vessels, *k, l, m,* containing spirits of wine. The conductor between the coolers was wrapped in a large quantity of cotton wool represented by the space within the dotted line.

To send a current through the conductor thus prepared, the whole battery, arranged, as described below, in two elements, each exposing ten square feet of zinc surface to seventeen square feet of copper, was employed :—

Description and Drawing of Battery with Connexions.

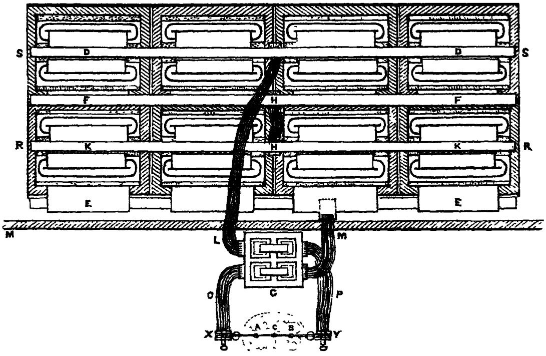
R, R and S, S, two series of cells, each containing eight porous cells and eight zinc plates.

K, K and D, D, thick copper supports for the zinc plates, the zincs of R, R being firmly clamped to K, K, and those of S, S to D, D.

E, E, a thick conductor connecting the coppers of series R, R together.

F, F, a similar conductor connecting the coppers of S, S.

Fig. 7.



H, H, a bundle of wires connecting the coppers of S, S with the zincs of R, R.

M, M, a wooden partition separating the battery-room from the experimenting-room. L and M are two bundles of wire which pass through holes in the partition and connect the commutator G respectively with the coppers of series R, R, and the zincs of series S, S. The bundles of wires O and P complete the circuit through XY, the conductor to be tested.

The dotted spaces round the porous cells represent shelves for holding crystals oı sulphate of copper.

65. After about an hour and a half, the thermometer at the middle of the conductor indicated 170° Fahr. (76°·7 Cent.); and one of the brass bridges of the commutator was then lifted so as to break the circuit. Immediately the liquid mounted rapidly in each of the three glass tubes of the air-thermometers, and it was prevented from rising above a certain point in the middle one by completing the circuit again. The column of liquid was kept as steady as possible at this point in the middle air-thermometer by a person observing it, and making and breaking the circuit by means of the brass bridge, while two other persons noted the indications of the two lateral air-thermometers. The current was reversed every three or four minutes, and the liquid in the middle air-thermometer brought back to the same point, and kept as nearly as possible to it. The imperfection of the regulating system was such as to make it very difficult to prevent great oscillations in the thermometers, but the instantaneous manner in which their indication followed the operations of the break made it certain that the plan would be perfectly successful when a continuously acting regulator should be introduced.

66. As it was, the result afforded a most striking and immediate confirmation of the conclusion previously arrived at regarding the electrical convection of heat in iron. Every time the current was reversed, the liquid fell rapidly (showing a rise of temperature) in the thermometer next the end, by which the current nominally entered, and rose rapidly (showing a fall of temperature) in the other.

Mr. JOULE assisted in this experiment, and was satisfied with the evidence it afforded in favour of the conclusion that *the Resinous Electricity carries heat with it in iron.*

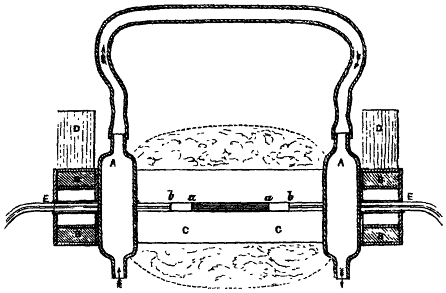
67. Unsuccessful attempts were next made with tubular conductors of different metals; and in endeavouring to get decisive results regarding the qualities of copper and brass, I again had recourse to the form of conductor used in the preceding experiment. The new conductors were, however, made of much thinner sheet metal than those of the iron, to admit of a less powerful battery being used; and consequently, in each case, a frame-work had to be arranged to hold the conductor steady. Great difficulties were met with in continually repeated failures of the air-thermometers. It was therefore found necessary to have metallic tubes continued downwards several inches from the bulbs, so as to prevent the wax by which the glass was cemented from being melted by the heat. The battery, however, had also to be reduced to a single zinc plate in one of the wooden cells, as with more of the battery than this, the heating action had been found to be so sudden in the thin copper and brass conductors, as almost immediately to melt the solder about some of the bulbs, and so make one or more of the thermometers fail before the regulating action of the break was applied. Notwithstanding all precautions, the central thermometer failed in each case, and the action of the lateral thermometers was very unsatisfactory both in the copper and in the brass conductor. The central thermometer could, however, be well dispensed with by regulating by the break one or other of the lateral thermometers; and thus, after many unsuccessful attempts, experiments were made on copper and brass conductors, which, although still unsatisfactory, showed decidedly the looked-for convective effect. In each case, the thermometer which was not kept to one point by the regulator, always showed an increase of temperature, both in the copper and in the brass conductor, when the current was reversed so as to enter by the end remote from it, and showed a diminution of temperature when the current was again reversed so as to enter by the end next it. Hence it appeared that *the Vitreous Electricity carried heat with it in both copper and brass.*

68. The lateral metallic tubes branching down from the conductor to carry the glass tubes of the air-thermometer, constituted a great defect in the plan of apparatus used in the experiments just described; and the only way of avoiding it appearing to be to make the glass tubes pass through the body of the conductor itself, so as to admit of their being cemented air-tight at its cool ends, I again had recourse to the tubular form of conductor which had been tried unsuccessfully before.

69. A tube made of very thin sheet platinum, soldered with gold, was arranged in the following manner:—A glass rod, $2\frac{1}{8}$ inches long, wrapped closely round with thin

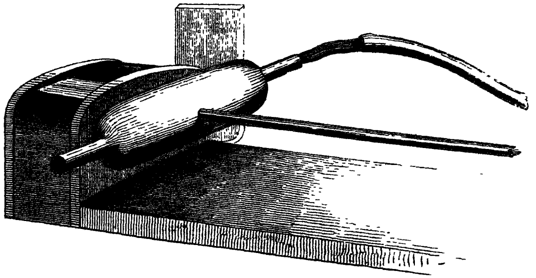
cotton-thread, was pushed into the central part of the tube, in which it fitted closely, and was carefully luted with red lead. After keeping it for several days heated by a stove, gutta-percha coolers, A, A, were fitted on it, leaving a length of 6 inches of tube between them. Wooden troughs, B, B, were then fitted on outside the coolers, and fastened to the ends of a piece of wood, C, C; straps of thick copper, about an inch broad, were bent to form conducting linings for the troughs, their ends turned round, firmly fastened to C, C, and brought together at D, D, thus forming con-

Fig. 8.



nexions with the electrodes of the commutator (for this part of the arrangement see also fig. 9). Two pieces of thermometer-tube, bent to right angles, had their short arms rolled with thread, and were pushed into the tube from its ends, as far as *b*, *b*, leaving spaces *ab*, *ab*, each two-thirds of an inch, between them and the stopper *aa* in the middle of the tube, and made air-tight by cement applied at E, E. The dotted line represents the space round the tube and its wooden stand C, C, filled with cotton-wool. A conducting communication was established between the platinum tube and D, D, by pouring mercury into the troughs (see also fig. 9).

Fig. 9.



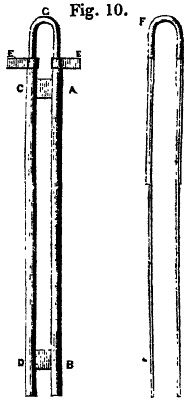
70. The system of regulating the temperature in one part of the conductor by breaking and making the circuit, had been adopted only as a temporary expedient in the experiment on the iron conductor (§ 65), in consequence of the failure of a continuous regulator which had been fitted up for that experiment. It had the advantage

of requiring no other apparatus than the commutator, in regular use in all applications of the battery, and it had been found to answer the purpose tolerably well in the first trial. It proved, however, very inconvenient with the finer conductors, from the too great abruptness of its action. Besides, it was open to this very serious objection, that it kept up the required heating effect by an intermittent current, and therefore by the passage of a much less quantity of electricity than would be required to produce the same heating effect if flowing in a nearly constant current (the rate of generation of heat being proportional to the square of the strength of the current at each instant, while the looked-for convective effect is proportional simply to the strength of the current at each instant, and is therefore, on the whole, proportional to the whole quantity of electricity that passes).

In order, therefore, that the current might be kept as nearly as possible constant at the particular strength required to maintain the heating effect used, I had the following regulator constructed.

71. Two iron tubes, AB, CD, 20 inches long and $\frac{3}{4}$ths of an inch in diameter, open at the top but closed at the bottom, are bound firmly together with insulating blocks of wood, AC and BD, so as to be parallel to one another. Pieces of thin sheet copper are bent into cylinders; to their tops pieces of thick copper, E, E, are soldered, and the copper cylinders are put into the iron tubes. To each end of a piece of thick copper wire, shown separately at F, two pieces of No. 18 iron wire are fixed, one of the same length as the iron tubes, and the other less than half that length, and the two branches are parallel, and at such a distance that when their ends are introduced into the two tubes, they move along their axes. To use the regulator, the tubes are filled with mercury, the apparatus is put into the circuit by connecting with EE, and the requisite amount of resistance is introduced by raising G, which is kept in any position by having one end of a cord fixed to its upper part, carried over a pulley, and stretched by a counterpoise hung at its other end. [Great improvement has been since made in the regulator, by using, instead of No. 18 iron wire, thick copper wire tapering to points at the lower ends; and by attaching cups of gutta percha to the tops of the iron tubes allowed to communicate with the interior by small holes, to serve as overflow cisterns for the mercury. By this arrangement the tubes were kept always full of mercury, and irregular contacts between the connecting conductor and the interior of empty parts of the tubes were prevented.—*Nov.* 1856.]

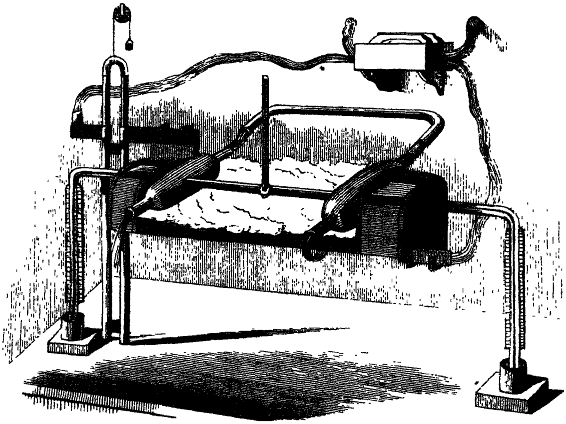
Fig. 10.



72. The apparatus was set up as shown in the accompanying view. The battery connexions were completed with the regulating break partly up, so as to check the current somewhat, and prevent injury from sudden overheating in any part of the conductor.

After a few minutes the break was raised further so as to reduce the current very much, and the liquid began to rise in the stem of each of the glass tubes, showing that both air-thermometers at first acted perfectly. One of the thermometers was then steadied with great ease to a small fraction of a scale-division by using the regulator. The liquid in the other thermometer was observed, and its position occasionally noted. The direction of the current was reversed every few minutes, as before, by means of the ordinary commutator.

Fig. 11.



73. Slight differences were observed in the free thermometer after the reversals, but as yet no very decisive indications of the looked-for effect appeared. The mercurial thermometer beside the central conductor indicated less than 80° FAHR. (27° Cent.), its column of mercury having not yet become visible, after the experiment had been continued in this way for several reversals.

74. The regulating break was then pushed down until a somewhat further elevation in the temperature of the platinum was indicated by a considerable escape of air in bubbles from the open ends of the thermometer-tube. The break was again drawn up until the liquids again mounted in the stems. One of the thermometers was again steadied by the regulator, and, the other being observed, the experiment was continued as before. A decided effect now appeared almost immediately after each reversal. The free thermometer regularly indicated a higher temperature when the current nominally entered by the end next it, and a lower temperature when the current nominally entered by the remote end. After four reversals this part of the experiment had lasted about twenty minutes, and the mercury thermometer beside its middle showed 104° FAHR. (40° Cent.).

75. The break was again pushed down for some time, and again raised till the liquid rose in each thermometer-tube, and the experiment was continued as before

for four reversals, the central mercury thermometer rising to about 150° FAHR. (66° Cent.). The free thermometer rose and fell alternately through several scale-divisions almost immediately after each reversal, and showed the same convective effect as had previously been observed by smaller indications.

76. The bridge was again pushed down and air again escaped copiously from the thermometers, but very soon beads of liquid began to appear following one another rapidly down the capillary tubes from the interior of the conductor.

As the spirits of wine had not once been allowed to run up into the bulb of either thermometer, these beads of liquid could be nothing but products of the distillation of the oil which had been used in the luting of the central plug; and on taking away the cups of spirits of wine from below the tubes, the smell and taste of the small quantities of liquid which continued to descend gave unmistakeable evidence of their origin. After this it was scarcely possible to get any satisfactory indication from either of the air-thermometers; but the experiment was continued, and one or other of them, when by any means the beads of disturbing liquid could be sufficiently got rid of for a time, was steadied to a constant temperature; the other thermometer being observed when possible, and the reversals repeated as before.

The same result was still obtained; and on the whole, notwithstanding the defect which caused so much inconvenience, it was very decidedly established by the experiment that *the Resinous Electricity carries heat with it in platinum.*

77. [*Added Dec.* 1856.]—After many unsuccessful trials on short brass tubes, first with air-thermometers of the metal itself and capillary glass tubes arranged as in the platinum tube (§ 69), and latterly with glass air-thermometers (§ 62) having very small cylindrical bulbs, the following conclusive experiment was made a few days ago. Four of the large double cells, connected to form a single DANIELL's element, exposing 10 square feet of zinc to 17 square feet of copper, were used to send a current through a piece of brass telescope tube 6 inches long, $\frac{1}{4}$ of an inch diameter, and ground as thin as it could be without breaking it up by emery-paper, over the length of $3\frac{1}{2}$ inches which was left between the near sides of gutta-percha coolers, fitted to it in the manner represented above (see fig. 11, § 72). Streams of water being, as in other experiments, kept running through the coolers, and the regulating break (§ 71) being used to keep the liquids within range in the tubes of the air-thermometers, a small mercurial thermometer pressed against the middle of the brass tube, with its stem and scale projecting out through the cotton wool, indicated from 190° to 195° FAHR. (90°·6 Cent.).

The regulator was not used so much as it might have been with advantage; but, notwithstanding great unsteadiness in the indications of the two air-thermometers, the observations showed decidedly, after each reversal of the current, a cooling effect on the thermometer next the entering stream, in every case in which the irregularities were not so great as to make a comparison impossible. This effect is manifest from the following four cases, selected merely as being those in which one of the

thermometers was most nearly steady during a few minutes of flow of the current, first in one direction and then in the other.

Current entering by end next		Readings, in arbitrary scale-divisions, of Thermometer A.	Thermometer B.
I.	A	43	57½
	B	42	44½
II.	A	41	41
	B	41	34½
III.	A	31½	26½
	B	29½	16½
IV.	A	27½	22
	B	20½	12½

Hence the conclusion (see below, §§ 102 and 103), that *the Vitreous Electricity carries heat with it in brass*, which I anticipated three years ago from the mechanical theory *, is now established by a direct experimental demonstration.

PART II. ON THERMO-ELECTRIC INVERSIONS.

78. CUMMING's discovery of thermo-electric inversions having afforded the special foundation of that part of the theory by which I ascertained the general fact of electric convection in metals, and every observation of a thermo-electric inversion being a perfect test as to the relative positions of the two metals between which it is observed in the Table of Convections (see below, § 103), I was induced to make experiments with a view to finding new instances of inversion, and to determine in each case, with some degree of precision, the temperature at which the two metals are thermo-electrically neutral to one another.

79. In the experiments on thermo-electric inversion described by CUMMING, and by BECQUEREL, the only other experimenter, so far as I am aware, who has published observations on the subject, one junction between the two metals is generally kept cool, while the other is raised until the current indicated by the galvanometer, instead of going on increasing, begins to diminish, comes to a stop, and then sets in the reverse direction†.

80. In this way CUMMING found that "if gold, silver, copper, brass, or zinc wires be heated in connexion with iron, the deviation [indicating the current], which is at first positive, becomes negative at a red heat‡." Many other experimenters have professed themselves unable to verify these extraordinary results, and have attempted

* See "Dynamical Theory of Heat," § 132.

† CUMMING's 'Electro-Dynamics,' section 104, p. 193. Cambridge, 1827.

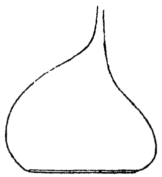
‡ Cambridge Philosophical Transactions, 1823, addition to p. 61.

to explain them away by attributing them to coatings of oxide formed on the metals, or to other causes supposed with equally little reason to exercise sensible disturbing influences; but the descriptions, given by the original observers, of their experiments leave no room for such doubts. It is certainly not easy to get the inversion between copper and iron (with such specimens as I have tried) by the heat of a spirit-lamp, applied as described by BECQUEREL to one junction while the other is left cool; but I readily obtained it by raising the other junction somewhat in temperature, with the first still kept at a red heat. Probably if the atmospheric temperature had b.en higher, or if a somewhat more intense red heat had been obtained from the spirit-lamp, I should at once have obtained the result simply in the manner described by the previous observers.

81. The easiest way to verify the thermo-electric inversion of iron and copper is to take a piece of iron wire a foot or two long, and twist firmly round its ends two copper wires connected with the *electrodes* of any ordinary astatic needle-galvanometer. Then first heat one of the junctions with the hand, or by holding it at some height over a flame; and note the deflection, which will be found such as to indicate a current from copper to iron through the hot junction. Again, heat both junctions in flame, or in sand at any temperature above 300° Cent., and withdraw one a little from the hottest place, so that while both junctions are at temperatures above 300° Cent., that which was heated in the first experiment may be still decidedly hotter than the other. The deflection will now be found to be the reverse of what it was before, and will be such as to indicate a current from iron to copper through hot. The reversal of the current may be very strikingly exhibited by allowing the two junctions gradually to cool, while ensuring that the same one remains always somewhat above the other in temperature. When the mean of the temperatures of the two junctions falls below 280° Cent. or thereabouts, the primitive deflection will be again observed. All these phenomena are observed indifferently whether the copper wires be simply twisted on the ends of the iron wire, or brazed to them, or tied to them by thin platinum or iron wire.

82. Similar phenomena may be observed without the necessity of going to so high temperatures, by soldering galvanometer electrodes of copper to the ends of a double platinum and iron wire, and treating this compound circuit in the manner just described, only with a more moderate application of heat. If the platinum wire be very thin in comparison with the iron one connected along with it, the circumstances will be but little altered from those observed when iron simply is used. By taking a thicker platinum wire, or several thin ones together, in connexion with the same iron wire, or by using a thinner iron wire and the same platinum, the neutralization and reversal may be shown with temperatures below the boiling-point. Most specimens of platinum wire thus applied reduce the neutral point of copper and the compound platinum and iron

Fig. 12.



wire much below the temperature of melting ice, when the proportion of platinum to iron in the bundle is sufficiently increased (the limit, of course, being the neutral point of copper to the platinum itself. See below, §§ 83, 84).

83. A certain specimen of platinum wire in my possession, when tested by such elevations of temperature as could be produced by the hand, was found to lie in the thermo-electric series, on the other side of copper from the position in which platinum is placed in all statements of the thermo-electric qualities of metals previously published. That is to say, when connected by copper electrodes with the circuit of a galvanometer, and when heated at one junction up to ten or twenty degrees above the atmospheric temperature, a current set from copper to platinum through hot. On raising the temperature of the hot junction towards the boiling-point of water, the strength of the current began to diminish; came to a stop when a temperature I suppose little above that of boiling water was reached; and set in the reverse direction with increasing strength when the temperature of the hot junction was further raised, the other junction being kept all the time at the atmospheric temperature. I afterwards found that this specimen of platinum wire (referred to under the designation P_1 in what follows) became neutral to ordinary copper wire at the temperature 64° Cent.

84. Of two other specimens of platinum wire which I tried with copper, one (marked P_2) gave indications of a neutral point about the zero of FAHRENHEIT's scale, but the other (P_3) remained, for the lowest temperatures I reached, always on the same side of copper as that on which platinum appears, at ordinary and at high temperatures, generally to lie. When these three platinum wires were tried with one another thermo-electrically, they gave, as was to be expected, the mutual thermo-electric indications of different metals lying in the order Bismuth, P_3, P_2, P_1, Iron, Antimony. They retained all the same qualities after being heated to redness; and in a great many experiments performed upon them, in which I have found them extremely convenient as thermo-electric standards, have exhibited perfect constancy in their thermo-electric bearings.

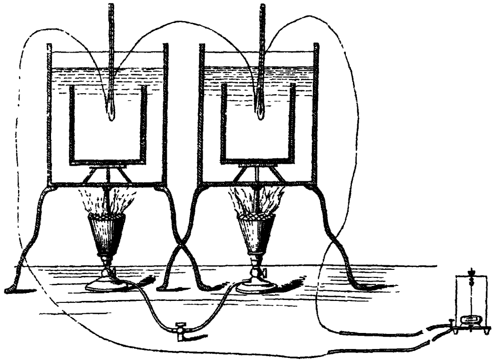
I have not yet discovered on what their differences depend, but in all probability it is on the degrees to which they are alloyed with other metals.

85. The fact of copper changing in the thermo-electric series from below the position of the platinum specimen P_2 to above that of iron, when the temperature is raised from $-30°$ or $-20°$ Cent. to 300°, proves that every metal which lies between P_2 and iron for any intermediate temperature, must become neutral to either P_2 or copper, or iron at some temperature between these limits. Now nearly all the common metals, for instance, lead, tin, brass, zinc, silver, cadmium, gold, lie between platinum and iron in the thermo-electric series at ordinary temperatures, and no doubt many of the rarer metals (I have found aluminium to lie between P_3 and P_2) are to be ranked within the same limits. Hence at temperatures easily reached and tested, neutral points may be looked for with the certainty of finding them, between each of those metals and one or other, if not several, of the metals and metallic

specimens (P_2, P_1, Copper, Iron) referred to above. Taking then the platinum specimens P_1, P_2, P_3 as standards, and using besides ordinary copper and iron wires, I commenced investigating their thermo-electric relations to as many other metals as I could obtain.

86. In experiments to determine temperatures of neutrality, the first apparatus which I employed for regulating the temperatures of the two junctions, consisted of copper vessels placed side by side in which oil could be raised by gas-burners as high in temperature as the mercurial thermometer can be used, that is to 340° or 350° Cent., or somewhat above the boiling-point of mercury. To do away with irregularities from the flame and cold air playing unequally on the sides of these vessels, smaller ones were placed on wire stands within them, and were completely filled and surrounded with oil. In each experiment a wire or slip, about 18 inches long, of one of the metals to be tested, had somewhat longer wires or slips of the other soldered to its ends. The compound conductor thus constituted was bent into such a shape that the two junctions of the metals could be placed near the centres of the oil-baths; it was supported in this position, carefully insulated from touching the

Fig. 13.



copper vessels and from all other metallic contacts; and thermometers were put with their bulbs in the oil as close to the junctions as possible. The gas-furnaces were applied below and round the sides of the large copper vessels, so that they could be regulated to any desired temperatures.

87. After this apparatus had been used in several experiments, and neutral points between copper and iron, copper and P_1, lead wires and P_1, and brass and P_1 had been determined, I saw reason to alter the arrangements in various respects, and had another apparatus constructed, according to the following description.

88. Two small oil-baths were made, each of an outside partly cylindrical and partly plane sheet of copper, and a concentric copper tube 5 inches long and $\frac{6}{10}$ths of

an inch diameter brazed to it by ends of sheet copper, shaped as shown in the diagram. The space round the inner tube and within the outer sheet and ends was filled with oil completely covering the inner tube, and, when heated, rising into the space between the upper parallel plane parts of the outer sheet. A narrow ring of sheet metal with a long slip projecting from one side for holding it by, was put in the inner tube before the other parts were brazed on, and during an experiment was kept as constantly moving from one end of the bath to the other and back as was required to keep the whole mass of oil at one temperature.

Fig. 14.

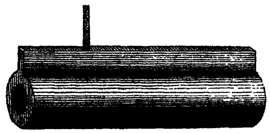
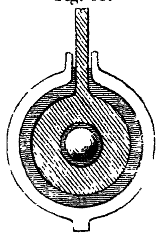


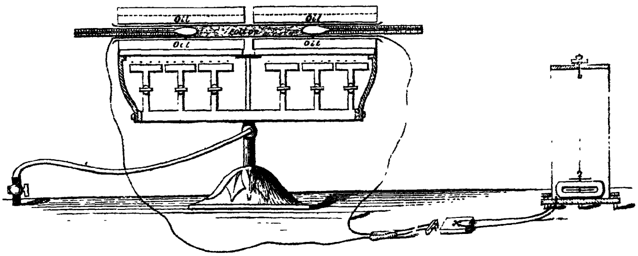
Fig. 15.



The second drawing represents, on the actual scale, a section of either bath through the position occupied by its stirrer. This diagram also shows a section of an external case of sheet metal which supports the bath, and serves as a flue to carry the flame and products of combustion round its sides. The rows of gas-burners for the two baths were fixed in a line, and each burner regulated by a separate stopcock. The outer cases are screwed to the same stand, and the copper vessels holding the oil are pushed into them and rest with their axes in a line over the burners. The ends of the baths and of the outer cases are kept about one-fourth of an inch apart, and their supports are also made quite separate, which was found to be necessary to allow one of the baths to be kept cool, while the other is raised to a high temperature. (In the third drawing, the stems by which the stirrers are held are accidentally omitted.)

89. When the baths and their furnaces are all fixed in their proper positions, a tube of thin glass, about $10\frac{1}{2}$ inches long, just small enough to enter easily, is pushed

Fig. 16.



into the inner tubes of the baths, and is left resting there, with its ends projecting a little outside their remote ends. In recent experiments I have substituted a simple roll of paper for the glass tube, and have found it answer quite as well.

90. A compound conductor, to be tested thermo-electrically in this apparatus, con-

sists of a wire or a thin bar of one metal, or a bundle of wires of two different metals $5\frac{1}{2}$ inches long, with wires of from 18 to 30 inches long of another metal soldered to its ends. To avoid circumlocution I shall call the former the *mean conductor*, and the wires soldered to it the *electrodes*, of the thermo-electric arrangement. The connexions between the mean and the electrodes are generally made by brazing, or by hard silver solder, when temperatures much above the boiling-point of water are to be used.

91. A conductor thus prepared of two metals to be tested is drawn through the glass tube till the *mean* occupies a position, lying on the glass or paper tube, with its centre under the centre of the tube, and consequently with its ends about the middle of the hollow spaces surrounded by the oil-baths.

92. The electrodes are carried from the ends of the insulating tube to the connexions required for completing the circuit through the coil of a galvanometer. These must essentially be maintained at the same temperature, unless the electrodes of the thermo-electric arrangement be copper, the same as those of the galvanometer. After trying several obvious, more or less troublesome plans to secure the fulfilment of this condition, I found a perfectly effective way simply to tie the connexions firmly together as close to one another as possible, only separated from contact by a fold or two of paper wrapped round each, and to tie a quantity of paper, or to make up a bundle of cotton wool, or some other bad conductor, round the two, for two or three inches on each side of the junctions. The junctions themselves, except when they are between homogeneous metals, are not made by binding-screws, but either by soldering, or by cleaning the surfaces and then tying the metal firmly together by fine twine. To avoid mistakes and prevent the necessity of disturbing the bundle round the junctions, in tracing the courses of the conductor on the two sides of it, a thread or mark of some kind is attached to one galvanometer electrode, and a corresponding mark on the electrode of the thermo-electric apparatus to which it is joined. This system of electric insulation and thermal connexion between junctions of dissimilar metals, I have found very convenient in a great variety of thermo-electric and other electro-dynamic experiments, and when it was used I have never observed the slightest trace of a current attributable to any difference of temperatures in the parts of the circuit to which it is applied.

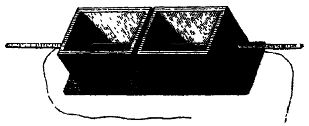
93. The conductor being thus arranged, two thermometers are pushed into the glass or paper tube from its ends and placed with the centres of their bulbs as close as possible to the metallic junctions, and with their graduated tubes extending nearly horizontally outside the apparatus, but inclined upwards as much as the inner diameter of the insulating tube and their dimensions permit, so as to check as much as possible the tendency (in some of the thermometers found very inconvenient) of the column of mercury to divide when sinking rapidly. All the space inside the glass or paper tube left vacant by the thermometers and the conductor is filled with cotton wool, well pressed in to prevent currents of air.

94. This apparatus has many advantages over that first used and described in

(§ 86) above: the temperatures of the baths can be changed with great rapidity, in consequence of the smallness of the quantities of oil which they contain; and by watching the thermometers and adjusting the gas-burners, can be regulated as desired with great ease. I have found it not a small practical advantage to be freed from the necessity of bending the mean part of the conductor to be tested, and of making the arrangements to prevent irregular contacts and to keep the junctions and the thermometers in their proper positions immersed in the oil. When a rare metal is to be tested, or one, such as sodium or potassium, which cannot be kept in air, it will be of great consequence to be able to apply the tests to a little straight bar or slip only a few inches long, or to a small column filling a glass tube.

95. For experimenting at low temperatures a modified apparatus was made, consisting of a double wooden box, each compartment, nearly a cube of 4 inches side, fixed to a common base with a space of about $\frac{1}{4}$ inch between their sides, and a glass tube running through them and cemented at the apertures in the sides so as to hold water-tight and resist the action of acids which might be employed in freezing mixtures. The conductor to be tested and the thermometers are arranged in this glass tube, as in that of the other apparatus (§ 89 to 93); and while a freezing mixture is kept in one compartment, the other is either allowed to take the atmospheric temperature, or is heated by hot water or steam.

Fig. 17.



96. The way of experimenting which I followed, was to raise the temperature of one bath until a deflection of the galvanometer-needle became sensible; then to go on raising it, and letting that of the other follow, so that the two thermometers may indicate as nearly as may be a constant difference of temperatures; and to watch the needle until a reversal is observed, or until the limit of temperature which the arrangement admits of is reached.

As soon as a reversal is obtained, the two thermometers are allowed to sink until the needle begins to return from its reverse deflection. When it approaches zero the thermometers are kept from any rapid changes, but allowed to sink very slowly, with always the same difference, or at least with a quite decided difference of the same kind as that raised between them at the beginning. The last readings of the sinking thermometers which give a sensible deflection before the original deflection is recovered, several readings when the needle appears perfectly at zero, and the first readings when the needle is discovered to deviate again in the original direction, are carefully noted. The arithmetical mean of the temperatures of the two thermometers for each of these simultaneous or nearly simultaneous readings is taken; and it is generally found that the means derived from the readings taken when no deflection can be discerned, lie within a fraction of a degree of the mean of the last sinking mean temperature of the junctions which show one deviation, and the first which shows the deviation in the other direction. The mean of either the readings which

give no deviation, or of the last and first which give the contrary deviations, or of all these readings together, according to the nature of the memoranda made by the observer, is taken as a determination of the neutral point of the two metals, that is, the temperature at which they are thermo-electrically like one metal, or thermo-electrically neutral to one another. In the course of one experiment several such determinations, both with descending and with ascending mean temperature, are made, and if possible also, with first one and then the other junction higher.

97. Either in one experiment, or with the same apparatus on successive days, determinations are sometimes made with as considerable a variety of differences of temperature between the two junctions as is attainable. Sometimes the difference of temperatures used is so small as to give very slight indications of electro-motive force, even when the mean of the temperatures of the junctions differs widely from the neutral point, in which cases, of course, the test is deficient in sensibility. The best determinations are generally those derived from observations showing the galvanometer at zero, with the widest difference between the temperatures of the junctions, to which the thermometers are applicable with trustworthy indications: as, for instance, 100° or 150° Cent., which are attainable in the most favourable cases, being those in which the neutral point is at about midway between the temperatures of freezing and boiling water. The differences between these determinations sometimes amount to a degree or two, and even to several degrees when zinc was one of the metals; but generally the final mean for the neutral point does not differ by more than a degree from any single determination considered as satisfactory at the time it was made.

98. The mutual interchanges of thermo-electric order observed in various specimens of zinc, gold and silver, occasioned considerable perplexity, which has only been cleared up by observations made subsequently to the communication of this paper. The following determinations were made at different times and by different observers, as noted:—

Observer.	Date.	Metals.	Neutral points.
Mr. C. A. Smith	Sept. 27, 1854	P_1 mean / Gold electrodes	− 3°·06
Mr. C. A. Smith	Aug. 18, 1854	P_1 mean / Silver electrodes	− 1·5
Mr. C. A. Smith	Sept. 8, 1854	P_1 mean / Zinc electrodes	+ 8·2
Mr. C. A. Smith	Sept. 20, 1854	Silver mean / Zinc specimen (1) electrodes	43·9
Mr. G. Chapman and Mr. J. Cranston	Jan. 29, 1856, and Feb. 5.	Silver electrodes / Zinc specimen (2) mean	51·5
Mr. G. Chapman and Mr. J. Cranston	Feb. 1856	Silver mean / Zinc (1) electrodes	46·55
Mr. G. Chapman and Mr. J. Cranston	Feb. 1856	Silver mean / Zinc (2) electrodes	58·18
Mr. J. Murray	Aug. 1856	Silver mean / Zinc electrodes	56·95
Mr. G. Chapman and Mr. J. Cranston	Feb. 1856	Gold mean / Zinc electrodes	71
Mr. G. Chapman and Mr. J. Cranston	Feb. 1856	Gold mean / Zinc electrodes	69·76
Mr. G. Chapman and Mr. J. Cranston	Feb. 27, 1856	Silver electrodes / Gold mean	70·8
Mr. J. Murray	Aug. 21, 1856	Silver mean / Gold electrodes	− 5·7

Of the two results for the neutral point between silver and gold, only the last can be reconciled with the indications derived from the previous results as to the relative positions of these and the other metals tried along with them; and accordingly $-5^\circ{\cdot}7$ has been taken as the neutral point of gold and silver in the thermo-electric diagram given below (§ 101). The first result, $70^\circ{\cdot}8$, was found as the mean of several determinations, from none of which it differed by more than $0^\circ{\cdot}7$, and the discrepance can scarcely be attributed to errors of observation, but is probably due to slight differences in the specimens of gold and silver used in the different experiments. That very slight chemical differences in specimens of gold and silver wire may make great alterations in the temperature at which they become thermo-electrically neutral to one another, is readily understood by glancing at the diagram given below (§ 101), and observing how close together the lines for gold and silver lie.

99. The question, *Does the difference between the specific heat of electricity in two metals vary with the temperature**? may be answered by experiments showing the law according to which the means of widely different temperatures of the junctions giving no electro-motive force deviate from the true neutral point, which is the mean of any infinitely small difference of temperature giving no electro-motive force.

I have not yet obtained indications of such a deviation in any case, having been prevented from prosecuting the inquiry by delays in the construction of a suitable air-thermometer. The examination I have been able to give the subject is only sufficient to show that the arithmetical mean of the temperatures of the two junctions giving no current, is probably in general within a degree of the true neutral point, when the difference between those temperatures does not exceed 100° Cent.

The following summary of a series of experiments made on two consecutive days may serve as an example of the degree of consistence of the results obtained by the method which has been explained, in a case in which the two metals deviate rapidly from one another above and below their neutral point.

Sheet-lead Electrodes; P_1 mean.

Determinations by Mr. C. A. Smith, May 17 & 18, 1854.

Difference of temperatures.	Half sum of mercurial thermometer temperatures giving no current.	
$77\frac{1}{2}^\circ$	121°	Mean $121^\circ{\cdot}5$
71	$121\frac{1}{4}$	
$71\frac{1}{2}$	$121\frac{3}{4}$	
$71\frac{1}{2}$	$121\frac{3}{4}$	
70	122	
$185\frac{1}{2}$	$120\frac{3}{4}$	Mean $121^\circ{\cdot}4$
$158\frac{1}{2}$	$121\frac{3}{4}$	
143	$121\frac{1}{2}$	
133	$121\frac{1}{2}$	
$125\frac{1}{2}$	$121\frac{1}{2}$	
$68\frac{1}{2}$	122	Mean $122^\circ{\cdot}5$
50	123	

	Mean of temperatures by mercurial thermometer giving no current.
Differences from 50° to 77°......	$122^\circ{\cdot}15$
Differences from 125° to 185°...	$121^\circ{\cdot}4$

* See "Dynamical Theory of Heat," § 115, equations (15) and (17).

These results seem on the whole to show that the mean of apparent temperatures giving no current is rather less for the wide than for the narrow ranges, in the case of the two metals concerned; that is, that the mean of the apparent temperatures giving no current is somewhat below the true neutral point. I need scarcely remark, however, that even if this indication could be relied on, it would be necessary to compare the actual mercurial thermometers which were used, with an air-thermometer, before any conclusions of value could be drawn from it regarding the constancy of the difference of specific heats of electricity in lead and platinum.

100. The following Table shows the results of observations leading to actual determinations of neutral points between various pairs of metals :—

−14° C.	−12°·2.	−5°·7.	−3°·06.	−1°·5.	8°·2.	33°.	36°.	38°.	44°.	44°.	47°...71°.
P_3 Brass.	P_1 Cadmium.	Silver. Gold.	P_1 Gold.	P_1 Silver.	P_1 Zinc.	Tin. Brass.	P_2 Lead.	P_2 Brass.	P_2 Tin.	Lead. Brass.	Different specimens of Silver. Different specimens of Zinc.

53°.	57°*.	64°.	71°.	99°.	121°.	130°.	162°·5.	Some temperature between 223° & 253°·5.	237°.	280°.
P_2 Double wire of Palladium 11·31 grs., and Copper 19·41 grs.	Hard steel. Cadmium.	P_1 Copper.	Gold. Zinc.	P_1 Brass.	P_1 Lead.	P_1 Tin.	Iron. Cadmium.	Iron. Gold.	Iron. Silver.	Iron. Copper.

The number at the head of each column expresses the temperature Centigrade by mercurial thermometers, at which the two metals written below it are thermo-electrically neutral to one another; and the lower metal in each column is that which passes the other from *bismuth towards antimony as the temperature rises.*

It was also found that Aluminium must be neutral to either P_3 or Brass, or P_2, at some temperature between −14° C. and 38° C.; that Brass becomes neutral to Copper at some high temperature, probably between 800° and 1400°; Copper to Silver, a little below the melting-point of silver; Nickel to Palladium, at some high temperature, perhaps about a low red heat; and P_3 to impure mercury (that had been used for amalgamating zinc plates), at a temperature between −10° and 0°. P_3 appears to become neutral to pure mercury at some temperature below −25° Cent.

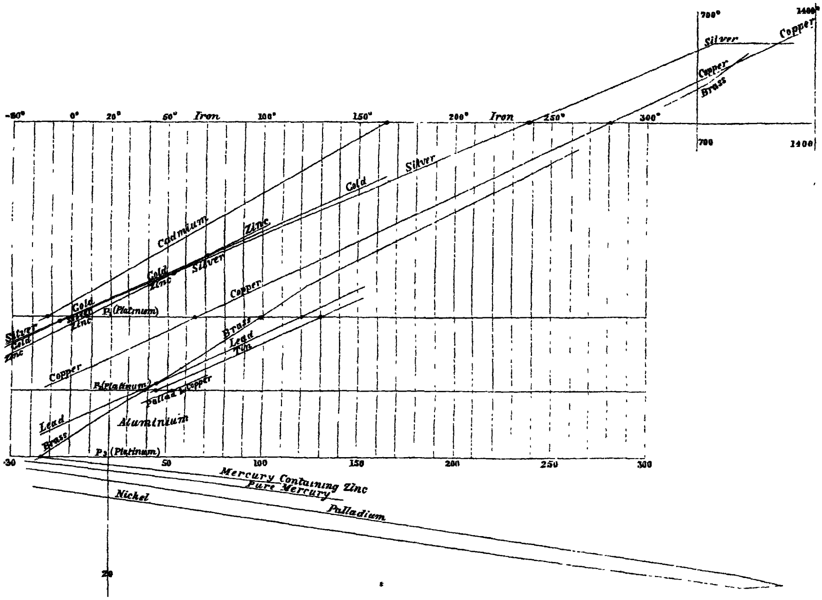
101. The following Diagram exhibits graphically the relative thermo-electric bearings of the different metals, and may in fact be regarded as a series of tables of the thermo-electric order of metals at different temperatures from −30° to 300° Cent.

* This determination has been added in consequence of information given by Mr. Joule (December 1856), that hardened steel at ordinary temperatures differs thermo-electrically from copper by about one-tenth of the thermo-electric difference of iron from copper.

Explanation of Thermo-electric Diagram.

The orders of the metals in the thermo-electric series, at different temperatures, are shown by the points in which the vertical lines marked with the temperatures Centigrade, are cut by the horizontal and inclined lines named for the different metallic specimens.

Fig. 18.



The object to be aimed at in perfecting a thermo-electric diagram, is to make the ordinates of the lines (which will in general be curves) corresponding to the different metallic specimens be exactly proportional to their *thermo-electric differences* * from a standard metal (P_3 in the actual diagram).

§§ 102, 103. *Theoretical inferences regarding Electrical Convections of Heat, from facts of Thermo-electric Inversion.*

102. The thermo-dynamic reasoning adduced above (§§ 10 to 15) leads to the conclusion (§§ 14, 15), that the *convective power of the vitreous electricity is greater*, or (which is the same thing) *the convective power of the resinous electricity is less, in each metal for which the line in the diagram cuts the line for another metal from below it on the left to above it on the right,* than *in this other metal.* Now it was established

* See "Dynamical Theory of Heat," § 140.

in Part I., that the vitreous electricity carries heat with it in copper (§ 54), or, as it may be expressed, the *electric convection of heat is positive in copper*. From the diagram we infer that it is greater, and consequently positive, in Brass. That it is positive in brass has been proved also by direct experiment (§§ 67 and 77). We infer also with certainty from the diagram, that the electric convection of heat (whether positive or negative) is greater in Zinc than in Gold, and greater in Gold than in Silver; that it is greater in Brass, Tin, Lead, Copper, Zinc, Gold, Silver, and Cadmium than in Platinum; that it is greater in Brass, Copper, Gold, Silver, and Cadmium than in Iron; that it is greater (that is to say, since it has been proved, § 76, to be negative, less negative) in Platinum than in Mercury; and that it is greater in Nickel than in Palladium. In Cadmium, as we may judge by the eye from the diagram, the convection is probably greater than in Copper; and in Palladium probably less (that is, greater negatively) than in Platinum.

103. These conclusions, certain and probable, are collected in the following Table of Convections, in which the different metals are arranged in order of the *amounts of the electric convection of heat* which they experience, or in the order of the values of "the specific heat of electricity in them."

Electrical Convection of Heat

	In Cadmium	Positive.
	Brass	Positive.
Order doubtful.	Copper	Positive.
	Lead, Tin (equal)	Positive.
	Zinc	Positive, Zero, or Negative.
	Gold	Positive, Zero, or Negative.
	Silver	Positive, Zero, or Negative.
Order doubtful.	Iron	Negative.
	Platinum	Negative.
	Nickel	Probably Negative.
Probably nearly equal.	Palladium	Probably Negative.
	Mercury	Negative.

PART III. EFFECTS OF MECHANICAL STRAIN AND OF MAGNETIZATION ON THE THERMO-ELECTRIC QUALITIES OF METALS.

104. Physical agencies having directional attributes and depending (as all physical agencies we know of except gravitation appear to do) on particular qualities of the substance occupying the space across or in which they are exerted, are transmitted or permitted with different degrees of facility in different directions if the substance is crystalline. The phenomenon of crystallization, exhibiting different chemical

affinities on different bounding planes, between a growing crystal and the fluid from which it is being formed, and the cleavage properties (different specific capacities for resisting stress in different directions) afford the primary illustrations of this statement. It is probable that the proposition asserted is a universal proposition in the sense, that there is no kind of physical agency falling under the category referred to, which does not meet with different capacities for receiving it in different directions in some crystals. There certainly may be, and probably are, crystals which transmit certain physical agencies equally in all directions. Crystals of the cubical system, for instance (unless possessing the conceivable dipolar rotatory property*, from which some, if not all, are certainly exempt), conduct heat and electricity equally in all directions, and have equal magnetic inductive capacities and equal thermo-electric powers. But thermal and electric conductivity, magnetic inductive capacity, and "thermo-electric power†" are undoubtedly different in different directions in many, if not in all, crystals not of the cubical system. Many crystals have not shown any marked difference in their absorption of light according to the direction of its propagation through them; but some undoubtedly do show a difference of this kind, to such a degree as to give sensibly different colour to light passing short distances through them in different directions‡. FARADAY had good reason, after making the discovery of the induction of electro-polarization in non-conducting substances§, to try the specific directional qualities of crystals used as dielectrics; and although he found no sensible differences in the inductive capacities of the crystals (rock crystals and Iceland spar) which he tried for this kind of action, in different directions, it appears highly probable that induced electro-polarization will sooner or later be ascertained to be no exception to the general rule.

105. Another very general principle is, that any directional agency applied to a substance may give it different capacities in different directions for all others. Whether or not this is true as a universal proposition, events have proved that the probability of its being true in any particular case is quite sufficient to warrant an experimental inquiry. BREWSTER discovered that mechanical stress induces in glass directional properties with reference to polarized light, which are lost as soon as the stress originating them is removed. These properties were shown by FRESNEL to be of the same kind as the property of double refraction possessed by a natural crystal. Experiments made by Sir DAVID BREWSTER and Mr. CLERK MAXWELL prove that isinglass and other gelatinous substances dried under stress, thin sheet gutta-percha

* See "Dynamical Theory of Heat," § 168; also §§ 163, 166, 167, 169 to 171, Transactions of the Royal Society of Edinburgh, May 1854. See also Professor STOKES "On the Conduction of Heat in Crystals," Cambridge and Dublin Mathematical Journal, Nov. 1851.

† Or thermo-electric difference from a standard metal. See "Dynamical Theory of Heat," § 140.

‡ Most crystals not of the cubic system, even when nearly colourless, exhibit difference. See HAIDINGER's 'Researches.'

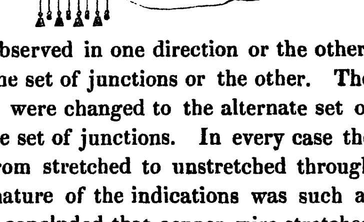
§ Experimental Researches in Electricity, Series XIV. §§ 1688, 1689, 1692 to 1698. June 1838.

permanently strained by traction, and probably all non-brittle (or plastic) transparent solids when permanently strained otherwise than by uniform condensation or dilatation in all directions, possess double refraction as a property of the molecular alteration which they acquire under the stress and retain after the stress is removed. Again, magnetization, as JOULE discovered, causes an elongation of iron in one direction (that of the magnetization) and a contraction in all directions perpendicular to it, with no sensible change of volume. FARADAY discovered the wonderful dipolar optical property of transparent bodies in a magnetic field (the first and only case known of any dipolar qualities, other than those of magnetic and electric reactive forces, called into existence by induction): MAGGI discovered that magnetized iron conducts heat with a greater facility across than along the lines of magnetization*.

106. In applying the dynamical theory of heat to thermo-electric currents in conducting crystals, I was led to consider the probable effects of mechanical strain, and of magnetization on the thermo-electric properties of non-crystalline metals, and in consequence entered on the investigation, of which the results, so far as I have yet advanced in it, are now laid before the Royal Society.

107. To find the effect of longitudinal tension on the thermo-electric quality of a metal, I first took eight thin copper wires each capable of bearing about 10 lbs., and, attaching their upper ends to a horizontal wooden arm at distances of about $\frac{1}{4}$ of an inch from one another, allowed them to hang down, each kept stretched by a weight of about $\frac{1}{4}$ lb. They were connected with one another in order, and the first and last with the electrodes of a galvanometer, by nine wires soldered to them, as shown in the diagram; the junctions between the successive wires being alternately in the upper and lower of two horizontal lines 4 inches apart. Every alternate wire was then stretched with a weight of about 3 lbs., and a slip of hot plate glass was applied, sometimes to the upper and sometimes to the lower row of junctions.

Fig. 19.



A deflection of the galvanometer needle was observed in one direction or the other, according as the glass heater was applied to one set of junctions or the other. The deflection was also reversed when the weights were changed to the alternate set of wires, and the heater kept applied to the same set of junctions. In every case the deflection was such as to indicate a current from stretched to unstretched through hot junctions. The uniform and consistent nature of the indications was such as could leave no doubt as to the result; and I concluded that copper wire stretched

* Doubts have been thrown on this result, I believe, by other experimenters, who have not succeeded in verifying it by their own observation, but its close correspondence with a result I have recently discovered by experiments on the electric conductivity of magnetized iron, have diminished the impression such doubts produced on my own mind; and I look with much interest to a repetition of MAGGI's experiment.

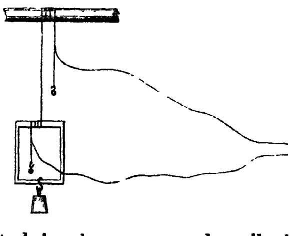
by a longitudinal force bears to copper wire of the same substance unstretched, the same thermo-electric relation as that of bismuth to antimony.

108. I next made a similar experiment on iron wire, varying the arrangement so that the weights could be rapidly shifted; and again so that equal sets of forces could be applied to one or to the other of the two sets of wires, merely by pressing with the foot upon one or another of two levers. A perfectly decided result was at once obtained; and I ascertained that the thermo-electric effect was induced and lost quite suddenly on the pressure being applied and removed. In this case the nature of the effect was the reverse of that found in the experiment on copper, the deflections being always such as to indicate a current in the iron wires from unstretched to stretched through the hot junctions.

109. The thermo-electric effect which these experiments demonstrated to accompany temporary strain produced by a longitudinal force, was, in each of the metals, the reverse of that which MAGNUS * had previously discovered in the same metal hardened by the process of wire-drawing, and which I ascertained for myself to be produced in each case when the metal is hardened by simple longitudinal stress without any of the lateral action inseparable from the use of the draw plate. I thus arrived at the remarkable conclusion, that when a permanent elongation is left after the withdrawal of a longitudinal force which has been applied to an iron or copper wire, the residual thermo-electric effect is the reverse of the thermo-electric effect which is induced by the force, and which subsists as long as the force acts.

110. I have made a single experiment demonstrating this conclusion for iron by means of a multiple tension apparatus, similar in principle to that described above (§ 105). But with a somewhat more sensitive galvanometer than the one I used, the result may be shown in a perfectly decided manner (for iron at least) without any multiplication of the thermo-electric elements; and a very striking experiment may be made on the following plan:—A thin iron wire is wrapped three or four times round a wooden peg held firmly in a horizontal position, and again two or three times round another parallel peg, about 4 inches lower. A frame is rigidly connected to this second peg, so that it may remain stably in a horizontal position; hanging from the wire and pulled down by the frame with either a light or a heavy weight attached to its lowest point. To keep the wire from slipping, the parts of it running from the pegs towards the ends are kept stretched by light weights tied to them; and the slack parts below these weights are carried away to the galvanometer electrodes, with which they are connected in the manner described above (§ 92). Any convenient source of heat is applied to the part of the wire bent round either peg, so as to keep it at some temperature, perhaps about as high as that

Fig. 20.



* POGGENDORFF's 'Annalen,' Aug. 1851.

of boiling water. If the wire be well annealed at the commencement of the experiment, and if weights be gradually added to the lower side of the frame, the galvanometer needle gradually moves to one side, indicating a current from the unstretched to the stretched round the hot peg; and the deflection goes on increasing as long as weights are added, up to the breaking of the wire. If, however, before the wire breaks, the weights are gradually removed, the needle comes back towards its zero-point, reaches zero, and remains there when a certain part of the weight is kept suspended. If this is removed the needle immediately goes to the other side of zero, and remains, indicating a current from the strained part into the unstrained part of the iron wire round the part wrapped on the hot peg; that is, from strained to unstrained through hot, or as MAGNUS found, "from hard to soft through hot."

111. If weights be added again, as at first, this deflection is done away with, and the deflection that first appeared is regained, when the weight which previously allowed the needle to return to zero is exceeded. We thus conclude that iron wire hardened by longitudinal tension, may, by the application of a certain longitudinal force, have its thermo-electric quality reduced to that of unstrained soft iron, and by a greater force may be made to deviate in the other direction; or *that hardened iron under a heavy stress, of the kind by which it has been hardened, and hardened iron left free from stress, are on different sides of unstrained soft iron in the thermo-electric series.* There can be no doubt but that the same property holds for copper wire, being in fact demonstrated by the experimental results described above in §§ 107 and 109.

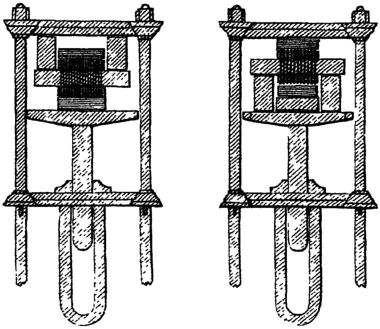
112. I have not yet investigated the thermo-electric effects of stress (that is, the effects accompanying temporary strain) in other metals than iron and copper; but it appears probable that the same law of relation to the thermo-electric effects of permanent strain without stress will be found to hold in each case, since it has been established for two metals in which the absolute thermo-electric effects are of contrary kinds. I hope, however, before long to be able to adduce experimental evidence which will supersede conjectures on the subject. [Since this paper was read I have verified the same law for platinum wire.]

113. The object which was proposed in entering on the investigation, being to test the thermo-electric properties of a strained metal, in different directions with reference to the direction of the strain, was not attained by comparing the thermo-electric properties of a longitudinally strained metal with those of the same metal in its natural state; but it would certainly be promoted by discovering the effect of lateral pressure on a wire in modifying its longitudinal thermo-electric action. I therefore made the following experiments on the thermo-electric effects experienced during the application of a *moderate* lateral pressure, and of permanent strain after the cessation of *excessive lateral pressure*, in various wires.

114. Experiment to discover the temporary effect of lateral pressure on the thermo-electric quality of iron wire:—A rectangular bar of iron ($1\frac{2}{3}$ inch square), with pieces of thin hard wood placed on two opposite sides, had fine iron wire laid in a coil of about twenty turns round it. The wood perfectly insulated the wire from the iron bar, and

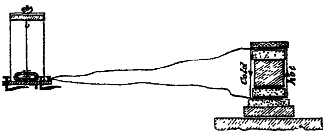
the different turns of the wire were kept from touching one another, by little notches cut in the edges of the pieces of wood. The whole coil was made firm, and its extreme turns tied down to the wood to prevent slipping. The ends of the wire, extending a foot or two on each side of the coil, were connected in the usual way (§ 92) with a galvanometer. The bar bearing the coil was laid with its two wooden faces horizontal, and one of them supported on a thin piece of hard wood lying on the stage of a BRAMAH's press. Another thin piece of hard wood was laid upon the top of the coil, to prevent the upper part of it (when, in the course of the experiment, it is forced upwards,) from touching the roof of the press. Blocks of iron were placed on the ends of the bar, so that when the stage is pushed up they may be resisted by the roof, cause a heavy stress to act on the bar, and press the lower horizontal parts of the wire coil between the two pieces of hard wood touching them above and below. The same blocks are afterwards shifted to rest on the stage and bear the ends of the bar upon them, so that, when the stage is forced up, the upper parts of the wire coil may be pressed against the piece of hard wood above them, which will then be resisted by the roof of the press.

Fig. 21.



Pieces of plate glass highly heated were applied to the vertical parts of the wire on one side of the bar, those on the other side being left cool, and the galvanometer was observed. Some slight deviation of the needle was generally noticed. Then the press was worked, and immediately a strong deflection took place, indicating a current in the iron coil, from the uncompressed portions through the heated vertical portions, into the compressed portions. The pressure was relieved, and the galvanometer needle returned nearly to zero. It was reapplied, and the same powerful deflection was observed. The glass heaters were shifted to the other side, and, the pressure being continued, the deflection of the needle became reversed. The pressure was removed, and by shifting the iron blocks, and working the press again, was applied on the other horizontal side of the coil. The heating being kept unchanged, a reverse deflection was observed, powerful as at first. The current indicated was in every case from *free iron wire* to *pressed iron wire through hot*, as is illustrated in the diagram, for a case in which the upper parts of the wire are compressed.

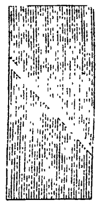
Fig. 22.



115. From this, in conjunction with the result regarding the effect of longitudinal stress previously obtained, we may nearly conclude that a longitudinal strain in iron

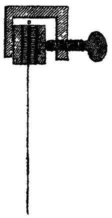
developes reverse thermo-electric qualities in the axial direction and in directions perpendicular to it; for there can be little doubt but that a lateral traction would produce the reverse effect of a lateral pressure, or that a portion of a linear conductor of iron pulled out on two opposite sides in a direction at right angles to its length, would acquire such a thermo-electric quality as to give rise to currents from *stretched to free through hot.* But in the former experiment (§ 108) it was demonstrated, that when part of an iron conductor is pulled out longitudinally, the thermo-electric effect gives currents from *free to stretched through hot.* The crystalline characteristic is therefore established for the thermo-electric effect of mechanical stress applied to iron, if it be true that traction produces the reverse temporary effect to that of pressure in the same direction. There seems so strong a probability in favour of this supposition, that it may almost be accepted without experimental proof; but I intend, notwithstanding, to make experiments, for the purpose of explicitly testing it, as soon as some preparations at present in progress enable me to do so. In the mean time I have made the following decisive experiment on the difference of thermo-electric quality in different directions in iron subjected to stress.

Fig. 23.



116. A piece of sheet-iron 36 inches long and 16 inches broad, was rolled round two thick iron wires ($\frac{1}{4}$-inch diam.), along its breadth at its two ends, and soldered to them. It was cut into narrow slips, each about $\frac{1}{4}$ of an inch broad and of different lengths, as shown in the diagram, so as to prevent electric conduction, except along a band about half an inch broad running across the sheet at an angle of 45° through its centre. The ends of the slips on each side of this band were clamped (as shown in the annexed sketch) between two flat iron bars, but insulated from them by thin pieces of hard wood *, and from one another, where necessary, by pieces of cotton cloth. These bars were each $\frac{1}{2}$ an inch thick, 3 inches broad, and 30 inches long; and the two at each side clamped together upon the pieces of hard wood, with the iron slips between them, formed a firm beam, by means of which a considerable stress would be brought to bear on the sheet iron to stretch it in the direction of the slips. The upper of these beams was laid resting with its two ends on the tops of stout wooden pillars, supported below on a very strong wooden bar laid on the stage of a BRAMAH's press. The lower double iron beam hanging down and straightening the sheet iron by its weight had strong iron links put over its ends, and an iron bar of about $1\frac{3}{8}$-inch square section slipped through them below, so as to hang down a small distance below the roof of the press. Thus, when the press is worked, the upper double iron beam is forced up, and the sheet iron is stretched between it and

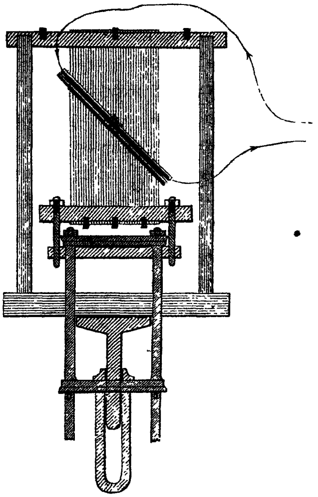
Fig. 24.



* The thinner the better, I believe, as a partial failure was experienced from these pieces of wood breaking at one side and allowing the ends of the iron slips to get drawn in between the iron bars.

the lower double iron beam, which is held down by the links and the bar under the roof of the press. Before working the press, the rectangular wooden frame with its iron cross-head is steadied by cords from hooks in the ceiling, and the following arrangements are made:—Two slips of sheet iron, each about 18 inches long, are soldered to the upper and lower ends of the oblique conducting channel, and their other ends are soldered to copper wires and put into the circuit of a galvanometer, with the usual precautions (§ 92) to ensure equality of temperature and electrical insulation between the two junctions of the dissimilar metals. Four tin-plate tubes, of semicircular section, each about $\frac{2}{5}$-inch diameter, and coated with a single fold of paper pasted round it, are pressed with their flat sides on the two sides of the sheet iron against the upper and lower edges of the oblique conducting band; and are connected by india-rubber junctions, so that steam may be blown through two of them to heat one edge of the conducting band, and cold water sent through the other pair to keep the other edge of the band cold. The arrangements being thus made, a small boiler, heated by a common wire-gauze gas-lamp, is used to send steam through one pair of the tubes, and the town-supply water-pipes give a continued stream of cold water through the other pair. When the galvanometer was observed, there was at first no sensible indication of a current. The press was then worked, and the galvanometer immediately exhibited a slight deflection. The press was released, and a careful observation gave again little or no evidence of a current. Then, by an arrangement of double-branched stop-cocks, the steam and cold water were quickly reversed, so that the edge of the conducting band which was hot became cooled, and the other one became heated. Still the galvanometer showed no sign of current until the press was worked, when a reverse deflection to the former was manifested. While the press was kept up the steam and cold water were again sent along the same edges as at first. After a short time the deflection of the needle was reversed, and the same current as at first was indicated. The deflections were very slight in each case, but were unmistakeably demonstrated by the use of the reversing break (commutator) connected with the galvanometer. Had it not been for the accident noted above, a much more powerful stress would have been applied to the iron, and I have no doubt but that conspicuous deflections of the needle would have been produced.

Fig. 25.



117. The current in every case was *down the inclined channel of sheet iron when the upper edge was heated, and up the incline when the lower edge was heated.* That is, if

we imagine a rectangular zigzag, from side to side of the bar, instead of the true rectilinear course of the current, the current would be from *transversely stretched to longitudinally stretched through hot.* Hence it is established by this experiment, that iron, under a simple longitudinal stress, has *different thermo-electric qualities in different directions.*

Knowing, as we do, from the first experiment on copper, described above (§ 107), that iron is not the only metal thermo-electrically affected by stress, we may conclude with much probability that, in general, metals subjected to stresses not equal in all directions will acquire the crystalline characteristic of having different qualities, as regards thermo-electricity, in different directions.

118. The qualitative investigation of the thermo-electric effects of stress, unaccompanied by permanent strain, that is, the elastic thermo-electric effects of stress, would be complete for iron if the thermo-electric effect of a uniform dilatation or condensation in all directions had been ascertained. I hope before long to be able to carry into effect various plans I have formed with this object in view; but in the mean time it would be the merest guessing to speculate as to the result.

119. The establishment of the crystalline characteristic for the thermo-electric effects of stress not equal in all directions, would make it probable that any thermo-electric effects which a metal permanently strained by such a stress can retain after the stress is removed, must also possess the crystalline characteristic. That this is really the case I had in fact proved, before performing the decisive experiment, just described, regarding the nature of the elastic effect, which was only made a few weeks since. The following experiments on the thermo-electric effects of permanent strains in metals were all made more than a year ago.

120. Well-annealed iron wire was rolled in a coil of about twenty turns on a flat bar of iron $\frac{1}{4}$-inch thick and 2 inches broad. The bar was laid on an anvil, with little pieces of thicker wire laid upon it to support the iron core and prevent the lower parts of the coil from being pressed. The upper parts of the coil lying on the upper flat side of the core were hammered till they were all very much flattened. The coil was then a little loosened and drawn off the bar of iron, and a similar wooden core was pushed into it. The ends of the iron wire were arranged, with the usual precautions (§ 92), in connexion with the electrodes of a galvanometer. A piece of hot glass (not above the boiling-point of water) was laid along one edge of the coil, so as to heat the iron wire at one set of the points separating hammered from unhammered portions. The galvanometer showed by a great deflection of its needle a current through the iron coil *from hammered* to *unhammered through hot.* When the heater was applied at the other edge of the flat coil, the deflection soon became reversed; still, and always in subsequent repetitions, *indicating a current from the strained to the soft metal through the hot junctions.*

121. The coil was next replaced on its iron core, heated to redness in the fire, and cooled slowly. It was then insulated by slipping in paper between it and the iron

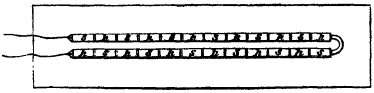
bar, or by putting it once more on its wooden core; and it was tested in the galvanometer circuit with the application of glass heaters as before. Not the slightest trace of a current was now found; a result verifying the conclusion arrived at by MAGNUS, that it is not peculiarities of form in different parts of a circuit of one uncrystallized metal, but variations in its quality as to mechanical strain, that can ever give it continuous thermo-electric action.

122. It has thus been proved that a circuit of iron permanently strained by pressure across the lines of conduction acquires the same kind of thermo-electric quality as that which MAGNUS first discovered to be produced by the lateral pressure compounded with longitudinal traction, which the process of wire-drawing calls into play, or as that which I had myself found to result from a simple traction, leaving a permanent elongation after the force is removed. In all these cases the iron is found to be harder than it was before acquiring the strain, or than it becomes again after being annealed. Hence the nature of the thermo-electric effect in each of the three cases falls under the designation "*current from hard to soft through hot*," by which MAGNUS stated his result as regards iron. This is just as is to be expected from the crystalline theory; since longitudinal extension has a common characteristic with lateral condensation in the theory of strains, and only differs from condensation uniform in all transverse directions, by a certain degree of absolute dilatation which accompanies it, instead of the slight absolute condensation accompanying the lateral condensation as an effect of pressure all round the sides. In fact the agreement between the characters of the thermo-electric effects due to longitudinal traction and lateral pressure, and again between the reverse characters of the effects of permanent longitudinal extension and those of permanent lateral compression established by the experiments which have been described, proves that these effects are due to distorting stress, and to permanent distortion, in the main, and leaves it quite an open question, only to be decided by further experimental investigation, what may be the effects of uniform pressure and of permanent uniform condensations or dilatations.

123. The crystalline theory is really unavoidable when it is thus established that the effect discovered is due to distortion; but still, as the one designation "current from hard to soft through hot" applies to all the cases of permanent strain in iron as yet experimented on, I thought it necessary, for removing the possibility of objections, that an iron conductor giving a current from soft to hard through hot, should be constructed. I therefore took twenty-four small soft iron bars turned in a lathe to a cylindrical form $\frac{1}{4}$th of an inch diameter, and each an inch long, with flat ends; and compressed twelve of them longitudinally in a BRAMAH's press, so as to permanently shorten each by about $\frac{1}{8}$th of an inch. They were then set in a wooden board cut to hold them firmly lengthwise in two rows, those hardened by compression and those left soft, being placed alternately with their ends in contact. The end pieces towards one side were connected with one another by a little slip of iron touching

each, and the other ends of the rows were connected with the electrodes of a galvanometer by slips of iron touching them. Each row was firmly wedged up between its terminal iron slips to ensure metallic contact; but after several attempts, and with all care in cleaning the surfaces meant to touch, no sufficient completeness of contact throughout the circuit could be obtained until mercury was introduced as a liquid solder to connect the pieces of iron. This was done simply by pressing them together as at first, pasting paper round the junctions, and pushing little drops of liquid mercury or small quantities of soft mercurial amalgam into apertures in the tops of these paper coverings. Twelve hollows were cut in the board under and round the junction of the iron bars, each except the last including a pair of ends of the bars in contact in each row, and the last including the ends of the extreme bars on that side and the slip of iron by which they are connected. These hollows were filled alternately with hot sand and cold sand, which was everywhere piled over the junctions; and the galvanometer gave slight indications of a current, the direction of which through the iron appeared to be generally from uncompressed to compressed through hot.

Fig. 26.



124. The result, however, was not satisfactory; and it was obvious that the plan which had been adopted for heating and cooling was quite insufficient to sustain the required differences of temperature through so considerable masses of iron; I therefore had an apparatus constructed for the purpose, consisting of two main pipes of tin-plate, each carrying six smaller pipes and leading to small cells, also of tin-plate, with cylindrical passages through them to admit the iron bars, and with short discharge pipes attached to them on the other side from that by which the former enters. These cells were fitted into the hollows cut for the sand in the board formerly used, the main pipes occupying parallel positions above them on each side several inches from one another. The iron bars, each coated with paper and united as before one to another with mercury solder, were pushed through the hollows of the cells, and were fixed in two rows, with a junction in the centre of each of these hollows, and with the terminals adjusted as before. Cold water from the town supply-pipes was then run into one of the main pipes, so as to flow through the branch pipes and cells connected with it; and steam from a boiler heated by an ordinary wire-gauze gas-burner was sent through the other system, so as to cool and heat alternately in their

Fig. 27.

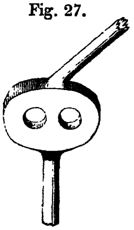
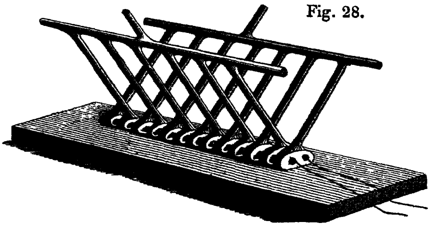


Fig. 28.



order of position the twelve cells with the junctions which they surround. A deflection of the galvanometer needle, amounting to about 4°, was now observed; and when the cold water and steam supplies were interchanged in the two sets of tubes, an equal reverse deflection almost immediately took place. The current indicated was always in many trials from *uncompressed to compressed through hot* in the iron of the circuit.

125. Here then we have a case of thermo-electric action in iron giving a current *from soft to hard through hot*; not as found before, "from hard to soft through hot." Hence it is not pieces of hardened iron in general, but *the direction of extension* or *directions perpendicular to the direction of compression*, in iron hardened by extension or by compression, that have the thermo-electric quality of deviating from soft iron towards bismuth; and a line of compression, or (as we may now safely conclude) *lines perpendicular to a line of extension*, have the reverse deviation, that is deviate from soft iron towards antimony, in the thermo-electric series. [*Addition, Dec.* 1856.—Subsequently to the reading of the paper, I have, in verification of this conclusion, found, by a direct experiment, that a conductor of sheet iron, hardened by lateral extension and softened in parts, has the thermo-electric property of giving a current *from soft to hard through hot.*] The crystalline theory being thus fully established for the thermo-electric effects of mechanical strain in iron, whether temporarily induced during the application of stress, or remaining with molecular displacement after the stress is removed, we may readily suppose it will be found to hold equally for all thermo-electric effects any metal can experience from mechanical action, except the hitherto undiscovered effects of condensations or dilatations equal in all directions. The experiments I have already made on other metals than iron, do not go further in verifying the crystalline theory than to show for copper and tin wires what I had previously shown for iron, that the same thermo-electric effect in a linear conductor is produced by permanent longitudinal extension and permanent lateral compression.

126. The process of raising to a high temperature and then cooling very suddenly, produces a marked effect on the mechanical qualities of most metals, especially on their hardness; and generally all that is necessary to do away with this effect and restore the metal to its primitive condition, is to keep it for some time at a high temperature and let it cool slowly. This process being called annealing, I shall for brevity designate as *unannealed*, any substance which has been subjected to the former process (sudden cooling) and which has not been subsequently annealed. It is not easy to judge exactly of the relation of the strains in the different parts of an unannealed piece of metal, to simple mechanical strains; but *some thermo-electric effect*, whatever its exact nature and explanation may be, is to be anticipated, with so great a change of other qualities as many metals experience in the process of sudden cooling; and it may be readily supposed that different thermo-electric qualities will be found in unannealed pieces of different shapes. I have therefore made experiments on the thermo-electric differences between unannealed and annealed linear conductors

consisting of round wires, of wires flattened by hammering, and of flat slips, of one metallic substance.

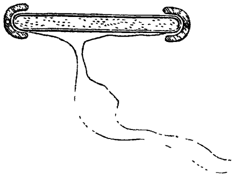
127. Twisting a wire beyond its limits of elasticity hardens it perhaps as much as traction or hammering, and certainly in every case, when continued far enough, makes the metal very brittle. The nature of the mechanical strain here operative is easily expressed and explained in the theory of elasticity in terms of simple strains different in magnitude and direction in different parts of the wire; but it is not very easy to judge by theory from the effects of simple strains supposed known, what kind of thermo-electric effect, if any, is to be expected in a metallic wire, with strain thus heterogeneously distributed through it. I have therefore made experiments to determine this effect in various metals.

128. For experimenting on the thermo-electric differences between annealed and unannealed metallic conductors, a wire, round or flattened, or a slip of the metal was wrapped in a coil of from ten to thirty turns on a wooden core, about 2 inches broad and $\frac{1}{4}$ of an inch thick, or sometimes only an inch broad, with a flat slip of thin sheet-iron laid on one side of it. The wooden core was then drawn away, and the coil, held in form by the thin iron core, was heated to redness in the fire, or to some temperature short of its melting-point, in hot oil, and was then suddenly plunged in cold water. After that, one side of the iron core was held over a flame, so as to heat the parts of the coil next it, while the parts of the coil on the other side were carefully kept cool, by the constant application of cold water with a sponge. The wooden core was then slipped in and the sheet-iron removed; and the coil was ready for testing by the galvanometer.

129. The preparations for an experiment on the thermo-electric effect of permanent torsion, were commenced by bending a short portion at each end of a length of two or three yards of the wire to be examined, holding these end portions so as to keep the wire between them firmly stretched, and twisting it till it became brittle. It was then wound on a flat iron core (unless it was too brittle, as often proved to be the case, and then another wire was similarly prepared but not twisted quite so much); the parts of the coil on one side were carefully annealed by flame or hot oil, while those on the other side were kept cool by sponging with cold water. The iron core was then drawn out and the wooden core slipped into its place; and the coil was ready for testing by the galvanometer.

130. In making the thermo-electric experiments on the coils prepared in these various ways, glass heaters were first used, but I afterwards substituted two tubes of horse-shoe section made of tin-plate and coated with paper, which were applied with their concave parts touching the coil round its two edges. Steam from the small boiler was sent through one of these, and cold water from the town supply-pipes through the other.

Fig. 29.



131. The wires used, with the exception of the iron, steel and brass, were all supplied by Messrs. MATTHEY and JOHNSON, as chemically pure. The results of the experiments (made as described in §§ 120 and 121) on the effects of lateral hammering were, in every other kind of wire tried, the reverse of those found for iron. Thus in steel, copper, tin, brass, lead, cadmium, platinum, zinc, the current was always found to be from the unhammered to the hammered portions through hot. All the wires except zinc were carefully annealed by myself, before they were coiled and hammered (§ 120); but the process of annealing by heating in oil and cooling slowly made the zinc very brittle and crystalline, instead of softening it as in the other cases, and it was therefore taken as supplied by the manufacturers, and coiled on the core and hammered in the manner described.

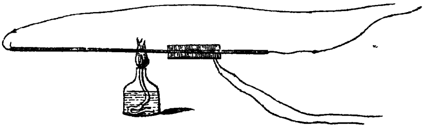
132. The experiments on the coils differently tempered in their different parts (§ 126), in the cases of tin and cadmium, gave only doubtful galvanometer indications; zinc wire proved so brittle *in the annealed parts* as to defeat some attempts to test the thermo-electric effects of temper. I have little doubt but that results may be obtained in all these cases by a careful repetition of the experiments, with perhaps some modification to meet the peculiarity of zinc. Slips of sheet iron and of sheet copper were tried without any thermo-electric indication being noticed. [*Addition, Dec.* 1856.—I have recently found in slips of sheet iron the same thermo-electric effect of temper as in round and flattened iron wires.] All the other conductors tried gave very decided results. In the cases of round iron wires of very different diameters, of iron wire flattened through its whole length by hammering, of round steel wire, and of steel wire flattened through its whole length by hammering, and of steel watch-spring, the thermo-electric effect of annealing portions of the coil after the whole had been suddenly cooled, was *a current from unannealed to annealed through hot.* In round wires of copper and brass, the thermo-electric effect of the same process was *a current from annealed to unannealed through hot.*

133. The effects of permanent torsion were decisively tested only for iron and copper wires; and they proved to be in each case the same as the effects of hardening by longitudinal extension, by lateral compression, or by rapid cooling, being quite decidedly *from brittle to soft through hot in the iron,* and *from soft to brittle through hot in the copper.*

134. The views explained above (§ 105), by which I was led to look for the thermo-electric qualities of a crystal in a non-crystalline metal subjected to mechanical strain, show the probability of finding such properties also developed along with magnetism, by external magnetic force, especially in the few metals, iron, nickel and cobalt, which have high capacities for magnetic induction. Towards verifying this idea I tried first the following simple experiment, analogous to the first experiment (§ 107) which I had made on the thermo-electric effects of tension. A little helix about 3 inches long, consisting of 220 turns of thin covered copper wire laid on in three strands on a cylindrical core of pasteboard, about $\frac{1}{4}$ of an inch internal diameter,

was slipped upon a piece of thick straight iron wire about 2 feet long, which was supported in a horizontal position by its ends, and through them put in the circuit of a galvanometer. A spirit-lamp was held under the middle of the wire so as to raise it to a high temperature, and then a current from a few of the iron cells was sent through the helix, which was kept a little on one side of the middle of the wire. Immediately the galvanometer needle, which was not at first disturbed by the application of the spirit-lamp, experienced a deflection. The little helix was slipped rapidly through the flame of the spirit-lamp to the other side of the hot part of the wire, and a reverse deflection was immediately produced.

Fig. 30.



It was easy, by moving the helix alternately to the two sides of the hot middle of the wire, to make the needle of the galvanometer to swing through an arc of 10° or more. When the needle was brought to rest there was always a most sensible permanent deflection, on one side or the other, according as the helix was left on one side or other of the heated parts. When the circuit of the galvanometer was broken, none of these effects followed from the motions of the helix. They were therefore not due to the direct force of the magnetism in the helix and iron wire, but to that of a current through the galvanometer coil. This always took place in such directions as to indicate a current *from unmagnetized* to *magnetized through hot.*

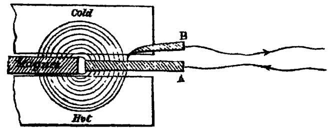
135. The decided character of the result of this experiment established it beyond doubt, that the thermo-electric quality of iron is altered by magnetization. Immediately the question arose (from the general considerations referred to above, §§ 104 and 105), *are the thermo-electric qualities equally or even similarly affected in all directions?* and the crystalline hypothesis suggested the answer:—no; probably even the reverse thermo-electric effect may be found across their lines of magnetization. As theory could give no more than a conjectural answer, I tried to find the truth by experiment; and, after various fruitless operations, obtained a very decided result, in the following way.

136. A piece of thin sheet iron was cut into the shape shown in the diagram, the breadth everywhere being about $\frac{1}{4}$ of an inch, the length of the longer branch 45 inches, and that of the shorter 6 inches. The longer branch was rolled into a plane spiral, on a cylindrical core $\frac{1}{2}$ an inch diameter, the different successive turns being prevented from touching one another by a piece of narrow tape wound on along with the iron slip. The shorter branch, which stood out from the inner end of the coil at right angles to the plane of the spiral, was bent round into this plane, and carried out along one side of the spiral several inches beyond its circumference. Along

Fig. 31.

with it, a portion of the slip next the other end which was left uncoiled, was carried out from the outer part of the spiral, and cut to such a length as to let the two ends be brought close together. Copper wires, to lead to the galvanometer electrodes, were soldered to these ends, and the junctions of dissimilar metals thus formed were arranged with the usual precautions (§ 92) to ensure equality of temperature and electrical insulation. Contrary poles of two steel bars, each about 3 feet long and of rectangular section, 4 inches by $\frac{1}{2}$ inch, were placed pressing on each side of the spiral, as shown by the dark shading in the diagram, but insulated from it of course. Four rectangular pieces of thick plate glass, two of them very hot (perhaps about 300° Cent.) and two cold, were applied, touching the coil on each side, and symmetrically arranged on the two sides of the steel magnets.

Fig. 32.

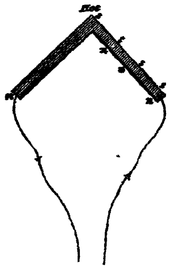


The galvanometer showed a current in the direction indicated by the arrow-heads. The pieces of hot and cold plate glass were interchanged, and the current became reversed. The magnets were removed, and their effects became scarcely perceptible, or altogether ceased. On repeated trials a current was found always in the direction, from parts of the coil between the magnets towards parts touched by the hot glasses. The experiment was repeated with a powerful electro-magnet, and gave the same result, but not with the same ease, because of difficulties in applying the heaters, &c.

137. The very strong tendency iron has to assume longitudinal rather than transverse magnetization, when of any form extended in one direction more than in others, was partially done away with by the mutual influence of the different turns of the spiral used in the experiment which has been described; and the symmetrical arrangement of the heaters was such as to nearly exclude all thermo-electric action, except what is due to the thermo-electric difference between that part of the coil touched on each side by the steel magnets, and the part diametrically opposite. Any thermo-electric effect there may have been from longitudinal magnetization in the parts of the iron ribbon on each side of the steel magnets, must, so far as I could judge, have been contrary to the effect observed. The result obtained, therefore, demonstrates an electro-motive force urging a current from *transversely magnetized parts of the iron conductor, through hot parts,* to *comparatively unmagnetized* parts. Hence a transversely *magnetized* iron conductor deviates from unmagnetized iron towards bismuth, or in the reverse direction to that of the deviation discovered in wire longitudinally magnetized, in the first experiment on the thermo-electric effects of magnetism. It may be concluded, *à fortiori*, that in uniformly magnetized iron, *directions transverse to the lines of magnetization* differ thermo-electrically from *directions along the lines of magnetization*; and differ in such a way, that if we could get

an iron conductor of the shape indicated in the diagram magnetized, with perfect uniformity everywhere, in the direction shown by the lines of shading, and if, when the two ends kept at the same temperature are put into the circuit of a galvanometer, the corner is heated, a current would be found to set in the direction shown by the arrow-heads, that is, *from transversely magnetized* to longitudinally magnetized *through hot.*

Fig. 33.



138. To test and illustrate this conclusion, I took a piece of sheet iron, cut to the shape shown in the diagram, and wound it spirally on a wooden cylinder, prepared with spiral grooves and pipes for steam and cold water, as described below. The oblique edge of the iron, shown on the left boundary in the diagram, being cut at angles of 45° and 135° to the long edges conterminous with it, was bent in a plane perpendicular to the axis of the cylinder, and thus the long edges of the iron, and the cut separating it into two branches, formed spirals, each at an angle of 45° to the axis of the cylinder. The two long edges themselves came very nearly to coincide, the circumference of the cylinder being a little greater in length than the oblique edge of the iron which thus nearly met round it. These two edges, as well as the two edges on each side of the cut between the branches, were prevented from touching one another by being, one at least in each of the contiguous pairs, bound with cotton tape. The projecting slips (shown on the right in the diagram) came to positions parallel to the axis of the cylinder, through two diametrically opposite parts of its circumference. Their ends had copper wires soldered to them, and were arranged with the usual precautions (§ 92) to ensure electric insulation and equality of temperature between them. The wooden cylinder had two diametrically opposite spiral grooves, each at the same inclination of 45° to the axis, and spiral sheet copper tubes, prepared of the proper shape, were slipped into these grooves, and nearly filled up the spaces to the surface of the cylinder. The outsides of these tubes were coated with paper, so as to maintain electric insulation between them, and the sheet iron wound on outside.

Fig. 34.

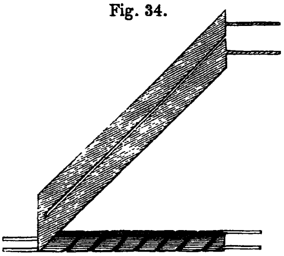
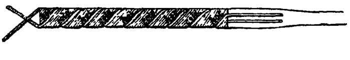


Fig. 35.

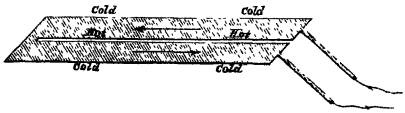


The wooden cylinder bearing the spiral tubes, and the sheet iron arranged in the manner described, was slipped into the hollow of an electro-dynamic helix, steam was sent through one of the spiral tubes and water through the other, and the copper wires soldered to the ends of the iron slips were connected with the electrodes of a galvanometer. No current was at first indicated. The galvanometer circuit

was broken by its own commutator, and a current was sent through the magnetizing helix. The galvanometer circuit was completed again, and immediately a strong indication of a current through it was manifested. The galvanometer circuit was broken, the magnetizing current reversed, and the galvanometer circuit again completed; again the same current as before was observed. The steam and cold water were interchanged in the spiral pipes, and the galvanometer current soon set in the reverse direction, with about the same force as before. The magnetizing current was stopped (the galvanometer circuit being broken for the time and closed again), and only slight traces of the current that had been so powerfully indicated could now be observed.

139. In this experiment the action of the electro-dynamic helix caused the double slip of iron to receive magnetization in lines nearly parallel to the axis of the cylinder (only a little disturbed in consequence of the gaps between the adjacent edges), that is to say, magnetization as nearly as may be in directions at an angle of 45° to its length. The sources of heat and cold applied along the two spirals, gave either heat along each of the outer edges of the double slip, and cold along the inner edges between the two branches, or cold along the outer edges and heat along the inner edges. When the ends were connected with the electrodes of the galvanometer, in the case illustrated in the diagram, the current was in the direction indicated by the arrow-heads; and it was always in such a direction, that if a zigzag line be traced through the two slips from side to side of each, on the whole in the same direction as the current, the changes of direction at the sides of the slips are from *transversely to longitudinally magnetized through hot, and from longitudinally to transversely magnetized* through cold; which is the conclusion that was anticipated.

Fig. 36.



140. I also experimented on the thermo-electric effects of retained magnetism in steel after the magnetizing force is removed, and obtained very decided results, showing that at least in the case of magnetization along the lines of current, the effect is of the same quality as in soft iron or in the steel itself while under a magnetic force which induces such a state of magnetization.

141. In one of these experiments, thirty-nine pieces of steel wire, each about $\frac{1}{18}$th of an inch diameter and 2 inches long, soft tempered, were connected by thirty-eight pieces of copper wire, each an inch long, placed between each two of the pieces of steel, and hard soldered to their ends. Pieces of copper wire of the same length were soldered to the outer ends of the first and last pieces of steel, and several feet of steel wire to the ends of each of these. A little electro-dynamic helix was made, 2 inches long and wide enough internally to slide freely over this compound steel and copper conductor; and by means of it every second piece of the 2-inch steel wires, commencing with the first and ending with the thirty-ninth, were magnetized alternately

with their poles in dissimilar directions, while the other short wires, and the longer steel terminals, were left as free from magnetism as possible. The magnetizing helix was then removed, and the compound conductor was made into a flat coil on a wooden core (2 inches broad and $\frac{1}{4}$-inch thick), by bending the short copper wires, and arranging the 2-inch steel wires alternately on the two sides of the wood. The terminals were joined, with the usual precautions (§ 92), to the galvanometer electrodes, and one edge of the coil was immersed nearly an inch below the surface of a vessel of oil at the temperature of about 100° Cent. Immediately a strong deflection of the needle showed a current, of which the direction in the coil was *from unmagnetized to magnetized through hot*. When the other edge of the coil was similarly heated, a contrary deflection of the needle as decidedly showed the same thermo-electric difference of quality between the magnetized and the unmagnetized steel wires.

142. The object of the peculiar arrangement just described, was to prevent the magnetism from spreading to those of the steel portions of the circuit which were to be kept as free from magnetism as possible in order to be compared with those which were magnetized. The introduction of the connecting pieces of a different metal from steel into the circuit cannot give rise to any thermo-electric disturbances *, provided the two ends of each are at the same temperature, a condition which was nearly enough fulfilled in the way the experiment was made, and which was very much favoured by the shortness and the high thermal conductivity of the little copper arcs.

The same result was demonstrated in an experiment made with a homogeneous coil of steel wire, of which parts had been magnetized, by ordinary steel magnets, before it was bent on the core.

[§ 143. Received May 10, 1856.]

§ 143. *Experiment.—On the Effect of Magnetization on the Thermo-electric Quality of Nickel.*

Through the kindness of Dr. George Wilson, I have been able to experiment on a bar of nickel, about $\frac{1}{2}$ an inch in diameter and about 8 inches long, in the form of a horse-shoe magnet, belonging to the Industrial Museum of Edinburgh. The accompanying sketch and description show the plan of the experiment.

Description of Sketch.

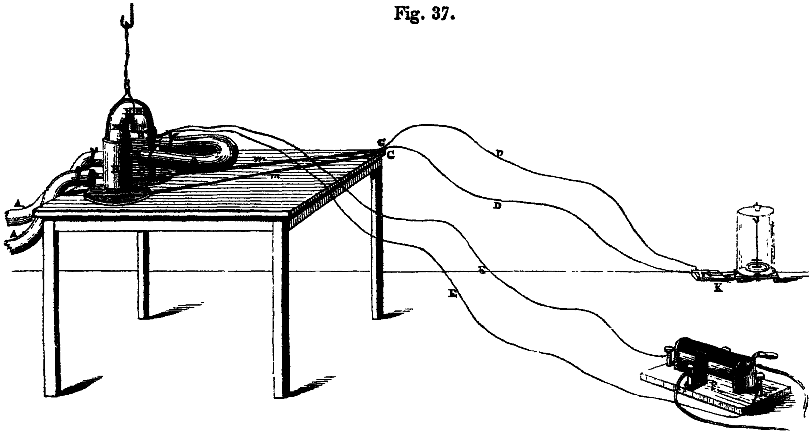
N, nickel horse-shoe.

B B, double tubes of sheet copper, electrically connected with one another by a copper band, and insulated from the nickel by silk paper, laid on with shell-lac varnish; serving to drain all electrical leakage from the magnetizing

* Dynamical Theory of Heat, § 138, Cor. 1.

coil, without causing the slightest sensible current through the nickel, and serving also to convey a stream of cold water to maintain the lower parts of the two branches of the horse-shoe at as nearly as possible equal temperatures.

Fig. 37.



A A A, india-rubber pipes to lead a stream of cold water through the coolers.

C, magnetizing coil, wrapped on one of the copper coolers.

E E, electrodes of magnetizing battery of twenty iron cells, charged with nitric acid, &c.

F, commutator for interrupting and reversing the connexion between the magnetizing battery and coil, or reversing the current.

M M, mercury cups, in which the extremities of the nickel were immersed (mercury being both very convenient for the purpose, and the metal least thermo-electrically removed from nickel of all that have been tried by any experimenter).

m m, mercury electrodes joining copper galvanometer electrodes D D, at G G.

K, commutator for interrupting and reversing the connexions of the galvanometer electrodes.

Heat was applied at H H by means of a gas-lamp and blowpipe. A current from magnetized to unmagnetized through hot, was indicated by a considerable galvanometer effect, which, by management of the galvanometer break, K, was readily directed to give oscillations of the needle through three or four degrees.

The same conclusion had been indicated in several previous attempts, with various defects of arrangement remedied in the experiment just described. In this last experiment the result was made most manifest; and, being completely separated from all effects of induced currents (which were quite insensible), of electrical leakage, and

of unequal heating of the junctions of mercury and nickel, and of the junctions of mercury and copper, was set beyond all doubt. I therefore conclude, *that longitudinally magnetized nickel in a thermo-electric circuit deviates from nickel not under magnetizing force, in the same direction as bismuth.* This is the reverse of the deviation which I formerly found to be produced in iron by longitudinal magnetization.

144. The results of the various experiments which have been described in Part III. are collected in the following Tables.

TABLE I.—Effects of Stresses and Strains on the Thermo-electric Qualities of Metals.

Description of Conductor.	Thermo-electric Order reckoned from Bismuth towards Antimony.		
Iron	...	Free	Under longitudinal traction.
Iron	...	Free	Under transverse compression.
Iron	Under transverse traction	...	Under longitudinal traction.
Iron	Permanently strained by longitudinal traction, and left free from stress.	Soft	Permanently strained by longitudinal compression, or by lateral extension, and left free from stress.
Iron	Hardened by transverse hammering	Soft	Hardened by longitudinal hammering.
Round iron wires of different diameters.	Made brittle by twisting	Annealed after being made brittle by twisting.	
Round and flattened iron wires.	Suddenly cooled	Annealed.	
Steel wire	Some specimens flattened by transverse hammering.	Soft	Other specimens flattened by transverse hammering.
Round and flattened steel wires.	Hardened by sudden cooling	Annealed.	
Steel watch-spring	Hardened by sudden cooling	Annealed.	
Copper	Under longitudinal traction	Free.	
Copper	...	Soft	Permanently elongated by longitudinal traction, and left free from stress.
Copper	...	Soft	Hammered transversely.
Round copper wire	...	Annealed after being made brittle by twisting.	Made brittle by twisting.
Round copper wire	...	Annealed	Suddenly cooled.
Platinum	Under longitudinal traction	Free.	
Platinum	...	Soft	Hammered transversely.
Tin	...	Soft	Permanently elongated by longitudinal traction, and left free from stress.
Tin	...	Soft	Hammered transversely.
Brass	...	Soft	Hammered transversely.
Round brass wire	...	Annealed	Suddenly cooled.
Cadmium	...	Soft	Hammered transversely.
Lead	...	Soft	Hammered transversely.
Zinc	...	Soft	Hammered transversely.

TABLE II.—Effects of Magnetism on the Thermo-electric Qualities of Iron and Nickel.

Description of Conductor.	Thermo-electric Order reckoned from Bismuth towards Antimony.		
Iron	Under transverse magnetizing force	Free	Under longitudinal magnetizing force.
Steel	....	Unmagnetized	Retaining longitudinal magnetization.
Nickel	Under longitudinal magnetizing force.	Free.	

PART IV. METHODS FOR COMPARING AND DETERMINING GALVANIC RESISTANCES, ILLUSTRATED BY PRELIMINARY EXPERIMENTS ON THE EFFECTS OF TENSION AND OF MAGNETIZATION ON THE ELECTRIC CONDUCTIVITY OF METALS.

145. In endeavouring to discover the effects of magnetization and of mechanical strain on the electric conductivity of iron and other metals, I was led, from trying various more or less obvious methods for testing resistances, to use a differential galvanometer of a very simple kind, which I constructed for the purpose. I shall give no description of this instrument, as I now (Nov. 1856) find it in one important quality inferior to the differential galvanometer first constructed and used by M. BECQUEREL*, and I do not know that its peculiarity has compensating advantages. I mention it only because it was with it that I made nearly the first of my trials to find the effects of magnetism on the electric conductivity of iron, and the very first by which I obtained a decided result.

146. In these experiments I used two covered iron wires, each several yards long, coiled into circles about 4 inches diameter, as the two resistance branches in the divided channel through the two conductors of the galvanometer. Magnetizing one of them tangentially by means of a coil of covered copper wire wound on a copper sheath soldered round it as an electric drain, I ascertained, on the 23rd of April, 1855, that the electric conductivity of iron wire is diminished by longitudinal magnetization. The arrangement however proved, as I anticipated, to be of a very unsatisfactory kind; and the needle kept moving across the field in one direction almost steadily, during the whole time the current was sustained through the tested conductors, which was for several hours. Continually more and more resistance had to be added to the conducting channel containing the iron wire round which there was no magnetizing coil, to keep the needle within range. After the magnetizing current had passed for some time, this variation of the needle went on more rapidly, and called for more frequent adjustment by the additions to the other branch. All this was just as must be expected; and my reason for not introducing currents of cold water round the two iron coils, to maintain them in precisely similar thermal circumstances, was that the tubular systems required for the purpose could not be easily made, and that I thought I might find out the nature of the result in the first

* Annales de Chimie et de Physique, tome xvii. 1846.

instance, notwithstanding the imperfection of the arrangements. In this hope I was not disappointed. The glass needle (carried by the little suspended magnet, which was only about ½ an inch long), while moving steadily across its field, would receive an impulse forward and make two or three very rapidly diminishing oscillations, when the current was started through the magnetizing coil: when the current was suddenly reversed, the needle would show little or no indication of any effect: when the current was broken, it would make a start backwards, and after two or three oscillations would continue advancing as before, perhaps rather more rapidly. Traces of induced currents in the iron coil under the influence of the magnetizing helix were exhibited by scarcely perceptible differences in the bearing of the needle, according as the current was made in one direction or the other, and by slight impulses it received when the magnetizing current was suddenly reversed. After the current had been kept up for some hours through the iron wires, and when, partly by the heat developed by the magnetizing current during the periods of its flow, and partly by heat conducted from the iron wire within, the outside of the magnetizing coil had become very sensibly hot to the touch, the variation of the needle in the galvanometer became much less rapid than at first; and tolerably satisfactory indications, amounting to a fraction of a degree of permanent deflection, showed with perfect consistence an increase of resistance in the iron wire under magnetic force when the magnetic current was sustained in either direction, and a diminution of resistance in the same iron wire following immediately a cessation of the magnetizing current.

147. I followed the same method in a first attempt to find the effect of transverse magnetization on the electric conductivity of iron; two spirals made on the plan described above (§ 136) being used as the resistance branches in the two channels conveying the divided current, and one of them placed between convex poles of a Ruhmkorff electro-magnet. The induced currents in making, reversing, and breaking the magnetizing current were of course most conspicuously indicated by the galvanometer needle, but the needle came to rest after a few oscillations; and then it did not exhibit any deviations of a sufficiently marked character, when the direct effect of the electro-magnet (which by a very troublesome process of shifting the position of the magnet, was reduced as much as possible in preliminary arrangements,) was eliminated by reversals, to allow me to draw any decided conclusion as to the effect of the magnetic force on the conductivity of the iron spiral across which it acted.

148. Before carrying into execution various obvious improvements in the experimental arrangements just described, or applying the system with the differential galvanometer to other investigations, I began to think of Maggi's experiment* on the relative thermal conductivities of a magnetized iron disc in directions across and along the lines of magnetization. As the electrical analogue, the method which

* De la Rive, 'Electricity,' vol. i. part 3. chap. iii. (p. 316, English edition, 1853).

MATTEUCCI, and I believe KIRCHHOFF and others, have used in tracing equipotential lines on the surface of a conductor traversed by an electric current, occurred to me. Six months later, I thought of the multiplying branch (first used in the experiment described in § 161 below) to render available the sensibility which a powerful current through the body to be tested, with the use of a moderately sensitive galvanometer, must obviously give to that method when applied to the investigation of differential effects on the electric conductivity of a body in different directions; and I succeeded with great ease in making very satisfactory experiments (§§ 161 to 165 below) by means of it, which first decided the question as to whether or not the effects of magnetization give different electric conductivity in different directions to a mass of iron. At first, however, I did not see this or any other way to render the method practicable with galvanometer electrodes, either moveable upon the sheet of metal to be tested (in which case a motion of $\frac{1}{100}$th of an inch would drive the needle from an extreme deflection on one side to an extreme reverse deflection), or by electrodes soldered to points on an equipotential line (in which case a slight alteration in temperature in different parts of the plate might drive the needle irrecoverably to an extreme deflection on one side or the other); but the experiments which I knew as having been made by MATTEUCCI suggested to me the following very simple plan, which I immediately commenced trying, and which I have since found applicable with the greatest ease to a variety (I believe now to every variety) of experiments on electric conductivities *.

149. Let AB be the conductor to be tested, and let CD be another of nearly equal resistance, either a piece of the same wire continuous with the other through an arc BC, or connected with it by a thicker arc of copper, or of another metal, as may appear convenient for the particular case treated. Sometimes the experiment is arranged to test differential effects experienced alternately or simultaneously by AB and CD. But when one of them, AB, alone is acted upon, with a view to varying its resistance, it alone may be regarded as the conductor which is tested; and the other, CD, will then be called the *reference conductor*. Let a wire, AOPD, which will be

* [*Note added Nov.* 1856.] An hour before the meeting of the Royal Society at which this paper was read, I learned that a method of testing resistances had been given by Mr. WHEATSTONE which would probably be found to be the same in principle as that to which I had been led in the manner described in the text. I have since ascertained that Mr. WHEATSTONE's "Differential Resistance Measurer" (described in § 15 of the Bakerian Lecture for 1843, see Transactions, June 15, 1843) is an instrument founded on precisely the same principle as all the various arrangements by which, with great and necessary alterations of detail, I have continued the investigation of effects of magnetism and of other influences, on the electric conductivity of metals, to the present time, and of which some are fully described in Parts IV. and V. of the text. Mr. WHEATSTONE refers to "Experimental Determinations of the Laws of Magneto-electric Induction," printed in the Philosophical Transactions for 1833, "as containing the description of a differential arrangement of which the principle is the same as that on which" his own instrument has been devised, and adds, "To Mr. CHRISTIE must therefore be attributed the first idea of this useful and accurate method of measuring resistances."

It is worth remarking, that the experiments of MATTEUCCI and KIRCHHOFF, alluded to in the text, are stated to have been first suggested from WHEATSTONE idea of applying the two electrodes of a galvanometer to points in separate channels through which two parts of the whole current from one battery are conducted.

called the *testing conductor*, be soldered by its ends to the ends A and D of the conductor to be tested and of the reference conductor, or to strong pieces of metal to which those ends are firmly attached. Let one electrode of a galvanometer be soldered to the connecting arc BC, at its middle, or at any other point of it, Q; and let the other galvanometer electrode be ready to be applied by the hand to any position on the testing conductor. A current is then sent from one or more cells of Daniell's battery through electrodes connected with A and D. This current flows through the divided channel ABCD and AP′OPD, in quantities inversely proportional to the resistances of the two parts. The moveable galvanometer electrode is then applied, first to one point and then to another of the testing conductor (care being taken not to reverse, nor even to diminish, the magnetism of the lower needle in the astatic system of the galvanometer*), until by trial the point O that may be touched without producing any deflection in the needle, is found. The influence to be tested, whether it be magnetization, or tension, or elevation of temperature, is then applied to AB, or the influences to be tested against one another are applied to AB and CD, and the moveable galvanometer electrode is (if it has been removed) again applied at O. If the needle remains undisturbed, no effect is indicated; that is, no alteration in the resistance of ABQ, or only an alteration in the same proportion as an alteration experienced by QCD, has been indicated. If, however, a deflection is observed, in such a direction that the moveable electrode must be moved to some point P in the part OD, it is inferred that the ratio of the resistance of ABQ to that of QCD has been increased; or on the other hand, if such a deflection as requires a motion of the moveable electrode to a point P′ in OA, the resistance of AB has been diminished relatively to that of CD.

Fig. 38.

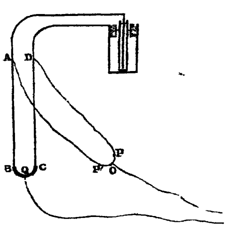
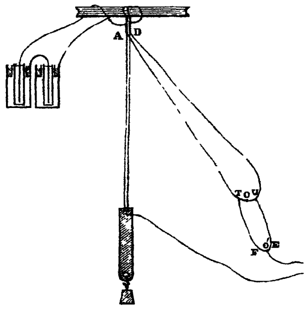


Fig. 39.



150. As an example, I shall describe an experiment on the relative effects of tension on electric conductivity in copper and iron wires. Two pieces of stout copper wire, A, D, were each twisted into a loop which was made fast by solder; a couple of inches towards one end of each wire being left free from the twisted part. These loops were put upon a strong hard wood peg about $\frac{3}{4}$ of an inch diameter, at a distance of about $\frac{1}{4}$ of an inch from one another; and to their lower ends were firmly soldered fine iron and copper wires (strong enough

* In the galvanometers which I have used, the two needles of the astatic combination are of similar material (pieces of the same steel wire, tempered brittle), and the lower one is a little longer (perhaps by about $\frac{1}{10}$) than the upper. Both are magnetized to saturation, and consequently the lower preponderates and gives its

to bear weights of about 8 lbs. and 5 lbs. respectively). These wires were cut to the same length of $4\frac{1}{2}$ feet, and their lower ends were put into slits about $\frac{1}{4}$ of an inch deep, cut in the top of a piece of stout copper slip of the form and dimensions shown in the diagram, and the copper pressed upon them to hold them fast by a pair of pincers. Solder was then applied, to make a complete and compact metallic connexion between the wires and the copper piece. A testing conductor, consisting of seven yards of No. 18 copper wire, was soldered by its ends to the upper copper pieces A, D; and a current from six small cells of DANIELL's was sent through the double channel by electrodes soldered a little higher up to the same copper pieces, A, D. One galvanometer electrode was soldered to the lower copper piece, and the other was applied to the testing conductor till the point O, equipotential with the point of attachment of the former, was found. As from previous experiments I knew that an accidental variation of $\frac{1}{100}$th of an inch in the position of the moveable electrode on the testing conductor might lose or overbalance the effect looked for, I added a *multiplying branch*, TFO′EU, consisting of a yard of No. 18 copper wire, with its ends soldered about half an inch on each side of O. This, of course, when touched by the moveable electrode, gave about thirty-six times the motion that would be required to produce or to correct any effect on the galvanometer if the simple testing conductor were used. The point O′, on the multiplying branch, that could be touched without giving any deflection was then found; and weights were hung from the lower end of the lower copper piece, so as to stretch the copper and iron wire equally. Immediately a deflection of the needle in the galvanometer showed a current. This was corrected by sliding the moveable electrode on the multiplying branch towards U, that is, towards the parts conterminous with the copper wire. When the weights were removed, immediately a reverse deflection was observed. The conclusion is, that iron and copper wire equally extended have their resistances altered differently when under the stress; that of the iron wire being more increased, should the absolute effect in each wire be an augmentation of resistance, as other experiments I have made give me reason to suppose it is, or less diminished should it turn out that the absolute effect in each wire is a diminution of resistance.

Fig. 40.

151. Again, a heavier weight was applied so as permanently to elongate the wires.

direction to the system. The strongest current through the coil only confirms the required state of magnetization, provided when it is started the index is either at zero, or on the side of zero towards which the deflection is to be. If by accident a powerful current is admitted through the coil when the index is on the wrong side of zero, the lower needle has its magnetism instantaneously reversed; but it may be as instantaneously put right again by suddenly reversing the current. If at any time, from the lower needle having either lost magnetic moment, or acquired a reverse magnetization, the astatic system is found reversed, it may be put in order with ease either by simply sending a powerful current through its coil, or by doing so and then suddenly reversing the current.

The deflection, which was in the same direction as at first, was noted, but not corrected by any motion of the moveable electrode, and the weight was again removed. The needle returned towards zero, but remained deviating in the same direction as it had done to a greater degree with the weight on. By applying the hand instead of weights and gradually pulling down the lower copper piece, at first slowly, and afterwards rather faster, the needle could be made to deviate to 7° and kept steadily there. After the wires had been stretched by rather more than an inch, the hand was removed with a gradual diminution of stress, which could easily be regulated to let the needle down without oscillation to whatever position it would rest in, with the stress entirely off. This in several repetitions of the experiment on the same wires was found to be somewhere about 3° or 4° in the same direction as the deviation which was kept at 7° for a few seconds during the stress. Hence it was further concluded, that, as regards electric conductivity of the substance, the effect of permanent elongation, remaining after the stress is removed, differed between iron and copper in the same way as the effect of longitudinal stress during its action; that is, that the galvanic resistance of iron is more increased by permanent elongation than that of copper. Irregular variations to a considerable extent, obviously due to thermo-electric effects from the copper and iron in the compound conducting circuit, made me not attempt to measure with much care, the distance the moveable electrode had to be shifted to counteract the effects of tension; but I intend repeating the experiment and making it for other pairs of metals, with this source of irregularity removed by a modification of the testing conductor.

152. In the kind of experiment which has been described, the channels through the two metals experienced exactly the same elongation, and, it may be said without committing any sensible error, the same narrowing, by the longitudinal extension. The effect observed, therefore, depends truly on variations in the conductivities of their substance. I had made previously various experiments on copper wire alone, and on iron wire alone, in which I attempted to eliminate the effects of elongation and narrowing, and had very nearly established, for the case of iron wire at least, that the augmented resistance due to tension, either temporary or permanent, is a very little more than can be accounted for by the change of form. As, however, I have other experiments in progress, by which I hope to be able to show for a single metal the absolute effect on its specific conductivity separated perfectly from any influence on the resistance of the conductor occasioned by a change of its form, I defer in the meantime giving more details of investigation on this subject.

153. The method which has now been described has many great advantages over that by the differential galvanometer, or any other that I know of for testing or measuring galvanic resistances. In the first place, the irregularities, dependent on the electrodes, connexions, and circular conductors, of the differential galvanometer, are entirely done away with, and only the tested and the testing conductors, all connected by compact solderings, can influence the indication from which the results are

to be drawn. In the second place, the galvanometer circuit may be broken and completed, and reversed, as often as is desired, by its own commutator, without affecting to the slightest sensible degree, the strength of the current through the tested and testing branches; while in the former mode of experimenting the indicating needle was always under the action of the divided current, unless the current in one or the other of the branches was broken, which introduced irregularities lasting for a considerable time, by the consequent changes of temperature through the conductors. This was an immense convenience in every experiment, and allowed small deflections, amounting to the tenth of a degree, to be tested with ease by using the commutator of the galvanometer, and getting oscillations. But it was of especial advantage in the experiments on the effects of transverse magnetization, since the galvanometer circuit had only to be kept broken for a few seconds during the making, breaking, or reversing of the magnetizing current, to get entirely rid of all disturbances of the needle due to induced currents; and in all experiments in which the RUHMKORFF magnet was used, since by breaking the galvanometer circuit and using a little steel magnet in the hand, the galvanometer needle could be let down in a few seconds into its position as affected by the direct action of the large magnet, before proceeding to test the current due to the change of resistance under investigation. In the third place, it is possessed of almost unlimited capacity for increase of sensibility. In some of the experiments on the influence of tension on electric conductivity, I have tested with the greatest ease effects amounting to only $\frac{1}{15000}$th of the whole resistance of the wire under examination, and I see no difficulty in testing effects amounting to only the tenth part of that, or even hundreds of times smaller effects, by using more powerful currents, and applying artificial means to keep the wires cool.

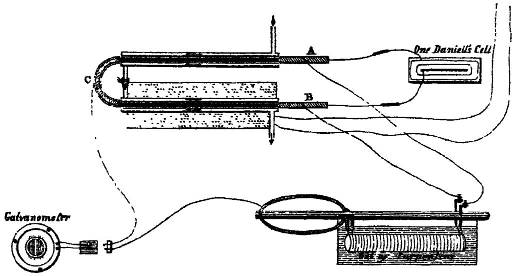
PART V. ON THE EFFECTS OF MAGNETIZATION ON THE ELECTRIC CONDUCTIVITY OF METALS.

154. The remarkable effects which I found produced in the thermo-electric quality of a metal by magnetization and by mechanical strain, appeared to render it highly probable that the same agencies would also influence their electric conductivities. To demonstrate this if I could, and to discover the nature of the anticipated effects, I commenced an experimental investigation of the subject, and, after various nugatory operations, arrived at a variety of positive results by the following processes.

155. Exp. 1. *On the longitudinal electric conductivity of longitudinally magnetized iron wire.*—A length of seventy-two yards of silk-covered copper wire was rolled in six strands, or altogether in about 860 turns on a core made up of two concentric brass tubes, connected at their ends by a ring of sheet brass, and arranged to have water sent through the space between them by suitable entrance and exit pipes soldered to apertures in the outer one; the external diameter of the brass tube was about $\frac{5}{12}$ inch, and the internal diameter of the inner one about $\frac{1}{6}$ inch; the metal of both outer and inner

tubes being as thin and as well smoothed as it could be got. The piece of iron wire to be tested was soldered at one end to a piece of thick copper wire, and then insulated by a thin coating of writing-paper, wrapped twice round it, and pushed into the inner brass tube, which was just large enough to admit it easily. A second iron wire of equal dimensions was similarly prepared and inserted in a second core, in all respects like the other, except that in this experiment it had no copper wire wrapped round it. The two cores being laid side by side, the free ends of the iron wires were connected as shown in the diagram, by an arc of thick copper wire, C, soldered to them. A current from a single large cell of DANIELL's was admitted and carried off by the electrodes A and B. Cold water was kept constantly flowing through the spaces between the concentric brass tubes round the iron wires. The testing conductor (§149) used in this experiment consisted principally of the following parts:—(1) Two pieces of No. 18 copper wire, each sixteen yards long, prevented from touching one another by a piece of twine between them, rolled together on a thin copper cylinder, 12 inches long and 3 inches diameter, from which they were insulated by a coating of two folds of silk cloth sewed round it. (2) Soldered to two of their contiguous ends, a connecting arc of thick copper wire, which was at first intended to be gradu-

Fig. 41.



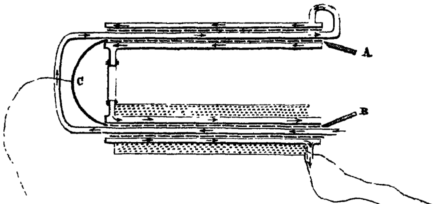
ated, and will be called *the scale of the testing conductor*. (3) Separate short thick wires soldered to the other ends of the wires coiled on the copper cylinder, to bear binding screws for making connexions with the electrodes A and B of the conductor to be tested. One electrode of the galvanometer was soldered to the middle of the connecting arc between the two iron wires, and the other was held in the hand, and applied about the middle of the scale of the testing conductor. A rather troublesome process was then required to bring the galvanometer to zero by adding resistance on one side or the other between the ends of the testing conductor and A or B. When this was done, it was found that great deviations of the galvanometer needle were produced by sliding its moveable electrode a few inches in either way on the scale, and a perfectly sensible deflection by sliding it as much as $\frac{1}{8}$th of an inch. The point of the testing scale to which the moveable electrode had to be brought, to give

no deflection of the galvanometer, was determined: the circuit of the galvanometer was broken, and a current from six of the small iron cells was sent through the magnetizing coil. Immediately on completing the galvanometer circuit again, with its electrode held on the same point of the testing scale as before, a very considerable deflection was observed. On breaking the galvanometer circuit, reversing the magnetizing current, and completing the galvanometer circuit again, the same deflection was observed; and when the magnetizing current was stopped the galvanometer again gave zero, or nearly so. On repeating the process as regards the magnetizing current, without breaking the galvanometer circuit, the same deflection was always observed, in whichever direction the current was sent through the magnetizing coil; and little or no either instantaneous or permanent effect was produced on suddenly reversing this current. It was found that the deflection occasioned by the magnetization was diminished by sliding the moveable electrode along the scale from its end communicating with B, towards its end communicating with A, and was corrected by such a motion through a space of about $\frac{3}{4}$ths of an inch; equivalent to $\frac{1}{10}$th of an inch of the No. 18 wire, constituting the chief part of the testing conductor. It was concluded that the iron wire had its electric resistance increased by magnetization, and that this augmentation amounted, in the particular experiment, to about $\frac{1}{3000}$ of the whole resistance of the magnetized piece.

156. Exp. 2. *On the effect of permanent magnetization on the electric conductivity of steel wire.*—The same apparatus as in Experiment 1 was used, and was in all respects similarly arranged, except that hardened steel wires as free from magnetism as possible were substituted in place of the soft iron cores in the brass tubes. On bringing the galvanometer to zero and sending a current through the magnetizing coil, the same deviation as before was observed, and a much smaller deviation in the same direction remained after the magnetizing current ceased. This experiment was repeated several times on fresh unmagnetic steel cores, and always with the same result. I concluded that steel when subjected to magnetic influence has, like iron, its electric conductivity diminished in the direction of the lines of force; and that it retains some of the same effect with the permanent magnetism subsisting after the magnetizing force is removed. At the same time I was not quite satisfied with the experiment, as the galvanometer needle was never very steady, and, to keep it about zero, the moveable electrode had to be shifted largely along the scale, sometimes quite to one end, when, to get it on the scale again, additional adjustment wires had to be added to the other branch of the testing conductor. This prevented me from using more powerful currents through the wires to be tested and so getting larger indications of the results; but I determined if possible to repeat the experiment afterwards with arrangements better adapted to do away with all variations in the conductivity of the circuit except those under investigation. I still keep it in view to do so, and I have no doubt now of being able to get rid of all the unsteadiness which I had found so troublesome.

157. Exp. 3. *Attempt to discover the effect of transverse magnetization on the longitudinal conductivity of a slip of sheet iron.*—Two brass cores like those described above, and of the same length (10 inches), but of larger inner and outer diameters, were prepared, and a quantity of covered copper wire rolled on one of them in four strands, or in all 570 turns. Two slips of sheet iron, each 7 feet long and $\frac{1}{8}$ inch broad, were wound upon single brass tubes coated with paper, and the successive spires of each were kept from contact by a piece of twine wound on between them. A length of 9 inches of each brass tube had 84 inches of the slip iron laid upon it, and therefore the inclination of the helix to a plane perpendicular to its axis was about 6°, being the angle whose sine is $\frac{9}{84}$. Each of these iron spirals was protected outside with a coating of paper, and pushed into the interior of one of the brass cores. A copper arc, C, was soldered to each of them so as to connect their extremities on one side, and powerful copper electrodes, A and B, were soldered to their other extremities. Then, a stream of water being kept constantly flowing through each of the inner tubes and through the spaces between the concentric brass tubes outside, a current from a large cell of DANIELL's (§ 63) (exposing 2·5 square feet of zinc to 4·4 square feet of copper) was sent through the iron spirals, and a testing conductor (the same one as before) was put in com-

Fig. 42.



munication with their electrodes, A and B. One electrode of the galvanometer being, as before, soldered to the middle of the copper arc connecting the iron spirals, the other was applied to the scale of the testing conductor. The galvanometer being brought to zero by the insertion of adjustment wires at one end or other of the testing conductor, it was found to be rather steadier than in the former experiments, probably because of the diminution of thermal effects by the stream of water through the cores, and the greater surface of iron exposed outside and inside to refrigeration. When a current was sent and maintained through the magnetizing helix, a very decided permanent deflection was occasioned in the galvanometer; and this the same with each direction of the magnetizing current. If the galvanometer circuit was kept complete, its needle experienced a powerful impulse, sending it through a great many degrees in one direction or the other at the instant of starting, or of reversing, or of stopping the magnetizing current, but quickly in each case showed the nature of the permanent deflection by oscillating about one position, when the current was steadily maintained, in either direction. These impulsive deflections were of course due to induced currents, and were entirely prevented by keeping the galvanometer

circuit broken during the starting, the reversal, or the stoppage of the current through the coil of the electro-magnet.

158. The deflection due to the effect of magnetic force on the substance of the iron was corrected in each case by sliding the moveable electrode towards the part of the testing scale remote from the end connected with the iron spiral which experienced that effect; and it therefore indicated a diminution of conductivity in the iron.

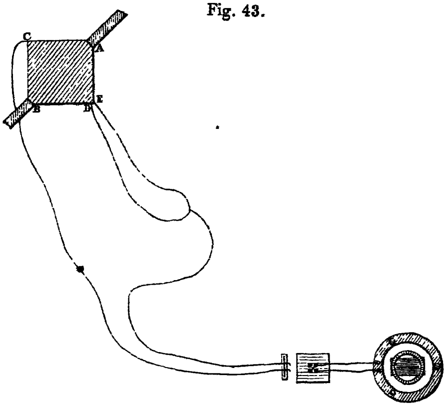
159. If the lines of magnetization had been exactly perpendicular to the lines of electric current through the iron, we should now conclude that transverse magnetization diminishes the conductivity of an iron conductor; that is, that it produces the same kind of effect on the conductivity as longitudinal magnetization. But the lines of current formed spirals inclined at an angle of 84° to the lines of the magnetizing force; and the mutual influence of the consecutive parts of the magnetized iron spiral would have an effect (not wholly compensated by the mutual influences between the successive spires because of the thickness of the twine between them,) contributing to longitudinal magnetization; and therefore the lines of magnetization must have been inclined, not at 90°, but at some angle less than 84°, to the direction of the lines of current. Hence all we can conclude is, that not only longitudinal magnetization but oblique magnetization up to some angle of obliquity less than 84° from the lines of current, diminishes the electric conductivity of iron.

160. It remains to be determined by experiment what is the effect of magnetization right across the lines of current: if a diminution of conductivity, whether a greater or a less diminution than is caused by an equal longitudinal magnetization? or if it is an increase of conductivity, what is the angle of obliquity of the magnetization which gives neither increase nor diminution of conductivity?

161. Exp. 4. *To discover the differential effect of magnetization on the conductivity of iron in different directions.*—A square of $1\frac{1}{2}$ inch each side was cut from thin sheet metal, and powerful electrodes were soldered to two corners, A and B. A reference electrode (§ 149) of No. 18 copper wire was soldered to C, one of the other corners, and the two extremities of a yard of the same kind of wire, to be used as a multiplying branch, were soldered to points D, E, about $\frac{1}{15}$th of an inch from one another on each side of the remaining corner. A current being conducted through the square by the principal electrodes A and B, the reference electrode was used to connect C permanently with the commutator belonging to the testing galvanometer. Another wire used as a testing electrode, was applied to connect any point of the plate, or of the multiplying branch, with the other galvanometer electrode. In the first place, it was found that a powerful current was raised in the galvanometer coil if the testing electrode was applied to any point of the multiplying branch; and it was necessary therefore, as was anticipated, to adjust the distribution of resistance through the square by filing, so that there might be some point on the testing branch which would give no current when touched by the testing electrode. (See below, § 176, where a less troublesome way of managing this part of the arrangement, in an analogous experiment, is described.) For this purpose, in

the first place the testing electrode was applied at different places along the edges BD, EA of the square till a point was found which gave no deflection of the galvano-

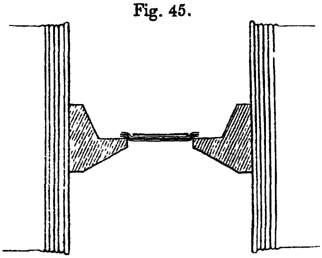
Fig. 43.



meter. If this was in BD, the plate had to be thinned in its middle parts parallel to CA and BD, or else to be thinned along the edges CB, AD, so as to increase the resistance to conduction parallel to the last-mentioned edges. Or if the neutral point was in EA, the plate had to be thinned in its middle parts parallel to CB and AD, or along its edges BD, CA. By using the file according to these directions, after a few trials the neutral point was brought upon the testing branch; that is to say, the resistance was so adjusted in the square that the line from C cutting right across the lines of conduction, or which is the same thing, the equipotential line through C, passed between D and E. A piece of sheet copper as broad as the iron square, but rather longer, was bent as shown in the diagram, so as to give a depressed space in which the iron, insulated from the copper simply by a piece of writing-paper, could rest steadily. This copper cradle was placed resting on the flat poles of a RUHMKORFF electro-magnet, which were pushed together so as to hold it firmly. Any leakage of electric currents from the coils of the electro-magnet was thus effectually drained by the copper, so that a simple sheet of paper was quite enough to do away with all sensible indications of currents in the iron acquired otherwise than through the electrodes A and B. [This electrical drainage would be made more nearly perfect by using paper or some other non-conductor to separate the cradle from the poles of the magnet.]

Fig. 44.

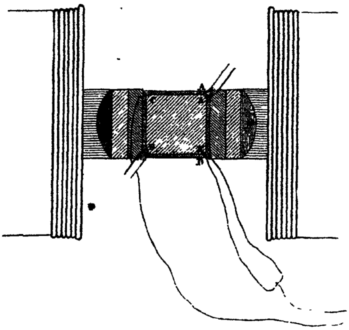
Fig. 45.



162. A large single element of DANIELL's (§ 63), consisting of seven zinc plates in

seven porous cells, contained in four large wooden cells, and exposing in all 8·75 square feet of zinc surface to 15·3 square feet of copper, was then used to send a current through the iron square, insulated between the poles of the electro-magnet, in the manner described.

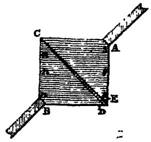
Fig. 46.



163. The neutral point on the testing branch being got by trial, it was found to remain tolerably steady, although no doubt during the first minutes of the flow of the current it may have varied much, as the iron got heated, which it soon did to a degree very sensible to the touch. Moving the electrode along the testing branch through a quarter of an inch on either side of the neutral point, gave a very marked deflection of the galvanometer. The galvanometer circuit was then broken, and a current from six of the small iron cells was started through the coils of the electro-magnet. When the galvanometer circuit was again, after a few seconds, closed, with its electrode on the same point of the multiplying branch as before, a very considerable deflection was observed in the needle. To correct this deflection and bring the needle to zero, the testing electrode had to be moved to a position 2 or 3 inches nearer D on the testing branch.

164. The new neutral point was unchanged when the electro-magnet was reversed, and when the magnetizing current was broken there was a permanent deflection in the galvanometer the reverse of that observed when the current was started in either direction. If the galvanometer circuit was completed within a second or two of any of the changes in the magnetizing current, the needle experienced, obviously from induced currents, powerful impulses in one direction or the other, according to the direction of the current made or unmade through coils of the electro-magnet. But in every case, although from various disturbing causes the neutral points gradually shifted largely along the testing branch, the permanent effects of making and of unmaking the electro-magnet were most marked, and were uniformly as stated above.

Fig. 47.



165. Thus it appears that magnetization shifts the equipotential line through C from its position running across to the opposite corner, to a position (dotted in the diagram) a little nearer CB; so much so that its end is shifted about $\frac{3}{36}\times\frac{1}{20}$, or $\frac{1}{240}$th of an inch from E towards D. This shows that the passage of electricity in the directions AE and CB has become less resisted than it was, relatively to the passage in the directions AC, DB; and it therefore follows that the electric conductivity of magnetized iron is greater across than along the lines of magnetization.

166. Still, as the preceding experiment (Exp. 3) had appeared (§ 159) to show that the absolute conductivity is diminished in all directions by magnetization, it seemed possible that the effect now observed might be caused by inequalities in the distribution of magnetism in the plate. Thus if from the character of the distribution of the magnetizing force, or because of non-uniformity in the plate, the parts between C and B and between A and E were less intensely magnetized, and those between C and A and between B and D more intensely magnetized, than the average, the observed effect could be accounted for without any difference in the electric conductivity of the substance in the different directions. To test this conceivable explanation, pieces of soft iron (cubes and little square bars nearly double cubes) were laid over the square plate, being kept insulated from electric communication with it by paper, so that while the conducting mass remained unchanged, the distribution of the magnetization of its substance might be altered. Before the magnetic force was applied, a great effect on the neutral point of the multiplying branch was observed, taking place gradually during several minutes, and obviously due, in a great measure, to variation of the distribution of temperature in the conducting square. (See below, § 177, for an illustration of this effect.) When a new neutral point was found, the magnet was made, reversed, unmade, &c., and always with the same effects as before. Different arrangements of the little masses of soft iron produced different absolute effects on the neutral point, causing it to shift sometimes as much as fifteen inches on the multiplying branch, but the effects of magnetism were invariably found to be consistent with the first-mentioned result. As the distribution of the magnetism in the square plate must have varied very much under these different circumstances, and in all probability must have been in some of the cases more intense in the quarters towards AE and CB than in those towards AC and DB, the conclusion could scarcely be avoided, that the conductivity of the magnetized substance was greater across than along the lines of magnetization. For the purpose of further testing and illustrating this conclusion I planned the following experiment, to compare directly the resistances of two equal and similar squares of sheet iron, equally and similarly magnetized, arranged in the same circuit to conduct electricity across the lines of magnetization of one and along those of the other.

167. Exp. 5. *To compare the conductivities of magnetized iron along and across the lines of magnetization.*—A piece of sheet copper, BCHK (fig. 48), 3 inches long, 2 broad and $\frac{1}{12}$ inch thick, was bent round the line FH into the form shown in fig. 49.

Fig. 48.

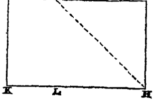


Fig. 49.

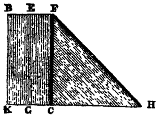
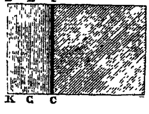


Fig. 50.



A square of thin sheet iron, 2 inches wide (weighing 103 grains), was soldered by one side to the edge CH of the copper in the position shown in fig 50. The pro-

jecting part, FBKL, of the copper slip was bent round its middle line EG, so as to bring its edge, BK, close over the edge of the iron square lying over FC; and to this edge, BK, in its new position (fig. 51), a second iron square, of the same dimensions and weight as the other, was soldered by one side, with its area lying in a position close to that of the former. The relative position of the two squares and the connecting piece of copper will be understood by looking at fig. 52, which represents the

Fig. 51.

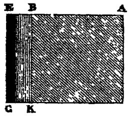
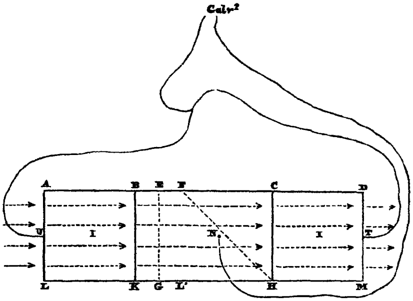


Fig. 52.

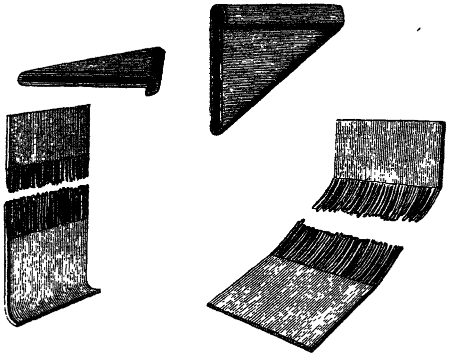


iron squares as if soldered to the piece of copper before it was bent, and the iron square CDMH turned round its side CH, from the position close to the plane of the copper adjoining it, into a position in this plane continued across CH. If, now, we suppose the iron square CDMH to be turned down so as to lie below a square, FL'HC, of the copper; this square of the copper to be bent sharp round its diagonal, FH, till the part FL'H lies over HCF; and, lastly, the part FL'KB projecting beyond FC to be bent downwards round EG with a less sharp bend; the iron square, ABKL, will be brought close under the other one, CDMH, with the edges of the two which are connected to the edges of the copper perpendicular to one another, and the whole compound conductor will have exactly the position shown below in fig. 54.

168. A convenient electrode was soldered along the edge of each iron square parallel to the edge of the same square soldered to the connecting piece of copper; so that a powerful electric current entering by one of those electrodes and carried away by the other would pass through the second-mentioned square of iron in lines exactly parallel to the side AB, through the connecting piece of copper in lines which were parallel to its length, BC, before it was bent; and through the first-mentioned square in lines exactly parallel to its side CD, and therefore perpendicular to the lines along which it traverses the second square. The course of the current will be understood by looking at fig. 52, where the two squares and the copper connecting them are supposed to be opened out so as to throw the course of the current into a straight line. The order followed in constructing the compound conductor was not exactly the order of the description given above; but the connecting piece of copper was first cut and bent, and other pieces, to serve as electrodes (shown in the accompanying

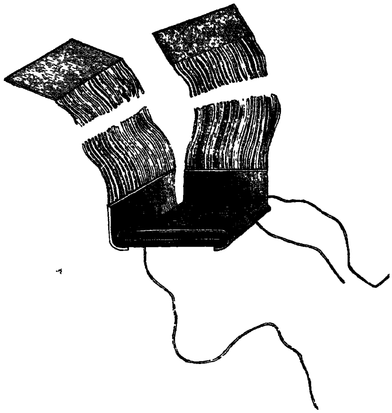
views, fig. 52), were prepared, and the iron squares, put in their proper places, were then soldered by their edges to the edges of the connecting piece, and the electrodes

Fig. 53.



were soldered to their opposite edges. A view of the whole thus put together, with the reference and testing wires described below, is given in fig. 54. A testing con-

Fig. 54.



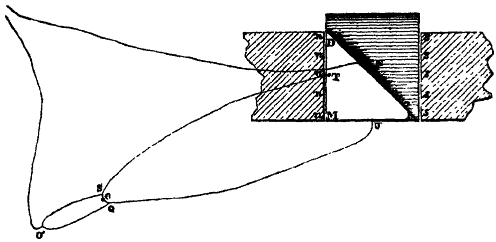
ductor of two yards of No. 18 copper wire was soldered, with its two extremities to the copper electrodes, close to the middle points of the edges AL, DM of the iron squares; and a fixed galvanometer electrode was soldered to the middle point, N, of the copper connecting piece.

169. The squares, their electrodes, the connecting piece, and the testing conductor

being then guarded against irregular contacts by a little square of pasteboard pressed between the iron squares, a half-square of pasteboard between the first-mentioned iron square and the portion FCH of the connecting copper (see fig. 49), and fragments of paper and pasteboard elsewhere, the whole was placed, with the second-mentioned square lowest, in a copper cradle lined with paper, and resting between the horizontal edges of the flat poles of the RUHMKORFF electro-magnet used in the preceding experiment.

170. The positions of the magnetic poles of the squares, of the bent connecting piece of copper, of the testing conductor, and of the galvanometer electrodes are indicated in fig. 55, but, to avoid confusion, the principal electrodes are not shown.

Fig. 55.



A current from the four large double cells, connected so as to constitute in all a single element of DANIELL'S, exposing 10 square feet of zinc surface to $17\frac{1}{2}$ square feet of copper, was then introduced by the principal electrode soldered to the edge MD of the upper square, and drawn off by the other principal electrode, namely, that soldered to the edge of the lower square lying exactly below the edge MH of the upper. The course of the current into the principal channel between these electrodes would be across the upper square from MD to HC, and across the lower square from the edge below CD to that below HM; also, in the secondary channel between the same electrodes, from T soldered to the first through the testing conductor, to its other end U soldered to the second.

171. A fixed galvanometer electrode being (§ 168) soldered to the middle point, N, of the connecting-copper, the other electrode of the galvanometer was moved along the testing conductor till a point, O, was found at which it might be applied without giving any deflection. By moving it $\frac{1}{50}$th of an inch on either side of O very sensible deflections were obtained, and therefore a yard of copper wire was soldered by its ends to points S and Q a quarter of an inch on each side of O, and was used instead of the "scale" of the testing conductor described as used in the first three experiments. The neutral point, O', on this multiplying branch having been found, the galvanometer circuit was broken, and the electro-magnet was excited by six of the small iron cells. On closing the galvanometer circuit again immediately, a considerable deflection was observed, to correct which the moveable electrode had to be

moved through about two or three inches from O′ towards Q. On unmaking the electro-magnet a reverse deflection in the galvanometer was observed, and was corrected by bringing back the electrode to O′. The same result was obtained when the magnet was made in the reverse way, and never failed to appear, to an unmistakeable extent and with perfect consistency, after the operation had been repeated many times and varied in every possible way.

172. It showed that the effect of the magnetization was to increase the resistance relatively in the upper square of iron, and to diminish it relatively in the lower square. I concluded with confidence that the electric conductivity of magnetized iron is greater across than along the lines of magnetization.

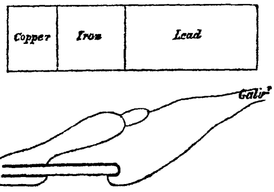
173. Exp. 6. *A double experiment, to test the absolute nature of the two effects of which the difference was shown in the preceding experiment.*—A divided current from the battery was made to pass through the two squares by electrodes, of which one was soldered to the middle of the copper band connecting them, and the other clamped to the now united extremities of the bundles of copper wire which had served before to lead in and out the whole current in the preceding experiment. As testing conductor was used the same piece of copper wire which had served as the fixed galvanometer electrode in the preceding experiment, with its end which had been connected with the galvanometer now soldered to the junction of the two copper branches of the divided channel (the resistance of each of which was found to be nearly equal to that of the iron square with which it is connected). The testing wire used in the preceding experiment was cut in two, one part to serve as fixed galvanometer electrode in one, and the other in the other of the two experiments which it was intended next to make. I first attempted to test the effect on the conductivity of the upper of the two squares produced by the magnetization which in it is along the lines of current. I found, however, on fixing the copper wire proceeding from one side of that square to one electrode of the galvanometer, and applying the other to the testing conductor in the usual way, that the circumstances were constantly varying, and that the point to be touched to give no deflection shifted rapidly along the testing conductor. Hence I gave up this part of the experiment, of which the result might be anticipated with certainty from the experiment on the effect of magnetization along the line of current described above (Exp. 1. § 155), and I gave the whole time during which the experiment could be continued, to an examination of the influence of the electro-magnet on the current in the branch leading through the lower square across its lines of magnetization. Accordingly, the galvanometer electrode, which had been united to the part of the old testing conductor terminating in an edge of the upper square, was transferred to the other part of the old testing conductor, that is, to the part terminating in a side of the lower iron square. The same new testing conductor was still used; and as soon as a point could be found on it which gave no current when touched by the moveable galvanometer electrode, points about $\frac{1}{4}$ of an inch on each side of it were

taken, and a multiplying branch of one yard No. 18 copper wire was soldered by its ends to them. Before, however, the effect of the magnetism could be decidedly tested, the zero-point had moved off the multiplying branch, which had accordingly to be shifted along the testing conductor to get into range again. The same process had to be gone through a great many times, and at last, after the current had been flowing continuously through the two squares and the divided copper channel for about five hours, the zero-point became sufficiently steady to remain on the multiplying branch when fixed at the right place on the testing conductor, and to allow a decisive experiment to be made. The result was a very slight effect, proving *a diminution of resistance in the iron square.*

174. The cause of the long-continued variation in the conditions of electric equilibrium between the testing conductor and the fixed point on the edge of the lower square, was clearly the gradual warming of the long copper wires extending up from this point, due to the conduction of heat generated in the iron squares by the electric current; and it would obviously be much diminished by using a simple form of conductor with only one iron square at a time, and with the reference conductor kept near it, so as to acquire quickly whatever temperature it would rest with during the flow of the current. I accordingly made the following experiment, choosing first the effect of transverse magnetization, as the experiment just described had not been of a satisfactory kind, although apparently conclusive, while the first experiment of the series (Exp. 1. § 155) had been less unsatisfactory in point of steadiness, and had led decisively to a conclusion regarding the effect of longitudinal magnetization on the resistance of a conductor.

175. Exp. 7. A square of sheet iron like those used in the last experiment (four square inches, weighing 103 grains, and consequently about $\frac{1}{78}$th of an inch thick,) was soldered along one edge to a slip of lead of the same width, about twice as thick and about one-half longer. To the opposite edge of the iron square was soldered a stout copper slip an inch broad and equal in length to the side. The piece of lead was bent round, so as to give a straight part lying about $\frac{1}{4}$ of an inch from the plane of the iron, and to extend about as far as the copper slip soldered to the other edge of the square. A current from an arrangement of the cells (§§ 63 and 64) constituting a powerful single element of DANIELL's was sent through the iron square and the lead band, by electrodes clamped to one end of the lead and to the copper slip fixed to the other edge of the iron. A point in the lead slip having been found, such that the galvanic resistance between it and the edge next the iron was nearly equal to the resistance in the iron square itself, a testing conductor (two yards of No. 18 copper) was soldered by one end to that point in the lead, and by its other end to the middle of the edge of the iron

Fig. 56.

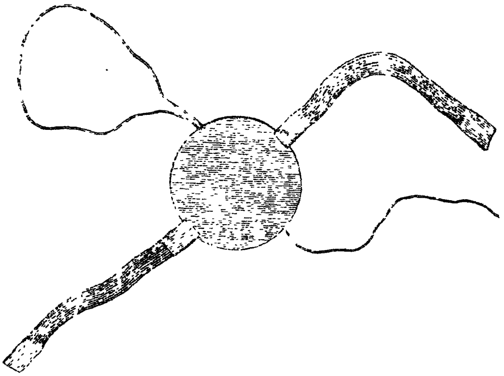


square to which the copper slip is attached. A copper wire, to serve as fixed galvanometer electrode, was soldered to the lead band, at a point in the middle of its breadth close to its edge of attachment to the iron. A copper cradle was put between the flat poles of the electro-magnet, as before (see above, § 161), and covered with a piece of paper. The iron square was supported upon it in a position with the line joining the poles perpendicular to the line of the current through it. Then, the current being kept steadily flowing through the iron and lead band, a zero-point was found on the testing conductor, and a multiplying branch (one yard of No. 18 copper) was soldered with its ends $\frac{1}{4}$ of an inch on each side of this point, in the usual way. The zero-point on this multiplying branch was almost immediately found, and continued on the whole very steady from the first. The galvanometer circuit being broken, a magnetizing current from six small iron cells was sent through the coils of the electro-magnet, and the needle of the galvanometer was let settle (as it could be in a few seconds by the aid of a little magnet held in the hand) into its position of equilibrium as affected by the direct force of the magnet. On completing the galvanometer circuit again, with its moveable electrode on the same point of the multiplying branch as before, a current was made sensible by an excessively slight deflection. The galvanometer circuit being broken, and the electro-magnet reversed, a similar deflection was found in the galvanometer on again completing its circuit. It ceased, as nearly as could be discovered, when the electro-magnet was unmade, and was uniformly observed when the magnet was made again either way, in a great many repetitions. The current indicated by the galvanometer when the magnet was made was always such as to be corrected by carrying the moveable electrode from its previous zero-point, along the multiplying branch, towards the part of the testing conductor terminating at the iron square, and therefore indicated an *increase of conductivity in the iron.* The effect was so very slight, that I could scarcely determine how much the moveable conductor had to be shifted to correct it. I intend to repeat the experiment with similar arrangements, but with two or three times as powerful a current through the electro-magnet, which ought to give about four or nine times the amount of effect. In the mean time, however, I am quite convinced that I have observed the true result, and I conclude that *the electric conductivity of iron is increased by magnetic force across the lines of current.*

176. Exp. 8. *To show the variation of a line of electric equilibrium in a circular disc of iron conducting electricity between two opposite points of its circumference, when subjected to magnetic force in a direction at an angle of* 45° *to the line joining these points.* —A circle 2·3 inches diameter was cut from a piece of sheet iron, and ground down to a thickness which must have been about $\frac{1}{69}$th of an inch, as the prepared disc was found to weigh 114 grains. Two stout copper electrodes were soldered to its circumference at opposite points. A point at 90° on the circumference from one of these was taken, and at about $\frac{1}{40}$th of an inch on each side of it were soldered the ends of a piece of No. 18 copper wire two yards long, to serve as a multiplying branch. The

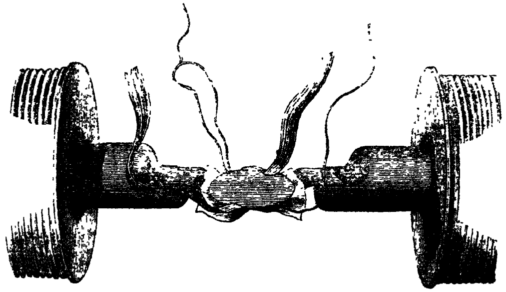
disc was put on a copper cradle covered with paper, supported between the flat poles of the RUHMKORFF electro-magnet, with the line joining its principal electrodes at an

Fig. 57.



angle of 45° to the magnetic axes of the field, and a current from a large single element of DANIELL's was sent through it by these electrodes. One electrode of the galvanometer was applied to the middle of the multiplying branch, and the other was moved about on the opposite parts of the circumference of the disc till a position giving no current was found, where it was then soldered. The moveable electrode applied to different points of the multiplying branch was then found to give sensible galvanometer indications with a motion of a quarter of an inch, and after a very short time the zero-point became tolerably steady. The electro-magnet was then made, with

Fig. 58.



the galvanometer circuit broken, and when it was closed again a decided indication of a current was observed in the galvanometer. This current was checked by sliding the moveable electrode towards the end of the multiplying branch next the equatoreal

part of the magnetic field; and the conclusion was, that the conducting power of the plate, when magnetized, became greater across than along the lines of magnetization, which was confirmed by every repetition and variation of the experiment. Now it is obvious that the intensity of magnetization must have been on the whole greater in the parts of the disc next the poles: hence a diminution of conductivity *across* the lines of magnetization, to the same extent as that which we know from Experiment 1. exists along them, would give a contrary effect to that now observed; and it follows that the electric conductivity is in reality greater across than along the lines of magnetization in magnetized iron.

177. This experiment was witnessed by Mr. Joule, and afforded a full confirmation of the conclusion (§ 172) which had been established by Experiment 5. above, and which follows from Experiment 1. and Experiment 6., considered together. The effects of applying pieces of hot wood equatoreally or axially to the disc were very clearly observed, and were always similar to those described above (§ 166), indicating a greater resistance to the parts of the current crossing the hot region than to those passing through the comparatively cool parts of the iron.

XXXII. *On the Construction of the New Imperial Standard Pound, and its Copies of Platinum; and on the Comparison of the Imperial Standard Pound with the Kilogramme des Archives.* By W. H. Miller, *M.A., F.R.S., Professor of Mineralogy in the University of Cambridge.*

Received up to p. 895 April 16,—Read April 24, 1856; from p. 895 Received June 7,—Read June 12, 1856.

History of the Standards of English Weight.

THE earliest legal standard of English weight, of which any very authentic account is preserved, is the weight called the pound of the Tower of London. According to Folkes*, it was the old pound of the Saxon Moneyers before the Conquest. This pound was lighter than the troy pound by three-quarters of an ounce troy, and did not very sensibly differ from twelve ounces of the weight still used in the money affairs of Germany, and there known by the name of the Cologne weight. It is most probable that the pound of the Tower of standard silver was then cut up into 240 pennies; whence the weight of the penny will be 22·5 troy grains. The silver pennies of the first two kings after the Conquest agree, as near as can be judged, in weight and goodness, with the pennies of the Saxon kings their immediate predecessors. It is therefore reasonable to think that King William introduced no new weight into his Mints. Clarke, in his Treatise on the connexion of Roman, Saxon and English Coins, p. 97, considers this evident from the words of William I.: 'Statuimus et præcipimus, quod habeant per universum regnum mensuras fidelissimas, et signatas; et pondera fidelissima, et signata, sicut boni prædecessores nostri statuerunt.' And also (p. 152) from one of the Conqueror's laws, where it is said, that the Saxon shilling was four pence (from the time of Athelstane), the preamble of which informs us, that these laws were in force during the Confessor's reign: 'Ice les meismes, que le Reis Edward sun Cosin tint devant lui.'

That the Tower pound was lighter than the troy pound by three-quarters of an ounce troy, appears by a verdict relating to the coinage dated 30th October, 1527, 18 Hen. VIII., in the Exchequer, in which are the following words: 'And whereas heretofore the merchaunte paid for coynage of every pounde Towre of fyne gold, weighing xi oz. quarter Troye, ii s. vi d. Nowe it is determyned by the King's highness, and his said councelle, that the foresaid pounde Towre shall be no more

* Table of English Coins, p. 1.

used and occupied, but al maner of golde and sylver shall be wayed by the pounde Troye, which maketh xii oz. Troye, which excedith the pounde Towre in weight iii quarters of the oz.' Hence it follows that the weight of the Tower pound was 5400 troy grains, and that of the ounce or the twelfth part thereof 450 like grains. He quotes a passage taken from the Register of Accounts in Paris, to prove that the Tower pound was also known in France, where it was called the Rochelle or English weight. The difference of the several pounds then made use of in France is there computed, and the proportion between the troy and English weights is thus estimated: 'Ou royaume souloit avoir iv marcs: c'est assavoir le marc de Troyes, qui poise xiv sols, ii den. Esterlins de poix.... le marc de la Rochelle, dit d'Angleterre, qui poise xiii s. iv den. Esterlins de poix.' It is supposed that this account was taken about the beginning of the reign of Edward III., not long after 1329*. Since the sol=12 esterlings, the ratio of the standards of Troyes and Rochelle is as 17 to 16; whence, supposing the weight of Troyes to be the same as the English troy weight, the Rochelle ounce =451·76 troy grains. He refers to a statute of the 51st of Henry III., called 'Assisa panis et cerevisiæ,' to show that the weights in use at that time, though commonly taken to have been troy weights, were not really so, but the money weights: 'By consent of the whole realm of England the king's measure was made, that is to say that an English pennie, which is called a sterling, round without clipping, shall weigh xxxii graines of wheat dry in the middest of the eare; and xx pence make an ounce, and xii ounces make a pound.' For otherwise the pennyweight here described, could never be, as the statute plainly implies, the true weight of the English coined penny.

Folkes determined the weights of a number of silver coins well preserved, or but little impaired, in troy grains (p. 159). Five pennies of Henry III. weighed 22·5 grains each, and one 22·25 grains. Of four pennies of Edward I., two weighed 22·5 grains each, and two others 22 grains each. Assuming the true weight of the penny at this time to be 22·5 grains, which was also that of the Saxon penny†, the weight of the pound will be 5400 grains. It is, however, just possible that the weights were adjusted in conformity with the words of the Act, but that the coin called the sterling fell 1·5 grain short of the full weight of 32 corns of wheat, or 24 grains troy, the weight of 4 corns of wheat being usually considered equivalent to 3 grains troy. On this supposition the pound defined in the statute of the 51st of Henry III., and in the 31st of Edward I., in precisely the same words, would be the pound of 5760 troy grains.

That another pound, the libra mercatoria, was in use at this time, is shown by the following extract from the Treatise on Arithmetic, by Dr. Peacock, in the Encyclopædia Metropolitana, Art. 166:—'Though this weight was the favourite of the legislature, there was another pound, one-fourth greater, which was in more general

* Clarke, p. 15. † Ibid. p. 428.

use; it is mentioned in a Tractatus de Ponderibus of the same age (the time of Edward I.), where the two pounds are said to consist of 20 and 25 shillings respectively: in the statute of the 54th of Henry III., where the composition of the gallon and pound (troy?) are given, there is mentioned also una libra, pondus vigintiquinque solidorum legalium sterlingorum. On many other occasions this libra mercatoria is referred to, and we may consider its use in mercantile transactions and ordinary sales as nearly universal.'

If the pound mentioned in the 51st and 54th of Henry III., and the 31st of Edward I., be supposed to contain 5400 grains troy, the libra mercatoria will contain 6750 grains, which does not differ very much from the old pounds of Villefranche (6741·9 grains), Zieriksee (6736 grains), Dresden stahlgewicht (6726·2 grains), Dantzig (6722 grains), Embrun (6714 grains), Murcia (6711 grains). This supposition is rendered probable by a passage from FLETA quoted by CLARKE, p. 96, who says, 'Quindecim unciæ faciunt libram mercatoriam.' Fifteen Tower ounces of 450 troy grains, twelve of which make the Tower pound, are equal to 6750 troy grains. Sixteen of these ounces make 7200 troy grains, a weight which approaches very closely to the Ptolemaic mina of 7199·96 troy grains, the 100th part of the large Alexandrian talent, also divided into 16 ounces. This weight appears to have survived in the old pounds of Namur (7201·1 grains), Altenburg (7202 grains), Ciney (7202·5 grains), Valenciennes (7195·2 grains), Duerstadt (7206·1 grains), Wittenberg (7207 grains), Heidelberg (7207·22 grains), Aix la Chapelle (7208 grains), Liege (7209·1 grains), Bruchsal (7190 grains), Brunswick (7212·3 grains), Mons (7185·2 grains), Dresden (7215·4 grains), Binche (7185 grains), Gotha (7213·85 grains), Jemappe (7185·2 grains), in the well-known Cologne pound of 7216 grains, and in many others differing rather more largely from 7200 grains. If, on the other hand, the pound of Henry III. and Edward I. contained 5760 troy grains, the libra mercatoria would weigh 7200 troy grains.

In the Acts of the 2nd of Henry V. st. 2. c. 4, and of the 2nd of Henry VI. c. 13, relating to Goldsmiths, mention is made of the 'Pound Troy*.' Either the Tower pound was abolished, or the use of the troy pound as a legal standard confirmed in 1498, the 12th of Henry VII. A statute made in that year enacts 'That every gallon contain viii li of wheat of Troy weight....and every pound xii ounces of Troy weight, and every ounce contain xx sterlings.' The sterling here mentioned must have been a weight, and not the coin of that name; for, during the reign of Henry VII., the weight of the groat was 48 grains, and that of the shilling 144 grains, which gives only 12 grains for the weight of the coin called the penny or sterling†.

The troy pound appears to have been derived from the Roman weight of 5759·2 grains, the 125th part of the large Alexandrian talent, and which, like the troy

* REYNARDSON, Philosophical Transactions, vol. xlvi. p. 61.

† FOLKES, p. 16.

pound, was divided by the Romans into 12 ounces. So also the avoirdupois pound was probably derived from the large Attic mina of 6945·3 grains troy, the 60th part of the large Attic talent, divided by the Romans, as the pound avoirdupois is divided, into 16 ounces, of nearly the same weight as the modern Roman ounce*. According to GREAVES, it approximates much more closely to the ancient Roman ounce.

The word 'avoirdepois,' applied to commodities, occurs in statutes of the 9th and 27th of Edward III. By a statute of the 24th of Henry VIII., butchers were obliged to provide themselves with beams, scales and weights sealed, called 'haberdepois.' It is not known when the avoirdupois weight was first introduced. Two weights, one of which, in its present state, is about 650 grains less than seven pounds avoirdupois, in the possession of C. C. BABINGTON, Esq., of St. John's College, Cambridge, are marked with a crowned H, which is supposed to be of the time of Henry VII. Two weights, one of 2 pounds, the other of 4 pounds avoirdupois, ornamented with the Tudor rose, and marked with the letter H, and therefore probably of the reign of either Henry VII. or Henry VIII., are preserved in the University Library, Cambridge. In the reign of Elizabeth avoirdupois weights were deposited in the Exchequer, as appears from the name and inscription thereon. The Transactions of the Royal Society, for 1742 and 1743 (vol. xlii. p. 541), contain an account of the comparison of various standards of measure and weight, from which the following extracts are made:—'The weights in His Majesty's Exchequer, and which are esteemed the standards, are a pile, or box, of hollow brass weights, from 256 ounces (troy) downwards, to the 16th part of an ounce, all severally marked with a crowned E.' 'The weight mentioned in all our old Acts of Parliament, from the time of King Edward I., is universally allowed to be the troy weight, whose pound consisted of 12 ounces, each of which contained 20 pennyweights. And as the pound is the weight of the largest single denomination commonly mentioned in those Acts, 12 ounces taken from the pile of troy weights above mentioned, as there is no single troy pound weight, must now be reputed to be the true standard of the troy pound, used at this day in England.

'Besides which troy standards, there are also kept in the Exchequer the following standards for averdupois weights; that is to say, a fourteen pound bell-weight of brass, marked with a crowned E, and inscribed

XIIII. POVNDE AVERDEPOIZ. ELIZABETH. REGINA. 1582.

As also a seven pound, a four pound, a two pound, and a single pound, like averdupois bell-weights, and severally marked as follows, excepting the variations for the number of pounds in each respective weight.'

(The marks are: VII. A., AN° DO, a crowned E. L., 1588, A° REG. XXX.)

* DOURSTHER, Dictionnaire des Poids et Mesures, p. 281.

'With which are also kept a pile of flat averdupois weights, from 14 pounds to the 64th part of a pound.'

The comparisons were made with considerable care, the weights being interchanged so as to eliminate the error produced by the inequality of the arms of the balance.

The results, in ounces troy, and grains of which the ounce contains 480, are—

14 lb.+1 lb. (bell-weights)=218 oz. 335·25 grains=104975·25 grains.
7 lb. (bell-weight)=102 oz. 45 grains=49005 grains.
1 lb. (bell-weight)= 14 oz. 282 grains= 7002 grains.
1 lb. (flat weight) = 1 lb. (bell-weight)—2·5 grains=6999·5 grains.

In the year 1758, a Committee of the House of Commons, appointed to inquire into the standards of weight and measure, recommended that the troy pound should be made the unit or standard by which the avoirdupois and other weights should be regulated. By order of the Committee, three several troy pounds were adjusted with great care, under the direction of Mr. HARRIS, the then Assay Master of the Mint, by a mean of a great number of comparisons with the old weights in the Exchequer. The details of the comparisons are given at length in the Report of the Committee, presented by Lord CARYSFORT on the 26th of May, 1758. One of these weights (the imperial standard troy pound which was destroyed by the burning of the Houses of Parliament in 1834) was placed in the custody of the Clerk of the House of Commons. The bill, however, brought in by Lord CARYSFORT in 1760 to give effect to the recommendations of the Committee, in the pressure of business attending the death of GEORGE II. and the accession of GEORGE III., and the dissolution of Parliament soon following, was not carried through all its stages.

In the year 1818, Sir JOSEPH BANKS, Sir GEORGE CLERK, Mr. DAVIES GILBERT, Dr. W. H. WOLLASTON, Dr. THOMAS YOUNG, and Captain KATER, were appointed Commissioners for considering how far it might be practicable to establish a more uniform system of weights and measures. In that part of their Report which relates to the subject of Weights, they recommend that the Parliamentary standard of 1758 should remain unaltered: they state that the avoirdupois pound, which has long been in general use, though not established by any act of the Legislature, is so nearly 7000 troy grains, that they recommend that 7000 such troy grains be declared to constitute a pound avoirdupois: also, that from Sir GEORGE SHUCKBURGH'S weighings of a cube, cylinder and sphere in air and in water in 1797, and Captain KATER'S measurements of the linear dimensions of the same in 1821, they determined the weight of a cubic inch of distilled water, weighed in air by brass weights, at the temperature of 62° FAHR., the barometer being at 30 inches, to be equal to 252·458 grains, of which the Imperial standard troy pound contains 5760.

The chief recommendations of this Committee passed into law by an Act of Par-

liament on the 17th of June, 1824. In the fourth clause of this Act it is enacted, that the old troy pound of 1758, now in the custody of the Clerk of the House of Commons, shall continue to be the original unit or only standard of weight from which all other weights shall be derived; and that it is to be denominated 'The Imperial Standard Troy Pound;' and that the avoirdupois pound, now in use, shall contain 7000 grains, of which the troy pound contains 5760. In the sixth clause it is enacted, that if the standard troy pound should be lost or destroyed, it is to be restored by a reference to the weight of a cubic inch of distilled water, which has been found and is declared to be 252·458 troy grains, weighed in air with brass weights, at the temperature of 62° FAHR., the barometer being at 30 inches.

The Imperial standard troy pound was compared with five troy pounds of gun-metal, destined for the use of the Exchequer, the Royal Mint, and the cities of London, Edinburgh and Dublin, by Captain KATER in 1824 or 1825*. Denoting the standard troy pound by U, and the troy pounds of the Exchequer, the cities of London, Edinburgh, Dublin, and the Royal Mint, by Ex, L, Ed, D, RM respectively, it was found that

No. of Comps.	grain.
16	$Ex = U + 0{\cdot}0010$
12	$L = U + 0{\cdot}0005$
15	$Ed = U - 0{\cdot}0015$
18	$D = U + 0{\cdot}0022$
20	$RM = U + 0{\cdot}0021$

In the year 1829 the standard troy pound was compared with extraordinary care by Captain v. NEHUS with two brass troy pounds and a platinum troy pound, all in the custody of Professor SCHUMACHER, and with a platinum troy pound, the property of the Royal Society.

Let Sb, K denote the two brass troy pounds, Sp the platinum troy pound in the custody of Professor SCHUMACHER, RS the platinum troy pound the property of the Royal Society, t the temperature of the air in degrees of FAHRENHEIT's scale, b the height of the mercury in the barometer in English inches, and reduced to the temperature of melting snow. Let Δ prefixed to the symbol of any weight denote the ratio of the density of the weight at the temperature of melting snow to the maximum density of water. The symbol $\triangleq$ placed between the symbols of two weights will be used to denote that they appear to be equal when weighed in air. The two weights in this case will not be equal unless their volumes are equal. When the weighings have been made in air of constant density, or have been reduced to what they would have been in air of given density; or when, the volumes of the weights and the temperature and pressure of the air being unknown, we are compelled to assume the equality of their volumes, the symbol $=$ may be substituted for $\triangleq$.

* Philosophical Transactions for 1826, Part II. p. 18.

According to the observations of Captain v. NEHUS,—

No. of Comps.	gr.	b.	t.
300	Sp $\triangleq$U−0·00857	29·722	65·62
140	RS $\triangleq$U−0·00205	29·806	65·73
60	Sb $\triangleq$U−0·01034	29·965	64·50
92	K $\triangleq$U+0·03389	29·646	65·09
16	RM$\triangleq$U+0·00887	29·679	65·91

$10-\log\Delta Sp=8{\cdot}67392$, $10-\log\Delta RS=8{\cdot}67392$, $10-\log\Delta Sb=9{\cdot}08471$, $10-\log\Delta K=9{\cdot}09724$ *.

In the burning of the Houses of Parliament in 1834, all the standards of measure and weight were either totally destroyed, or injured to such an extent as to render them quite useless as standards. The Imperial standard troy pound was never recovered from the ruins.

In the year 1838, the Astronomer Royal, Mr. F. BAILY, Mr. J. E. D. BETHUNE, Mr. DAVIES GILBERT, Sir J. F. W. HERSCHEL, Mr. J. S. LEFEVRE, Mr. J. W. LUBBOCK, the Rev. GEORGE PEACOCK, and the Rev. R. SHEEPSHANKS, were appointed Commissioners to consider the steps to be taken for the restoration of the standards of weight and measure, to replace those which were destroyed by the burning of the Houses of Parliament. They found provisions for the restoration of the lost standards prescribed to them by Sections 3 and 5 of the Act 5th George IV., whereby it is directed that, in case of the loss of the standards, the yard shall be restored by taking the length which shall bear a certain proportion to the length of the pendulum, vibrating seconds of mean time in the latitude of London, in a vacuum, at the level of the sea; and that the pound shall be restored by taking the weight which bears a certain proportion to the weight of a cubic inch of water weighed in a certain manner. The Commissioners, however, in their Report, dated December 21, 1841, decline to recommend the adoption of these provisions for the following reasons: 'Since the passing of the said Act, it has been ascertained that several elements of reduction of the pendulum experiments therein referred to are doubtful or erroneous.... It is evident, therefore, that the course prescribed by the Act would not necessarily reproduce the length of the original yard. It appears also that the determination of the weight of a cubic inch of water is yet doubtful (the greatest difference between the best English, French, Austrian, Swedish and Russian determinations being about $\frac{1}{1200}$ of the whole weight, whereas the mere operation of weighing may be performed to the accuracy of $\frac{1}{1,000,000}$ of the whole weight). Several measures, however, exist, which were most carefully compared with the former standard yard; and several metallic weights exist, which were most accurately compared with the former standard pound; and by the use of these, the values of the original standards can be respectively restored without sensible error. And we are fully persuaded that, with reasonable precautions, it will always be possible to provide for the accurate restoration of standards by means

* SCHUMACHER, Philosophical Transactions for 1836, p. 437.

of material copies which have been carefully compared with them, more securely than by reference to experiments referring to natural constants.'

The weight of a given volume of water at 62° FAHR., by means of which the Act of Geo. IV. directs the pound to be restored, was deduced from the weighings in air and in water of a brass cube of 5 inches, of a cylinder of 4 inches diameter and 6 inches long, and of a sphere of 6 inches diameter, by Sir GEORGE SHUCKBURGH in 1797, and the measurements of their linear dimensions by Captain KATER in 1821. The resulting values of the weight of a cubic inch of water at 62° FAHR. in vacuum, in grains of which the lost standard pound contained 5760, were,—by the cube 252·741, by the cylinder 252·685, by the sphere 252·741. Mean 252·722*. Similar observations had been made in France by MM. LEFÈVRE-GINEAU and FABBRONI for the purpose of establishing the value of the kilogramme, which was intended to be the weight of a cubic décimètre of water at its maximum density, in a vacuum: the solid used on this occasion was a cylinder the diameter and axis of which were nearly 243·5 millimètres each†. In Sweden, MM. BERZELIUS, SVANBERG and AKERMANN, who employed a cylinder 4 inches in diameter and 6 inches long, found the weight of a cubic décimètre of water, at 62° FAHR. in a vacuum, to be 2·350595 Swedish pounds‡. In Austria, Professor STAMPFER, who used a cylinder of about 3·11 inches diameter and 3·11 inches long, found the weight of a Vienna cubic inch of water, at 62° FAHR. in a vacuum, equal to 18·2492 grammes§. Lastly, in Russia, Professor KUPFFER has determined the weight of an English cubic inch of water in vacuum at 62° FAHR., in doli of which a kilogramme contains 22504·86, to be 368·380 by a cylinder the axis and diameter of which were nearly 80 millimètres each, and 368·341 by a cylinder the axis and diameter of which were 4 English inches each. Mean 368·361 doli. At the end of the work entitled 'Travaux de la Commission pour fixer les Mesures et les Poids de l'Empire de Russie,' Professor KUPFFER has collected the different results expressed in doli. The English observations are affected by a small error arising from the uncertainty of the value of Professor SCHUMACHER's troy pound K, which was used by Professor KUPFFER in finding the relation between the English and French standards of weight. This error, however, is quite insignificant compared with the differences between the results obtained by the several observers.

French observations	368·365
English observations	368·542
Swedish observations	368·474
Austrian observations	368·237
Russian observations	368·361

* Philosophical Transactions for 1821, Part I. p. 326.

† Base du Système Métrique, t. iii. p. 558.

‡ Memoirs of the Royal Academy of Stockholm, 1825.

§ Jahrbücher des k. k. Polytech. Institutes zu Wien, B. 16, S. 53.

Assuming the Russian observations to be the most accurate, as they probably are, it will be seen that even if we leave entirely out of the question the injurious effect of the error likely to arise in establishing the standard of length, a troy pound deduced according to the method prescribed by the Act would be 2·829 grains too heavy; while, if the Austrian observations had been accepted as the best, the troy pound would have been 4·707 grains too heavy. On the other hand, it was possible to recover the weight of the lost standard in air to within a fraction of 0·001 grain, by means of the troy pounds which had been compared with it, and could be easily brought together for recomparison. Seeing, then, that the error of one of these two methods of restoring the lost standard is at least 2829 times as large as the error of the other method, the Committee could not hesitate to recommend the adoption of the latter. The Commissioners recommend also that the avoirdupois pound, being universally used through the kingdom, while the troy pound is wholly unknown to the great mass of the population, be adopted as the standard of weight; that the troy pound be no longer recognized; and that the use of the troy ounce be confined to gold, silver, and precious stones.

In the year 1843 a Committee was appointed to superintend the construction of the new Parliamentary standards of length and weight, to replace those which were destroyed by the burning of the Houses of Parliament. The members of this Committee were, the Astronomer Royal, the Marquis of NORTHAMPTON, the Earl of ROSSE, the Lord WROTTESLEY, Sir J. W. LUBBOCK, Bart., Sir J. F. W. HERSCHEL, Bart., the Rev. G. PEACOCK, Dean of Ely, the Rev. R. SHEEPSHANKS, F. BAILY, Esq., J. E. D. BETHUNE, Esq., J. G. S. LEFEVRE, Esq., and Professor W. H. MILLER. To the last of these was intrusted the construction of the new standards of weight.

The evidence for ascertaining the weight of the lost Standard Troy Pound, placed at the service of this Committee, consisted of the following weights:—The brass troy pound of the Exchequer Office. The brass troy pounds from the cities of London, Edinburgh and Dublin. The platinum troy pound and the two brass troy pounds then in the custody of Professor SCHUMACHER. The platinum troy pound of the Royal Society. The troy pound used by the late Mr. ROBINSON of Devonshire Street, Portland Place, purchased by the Committee. Two troy pounds, formerly in the possession of Mr. BINGLEY (one of which, lately in the possession of STANSBY ALCHORNE, Esq., of the Royal Mint, has been purchased by the Committee). The troy pound formerly the property of Mr. FREEMAN, now the property of Messrs. VANDOME and TITFORD. To these has very recently been added a troy pound the property of the Bank of England.

The results of the comparisons of the troy pounds of the Exchequer Office, of the cities of London, Edinburgh and Dublin, and of the three troy pounds in the custody of Professor SCHUMACHER, and the troy pound of the Royal Society, with the lost standard, have already been given. Mr. ROBINSON's troy pound is also said to have been compared by Captain KATER, but no record has been discovered of the

comparison. Mr. VANDOME's pound, Mr. BINGLEY's troy pounds, and the Bank of England troy pound, were all constructed, along with the lost standard, in 1758 by Mr. HARRIS, Assay Master of the Mint. These were referred to, at the suggestion of Professor SCHUMACHER, in the hope of arriving at a knowledge of the volume of the lost standard, which unfortunately had never been determined by weighing it in water. For, as long as the volume of the lost standard remains unknown, the weight of the air displaced by it, and, consequently, its absolute weight, is uncertain within limits far exceeding the errors of weighing.

The first step in the process of arriving at the weight of the lost standard, was obviously to compare among themselves the different troy pounds with which the lost standard had been compared by Captain KATER and Captain v. NEHUS. These comparisons were made with a balance of extreme delicacy procured from Mr. BARROW. In its construction it nearly resembles the balances of the late Mr. T. C. ROBINSON. The beam is made sufficiently strong to carry a kilogramme in each pan. The middle knife-edge is about 1·93 inch long, and rests, when the balance is in action, throughout its whole length on a single plane surface of quartz. The surfaces of quartz which rest upon the extreme knife-edges, and from which the pans are suspended, are also plane. The distance between the extreme knife-edges is about 15·06 inches, the length of each about 1·05 inch.

Instead of having an index pointing downwards, as is usual in balances of this description, the beam has a pointer at each end, and a graduated scale is carried by an arm attached to the pillar of the balance at a little distance behind the left-hand pointer. Affixed to the right-hand end of the beam is a thin slip of ivory, a little more than half an inch long, divided into spaces of about 0·01 inch each, or subtending an angle of about $5'$ each at the middle knife-edge. This scale is viewed through a compound microscope, having a single horizontal wire in the focus of the eye-piece. The distance between two divisions of the scale, as seen through the microscope, subtends an angle of about $37'$. This contrivance for determining the position of the beam at the extremity of an oscillation, was found so superior to a scale and pointer viewed with the naked eye, that after a trial of a few days, the scale at the left hand was found to be a useless incumbrance and was accordingly removed. A screen was interposed between the observer and the front of the balance case, having a small opening opposite to the eye-piece of the microscope, through which the scale could be seen.

In order to admit of the employment of a large vessel of water in observations for finding specific gravities, the base of the balance has an opening immediately under the right-hand pan, capable of being closed when not in use by a sliding plate of brass. A corresponding opening exists in the table on which the balance stands. The vessel of water is placed under the table, and the wire by which the object to be weighed in water is suspended from a hook under the right-hand pan, passes through the openings in the base of the balance under the table.

In the comparisons of weights, I at first employed the method of weighing invented by the Père AMIOT, more commonly known as BORDA'S*. The counterpoise was invariably placed in the left-hand pan, and the weights to be compared alternately in the right-hand pan. The reading of the divided scale was noted at the end of each of three consecutive oscillations. One-fourth of (first reading + third reading + 2 second reading) was taken as the reading of the scale in the position of equilibrium of the balance.

In all the more important weighings the reading of the scale was noted at the end of each of four consecutive oscillations, of which the last three only were used in finding the reading corresponding to the position of equilibrium of the beam. The first reading is apt to exhibit small irregularities, especially when it follows very soon after the interchange of the weights. Hence the employment of it in finding the position of equilibrium would not be likely to increase the accuracy of the result. The observation of an additional reading is not, however, without its use; for by comparing the first with the third, as well as the second with the fourth, the error of an integer in either of the readings, if it occurred, would be instantly detected.

Let P, Q be the apparent weights in air of two bodies P, Q, either of which in the right-hand pan is nearly in equilibrium with the counterpoise C in the left-hand pan; (C, P), (C, Q) the scale readings in the position of equilibrium of the balance when P, Q respectively are in the right-hand pan; and let m be the weight equivalent to one part of the scale, the readings increasing with an increase of the weight in the right-hand pan. Then $Q=P+m[(C,Q)-(C,P)]$.

Subsequently I used the method attributed to GAUSS†. Let P, Q be the apparent weights in air of two bodies P, Q, and (P, Q) the reading of the scale in the position of equilibrium of the balance, when P is in the left-hand pan, and Q is in the right-hand pan. Now let P be placed in the right-hand pan, and Q in the left-hand pan, and let P, Q become R, S respectively, by the addition of small weights, in order to bring the balance nearly into its former position of equilibrium. Let (S, R) be the reading of the scale in the position of equilibrium of the balance, when R is in the right-hand pan, and S is in the left-hand pan. Then, m being the weight equivalent to one division of the scale, the reading increasing with an increase of weight in the right-hand pan, $Q+S=P+R+m[(P,Q)-(S,R)]$.

When the weights P, Q are very nearly equal, the balance may be so adjusted by placing a small constant weight in one of the pans or hanging it on the beam, that, on interchanging the weights P, Q, the position of equilibrium may still be near the middle of the scale. Supposing the balance to be so adjusted, let (P, Q) be the reading of the scale in the position of equilibrium of the balance, when P is in the left-hand pan and Q is in the right-hand pan; and let (Q, P) be the reading of the scale

* PÉCLET, Cours de Physique, p. 48.

† STEINHEIL, Denkschriften der K. Akademie der Wissenschaften zu München fur die Jahre 1844–46, B. iv. S. 222.

when the balance is in its position of equilibrium, with Q in the left-hand pan and P in the right-hand pan. Then $2Q=2P+m[(P,Q)-(Q,P)]$.

In making a large number of comparisons, the weights are exposed to the risk of being injured by wear. In order to obviate this danger, two light pans were procured of very nearly equal weight, each of which has a loop of wire forming an arch the ends of which are attached to the pan at opposite extremities of a diameter of the pan, by which the pan could be lifted with the hook at the end of a long handle, into or out of either of the pans of the balance. Calling the pans X and Y, and the weights to be compared P and Q, P was placed in X and Q in Y, and $P+X$ compared with $Q+Y$ n times; then P was placed in Y and Q in X, and $P+Y$ compared with $Q+X$ n times. The weights were thus exposed to the wear of two ordinary comparisons only in the course of $2n$ comparisons. The mean of the $2n$ comparisons gives the difference between P and Q, unaffected by the very small but unknown difference between the weights of the pans X and Y. This contrivance was found to be especially useful when either of the weights to be compared consisted of several parts.

An improvement upon this was made by using the pans of one of the balances employed by the French Pharmaciens, which resemble those above described, with the addition of an iron hook at the highest point of the wire loop. Either pan is suspended by a wire of suitable length bent into a hook at each end, from the ring attached to the agate-plane. In using the method of double weighing, the original left-hand pan of the balance was suffered to remain with the counterpoise in it, and the pan X containing the weight P, and the pan Y containing the weight Q, alternately suspended from the right-hand end of the beam, and the positions of equilibrium observed (usually about twenty times). The weights were then interchanged, and pan Y containing the weight P, and pan X containing the weight Q, suspended from the right-hand end of the beam, and the positions of equilibrium observed the same number of times. The weights of X and Y were frequently reduced in order to make them as nearly equal as possible, and sometimes in order to remove rust from the iron hooks.

In using GAUSS's method, it was desirable to be able to transfer the pans, and the weights contained in them, from one end of the beam to the other without opening the doors of the balance-case, and thus avoid sudden changes of temperature of air within the balance-case, and consequent production of currents of air. In order to effect this, various contrivances were tried. Of these the following proved the most successful. A slender brass tube, 38 inches long, passes freely through two holes in the ends of the balance-case, which is 22·75 inches long, near the top of the case, and half-way between the balance and the front of the case. To the middle of the tube is attached a descending loop of wire. Suppose that by sliding the rod, the loop is brought near to the right-hand end of the beam, and a pan with a weight in it transferred from the end of the beam to the wire loop by a brass rod having a deep groove filed round

it near the end, which is inserted through a hole in the middle of the right-hand end of the balance-case. By sliding the rod in the opposite direction, the loop with the pan and weight suspended from it, is brought near to the left-hand end of the beam, to which the pan is transferred by a brass rod passing through a hole in the left-hand end of the balance-case. Pins inserted in holes at each end of the tube at right angles to it, prevent it from being pushed too far. A similar tube half-way between the balance and the back of the case, serves to transfer the other pan and weight from one end of the beam to the other. In this manner any number of comparisons may be made without opening the balance-case, except in the middle of the series, for the purpose of changing the pans. The transfer of the pans and weights from one end of the beam to the other, might be effected still more conveniently by means of two coarse screws of the same length as the balance-case, turned by small winches at each end, and provided with loosely-fitting nuts with wire loops from which to suspend the pans and weights.

In the course of making the preliminary observations some peculiarities of the instrument were discovered, which, though they probably exist in other balances, do not appear to have been hitherto noticed. One of these is, that the expansion of one arm by heat, the left in the present case, is a little greater than that of the other arm. Hence, when the weights in the two pans are nearly equal and of equal volume, the reading of the scale in the position of equilibrium diminishes as the temperature of the beam increases. Another is, that the sensibility of the balance, as measured by the number of parts of the scale equivalent to a given weight, was found to diminish with an increase of temperature. The cause of this is obvious. The beam being of bronze and the knife-edges of steel, the balance-beam becomes an over-compensated pendulum, and an increase of temperature increases the distance between the middle knife-edge and the centre of gravity of the beam and weights, supposing the latter concentrated in the extreme knife-edges. Possibly also, the flexure of the beam may increase with the temperature, or the mean expansion of the upper bar of the beam may be greater than that of the under bar. The variation of the sensibility of the balance is so large, that it is necessary to determine the weight equivalent to a given number of parts of the scale for each set of observations, except in cases where the temperature is very nearly the same.

For the comparison of the smaller weights two excellent balances by Robinson were used, one having a beam 10·5, the other a beam 5·5 inches long. The reading of the scale of these balances increases on the addition of a small weight to the weight in the left-hand pan.

Comparisons of Sp, RS, Sb, K, Ex, L, Ed, R.

T is a platinum troy pound left a little in excess; P the sum of five weights of platinum making together a troy pound; *b* is the height of the mercury in the

barometer in inches, reduced to the freezing-point; t the temperature of the air in the balance-case in degrees of FAHRENHEIT's scale.

1844, March 29.		100 parts = 0·1810 grain.
gr.	gr.	gr. pt.
T=Sp+0·0147	T=RS+0·0092	RS=Sp+0·010−3·70
+0·0122	+0·0124	+0·010−2·60
+0·0181	+0·0128	+0·010−3·00
+0·0189	+0·0090	+0·010−3·15
+0·0217	+0·0085	+0·006+1·48
+0·0155	+0·0078	+0·006+0·60
+0·0147	+0·0072	+0·006+1·40
+0·0136	+0·0103	+0·006+0·25
+0·0161	+0·0108	+0·006+1·95
+0·0156	+0·0110	Mean RS=Sp+0·00640 grain.
+0·0124	+0·0081	
+0·0174	+0·0087	
+0·0209	+0·0138	
+0·0194	+0·0107	
+0·0175	+0·0111	
+0·0185	+0·0090	
Mean T=Sp+0·01670 grain.	Mean T=RS+0·01002 grain.	

The numbers in the columns headed Sp+X, P+Y, Sp+Y, P+X are the readings of the scale in the positions of equilibrium of the beam with the counterpoise in the left-hand pan, and the weights Sp+X, P+Y, Sp+Y, P+X respectively in the right-hand pan. The scale readings of the alternate weighings are arranged in separate columns for the sake of greater convenience in adding up the results.

June 18. 100 parts=0·22082 grain.

Sp+X.	P+Y.	Sp+Y.	P+X.
25·30	27·80	22·50	22·90
26·10	27·00	21·90	22·90
24·05	25·20	22·05	21·40
24·60	25·30	26·50	26·45
24·40	24·60	27·15	29·50
23·50	24·40	28·50	28·65
22·75	23·55	28·50	27·30
23·00	23·00	28·90	29·30
22·20	22·55	29·65	30·10
21·90	22·40	29·75	30·40
237·80	245·80	265·40	268·90

10(Sp+X)=10(P+Y)−8 parts. 10(Sp+Y)=10(P+X)−3·5 parts.

Mean Sp=P−0·00127 grain.

June 12. 100 parts =0·23641 grain.

RS+X.	P+Y.	RS+Y.	P+X.
19·60	18·60	20·70	21·20
20·20	18·10	21·10	19·00
20·00	18·70	21·60	19·30
20·00	18·50	20·80	19·50
19·30	19·10	21·30	19·40
19·85	18·00	20·80	18·70
118·95	111·00	126·30	117·10

6(RS+X)=6(P+Y)+7·95 parts. 6(RS+Y)=6(P+X)+9·2 parts.

Mean RS=P+0·00318 grain.

April 22, 23. 100 parts = 0·14798 grain.

gr. pt.
Sp≙Sb + 0·01 − 0·45
+ 0·01 − 0·45
+ 0·01 − 0·60
+ 0·01 − 1·90
+ 0·01 − 0·70
+ 0·01 − 0·95
+ 0·01 − 1·05
+ 0·01 − 3·10
+ 0·01 − 3·00
+ 0·01 − 2·90

Mean Sp≙Sb + 0·00777 gr. in air (t=62·2, b=30·145).

April 24. 100 parts = 0·14009 grain.

gr. pt.
K≙Sp + 0·027 − 0·10
+ 0·027 − 1·15
+ 0·027 − 0·90
+ 0·024 + 0·20
+ 0·024 + 0·50
+ 0·024 + 0·25
+ 0·024 − 0·10
+ 0·024 + 0·30
+ 0·024 − 0·30

Mean K≙Sp + 0·02479 gr. in air (t=62·2, b=30·186).

April 23. 100 parts = 0·14798 grain.

gr. pt.
RS≙Sb + 0·016 − 0·35
+ 0·016 − 1·45
+ 0·016 − 0·90
+ 0·016 − 1·45
+ 0·016 − 1·70
+ 0·016 − 1·10
+ 0·016 − 1·05
+ 0·016 − 1·10
+ 0·016 − 0·65
+ 0·016 − 1·70
+ 0·016 − 0·95

Mean RS≙Sb + 0·01433 gr. in air (t=62·33, b=30·235).

gr. pt.
K≙RS + 0·016 + 1·00
+ 0·016 + 1·85
+ 0·016 + 1·80
+ 0·016 + 1·70
+ 0·016 + 1·45
+ 0·016 + 2·00
+ 0·016 + 1·60
+ 0·016 + 1·45

Mean K≙RS + 0·01837 gr. in air (t=62·33, b=30·196).

April 23. 100 parts = 0·144 grain.

gr. pt.
K = Sb + 0·040 − 4·95
+ 0·030 + 0·12
+ 0·034 − 2·37
+ 0·030 + 6·40
+ 0·030 + 3·80
+ 0·030 + 4·05
+ 0·030 + 3·45
+ 0·030 + 2·40
+ 0·020 + 8·20
+ 0·030 + 2·40
+ 0·030 + 0·95
+ 0·030 + 1·80
+ 0·030 + 0·70
+ 0·030 + 1·00

Mean K = Sb + 0·03316 grain.

April 8. 100 parts = 0·2244 grain.

gr. pt.
K = Sb + 0·031 + 1·00
+ 0·031 + 1·65
+ 0·031 + 1·95
+ 0·031 + 1·80
+ 0·031 + 1·20
+ 0·031 + 1·40
+ 0·031 + 1·90

Mean K = Sb + 0·03449 grain.

April 22. 100 parts = 0·148 grain.

gr. pt.
Ex = Sb + 0·016 − 0·25
+ 0·020 − 2·72
+ 0·013 − 2·00
+ 0·010 + 0·05
+ 0·010 − 0·63
+ 0·010 + 0·60
+ 0·010 + 1·45
+ 0·010 + 0·40
+ 0·010 + 0·95
+ 0·010 + 1·00
+ 0·010 + 1·20
+ 0·010 + 1·10

Mean Ex = Sb + 0·01172 grain.

gr. pt.
K = Ex + 0·010 + 3·13
+ 0·014 + 0·37
+ 0·014 + 0·35
+ 0·015 − 1·95
+ 0·015 − 1·70
+ 0·015 − 0·65
+ 0·015 + 0·05
+ 0·015 − 0·50
+ 0·015 − 0·05
+ 0·015 − 0·25
+ 0·015 − 0·20
+ 0·015 − 0·10

Mean K = Ex + 0·01406 grain.

gr.
L = Sb + 0·0233
+ 0·0206
+ 0·0203
+ 0·0158
+ 0·0171
+ 0·0169
+ 0·0181
+ 0·0178
+ 0·0196

Mean L = Sb + 0·01883 grain.

100 parts = 0·116 grain.

gr. pt.
K = L + 0·007 − 0·35
+ 0·007 + 0·25
+ 0·007 + 1·40
+ 0·007 + 2·80
+ 0·010 + 1·75
+ 0·010 + 2·70
+ 0·013 + 0·50
+ 0·013 − 0·35
+ 0·013 − 0·01

K = L + 0·01079 grain.

gr.
Ed = Sb + 0·0285
+ 0·0290
+ 0·0287
+ 0·0227
+ 0·0235
+ 0·0248
+ 0·0276
+ 0·0250
+ 0·0204
+ 0·0232

Mean Ed = Sb + 0·02534 grain.

gr.
K = Ed + 0·0065
+ 0·0082
+ 0·0062
+ 0·0059
+ 0·0054
+ 0·0064
+ 0·0037
+ 0·0078
+ 0·0060
+ 0·0059

Mean K = Ed + 0·00621 grain.

100 parts = 0·121 grain.

gr. pt.
D = Sb + 0·031 − 1·40
+ 0·031 − 1·90
+ 0·031 − 1·60
+ 0·030 + 0·65
+ 0·030 − 1·30
+ 0·030 − 1·80
+ 0·030 − 1·80
+ 0·030 − 1·40
+ 0·030 − 1·40

Mean D = Sb + 0·02873 grain.

March 27, April 13.

gr. pt.
K=D+0·006−2·65
+0·003−0·60
+0·003−0·20
−1·60
−2·30
−1·20
−0·70
+1·20
+3·50
+1·90
+0·60
+1·70
+2·20
+3·10

Mean K=D+0·00129 grain.

April 16. 100 parts=0·1310 grain.

gr. pt.
Sb=R+0·006+3·75
+0·010+0·70
+0·010−1·15
+0·010−0·97
+0·008+0·68
+0·008−1·63
+0·008−2·27
+0·008−1·97
+0·006−1·00
+0·006−0·75

Mean Sb=R+0·00740 grain.

gr. pt.
K=R+0·035−0·10
+0·035+2·60
+0·037+2·20
+0·038+0·60
+0·038−0·73
+0·038+2·50
+0·038+2·23
+0·038+2·20
+0·040+1·25
+0·040+0·27

Mean K=R+0·03941 grain.

April 25. 100 parts=0·1456 grain.

gr. pt.
L=Ex+0·008+0·20
+0·008−0·65
+0·008−1·35
+0·008−1·25
+0·006−0·90
+0·006−0·80
+0·006−0·65
+0·006−0·60
+0·006−1·75
+0·006−0·65
+0·006−0·55
+0·006−1·00

Mean L=Ex+0·00546 grain.

April 26. 100 parts=0·1456 grain.

gr. pt.
Ed=Ex+0·014−0·90
+0·014−2·10
+0·014−3·10
+0·014−1·50
+0·010+1·40
+0·010+0·85
+0·010+0·50
+0·010+0·55
+0·010+1·75
+0·010+0·10
+0·010+0·55
+0·010+0·90

Mean Ed=Ex+0·01121 grain.

April 25. 100 parts=0·1456 grain.

gr. pt.
D=Ex+0·014+0·30
+0·014+0·70
+0·014+0·50
+0·014+0·85
+0·014−0·80
+0·014+0·70
+0·014+0·40
+0·014−0·50
+0·014−0·40
+0·014−0·50
+0·014−0·55
+0·014−0·25

Mean D=Ex+0·01405 grain.

April 25.

gr. pt.
Ed=L+0·000+4·90
+0·010−2·70
+0·010−2·20
+0·000+5·90
+0·007+0·50
+0·007+0·15
+0·007+1·00
+0·007+0·75
+0·007+1·20
+0·007+0·20
+0·007+1·30
+0·007+1·40

Mean Ed=L+0·00784 grain.

April 27.

gr. pt.
D=L+0·007+2·30
+0·010+0·05
+0·010+0·35
+0·010+1·05
+0·010+0·35
+0·010+0·20
+0·010+0·70
+0·010+0·50
+0·010+0·60
+0·010+0·05
+0·010+0·05

Mean D=L+0·01055 grain.

April 24.

pt.
Ed=D+1·85
+0·80
+0·60
+0·30
+0·30
−0·80
−0·35
−0·50
−0·45
−1·20

Mean Ed=D+0·00008 grain.

Before any use can be made of these weighings, the results must be reduced to what they would have been either in a vacuum or in air of given density. The latter is to be preferred, for then the result will be less affected by any uncertainty in the densities of the weights, or the value of the density of air, and also because we are not in possession of the data requisite for the reduction of the weighings in 1824.

Let P appear to weigh as much as Q in air (thermometer$=t$, barometer$=b$), p, q being the weights of the air displaced by P and Q respectively. Let P appear to weigh as much as Q+R in air (thermometer$=t'$, barometer$=b'$), p', q' being the weights of the air displaced by P and Q respectively,—

$$P-p=Q-q \text{ and } P-p'=Q+R-q'.$$

Whence $P+p'-p$ appears to weigh as much as $Q+q'-q$ in air (thermometer$=t'$, barometer$=b'$).

In calculating the densities of Sp, K, Sb, Professor SCHUMACHER adopted the formulæ and constants given in BESSEL's paper on the reduction of weighings, in the Astronomische Nachrichten, B. vii. S. 373. It will therefore be proper to use his tables in reducing the weighings of 1844, even though the values of some of the constants, according to recent and more accurate observations, differ slightly from those employed by BESSEL.

Let vP be the volume of the weight P at the temperature of melting snow, the unit of volume being the volume of one grain of water at its maximum density; p the weight in grains of the air displaced by P; t the temperature of the air in degrees of FAHRENHEIT's scale; and b the height of the mercury in the barometer in English inches reduced to the temperature of melting snow. Then

$\log p = \log b + \log v\mathrm{P} + \log$ from Table A. $+ \log$ from Table B. or P., according as the weight is of brass or platinum, t being the argument in each of the tables.

Of the following tables, copied as far as they are wanted from the Philosophical Transactions for 1836, p. 486, A contains the logarithm of the ratio of the density of air temperature t, and under the pressure of one inch of mercury at the temperature of melting snow, to the maximum density of water. B and P contain the logarithms of the ratio of the density of brass and platinum, respectively, at the temperature of melting snow, to the density at the temperature t of FAHRENHEIT's scale.

t.	A.	t.	B.	t.	P.
61	5·61177	61	0·000394	61	0·000189
62	5·61092	62	0·000408	62	0·000195
63	5·61007	63	0·000421	63	0·000202
64	5·60922	64	0·000435	64	0·000209
65	5·60837	65	0·000449	65	0·000215
66	5·60752	66	0·000462	66	0·000222
67	5·60668	67	0·000476	67	0·000228

Let Δ prefixed to the symbol by which any weight is designated denote the ratio of the density of the weight at the temperature of melting ice to the maximum density of water. Then (Philosophical Transactions for 1836, pp. 490–493), ΔSp$=21·1874$, ΔRS$=21·1874$, ΔK$=7·994$, ΔSb$=8·228$. The density of U is unknown. Let it be assumed equal to a mean between the densities of K and Sb, or ΔU$=8·111$. The platinum weights contain about 5759·5 grains, the brass weights about 5760 grains each. Hence $\log v$Sp$=2·43430$, $\log v$RS$=2·43430$, $\log v$K$=2·85766$, $\log v$Sb$=2·84513$, $\log v$U$=2·85135$.

The temperature and pressure to which it will be most convenient to reduce the weighings, is the mean of the temperatures and pressures observed during the comparisons of Sp and RS with U. This, taking into account the number of observations in the two cases, is,—thermometer $=65·66$ FAHRENHEIT, barometer $=29·75$ English inches. When both the weights are of brass, or both of platinum, the reduction is so small as to be insensible.

Calculation of the weights of air displaced by Sp, RS, K, &c. when (t=65·66, b=29·75).

b	29·75	1·47349	b	29·75	1·47349
A	65·66	5·60781	A	65·66	5·60781
P	65·66	0·00022	B	65·66	0·00046
		7·08152			7·08176

	Log v.	Log air displaced.	Air displaced.
Sp	2·43430	9·51582	0·32796
RS	2·43430	9·51582	0·32796
K	2·85766	9·93942	0·86980
Sb	2·84513	9·92689	0·84506
U	2·85135	9·93311	0·85726

Reduction of the weighings of Sp, RS, K, &c. to air (t=65·66, b=29·75).

Sp≏U−0·00857 grain in air (t=65·62, b=29·722).

29·722	1·47308	29·722	1·47308
65·62	5·60784	65·62	5·60784
65·62	0·00022	65·62	0·00046
	2·43430		2·85135
0·32767	9·51544	0·85650	9·93273

Sp displaces 0·32767 gr. of air.
U displaces 0·85650 gr. of air.

Sp≏U−0·00810 grain in air (t=65·66, b=29·75).

RS≏U−0·00205 grain in air (t=65·73, b=29·806).

29·806	1·47430	29·806	1·47430
65·73	5·60775	65·73	5·60775
65·73	0·00022	65·73	0·00046
	2·43430		2·85135
0·32853	9·51657	0·85874	9·93386

RS displaces 0·32853 gr. of air.
U displaces 0·85874 gr. of air.

RS≏U−0·00296 grain in air (t=65·66, b=29·75).

Sp≏Sb+0·00777 grain in air (t=62·2, b=30·145).

30·145	1·47921	30·145	1·47921
62·2	5·61075	62·2	5·61075
62·2	0·00020	62·2	0·00041
	2·43430		2·84513
0·33455	9·52446	0·86198	9·93550

Sp displaces 0·33455 gr. of air.
Sb displaces 0·86198 gr. of air.

Sp≏Sb−0·00256 grain in air (t=65·66, b=29·75).

K≏Sp+0·02479 grain in air (t=62·2, b=30·186).

30·186	1·47980	30·186	1·47980
62·2	5·61075	62·2	5·61075
62·2	0·00041	62·2	0·00020
	2·85766		2·43430
0·88842	9·94862	0·33501	9·52505

K displaces 0·88842 gr. of air.
Sp displaces 0·33501 gr. of air.

K≏Sp+0·03636 grain in air (t=65·66, b=29·75).

RS≏Sb+0·01433 grain in air (t=62·4, b=30·235).

30·235	1·48051	30·235	1·48051
62·4	5·61058	62·4	5·61058
62·4	0·00020	62·4	0·00041
	2·43430		2·84513
0·33542	9·52559	0·86424	9·93663

RS displaces 0·33542 gr. of air.
Sb displaces 0·86424 gr. of air.

RS≏Sb+0·00261 grain in air (t=65·66. b=29·75).

K≏RS+0·01837 grain in air (t=62·4, b=30·196).

30·196	1·47995	30·196	1·47995
62·4	5·61058	62·4	5·61058
62·4	0·00041	62·4	0·00020
	2·85766		2·43430
0·88838	9·94860	0·33499	9·52503

K displaces 0·88838 gr. of air.
RS displaces 0·33499 gr. of air.

K≏RS+0·02992 grain in air (t=65·66, b=29·75).

Let U, Sp, RS, &c. denote the apparent weights in air ($t=65\cdot66$, $b=29\cdot75$) of the troy pounds U, Sp, RS, &c. Then,—

	In 1824.			gr.	No. of obs.
	gr.	No. of obs.	7	K =Sp +0·0364	9
a	Ex =U +0·0010	16	8	RS=Sb +0·0026	11
b	Ed =U −0·0015	15	9	K =RS+0·0299	8
c	L =U +0·0005	12	10	K =Sb +0·0336	21
d	D =U +0·0022	18	11	Ex=Sb +0·0117	12
e	RM=U +0·0021	20	12	K =Ex +0·0141	12
	In 1829.		13	L =Sb +0·0188	9
f	Sp =U −0·0081	300	14	K =L +0·0108	9
g	RS =U −0·0030	140	15	Ed=Sb +0·0253	10
h	Sb =U −0·0103	60	16	K =Ed +0·0062	10
k	K =U +0·0339	92	17	D =Sb +0·0287	9
l	RM=U +0·0089	16	18	K =D +0·0013	14
	In 1844.		19	Sb=R +0·0074	10
			20	K =R +0·0394	10
1	T =Sp +0·0167	16	21	L =Ex +0·0055	12
2	T =RS+0·0100	16	22	Ed=Ex +0·0112	12
3	RS =Sp +0·0064	9	23	D =Ex +0·0140	12
4	Sp =P −0·0013	20	24	Ed=L +0·0078	12
5	RS =P +0·0032	12	25	D =L +0·0106	11
6	Sp =Sb −0·0026	12	26	Ed=D +0·0001	10

The results 11 and 12 appear to be considerably in error.

The numbers in the first column in the following Table indicate the equations from which the results are deduced; those in the last column express roughly the weights of the results, supposing all the observations to be equally good.

	gr.	
1, 2	RS=Sp+0·0067	1
3	RS=Sp+0·0064	1
4, 5	RS=Sp+0·0045	1
6, 8	RS=Sp+0·0052	1
Mean ...	RS=Sp+0·0057	

	gr.	
6	Sp=Sb−0·0026	2
3, 8	Sp=Sb−0·0038	1
7, 10	Sp=Sb−0·0028	1
Mean ...	Sp=Sb−0·0030	

	gr.	
8	RS=Sb+0·0026	2
3, 6	RS=Sb+0·0038	1
9, 10	RS=Sb+0·0037	1
Mean ...	RS=Sb+0·0032	

	gr.	
7	K=Sp+0·0364	2
3, 9	K=Sp+0·0363	1
6, 10	K=Sp+0·0362	2
Mean ...	K=Sp+0·0363	

	gr.	
9	K=RS+0·0299	2
3, 7	K=RS+0·0300	1
8, 10	K=RS+0·0310	2
Mean ...	K=RS+0·0304	

	gr.	
6, 7	K=Sb+0·0338	1
8, 9	K=Sb+0·0325	1
10	K=Sb+0·0336	4
11, 12	K=Sb+0·0258	1
13, 14	K=Sb+0·0296	1
19, 20	K=Sb+0·0315	1
17, 18	K=Sb+0·0300	1
19, 20	K=Sb+0·0320	1

Mean, omitting the 4th result on account of the large difference between it and the others,

K=Sb+0·0324

	In 1829.	In 1844.
	gr.	gr.
f, g	RS=Sp +0·0051	RS=Sp +0·0057
f, h	Sp =Sb +0·0022	Sp =Sb −0·0030
g, h	RS=Sb +0·0073	RS=Sb +0·0032
f, k	K =Sp +0·0420	K =Sp +0·0363
g, k	K =RS+0·0369	K =RS+0·0304
h, k	K =Sb +0·0442	K =Sb +0·0324

In the interval between 1829 and 1844, the difference between the two platinum troy pounds Sp and RS had undergone no very sensible relative change. If, as appears highly probable, Sp and RS have undergone no sensible absolute change, Sb has gained 0·0046 grain, and K has lost 0·0061 grain.

In 1844, Sb+K=Sp+RS+0·0333 grain. Assuming Sp and RS to have experienced no change since 1829, Sp+RS=2U−0·0111 grain; whence Sb+K=2U +0·0222 grain.

The equations 11 ... 18 give—

$$
\begin{aligned}
2\mathrm{Ex} &= \mathrm{Sb}+\mathrm{K}-\overset{\text{gr.}}{0\cdot0024}=\mathrm{Sp}+\mathrm{RS}+\overset{\text{gr.}}{0\cdot0309}\\
2\mathrm{L} &= \mathrm{Sb}+\mathrm{K}+0\cdot0080=\mathrm{Sp}+\mathrm{RS}+0\cdot0413\\
2\mathrm{Ed} &= \mathrm{Sb}+\mathrm{K}+0\cdot0191=\mathrm{Sp}+\mathrm{RS}+0\cdot0525\\
2\mathrm{D} &= \mathrm{Sb}+\mathrm{K}+0\cdot0274=\mathrm{Sp}+\mathrm{RS}+0\cdot0607
\end{aligned}
$$

The first column of the following Table exhibits the errors of Ex, L, Ed, D, as deduced from the above equations; the second column exhibits the errors of the same weights, as determined by Captain KATER, in 1824, by direct comparison with U; the third shows the increase of weight of the several troy pounds in the course of twenty years.

In 1844.	In 1824.	
gr.	gr.	gr.
Ex−U=+0·0099	Ex−U=+0·0010	0·0089
L −U=+0·0151	L −U=+0·0005	0·0146
Ed−U=+0·0206	Ed−U=−0·0015	0·0221
D −U=+0·0248	D −U=+0·0022	0·0226

In 1824, RM=U+0·0021 grain; in 1829, RM=U+0·0089; consequently RM gained 0·0068 grain in five years. With the single exception of K, all the new brass weights have become heavier since they were first compared with U, in consequence probably of the oxidation of their surfaces, while U, which was made in 1758, was preserved from further change by the coat of oxide already formed. One of these, Sb, appeared to have been protected by gilding, though imperfectly, as parts of its surface were slightly tarnished. Ex and L were brighter than Ed and D. K, though it had become lighter, was much tarnished, yet exhibited no traces of abrasion. The discordances presented by the different weighings of K previous to 1844 were highly perplexing, and were probably the cause of the very numerous and accurate comparisons of the various troy pounds placed at the disposal of the Committee, with the lost standard, on which alone the possibility of restoring it with sufficient accuracy depends.

Professor SCHUMACHER received K in March 1827, accompanied by a statement that it had been found by Captain KATER to exceed the standard very little, not more than 0·006 grain*. In June 1828, Captain KATER compared a second weight Kn with each of two troy pounds in his possession, the errors of which were well determined. One of these was 0·0122 grain too heavy, the other 0·0267 grain too heavy. Let 2W denote the sum of these two troy pounds. Then W=U+0·0194 grain. By a

* Philosophical Transactions, 1836, p. 457.

mean of eight comparisons Kn=W−0·0170 grain. In September 1828, by a mean of twenty comparisons, Professor SCHUMACHER found K=Kn+0·0198 grain; whence K=U+0·0223 grain. In February and March 1829, by a mean of twenty comparisons, Captain KATER found K=W+0·0065; whence K=U+0·0259 grain. This differs from the first result, 0·0199 grain (not 0·0299 grain as it is erroneously printed). By ninety-two direct comparisons of K with the standard by Captain v. NEHUS in June and July 1829, K=U+0·0339 grain. In the autumn of 1829 Professor SCHUMACHER compared K again with the sum of three brass weights of 5000 grains, 400 grains, 300 grains and 60 grains of platinum, with which it had been compared on its arrival at Altona in 1827, and there was no sensible difference from the first comparison. By thirty comparisons in October 1829 and February 1830, Professor SCHUMACHER found K=Kn+0·0200 grain. This differs but 0·0002 grain from the result obtained in 1828. In April 1844, 2K=Sp+RS+0·0667 grain, Sp+RS=2U−0·0111 grain. Therefore K=U+0·0278 grain.

On taking K out of its case after I had received it from Professor SCHUMACHER in March 1844, I observed a small fragment of wood, like a grain of coarse sawdust, adhering to the under surface of the weight so firmly that I was unable to brush it off with a feather, and had some difficulty in removing it with a pointed bit of quill. The adhesion of the bit of wood to the weight is due apparently to the pressure produced by screwing down very tightly the lid of the box in which it was contained. Two or three similar grains were imbedded in the velvet lining of the case. In all probability this bit of wood had been attached to K immediately after its first comparison by Captain KATER, when it appeared to be 0·006 grain too heavy, and previous to its comparison by Professor SCHUMACHER with the brass weights of 5000 grains, 400 grains and 300 grains, and the platinum weights of 60 grains.

By the observations of February and March 1829, K=W+0·0065 grain, and by those of June and July 1829, K=U+0·0339 grain; whence W=U+0·0274 grain. But W=U+0·0194 grain when first compared. Therefore in 1829 W had gained 0·0080 grain. At the same time RM, which has been very carefully preserved, had gained 0·0068 grain. In 1844 the well-preserved troy pounds Ex and L had gained 0·0089 grain and 0·0146 grain respectively, and Ed and D, which were in a less perfect state of preservation, had gained 0·0221 grain and 0·0226 grain respectively. The whole gain of K up to 1844 appears to be 0·0218 grain, about the same as that of Ed or D. If, as seems probable, K was compared with W about the end of 1826 or the beginning of 1827, this error of +0·0218 must include the gain of W up to that period. The discordances in the weighings of K may be explained by supposing the gain of K, including that of W up to 1829, to be 0·014 grain, and the gain from 1829 to 1844 to be 0·008 grain, since it may be assumed that brass having a recently polished surface gains weight faster than when its surface is protected by a film of oxide; also, that in the same interval, Sb, which was in some measure protected by gilding, had gained rather less than K, and that the bit of wood weighed 0·014 grain.

Then, original error of K=+0·006 grain. Error of K in 1829=original error +0·014 grain (wood) +0·014 grain (oxygen)=+0·034 grain. Error of K in 1844 = error of K in 1829 −0·014 grain (wood) +0·008 grain (oxygen)=+0·028 grain.

The comparisons of the troy pounds Ex, L, Ed, D with each other in 1814 give

$$Ex+0{\cdot}0102 \text{ grain } = L+0{\cdot}0061 \text{ grain } = Ed = D.$$

This result agrees with the conclusion already derived from the comparison of Ex, L, Ed, D with Sb and K, in showing that the differences between Ex, L, Ed, and D have very sensibly changed in the course of twenty years.

The troy pound R, which is much tarnished, is about 0·012 grain lighter than U, and therefore cannot be either of the weights used by Captain Kater in finding the errors of K and Kn.

The discrepancies presented by the weighings of the brass troy pounds at different times, due to the effect of oxidation or other causes, are so large, that I resolved, with the consent of the Astronomer Royal, to rest for the evidence of the weight of the lost standard entirely on the comparisons of the two platinum troy pounds Sp and RS. In a note appended to Professor Schumacher's paper in the Transactions of the Royal Society for 1836, p. 471, Mr. Baily observes, that, for some unexplained reason, Mr. Cary, who was commissioned to construct the troy pound RS, used for this purpose some platinum of his own instead of that which was supplied to him by the Royal Society. The exchange, whatever may have been the cause of it, does not appear to have been detrimental, for the surface of RS, though certainly inferior to that of the newly made platinum kilogrammes and mètre bars which I saw in Paris in 1844, is superior to that of Sp, in which plugs have been inserted to fill up holes left by drilling out defective places, and is much better than that of the other pound weights made since of platinum prepared in England.

If we consider the discordances presented by the weighings of the brass troy pounds simply as errors of observation, without paying any regard to their probable causes, the resulting value of U will not be very different from that given by the platinum troy pounds alone.

By the observations of 1824 and 1829,—

	gr.	weight.
U=	Sp +0·0081	30
U=	RS+0·0030	14
U=	Sb +0·0103	6
U=	K −0·0339	9
U=¼(Ex+L+Ed+D)	−0·0022	6

By the observations of 1844,—

		gr.
RS	=Sp	+0·0057
Sb	=Sp	+0·0030
K	=Sp	+0·0363
Ex+L+Ed+D	=2(Sb+K)	+0·0260

Whence, supposing the errors of the weighings to be insensible compared with the discordances of the brass troy pounds,—

	gr.	weight.
1	U=Sp+0·0081	30
2	U=Sp+0·0087	14
3	U=Sp+0·0133	6
4	U=Sp+0·0024	9
5	U=Sp+0·0261	6

The mean of all the equations gives

U=Sp+0·0096 grain.

Excluding the last, which depends upon the weighings in 1824,

U=Sp+0·0079 grain.

Excluding all except the result of the comparisons of U with the two platinum troy pounds,

U=Sp+0·0083 grain.

Comparison of Thermometers.

The thermometer K was supplied by the Committee of the Kew Observatory. It bears the inscription "No. 43, Kew Observatory, July 1853." It is graduated by lines etched upon the tube at every fifth of a centesimal degree. The distance between the freezing- and boiling-points is about 18·1 inch. Mr. WELSH, under whose superintendence it was constructed, examined it by the method employed by Mr. SHEEPSHANKS, and concluded that the graduation was correct throughout the scale to one-tenth of a small division, or 0°·02 C. He obtained the following data for determining its boiling-point at the Kew Observatory, the stem being vertical:—

1853.	Barom.	Att. therm.	Barom. in millims. of mercury at 0° C.	Reading of K.	Temp. by REGNAULT's Tables.	Error.
July 27.	30·039	65·5	760·47	99·98	100·02	−0·04
August 16.	29·726	64·3	752·61	99·70	99·73	−0·03
August 17.	29·640	63·2	750·51	99·58	99·65	−0·07
					Mean......	−0·047

Hence the boiling-point with the stem vertical under the pressure of 760 millimètres of mercury at 0° C., is 99°·953. The freezing-point, with the stem vertical, was −0°·04, before boiling, and −0°·12, after boiling.

Assuming 100° C. to be the temperature of steam under LAPLACE's standard atmospheric pressure, or the pressure of a column of mercury at 0° C., the height of which in millimètres is 760+1·946 cos 2 latitude +0·0001492 height in mètres above the sea, the temperature of steam at Kew under the pressure of 760 millimètres of mercury at 0° C., will be 100°·016. But the reading of K was 99°·953. Hence, denoting by K the reading of No. 43 at the temperature t by a thermometer the freezing- and boiling-points of which are accurately determined, when $t=0°$, $t-K=+0°{\cdot}12$, and when $t=100°$, $t-K=+0°{\cdot}063$. In August 1853 it was heated to rather above 100° C. On December 15 the freezing-point had ascended to −0°·04. The reading was 0°·00 when the thermometer was surrounded with broken ice, May 26, 1855, and also when immersed in pounded ice, July 10, 1855. The determination of the zero of a

thermometer by ice is said to be less accurate than when snow is used. In the present instance it appears from the following comparisons to have been sufficiently exact. In February 1844 the freezing-point of a thermometer G, having an arbitrary scale, was 37·4. In March 1845 it was 37·5, the thermometer having been immersed in snow in both cases. In May 1855, the bulbs of G and K being almost in contact with each other and surrounded with broken ice, the reading of G was 37·55, while that of K was 0°·00. In July 1855, the reading of G, when immersed in pounded ice, was again found to be 37·55. In March 1855, by a mean of ten comparisons, the reading of G was 48·541, when that of K was 2°·365. By a mean of ten other comparisons, the corresponding readings of G and K were 90°·797 and 11°·432 respectively. Hence, one part of G=0°·2145. If we suppose the zero of G unchanged since 1845, the freezing-point of K would be −0°·004. But if we suppose the zero of G to have been correctly determined by immersion in ice in May and July 1855, or that the zero had ascended 0·05 part =0°·011 since 1845, the freezing-point of K would be +0°·007.

Assuming the freezing-point of K in 1855 to be 0°·00, as given by observation, the corrections of K will be,—

t.	*t*−K.	*t.*	*t*−K.	*t.*	*t*−K.
0°	0·000	35°	−0·020	70°	−0·040
5	−0·003	40	−0·023	75	−0·043
10	−0·006	45	−0·026	80	−0·046
15	−0·009	50	−0·029	85	−0·048
20	−0·011	55	−0·031	90	−0·051
25	−0·014	60	−0·034	95	−0·054
30	−0·017	65	−0·037	100	−0·057

The thermometers B, C, D were made by the late M. Bunten of Paris. They are all divided into centesimal degrees by lines etched upon the tubes. The graduation of B extends from −23° up to +107°. It bears the inscription '25 Mai 1843. Divisé le 18 Mai 1844.' The graduation of C extends from −24° up to +41°. It is dated 1841. The graduation of D, which is also dated 1841, extends from −25° up to +53°.

The graduation of B was examined at certain points of the scale by the method described by Professor Forbes in the Transactions of the Royal Society for 1836, p. 578, and the boiling- and freezing-points determined, in order that the other thermometers might be referred to it as a standard. It was not, however, used as a standard, in consequence of the acquisition of the Kew thermometer, which has a scale of much larger dimensions, and is more accurately and closely divided, and also on account of the inconvenience in using it as a standard, arising from the large amount of the displacement of its zero. Immediately after boiling, February 1, 1845, the freezing-point of B was −0°·20; on February 4 it was −0°·15; on March 3 it was −0°·11; in December 1846 it was 0°·00; and in July 1855 it was +0°·11. The depression of the zero, which in the present case amounts to 0°·31 C., depends upon

the composition of the glass, and perhaps also upon the manner in which it is worked by the glass-blower. LEGRAND observed depressions of from 0°·3 to 0°·5 in thermometers by M. BUNTEN *. DESPRETZ found changes amounting to 0°·47, 0°·45, 0°·23, 0°·30, 0°·57, 0°·61, 0°·60, 0°·60; and that the freezing-point became stationary at the end of about four years †. Mr. WELSH has observed a depression in the thermometers constructed at Kew varying from 0°·09 to 0°·11. In the thermometers employed by Mr. SHEEPSHANKS it amounts to about 0°·17. In ten different thermometers examined by Dr. LAMONT, the depressions were 0°·31, 0°·28, 0°·45, 0°·25, 0°·31, 0°·37, 0°·62, 0°·25, 0°·31, 0°·27 respectively. He also found that it takes about five years for the zero to regain its permanent position after boiling ‡.

The depression of the freezing-point of B below +0°·11, from March 1845 up to January 1851, is represented with sufficient accuracy by $0{\cdot}0044m^2$, where m is the time in months up to the middle of January 1851. The computed depressions are,—

1845. March, April	0°·22	1846. May, June	0°·14	1847. December—February	0°·06
May	0·21	July, August	0·13	1848. March—June	0·05
June, July	0·20	September, October	0·12	July—September	0·04
August	0·19	November, December	0·11	October—February	0·03
September, October	0·18	1847. January—March	0·10	1849. March—July	0·02
November, December	0·17	April—June	0·09	August—February	0·01
1846. January, February	0·16	July, August	0·08	1850. March—December	0·00
March, April	0·15	September—November	0·07		

Means of Comparisons of B and K in February and March 1855.

No. of comparisons.	B.	K.	B−K.	K−t.	B−t.
10	4·895	4·712	0·183	0·003	0·186
10	5·344	5·177	0·167	0·003	0·170
10	6·319	6·143	0·176	0·003	0·179
10	9·219	9·016	0·203	0·006	0·209
10	10·740	10·493	0·247	0·006	0·253
10	12·893	12·652	0·241	0·008	0·249
10	14·532	14·279	0·253	0·009	0·262
10	15·336	15·095	0·241	0·009	0·250
10	16·379	16·126	0·253	0·009	0·262
10	17·581	17·329	0·252	0·010	0·262
10	18·000	17·749	0·251	0·010	0·261
10	18·000	17·750	0·250	0·010	0·260
10	18·180	17·917	0·263	0·010	0·273
10	21·382	21·116	0·266	0·011	0·277
10	23·202	22·919	0·283	0·013	0·296
10	25·331	25·045	0·286	0·014	0·300

t.	B−t.	t.	B−t.	t.	B−t.	t.	B−t.
0	0·11	7	0·19	14	0·25	21	0·28
1	0·12	8	0·20	15	0·25	22	0·29
2	0·13	9	0·21	16	0·26	23	0·30
3	0·14	10	0·23	17	0·26	24	0·30
4	0·16	11	0·25	18	0·26	25	0·30
5	0·17	12	0·25	19	0·27		
6	0·18	13	0·25	20	0·28		

* Annales de Chimie, 1836, tome lxiii. p. 368.

† Ibid. 1837, tome lxiv. p. 312.

‡ Jahresbericht der Münchener Sternwarte für 1852, S. 64, 93, 101; and Annalen für Meteorologie, 1842, Heft iv. S. x. xv.

The observations for determining the freezing-points of C and D gave,—

	C.	D.
1845, March	0·00	0·05 in snow.
1846, December	0·01	0·08 in snow.
1855, May..................	0·02	0·07 in ice.
1855, July..................	0·00	0·08 in ice.
Means in 1855	0·01	0·07

Comparisons of C and D with K in March and May 1855.

No. of comparisons.	C.	K.	D.
10	6·047	6·004	6·064
6	7·780	7·772	7·783
5	10·314	10·316	10·310
10	13·097	13·137	13·098
24	16·000	16·042	16·037
33	17·000	17·056	17·030
24	18·000	18·062	18·032
11	20·161	20·268	20·228
17	24·955	25·079	25·086

t.	t−K.	K−C.	t−C.	K−D.	t−D.
0	0·000		−0·010		−0·070
6	−0·004	−0·043	−0·047	−0·060	−0·064
8	−0·005	−0·008	−0·013	−0·011	−0·016
10	−0·006	+0·002	−0·004	+0·006	0·000
13	−0·008	+0·040	+0·032	+0·039	+0·031
16	−0·009	+0·042	+0·033	+0·005	−0·004
17	−0·010	+0·056	+0·046	+0·026	+0·016
18	−0·010	+0·062	+0·052	+0·030	+0·020
20	−0·011	+0·107	+0·096	+0·040	+0·029
25	−0·014	+0·124	+0·110	−0·007	−0·021

t.	t−C.	t−D.
0	−0·01	−0·07
1	−0·02	−0·07
2	−0·03	−0·07
3	−0·03	−0·07
4	−0·04	−0·07
5	−0·04	−0·06
6	−0·05	−0·06
7	−0·03	−0·04
8	−0·01	−0·02
9	−0·01	−0·01
10	0·00	0·00
11	+0·01	+0·01
12	+0·02	+0·02
13	+0·03	+0·03
14	+0·03	+0·02
15	+0·03	+0·01
16	+0·03	0·00
17	+0·05	+0·02
18	+0·05	+0·02
19	+0·07	+0·02
20	+0·10	+0·03
21	+0·10	+0·02
22	+0·10	+0·01
23	+0·10	0·00
24	+0·11	−0·01
25	+0·11	−0·02

The zero-points of C and D do not appear to have undergone any very appreciable change. The comparisons of B, C, D with each other at distant intervals may, therefore, be used to check the values of the depression of the zero of B deduced from the observations of the freezing-point in 1845, 1846 and 1855.

Comparisons of C, B, D, July 31, 1846.

No. of comparisons.	C.	B.	D.
10	20·492	20·769	20·620
10	19·697	19·969	19·805
10	20·707	20·977	20·841
10	18·347	18·589	18·425
Mean of 40 comparisons ...	19·810	20·076	19·923

Hence, at 20°, B−C=+0·266, B−D=+0·153.

Comparisons of C, B, D in July 1855.

No. of comparisons.	C.	B.	D.
10	19·884	20·258	19·967
10	20·535	20·885	20·616
Mean of 20 comparisons ...	20·209	20·571	20·291

Hence, at 20°, B−C=+0·362, B−D=+0·280.

Also, by a mean of 11 comparisons at 20°, K−C=+0·107, K−D=+0·040. By 30 comparisons at 18°, and 10 at 21°, B−K=+0·261. Therefore B−C=+0·368, B−D=+0·301. The resulting mean value of the depression of the zero of B is 0°·12. The value deduced from the observations of the freezing-point is 0°·13.

Comparisons of C, B, D in September 1846.

No. of comparisons.	C.	B.	D.
10	26·551	26·884	26·750
10	25·385	25·695	25·516
10	24·401	24·670	24·531
10	23·448	23·725	23·577
10	26·634	26·949	26·796
10	23·646	23·940	23·774
Mean of 60 comparisons ...	25·011	25·311	25·157

Hence, at 25°, B−C=+0·30, B−D=+0·153.

By a mean of 16 comparisons of C, B, D between 23°·9 and 26°·2, in July 1855, their corresponding readings were

C.	B.	D.
25·081	25·487	25·204

Hence, at 25°, B−C=+0·406, B−D=+0·283.

Also, 17 comparisons of C, K, D, gave K−C=+0·124, K−D=−0·007, and 10 comparisons of B, K, gave B−K=+0·286. Therefore B−C=+0·410, B−D=+0·279. The resulting mean value of the depression of the zero of B is 0°·12, the same as its calculated value.

The thermometers G, L have arbitrary scales, the divisions of which are traced on the tubes with a diamond point at every tenth of an inch. The parts of the scales of G, L, as well as those of B, C, D, K, are subdivided to hundredths of an inch by sliding scales of glass. The division of the sliding scale which is brought into apparent coincidence with the extremity of the thread of mercury, is viewed through a hole in a plate of brass attached to a very light brass frame which carries the scale, so that the direction of vision may be perpendicular to the axis of the tube. The hundredths of an inch are subdivided by estimation.

Means of Comparisons of G and L with K in February 1855.

No. of comparisons.	G.	K.	No. of comparisons.	L.	K.
10	48·541	2·365	10	51·465	15·753
10	90·797	11·432	10	54·545	16·089
10	111·350	15·753	10	73·010	18·389
10	124·050	18·389	10	92·384	20·828
			10	94·698	21·055
			10	94·869	21·079
			10	97·790	21·448
			10	101·515	21·911

By a mean of 30 comparisons of G and L between 17° and 20° in July 1846, their corresponding readings were

G.	L.
120·897	67·591

Means of Comparisons of G, K, L in March and May 1855.

No. of comparisons.	G.	K.	L.
11	117·79	17·08	62·38
10	124·24	18·43	73·36
	6·45	1·35	10·98

1 part of G=0°·2093, 1 part of L=0°·12295, 1 part of G=1·702 parts of L.

	G.	K.	L.
Mean of all	120·860	17·723	67·609

In July 1846 the corresponding readings of G, L were 120·897 and 67·591.

By the observations of 1855, K=17·731 when G=120·897, and K=17·721 when L=67·591.

Hence L stood 0°·01 higher, compared with G, in 1855 than it did in July 1846. Between March 1845 and May 1855 the zero of G appears to have ascended 0°·01. It is probable that the whole or, at any rate, the greater part of this ascent occurred before July 1846. On this supposition, in July 1846, the depression of the zero of G below its permanent place would be 0°·00, and that of L would be 0°·01.

The thermometers H, P were used only in the earlier observations. The divisions of H were on the tube. The scale of P was traced on paper and enclosed between the tube of the thermometer and an exterior tube of glass joined by fusion to the thermometer tube at the lower end, and sealed at the upper end. The freezing-point of H was +0°·50. The freezing-point of P was +0°·25.

By a mean of 13 comparisons of H, P, C, D, their corresponding readings were

H.	P.	C.	D.
3·31	3·13	2·93	2·99

Let 2M=C+D. Then the corresponding readings of H, P, M, t will be

H.	P.	M.	t.	t−H.	t−P.
3·31	3·13	2·96	2·91	−0·40	−0·22

Means of Comparisons of H, B, P, March 3, 1845.

No. of comparisons.	H.	B.	P.
8	6·09	5·66	6·02
10	11·45	11·37	11·84
9	17·06	17·19	17·94
10	19·67	19·82	20·73
10	23·40	23·71	24·82

Hence, observing that the zero of B was 0·22 lower in March 1845 than in 1855,—

t.	t−H.	t−P.
5·70	−0·39	−0·32
11·34	−0·11	−0·50
17·15	+0·09	−0·79
19·76	+0·09	−0·97
23·63	+0·23	−1·19

Means of Comparisons of H, P with G and L in July and August 1844.

No. of comparisons.	H.	P.	G.	L.
25	16·43	17·13	114·96	——
27	17·46	18·24	——	65·90
14	18·37	19·32	124·75	——
13	20·73	21·82	——	93·03

The corresponding readings of G, K, t; L, K, t in 1855 were,—

G.	K.	t.	L.	K.	t.
114·96	16·504	16·49	65·90	17·503	17·49
124·75	18·535	18·52	93·03	20·909	20·90

At a given temperature the readings of G and L were higher in 1855 than in July 1844 by 0°·02 C. and 0°·04 C. respectively. Hence the corresponding readings of G, L, t, in July 1844, would have been,—

G.	t.	L.	t.
114·96	16·51	65·90	17·53
124·75	18·54	93·03	20·94

The corresponding readings of H, P, t, in July 1844, would have been,—

H.	P.	t.	t−H.	t−P.
16·43	17·13	16·51	+0·08	−0·62
17·46	18·24	17·53	+0·07	−0·71
18·37	19·32	18·54	+0·17	−0·78
20·73	21·82	20·94	+0·21	−0·88

By a graphic interpolation the following values of t−H, t−P were obtained:—

t.	t−H.	t.	t−H.	t.	t−P.	t.	t−P.
0	−0·50	13	−0·07	0	−0·25	13	−0·53
1	−0·48	14	−0·02	1	−0·25	14	−0·57
2	−0·46	15	+0·01	2	−0·26	15	−0·62
3	−0·43	16	+0·04	3	−0·27	16	−0·67
4	−0·41	17	+0·07	4	−0·28	17	−0·72
5	−0·38	18	+0·10	5	−0·29	18	−0·78
6	−0·35	19	+0·14	6	−0·31	19	−0·84
7	−0·31	20	+0·17	7	−0·33	20	−0·90
8	−0·27	21	+0·19	8	−0·36	21	−0·97
9	−0·24	22	+0·21	9	−0·39	22	−1·04
10	−0·20	23	+0·22	10	−0·42	23	−1·12
11	−0·16	24	+0·23	11	−0·45	24	−1·20
12	−0·11			12	−0·49		

The thermometer R was used in some of the earlier observations. It has an ivory scale the dimensions of which vary with the quantity of moisture present in the atmosphere, and consequently the error is very sensibly different at different times.

By a mean of twenty comparisons of R, B in October 1846, their corresponding readings were:—

R.	B.	B—R.
19·31	18·90	—0·41

But in October 1846, at 19°, $t-B=-0{\cdot}15$. Therefore $t-R=-0^{\circ}{\cdot}56$.

Comparison of Barometers.

Up to the end of August 1844 a siphon barometer by the late Mr. Robinson was employed. It resembles Bunten's improved Gay-Lussac's barometer in all respects except that it is graduated in English inches, and the attached thermometer in degrees of Fahrenheit's scale. The observations were reduced, for the mercury to 32° Fahr., and for the scale to 62° Fahr., by the tables in Schumacher's Jahrbuch für 1837.

From the beginning of September 1844 a cistern barometer by Ernst of Paris was used. It is graduated in millimètres, and the attached thermometer in centesimal degrees. In the following comparisons with the barometer of the Paris Observatory, made by one of the Assistants of the Observatory, O, T denote the readings of the Observatory barometer, and of its attached thermometer; F, E those of Ernst, and of its attached thermometer.

O. mm.	T.	F. mm.	E.	O—F. mm.
753·16	21·4	752·90	21·0	0·26
753·64	21·8	753·30	21·3	0·34
754·00	22·3	753·60	21·7	0·40
754·14	22·3	753·70	21·7	0·44
754·32	22·5	753·90	21·9	0·42
757·40	21·1	757·00	20·9	0·40
756·90	22·4	756·50	21·7	0·40
758·64	19·5	758·25	19·3	0·39
758·20	21·7	757·70	21·1	0·50
			Mean	0·393

Ernst stands 0·393 millimètre lower than the Observatory barometer. The latter requires no correction. Ernst was suspended close to the Observatory barometer all night previous to the day on which the comparisons were made, their temperatures must, therefore, have been very nearly equal. Yet E is less than T, while, by a subsequent comparison with B, E was found to be 0°·45 too great. There is reason to believe that this discrepance is due to the lodgment of a small quantity of mercury in the upper end of the tube of the thermometer, which occurs sometimes after the instrument has been conveyed in a carriage over a rough pavement, as was done previous to its comparison with the Observatory barometer.

In March 1845 the thermometer B was suspended so that its bulb was in contact

with the brass covering of the bulb of E, and the corresponding readings of B and E observed. At this time the freezing-point of B was 0°·22 lower than in 1855. The readings of a thermometer free from error are denoted by *t*.

No. of comparisons.	B.	E.	B−E.	*t*−B.	*t*−E.
10	5·25	3·75	−0·50	+0·05	−0·45
10	7·46	7·94	−0·48	+0·03	−0·45
17	9·73	10·12	−0·39	−0·01	−0·40
9	11·65	12·09	−0·44	−0·03	−0·47
9	15·10	15·52	−0·42	−0·03	−0·45
7	21·58	22·03	−0·45	−0·06	−0·51
				Mean	−0·45

Since the reduction of F to 0° C., with a temperature E 0°·45 too great, would make the reduced value of F 0·055 mm. too small, the error of E will be corrected by adding 0·45 mm. instead of 0·39 mm. to the reduced value of F.

Subsequently, by the kindness of the Rev. W. T. Kingsley, an opportunity was afforded of comparing F with a standard barometer having a tube of very large bore, also constructed by Ernst, belonging to the Taylor Library in Sidney-Sussex College, Cambridge. According to the maker's statement, the reading of this barometer is 0·04 mm. too great.

	Taylor.	Att. therm.	F.	E.
April 11, 1851.	761·28	9·50	760·80	10·00
	761·28	9·60	760·85	10·10
	761·32	9·75	760·90	10·20
	761·22	9·75	760·87	10·15
	761·18	9·75	760·80	10·30
	761·26	9·69	760·84	10·15
April 12.	760·78	9·25	760·38	9·6
	760·80		760·42	
	760·83	9·5	760·40	9·8
	760·74	9·6	760·30	10·1
	760·74		760·36	
	760·74	9·75	760·30	10·2
	760·76	9·52	760·36	9·92

	T.	Att. therm.	F.	E.
Means ...	761·01	9·60	760·60	10·03

Taylor−F=0·41 mm. But Taylor stands 0·04 mm. too high. Therefore F is 0·37 mm. too low. E is 0°·43 higher than the attached thermometer of Taylor. The reduction due to 0°·43 is 0·053 mm. Therefore the error of E will be corrected by adding 0·423 mm. instead of 0·37 mm. to the reduced height of F.

The mean of the two corrections gives 0·44 millimètre to be added to the height of F reduced to 0° C. The reductions were made by the tables in Schumacher's Jahrbuch für 1838.

Weight of Air.

According to RITTER* the observations of REGNAULT† show that in Paris, lat. 48° 50′ 14″, 60 mètres above the level of the sea, a litre of dry atmospheric air at 0° C., under the pressure of 760 millimètres of mercury, weighs 1·2932227 gramme. Assuming that atmospheric air contains on an average 0·0004 of its volume of carbonic acid the density of which is 1·529 of that of atmospheric air, the weight of a litre of dry atmospheric air containing its average amount of carbonic acid, under the circumstances already stated, will be 1·2934963 gramme. It appears from the discussion of pendulum experiments by Mr. BAILY‡, that if we take G to denote the force of gravity at the mean level of the sea in lat. 45°, the force of gravity in lat. λ, at the mean level of the sea,

$$=G(1-0{\cdot}0025659\cos 2\lambda).$$

POISSON§ has proved that the force of gravity in a given latitude at a place on the surface of the earth at the height z above the mean level of the sea

$$=\left\{1-\left(2-\frac{3}{2}\frac{\varrho'}{\varrho}\right)\frac{z}{r}\right\}\times \text{(force of gravity at the level of the sea in the same latitude)},$$

where r is the radius of the earth, ϱ its mean density, and ϱ' the density of that part of the earth which is above the mean level of the sea. If, as is probable,

$$\varrho':\varrho=5:11,\quad 2-\frac{3}{2}\frac{\varrho'}{\rho}=1{\cdot}32 \text{ nearly},\quad r=6366198 \text{ mètres}.$$

Hence the weight in grammes of a litre of dry atmospheric air containing the average amount of carbonic acid, at 0°, and under the pressure of 760 millimètres of mercury at 0°, at the height z above the mean level of the sea in lat. λ, is

$$1{\cdot}2930693\left(1-1{\cdot}32\frac{z}{r}\right)(1-0{\cdot}0025659\cos 2\lambda).$$

REGNAULT found the expansion of air from 0° to 100°, under constant pressure, equal 0·36706 of its volume at 0°; also that, at 50°, the mercurial thermometer was a little in advance of the air thermometer ‖. The difference between the mercurial and air thermometers, at 50°, amounts to about 0°·2¶. Hence, the expansion of air between 0° and 50°·2 is 0·18353 of its volume at 0°; or, between 0° and 50°, the ratio of the density of air at 0° to its density at $t°$ is $1+0{\cdot}003656t$. The density of the vapour of water is 0·622 of that of air. Hence, if t be the temperature of the air, b the barometric pressure, v the pressure of the vapour present in the air, b and v being expressed in millimètres of mercury at 0°, at a place on the surface of the earth at a height z above the mean level of the sea, in lat. λ, the weight in grammes of a litre

* Mémoires de la Société de Physique et d'Histoire Naturelle de Genève, t. xiii. p. 361.

† Mémoires de l'Institut, t. xxi. p. 157.

‡ Memoirs of the Astronomical Society, vol. vii. p. 94.

§ Traité de Mécanique, t. ii. p. 629.

‖ Mémoires de l'Institut, tome xxi. pp. 91, 238.

¶ Annales de Chimie, 3me série, tome v. p. 99.

of air will be

$$\frac{1{\cdot}2930693}{1+0{\cdot}003656t}\,\frac{b-0{\cdot}378v}{760}\left(1-1{\cdot}32\frac{z}{r}\right)(1-0{\cdot}0025659\cos 2\lambda).$$

In the cellar under the Mineralogical Museum in Cambridge, where the weights were compared, in lat. 52° 12′ 18″, about 8 mètres above the mean level of the sea, this becomes

$$\frac{1{\cdot}293893}{1+0{\cdot}003656t}\,\frac{b-0{\cdot}378v}{760}.$$

Since a litre is the volume of 1000 grammes of water at its maximum density, the above expression, divided by 1000, gives the ratio of the density of air to the maximum density of water. The logarithm of the ratio of the density of air at t°, to the maximum density of water, is obtained by adding the logarithm of $b-0{\cdot}378v$ in millimètres to $\log A_t$.

TABLE I.

$$10+\log 1{\cdot}293893-3-\log 760-\log(1+0{\cdot}003656\,t).$$

t.	$10+\log A_t$.	Diff.	t.	$10+\log A_t$.	Diff.
0	4·231085		15	4·207898	
1	4·229500	1585	16	4·206396	1502
2	4·227921	1579	17	4·204898	1498
3	4·226347	1574	18	4·203406	1492
4	4·224780	1567	19	4·201919	1487
5	4·223217	1563	20	4·200436	1483
6	4·221661	1556	21	4·198959	1477
7	4·220110	1551	22	4·197488	1471
8	4·218565	1545	23	4·196020	1468
9	4·217025	1540	24	4·194558	1462
10	4·215490	1535	25	4·193101	1457
11	4·213961	1529	26	4·191648	1452
12	4·212437	1524	27	4·190201	1447
13	4·210919	1518	28	4·188758	1443
14	4·209406	1513	29	4·187320	1438
15	4·207898	1508	30	4·185887	1433

The logarithms in the preceding Table, when diminished by 0·000028, serve for the reduction of the weighings in Somerset House, lat. 51° 30′ 40″, 29·56 mètres above the mean level of the sea; and when diminished by 0·000132, they may be used for the reduction of weighings in Paris.

According to a document to which the names of BIOT, REGNAULT, and BIANCHI are appended*, the pressure of vapour in rooms that are not heated artificially, in Paris, is two-thirds of the maximum pressure due to the temperature. It is probable that in Cambridge the hygrometric condition of the air is very nearly the same.

* Memorie di Matematica e di Fisica della Società Italiana della Scienze residente in Modena, tomo xxv. p. 1.

Table II.

Values of $0·378\times\frac{2}{3}v_t$, where v_t is the maximum pressure of vapour at the temperature t, in millimètres of mercury at 0°, according to Regnault's observations*.

t.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	1·16	1·17	1·18	1·18	1·19	1·20	1·21	1·22	1·23	1·24
1	1·25	1·25	1·26	1·27	1·28	1·29	1·30	1·31	1·32	1·33
2	1·34	1·35	1·36	1·37	1·37	1·38	1·39	1·40	1·41	1·42
3	1·43	1·44	1·45	1·46	1·47	1·48	1·49	1·50	1·51	1·53
4	1·54	1·55	1·56	1·57	1·58	1·59	1·60	1·61	1·63	1·64
5	1·65	1·66	1·67	1·68	1·69	1·70	1·72	1·73	1·74	1·75
6	1·76	1·78	1·79	1·80	1·81	1·82	1·84	1·85	1·86	1·87
7	1·89	1·90	1·91	1·93	1·94	1·95	1·96	1·98	1·99	2·01
8	2·02	2·03	2·05	2·06	2·08	2·09	2·10	2·12	2·13	2·15
9	2·16	2·17	2·19	2·21	2·22	2·24	2·25	2·27	2·28	2·29
10	2·31	2·32	2·34	2·35	2·37	2·39	2·40	2·42	2·44	2·45
11	2·47	2·48	2·50	2·52	2·53	2·55	2·57	2·58	2·60	2·62
12	2·64	2·65	2·67	2·69	2·71	2·72	2·74	2·76	2·78	2·80
13	2·81	2·83	2·85	2·87	2·89	2·91	2·93	2·94	2·96	2·98
14	3·00	3·02	3·04	3·06	3·08	3·10	3·12	3·14	3·16	3·18
15	3·20	3·22	3·24	3·26	3·28	3·31	3·33	3·35	3·37	3·39
16	3·41	3·43	3·46	3·48	3·50	3·52	3·54	3·57	3·59	3·61
17	3·63	3·66	3·68	3·71	3·73	3·75	3·78	3·80	3·82	3·85
18	3·87	3·90	3·92	3·95	3·97	4·00	4·02	4·04	4·07	4·09
19	4·12	4·15	4·17	4·20	4·22	4·25	4·28	4·30	4·33	4·36
20	4·38	4·41	4·44	4·47	4·49	4·52	4·55	4·58	4·61	4·63
21	4·66	4·69	4·72	4·75	4·78	4·81	4·84	4·87	4·90	4·92
22	4·95	4·99	5·02	5·05	5·08	5·11	5·14	5·17	5·20	5·23
23	5·26	5·30	5·33	5·36	5·39	5·43	5·46	5·49	5·52	5·56
24	5·59	5·62	5·66	5·69	5·73	5·76	5·80	5·83	5·87	5·90
25	5·93	5·97	6·01	6·04	6·08	6·12	6·15	6·19	6·22	6·26
26	6·30	6·34	6·37	6·41	6·45	6·49	6·53	6·56	6·60	6·64
27	6·68	6·72	6·75	6·79	6·83	6·87	6·91	6·95	6·99	7·03
28	7·08	7·12	7·17	7·21	7·25	7·29	7·34	7·38	7·42	7·46
29	7·51	7·55	7·59	7·64	7·68	7·73	7·77	7·82	7·86	7·91
30	7·95	8·00	8·04	8·09	8·14	8·18	8·23	8·28	8·32	8·37

Expansion of Brass.

The mean rate of expansion of cast brass from 0° to 100°, usually assumed 0·0000187 of its length at 0° for 1°, is considerably larger than the rate at ordinary atmospheric temperatures, according to the observations of Mr. Sheepshanks, who found that at about 17° the coefficient of the linear expansion of cast brass =0·00001722.

* Annales de Chimie, 1845, 3me série, tome xv. p. 138.

Table III.

Logarithms of the ratio of the density of brass at 0° to its density at $t°$, and their arithmetical complements, assuming the coefficient of linear expansion =0·00001722.

t.	log Br.	10−log Br.	t.	log Br.	10−log Br.		
0	0·0000000	10·0000000	16	0·0003588	9·9996412		
1	0·0000224	9·9999776	17	0·0003812	9·9996188		
2	0·0000449	9·9999551	18	0·0004037	9·9995963		
3	0·0000673	9·9999327	19	0·0004261	9·9995739		
4	0·0000897	9·9999103	20	0·0004485	9·9995515	0·1	22
5	0·0001122	9·9998878	21	0·0004709	9·9995291	0·2	45
6	0·0001346	9·9998654	22	0·0004933	9·9995067	0·3	67
7	0·0001570	9·9998430	23	0·0005158	9·9994842	0·4	90
8	0·0001795	9·9998205	24	0·0005382	9·9994618	0·5	112
9	0·0002019	9·9997981	25	0·0005606	9·9994394	0·6	135
10	0·0002243	9·9997757	26	0·0005830	9·9994170	0·7	157
11	0·0002467	9·9997533	27	0·0006054	9·9993946	0·8	179
12	0·0002691	9·9997309	28	0·0006277	9·9993723	0·9	202
13	0·0002916	9·9997084	29	0·0006502	9·9993498		
14	0·0003140	9·9996860	30	0·0006726	9·9993274		
15	0·0003364	9·9996636					

Expansion of Bronze.

Table IV.

Logarithms of the ratio of the density of the alloy adopted by Mr. Baily for the standard bars at 0° to its density at $t°$, the coefficient of its linear expansion at 17° being 0·00001705, according to the observations of Mr. Sheepshanks.

t.	log Ba.	10−log Ba.	t.	log Ba.	10−log Ba.		
0	0·0000000	10·0000000	16	0·0003553	9·9996447		
1	0·0000222	9·9999778	17	0·0003775	9·9996225		
2	0·0000444	9·9999556	18	0·0003997	9·9996003	0·1	22
3	0·0000666	9·9999337	19	0·0004219	9·9995781	0·2	44
4	0·0000888	9·9999112	20	0·0004441	9·9995559	0·3	67
5	0·0001111	9·9998889	21	0·0004663	9·9995337	0·4	89
6	0·0001333	9·9998667	22	0·0004884	9·9995116	0·5	111
7	0·0001555	9·9998445	23	0·0005106	9·9994894	0·6	133
8	0·0001777	9·9998223	24	0·0005328	9·9994672	0·7	155
9	0·0002009	9·9997991	25	0·0005550	9·9994450	0·8	178
10	0·0002221	9·9997779	26	0·0005772	9·9994228	0·9	200
11	0·0002443	9·9997557	27	0·0005994	9·9994006		
12	0·0002665	9·9997335	28	0·0006216	9·9993784		
13	0·0002887	9·9997113	29	0·0006437	9·9993563		
14	0·0003109	9·9996891	30	0·0006659	9·9993341		
15	0·0003331	9·9996669					

Expansion of Platinum.

Schumacher has given two tables of the expansion of platinum, one calculated on the supposition that the coefficient of the linear expansion is 0·00000900, the other with the coefficient 0·00000909, as if in doubt which deserved the preference. Steinheil adopts 0·0000085655 in the reduction of the weighings of the platinum kilogramme des Archives. In comparing the standard platinum mètre with a glass mètre the expansion of which was known, at temperatures, however, not differing much from each other, he obtained 0·00000905 for the coefficient of expansion.

Table V.

Logarithms of the ratio of the density of platinum at 0° to its density at $t°$, and their arithmetical complements, assuming the coefficient of the linear expansion of platinum to be 0·00000900.

t.	log P.	10−log P.	t.	log P.	10−log P.		
0	0·0000000	10·0000000	16	0·0001876	9·9998124		
1	0·0000117	9·9999883	17	0·0001993	9·9998007		
2	0·0000235	9·9999765	18	0·0002110	9·9997880	0·1	12
3	0·0000352	9·9999648	19	0·0002227	9·9997773	0·2	23
4	0·0000469	9·9999531	20	0·0002345	9·9997655	0·3	35
5	0·0000586	9·9999414	21	0·0002462	9·9997538	0·4	47
6	0·0000704	9·9999296	22	0·0002579	9·9997421	0·5	59
7	0·0000821	9·9999179	23	0·0002696	9·9997304	0·6	70
8	0·0000938	9·9999062	24	0·0002813	9·9997187	0·7	82
9	0·0001055	9·9998945	25	0·0002931	9·9997069	0·8	94
10	0·0001173	9·9998828	26	0·0003048	9·9996952	0·9	105
11	0·0001290	9·9998710	27	0·0003165	9·9996835		
12	0·0001407	9·9998593	28	0·0003282	9·9996718		
13	0·0001524	9·9998476	29	0·0003399	9·9996601		
14	0·0001641	9·9998359	30	0·0003516	9·9996484		
15	0·0001759	9·9998241					

Expansion of Water.

The latest and best observations for determining the expansion of water by heat, are those of Stampfer*, Despretz†, Pierre‡, tabulated by Frankenheim§, Kopp‖, and Plücker and Geissler¶. The observations of Hallström and Muncke are not included in this list, on account of certain sources of error pointed out by Despretz.

The following Table exhibits the ratios of the maximum density of water to its density at $t°$ C, according to the different observers above mentioned.

t.	Stampfer.	Despretz.	Pierre.	Kopp.	Plucker and Geissler.
Min.	1·000000	1·0000000	1·0000000	1·000000	1·000000
5	1·000012	1·0000082	1·0000091	1·000006	1·000013
6	1·000038	1·0000309	1·0000336	1·000026	1·000038
7	1·000079	1·0000708	1·0000716	1·000061	1·000079
8	1·000135	1·0001216	1·0001232	1·000109	1·000134
9	1·000205	1·0001879	1·0001882	1·000171	1·000204
10	1·000289	1·0002684	1·0002670	1·000247	1·000287
11	1·000387	1·0003598	1·0003580	1·000336	1·000381
12	1·000497	1·0004723	1·0004608	1·000437	1·000495
13	1·000620	1·0005862	1·0005745	1·000552	
14	1·000757	1·0007146	1·0007065	1·000679	
15	1·000906	1·0008751	1·0008463	1·000818	
16	1·001066	1·0010215	1·0009972	1·000969	
17	1·001239	1·0012067	1·0011592	1·001133	
18	1·001422	1·00139	1·0013320	1·001307	
19	1·001617	1·00158	1·0015153	1·001493	
20	1·001822	1·00179	1·0017128	1·001690	
21	1·002039	1·00200	1·0019185	1·001899	
22	1·002265	1·00222	1·0021296	1·002118	
23	1·002502	1·00244	1·0023498	1·002348	
24	1·002749	1·00271	1·0025836	1·002588	
25	1·003005	1·00293	1·0028263	1·002838	

* Jahrbücher des k. k. polytechnischen Institutes in Wien, B. xvi. S. 1.

† Annales de Chimie et de Physique, 1839, 2me série, tome lxx. p. 47.

‡ Ibid. 1845, 3me série, tome xv. p. 350.

§ Poggendorff's Annalen, B. lxxxvi. S. 451.

‖ Ibid. B. lxxii. S. 1.

¶ Ibid. B. lxxxvi. S. 238.

The discordances which these tables exhibit are partly due to errors in the assumed expansions of brass, glass and mercury, on which, by the nature of the experiments, the value of the expansion of water is made to depend.

STAMPFER deduced the expansion of water from the apparent weight of a hollow cylinder of brass suspended in water. The coefficient of the linear expansion of the brass was found to be 0·0000192, by experiments in which the variations of temperature amounted to from 38° to 62° (the absolute temperatures are not given). At about 17° Mr. SHEEPSHANKS found the coefficient of expansion of cast brass equal to 0·00001722. This is 0·00000153 less than the mean coefficient of expansion from 0° to 100°, assuming the latter to be 0·00001875. The error of the assumed expansion of the cylinder at ordinary atmospheric temperatures will probably be not quite so large. If taken equal to 0·00000133, the correction will be —0·000004(t—4). DESPRETZ experimented with thermometers filled with water. The expansion of the glass was inferred from the apparent expansion of mercury in the thermometer from 0° to 28°, using for the coefficient of the expansion of mercury 0·00018018, the value obtained by DULONG and PETIT. But the expansion of mercury from 0° to 28° is 0·005032 according to REGNAULT*. The resulting mean coefficient of expansion is 0·00017971. Hence the expansions obtained by DESPRETZ must be diminished by 0·00000047(t—4). PIERRE and KOPP, who employed the same method, deduced the expansion of the glass from the apparent expansion of the mercury from 0° to 100°, assuming its absolute expansion between those points to be 0·018018. But the absolute expansion of mercury from 0° to 100° is 0·018153. The glass used by PIERRE contained oxide of lead, and probably had very nearly the same rate of expansion at both high and low temperatures. It is not known how far the glass used by KOPP possessed this property. Hence these expansions require the addition of 0·00000135(t—4). The observations of PLÜCKER and GEISSLER extend only to 12°. They were made with a thermometric apparatus the capacity of which is compensated by mercury so as to be invariable, or very nearly so. Assuming the expansion of mercury from 0° to 100° to be 0·018018, the cubic expansion of the glass from 0° to 100°, deduced from the apparent expansion of mercury, is 0·002818. But according to REGNAULT† the coefficient of the cubic expansion of a glass free from lead was 0·00002761 from 0° to 100°, and 0·00002628 from 0° to 10°. It is therefore probable that the coefficient of the cubic expansion of the glass has been taken 0·00000133 too great. Also the quantity of mercury used for compensating the expansion of the glass will be too small in the ratio of 0·000179714, the rate of expansion of mercury from 0° to 10°, to 0·00018153, the rate of expansion of mercury from 0° to 100°. Hence, upon the whole, the expansion must be diminished by 0·0000013(t—4).

The ratios of the maximum density of water to its density at t°, according to the

* Mémoires de l'Institut, tome xxi. p. 328. † Ibid. p. 237.

observations of Stampfer, Despretz, Pierre, Kopp, and Plücker and Geissler, respectively, after applying the corrections above indicated, become,—

t.	S.	D.	P.	K.	P. G.
Min.	1·000000	1·000000	1·000000	1·000000	1·000000
5	1·000008	1·000008	1·000010	1·000007	1·000013
6	1·000030	1·000030	1·000036	1·000029	1·000036
7	1·000067	1·000069	1·000076	1·000065	1·000076
8	1·000119	1·000120	1·000129	1·000114	1·000129
9	1·000185	1·000186	1·000195	1·000178	1·000198
10	1·000265	1·000266	1·000275	1·000255	1·000279
11	1·000359	1·000357	1·000368	1·000345	1·000372
12	1·000465	1·000469	1·000472	1·000448	1·000485
13	1·000584	1·000582	1·000587	1·000564	
14	1·000717	1·000710	1·000720	1·000693	
15	1·000862	1·000870	1·000861	1·000833	
16	1·001018	1·001016	1·001013	1·000985	
17	1·001187	1·001201	1·001177	1·001151	
18	1·001366	1·001383	1·001351	1·001326	
19	1·001557	1·001573	1·001536	1·001513	
20	1·001758	1·001783	1·001734	1·001712	
21	1·001971	1·001992	1·001942	1·001922	
22	1·002193	1·002212	1·002154	1·002142	
23	1·002426	1·002431	1·002376	1·002374	
24	1·002669	1·002701	1·002611	1·002615	
25	1·002921	1·002920	1·002855	1·002866	

On account of the uncertainty of the correction to be applied to Stampfer's observations, and the small range of those of Plücker and Geissler, it appears best to rely exclusively on the corrected observations of Despretz, Pierre and Kopp. In the details of the observations, I am unable to detect any particular that warrants the acceptance or rejection of any one in preference to the other two. The mean of all three will probably approximate more nearly to the truth than any one of them taken by itself. Assuming 3°·945 C. to be the temperature at which the density of water is a maximum, in accordance with the result obtained by Messrs. Playfair and Joule*, the logarithms of the means, to seven places of decimals considered as integers, are represented with sufficient accuracy, between 4° and 25°, by

$$32\cdot72(t-3\cdot945)^2-0\cdot215(t-3\cdot945)^3.$$

Comparison of the means of D, P, K with the formula.

t.	Means of D, P, K.	Formula.	t.	Means of D, P, K.	Formula.
3·945	1·000000	1·000000	15	1·000855	1·000854
5	1·000008	1·000008	16	1·001005	1·001009
6	1·000032	1·000031	17	1·001176	1·001175
7	1·000070	1·000069	18	1·001353	1·001352
8	1·000121	1·000121	19	1·001541	1·001540
9	1·000186	1·000186	20	1·001743	1·001739
10	1·000265	1·000265	21	1·001952	1·001948
11	1·000356	1·000357	22	1·002169	1·002167
12	1·000463	1·000460	23	1·002394	1·002396
13	1·000578	1·000581	24	1·002642	1·002634
14	1·000708	1·000712	25	1·002880	1·002882

* Memoirs of the Chemical Society, vol. iii. p. 199.

Table VI.

Logarithms of the ratios of the maximum density of water to its density at t, for every fifth of a centesimal degree, according to the formula.

t.	$\log W_t$.	Diff.
3·945	0·0000000	
4·2	0·0000002	
4·4	0·0000007	5
4·6	0·0000014	7
4·8	0·0000024	10
5·0	0·0000036	12
5·2	0·0000051	15
5·4	0·0000069	18
5·6	0·0000089	20
5·8	0·0000111	22
6·0	0·0000136	25
6·2	0·0000164	28
6·4	0·0000194	30
6·6	0·0000227	33
6·8	0·0000262	35
7·0	0·0000299	37
7·2	0·0000339	40
7·4	0·0000381	42
7·6	0·0000426	45
7·8	0·0000474	48
8·0	0·0000524	50
8·2	0·0000576	52
8·4	0·0000630	54
8·6	0·0000687	57
8·8	0·0000747	60
9·0	0·0000808	61
9·2	0·0000872	64
9·4	0·0000939	67
9·6	0·0001007	68
9·8	0·0001079	72
10·0	0·0001152	73
10·2	0·0001228	76
10·4	0·0001306	78
10·6	0·0001386	80
10·8	0·0001468	82
11·0	0·0001553	85
11·2	0·0001640	87
11·4	0·0001729	89
11·6	0·0001821	92
11·8	0·0001915	94
12·0	0·0002011	96
12·2	0·0002109	98
12·4	0·0002209	100
12·6	0·0002312	103
12·8	0·0002416	104
13·0	0·0002523	107
13·2	0·0002632	109
13·4	0·0002743	111
13·6	0·0002857	114
13·8	0·0002972	115
14·0	0·0003089	117
14·2	0·0003209	120
14·4	0·0003331	122

t.	$\log W_t$.	Diff.
14·4	0·0003331	
14·6	0·0003455	124
14·8	0·0003580	125
15·0	0·0003708	128
15·2	0·0003838	130
15·4	0·0003970	132
15·6	0·0004104	134
15·8	0·0004240	136
16·0	0·0004378	138
16·2	0·0004518	140
16·4	0·0004660	142
16·6	0·0004804	144
16·8	0·0004950	146
17·0	0·0005098	148
17·2	0·0005248	150
17·4	0·0005400	152
17·6	0·0005554	154
17·8	0·0005709	155
18·0	0·0005867	158
18·2	0·0006026	159
18·4	0·0006187	161
18·6	0·0006351	164
18·8	0·0006516	165
19·0	0·0006682	166
19·2	0·0006851	169
19·4	0·0007022	171
19·6	0·0007194	172
19·8	0·0007368	174
20·0	0·0007544	176
20·2	0·0007722	178
20·4	0·0007902	180
20·6	0·0008083	181
20·8	0·0008266	183
21·0	0·0008451	185
21·2	0·0008637	186
21·4	0 0008826	189
21·6	0·0009016	190
21·8	0·0009207	191
22·0	0·0009400	193
22·2	0·0009596	196
22·4	0·0009793	197
22·6	0·0009991	198
22·8	0·0010191	200
23·0	0·0010393	202
23·2	0·0010596	203
23·4	0·0010801	205
23·6	0·0011008	207
23·8	0·0011216	208
24·0	0·0011426	210
24·2	0·0011637	211
24·4	0·0011850	213
24·6	0·0012065	215
24·8	0·0012281	216
25·0	0·0012498	217

Reduction of Weighings.

If the weights P and Q appear to be equal when compared in air,—

weight of P— weight of air displaced by P=weight of Q— weight of air displaced by Q.

Let t be the temperature of the air in centesimal degrees, b its pressure in millimètres of mercury at 0°, v the pressure of the vapour contained in it, also in millimètres of mercury, $h=b-0{\cdot}378v$, ΔP, ΔQ ratios of the densities of P and Q at 0° to the maximum density of water; ePt, eQt the expansions in volume of P and Q. Then log weight in grains of the air displaced by P

$$=\log h+\log A_t+\log(1+ePt)+\log \text{ weight of P in grains}-\log \Delta P.$$

If vP be taken to denote the volume of P at 0°, the unit of volume being the volume of a grain of water at its maximum density,

$$\log vP=\log \text{ weight of P in grains}-\log \Delta P.$$

The expression for the weight of air displaced by Q differs from the above only in the substitution of Q for P.

The value of h is deduced from b by means of Table II., assuming that the amount of vapour in the air is two-thirds of the quantity in saturated air. Table I. gives the second term for the expression for the weight of the air displaced, and Tables III., IV., V. give the third term according as the weight is of brass, bronze or platinum.

Calculation of Densities.

Let P in water at t° appear to weigh as much as Q in air. Then weight of water at t° displaced by P= weight of P− weight of Q+ weight of air displaced by Q, $\log vP=\log$ weight in grains of the water displaced by $P+\log W_t-\log(1+ePt)$, where W_t is the ratio of the maximum density of water to its density at t° obtained from Table VI. Log $\Delta P=\log$ weight of P in grains $-\log vP$.

An approximate value of vP having been found by assuming the weight of P equal to its apparent weight in air, this value of vP may be used in reducing the weight of P, and thus a more accurate value of vP obtained, by means of which a closer approximation to the values of the absolute weight of P and of ΔP may be found. This process is to be repeated when greater exactness is required.

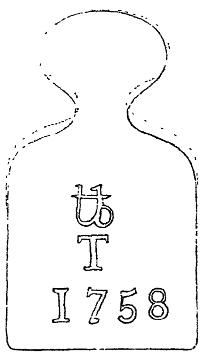
Densities of the Troy Pounds constructed in 1758.

Though it appears that only two of the five weights with which U was compared are in a state of unexceptionable preservation, and the number of trustworthy comparisons is reduced from 608 to 440, these are amply sufficient for the purpose of ascertaining the apparent weight of U in air (t=65·66 F, b=29·75 inches). But, in order to find the absolute weight of U, or indeed its apparent weight in air of a density different from that which it has when t=65·66, b=29·75, a knowledge of the volume of the lost standard is requisite. It is not probable that U was ever weighed in water, and certainly no record of any such weighing is known to exist. There is therefore no direct method of finding its volume. An indirect way of arriving at it was suggested to Professor SCHUMACHER by an examination of three Parliamentary Reports, the first presented May 26, 1758, the second April 11, 1759, and a third ordered to be printed March 2, 1824.

It appears from the first of these Reports that Mr. HARRIS, then Assay-Master of the Mint, presented to the first Committee three troy pounds made under his direction, one of which was the lost Imperial Standard Troy Pound. The third Report contains the evidence of Dr. KELLY, who in reply to the query, "What was effected with regard to weights and measures by the Committee of 1758?" answers, "They ordered three several troy pounds to be adjusted, under the direction of Mr. HARRIS, the then Assay-Master of the Mint. One of these was placed in the custody of the Clerk of the House of Commons; another was left with Mr. HARRIS, and is now in the possession of Mr. BINGLEY; and the third was, I understand, delivered to Mr. FREEMAN, weight-maker to the Mint, the Exchequer and the Bank of England, who used it as his standard, and it is still so employed by his successor Mr. VANDOME." The same page contains the following note:—This weight [Mr. BINGLEY's pound] was produced to the Committee, by Mr. BINGLEY, who said it had formerly belonged to Mr. HARRIS when he held the situation of Assay-Master. There was a memorandum on the lid of the box in which it was kept, stating that Mr. HARRIS had made from it the pound weight which was placed in the custody of the Clerk of the House of Commons by direction of the Committee of 1758, and which is commonly called the Parliamentary Pound.

Professor SCHUMACHER then observes that "if Dr. KELLY's statements be exact, as there is no doubt they are, and Messrs. BINGLEY's and VANDOME's pound be *really* the two remaining weights of the often mentioned three which Mr. HARRIS presented to the Committee of 1758, we can still either determine, with the highest degree of probability, the specific gravity of the lost Imperial standard troy pound, or know with certainty that all hope to arrive at this knowledge is lost. It will be only requisite to ascertain with the greatest care the specific gravities of both pounds, the one in the possession of Mr. BINGLEY, the other in the possession of Mr. VANDOME. If the specific gravity of both is found the *same*, we might from that circumstance draw the highly probable conclusion, that the three single pounds of Mr. HARRIS, according to my hypothesis, were really made of the same identical metal; and the specific gravities of the two remaining pounds might with safety be considered as that of the lost standard. If, on the contrary, the two remaining pounds prove to be of *different* specific gravities, the hypothesis that all three were made of the same metal is evidently erroneous, and nothing can be inferred from the specific gravity of either of the two remaining."

V. Fig. 1.



These two weights were found to be still in existence. Mr. VANDOME readily consented to allow the troy pound in his possession to be experimented upon by the Committee. In form and size this weight very closely resembles the figure of the lost standard given by Captain v. NEHUS. The

upright stroke of the 5 in the type appears to have been broken off, and the defect supplied in the inscription by a cut with a chisel. The letters SF are impressed diametrically opposite to the T. This weight, as well as the others of the same date, is of one piece of metal, without any cavity for adjustment by the addition of bits of wire. Mr. SIMMS, to whom it was shown, pronounced it to be of soft gun-metal, as hard as cast brass, but not so hard as hammered brass, and, for such an alloy, a very bad casting.

As the balance ordered of Mr. BARROW was not yet ready, Mr. VANDOME's troy pound (V) was weighed in water with a balance of 10½ inches beam by ROBINSON. The case of this was too small to admit a large cylinder of water, the use of which is considered essential to the accuracy of observations of this kind, and some unaccountable discordances in the weighings of V in air impair the probable accuracy of the result. For these reasons the result alone is given, omitting the details of the observations. By a mean of six weighings in water in July and August, 1843, the density of V at 0° C. appeared to be 8·15105 times the maximum density of water. This value, notwithstanding its uncertainty, was sufficiently exact for a preliminary comparison of the densities of the weights made in 1758. The accurate determination of the density of V and of the other weights of the same date presents considerable difficulty; for the pores in the metal are so deep, that the complete expulsion of the air contained in them is very questionable, even after prolonged immersion in boiling water.

The following observations were made with BARROW's balance under circumstances more favourable to accuracy. The glass jar containing the distilled water in which V was weighed, was 6·7 inches in diameter and 6·5 inches deep. V was suspended, from the pan of the balance, by a hook attached to a fine copper wire, 7·5 inches of which weighed about one grain. In order to expel the air adhering in bubbles to the weight, or contained in the cavities in the metal, it was placed, with the fine wire attached to it, in water in a bell-shaped jar of thin glass, just large enough to contain the weight. The jar was suspended over the flame of a spirit-lamp by a stout wire bent at its lower end into a ring into which the jar descended to its rim, and the water allowed to boil till it was supposed that the air was entirely expelled. The small jar containing the weight was then immersed in the water which very nearly filled the large jar, the suspending wire hooked on to the under side of the scale pan, and the small jar lowered till the weight hung clear of it, and then removed. The transfer of the weight from the small jar to the large one was thus effected without taking it out of the water. The counterpoise was placed in the left-hand pan of the balance; V was suspended in water from the right-hand pan. Small weights were placed in the right-hand pan till equilibrium was produced, and the readings of the scale observed. V was then removed, leaving the hook suspended in water, and a volume of water equal to that of V added to the water in the jar; the weights A, B, C, D, &c. were placed in the right-hand pan till equilibrium was again produced, and the

readings of the scale observed. The thermometer B was suspended with its bulb in a horizontal plane through the centre of gravity of V. The thermometer C was in the balance case. F denotes the reading of ERNST's barometer, E that of the attached thermometer. 100 parts of the micrometer scale =0·2 grain nearly, when V is suspended in water, or when the hook alone is suspended in water, and the right-hand pan contains 5053·3 grains.

Observations for finding the density of V.

Weighing of V in air.

1845, July 7. 100 parts=0·3302 grain.

$$S=A+B+C+D+F.$$

$V+\overset{gr.}{0·31}+X \simeq S+Y+\overset{pt.}{0·50}$	$V+\overset{gr.}{0·31}+Y \simeq S+X+\overset{pt.}{0·20}$
+0·25	+0·30
−0·50	+0·40
0·00	+0·65
+0·20	+1·00
−0·20	+0·80
−0·10	+1·15
0·00	+0·75
+0·20	+0·80
+0·20	+0·70
+0·055	+0·675

$$V+\overset{gr.}{0·31} \simeq A+B+C+D+F+\overset{gr.}{0·0012}.$$

V≏5759·160 grains of platinum in air (t=16·13, b=761·27).

Weighing of V in water.

1845, April 2.

	V and hook in water.			In right-hand pan.	
B.	C.	F.	E.	gr.	Scale.
				0·14	18·0
8·5	8	765·8	8·5	0·14	20·0
				0·14	20·0
				0·14	19·6
				0·18	26·5
				0·10	12·3
8·45				0·14	20·1
				0·14	20·0
				0·14	20·3
				0·14	20·2
8·47	8	765·8	8·5	0·140	19·7

Hook in water. In right-hand pan.	gr.	Scale.
A+B+C+D+L+(8)+(4)+(1)+	0·80	20·0
	0·80	18·5
	0·80	22·5
	0·80	19·2
	0·80	19·4
	0·80	19·6
	0·80	20·2
	0·800	19·9

V in water (B=8·47)≏A+B+C+D+L+(8)+(4)+(1)+$\overset{gr.}{0·7596}$ in air (C=8, F=765·8, E=8·5).

May 2.

	V and hook in water.				In right-hand pan.	
	B.	C.	F.	E.	gr.	Scale.
					0·24	22·8
					0·20	20·4
	10·5	11·1	758·3	11·5	0·20	18·0
					0·24	19·1
					0·24	25·0
					0·20	21·0
					0·20	21·5
					0·16	12·0
					0·20	23·7
					0·16	17·0
					0·20	22·5
	10·55				0·16	14·5
	10·52	11·1	758·3	11·5	0·200	19·81

Hook in water.	In right-hand pan. gr.	Scale.
A+B+C+D+N+(8)+(4)+(1)+	0·92	19·2
+	0·92	20·0
+	0·92	20·0
	0·920	19·7

V in water $(B=10{\cdot}52) \bumpeq A+B+C+D+N+(8)+(4)+(1)+\overset{gr.}{0{\cdot}7202}$ in air $(C=11{\cdot}1,\ F=758{\cdot}3,\ E=11{\cdot}5)$.

June 28.

	V and hook in water.				In right-hand pan.	
	B.	C.	F.	E.	gr.	Scale.
	14·45				1·08	20·6
					1·08	20·7
	14·5				1·08	22·0
	14·5	15·7			1·07	19·0
			744·5	16	1·08	22·3
	14·6	15·85			1·07	20·0
					1·07	20·3
					1·07	20·4
	14·51	15·77	744·5	16	1·075	20·66

Hook in water.	In right-hand pan. gr.	Scale.
A+B+C+D+N+(8)+(4)+(2)+	0·88	22·8
	0·87	19·7
	0·87	19·5
	0·88	22·5
	0·87	19·8
	0·87	19·2
	0·88	24·2
	0·87	20·0
	0·8737	20·96

June 28.

	V and hook in water.				In right-hand pan.	
	B.	C.	F.	E.	gr.	Scale.
	14·75	15·7			1·08	27·6
					1·06	21·8
					1·04	14·5
			747·9	15·7	1·06	22·0
					1·05	18·0
					1·06	21·5
	14·75				1·05	17·5
					1·06	21·1
					1·05	19·2
	14·75	15·7	747·9	15·7	1·0567	20·35

V in water $(B=14{\cdot}63) \bumpeq A+B+C+D+N+(8)+(4)+(2)-(1)+\overset{gr.}{0{\cdot}807}$ in air $(C=15{\cdot}73,\ F=746{\cdot}2,\ E=15{\cdot}85)$.

July 1. V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
	14·7	750·8	15·2	1·08	19·5
				1·08	19·4
14·35				1·08	20·2
				1·08	20·5
				1·08	20·3
				1·08	20·8
14·4				1·08	21·1
14·37	14·7	750·8	15·2	1·080	20·3

Hook in water. In right-hand pan.

	gr.	Scale.
A+B+C+D+N+(8)+(4)+(2)+	0·90	22·0
	0·87	19·5
	0·88	20·9
	0·87	19·7
	0·87	19·8
	0·87	18·7
	0·88	20·5
	0·87	18·1
	0·88	20·5
	0·88	20·6
	0·877	20·03

V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
14·7				1·08	18·8
	16	750·8	16·3	1·09	19·6
				1·09	20·1
14·75				1·09	21·6
				1·08	20·3
14·85				1·08	20·3
				1·08	20·8
				1·07	19·9
				1·07	20·5
14·95				1·07	21·5
14·83	16	750·8	16·3	1·080	20·34

V in water (B=14·6) ≏ A+B+C+D+N+(8)+(4)+(2)−(1)+0·798 gr. in air (C=15·35, F=750·8, E=15·75).

July 2. V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
14·3	14·6	759·2	15·1	1·07	18·2
				1·08	19·2
14·3				1·08	20·4
				1·07	19·1
				1·08	19·8
				1·08	20·0
14·4				1·08	20·3
				1·08	20·8
14·4	15·3	759·4	15·7	1·08	21·1
14·35	14·95	759·3	15·4	1·0778	19·9

Hook in water.	In right-hand pan.	
	gr.	Scale.
A+B+C+D+N+(8)+(4)+(2)+	0·88	23·1
	0·88	22·2
	0·87	20·1
	0·87	20·0
	0·87	19·6
	0·87	20·2
	0·87	19·7
	0·87	18·4
	0·87	18·0
	0·88	22·0
	0·87	18·8
	0·8727	20·19

V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
14·6	15·6	759·5	15·8	1·05	19·9
				1·05	19·9
				1·05	20·6
				1·05	20·0
14·65				1·05	20·0
				1·05	19·8
				1·05	20·0
14·65	15·6	759·6	15·9	1·05	20·7
14·63	15·6	759·55	15·85	1·050	20·1

V in water (B=14·49)≏A+B+C+D+N+(8)+(4)+(2)−(1)+0·808 gr. in air (C=15·27, F=759·42, E=15·62).

	t.	grs. of platinum.	t.	b.
V in water...	8·49	≏5053·194 in air	7·99	765·19
V in water...	10·49	≏5053·257 in air	11·09	757·34
V in water...	14·58	≏5053·344 in air	15·76	744·73
V in water...	14·55	≏5053·335 in air	15·38	749·33
V in water...	14·44	≏5053·345 in air	15·30	757·95

Where t is the temperature in centesimal degrees, b the height of the mercury in the barometer in millimètres corrected and reduced to 0° C.

For the platinum of which the weights, A, B, C, &c. are made log Δ=1·32564.

The resulting values of vV, ΔV and log ΔV are,—

vV.	ΔV.	log ΔV.
706·580	8·1515	0·911238
706·555	8·1518	0·911253
706·661	8·1505	0·911188
706·669	8·1505	0·911183
706·656	8·1506	0·911191

Means, giving to the third, fourth and fifth results twice the weight of the first and second, because each of these was deduced from two separate series of weighings in water, while the first and second were deduced each from one series of weighings in water,—

vV.	ΔV.	log ΔV.
706·638	8·15084	0·911202

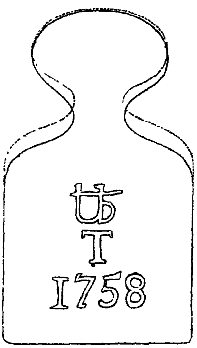
In air (t=16·13, b=761·27). V≏A+B+C+D+F−0·3088 gr. But A+B+C+D+F≏T−0·0017 gr. Therefore V≏T−0·3105 gr.

V displaces $\overset{\text{gr.}}{0\cdot8616}$, and T$-0\cdot310$ displaces $\overset{\text{gr.}}{0\cdot3317}$ of air ($t=16\cdot13$, $b=761\cdot27$). Hence V$=$T$+0\cdot2194$ grain.

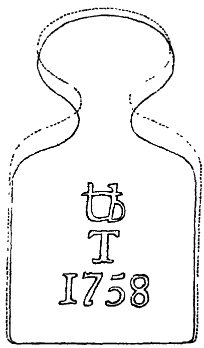
V displaces $\overset{\text{gr.}}{0\cdot8468}$, and T$-\overset{\text{gr.}}{0\cdot310}$ displaces $\overset{\text{gr.}}{0\cdot3261}$ of air ($t=18\cdot7$, $b=755\cdot64$). Hence, in air ($t=18\cdot7$, $b=755\cdot64$), V$\bumpeq$T$-0\cdot3013$ gr. But T$\bumpeq$U$-0\cdot0074$ gr. Therefore V$\bumpeq$U$-0\cdot3087$ grain in air ($t=18\cdot7$, $b=755\cdot64$).

Mr. Bingley had in his possession two troy pounds of the same date. One of these (O) is said to be the original weight from which the standard was made for the House of Commons in 1758. It is distinguished by a small dot under the T, and the imperfection in the type of the 5 is remedied by a cut with a chisel as in V. This weight has since (in 1851) been purchased by the Committee. The other (M), in which the 5 is left imperfect, and which has the mark ◎ ◎ impressed on its under surface, has since been presented to the Mint by its former possessor.

O. Fig. 2.



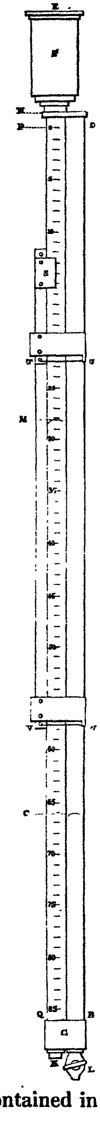
M. Fig. 3.



Mr. Bingley was unwilling to permit either of these troy pounds to be weighed in water; Messrs. Troughton and Simms were therefore commissioned to construct an instrument on the principle of the Stereometer invented by M. Say for the purpose of determining the specific gravity of gunpowder*, but with some improvements which I had described in the Philosophical Magazine for July to December, 1834, vol. v. p. 203. It consists of two glass tubes, PQ, DB (fig. 4), of equal diameter, cemented into cylindrical cavities communicating with each other at their lower ends, in an oblong piece of iron G. In the axes of the two cavities are holes concentric with the tubes. The hole under PQ is closed by a screw K, into the other is screwed an iron stopcock L. The upper end of the tube PQ is cemented into an iron cylinder N carrying a ring which surrounds the upper end of the tube DB. The inside of the cylinder is tapped to receive the screw of the stopcock, and the outside tapped so as to screw into the under end of a cup F, having its rim ground plane,

* Annales de Chimie, 1797, tome xxiii. p. 1.

Fig. 4.



and capable of being closed so as to be air-tight by a plate of glass E, smeared with lard. The tube PQ is graduated by lines traced upon the glass. The original tube, graduated in inches, having been broken, was replaced by a tube graduated in centimètres by M. BUNTEN. The subdivision is effected by an ivory scale S, of ten millimètres divided on the side next to the glass tube, to every fifth of a millimètre, attached to a rectangular rod of deal carrying frames on which filaments of silk TU, VW are stretched, and slips of brass having eye-holes so adjusted that the planes through the threads and the corresponding eye-holes may be perpendicular to the rod, the tubes being between the eye-holes and the threads, as shown in the section fig. 5. A weak brass spring attached to the rod keeps it in contact with the tubes, with the silk threads and ivory scale close to that part of PQ which is graduated, so that it can be easily moved up and down, and is retained in the position in which it is left by the pressure of the spring. The support of the stereometer is adjusted by three foot screws till a thread of unspun silk by which a small weight is suspended, hangs coinciding with the axis of the tube DB. Within E is a cup in which is placed the solid the volume of which is sought. Mercury having been poured into D till its surface rises to P, the first division of the graduation, the mouth of the cup is closed so as to be air-tight by the plate of glass. The stopcock is then opened and the mercury allowed to escape till the difference of the altitude of the mercury in the two tubes is nearly equal to half the height of the mercury in the barometer at the time of the observation. Let the point M of the graduation mark the height of the mercury in PQ, and C the height of the mercury in DB. Let u be the volume of the air in the cup F before the solid was placed in it; v the volume of the solid; b the altitude of the mercury in the barometer reduced to the temperature of the mercury in PQ and BD. Then

Fig. 5.

$$u-v=\frac{b-\mathrm{MC}}{\mathrm{MC}}\ \text{vol. PM}.$$

In order to find the capacity of the portion of the tube included between P and any point M in the graduation, the cup F is taken off, and the stopcock L screwed into the iron collar N. The screw K is taken out, and the tubes placed vertical in an inverted position. The tube PQ is then filled up to about 50 cm with mercury poured through a slender glass tube inserted into the opening at K. This precaution is necessary in order to prevent the formation of air-bubbles on the inner surface of the tube, which would interfere with the correct estimation of the capacity of the tube. The stopcock is then opened, and the mercury contained in a

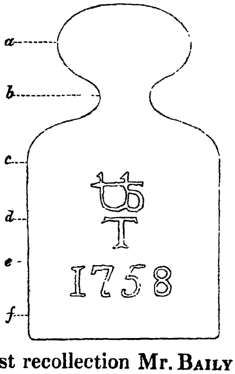
known number of divisions of the tube suffered to run into a light glass jar in which it is weighed. This process is to be repeated till the upper end of the column of mercury descends to the point P.

The stereometer was mounted in a room in Mr. BINGLEY's house at the Mint, September 12, 1843, and a few comparisons made of the volumes of V, O, M. The results, however, in consequence of the unequal heating of different parts of the stereometer in putting it together, did not prove satisfactory. On the 16th, the volumes of O, M and C, a hollow cylinder of brass, were compared with better success. The unit of volume being the volume of a grain of water at its maximum density, these observations gave $v\mathrm{O}+5{\cdot}3=v\mathrm{M}=v\mathrm{C}-0{\cdot}6$.

By observations made August 19, 1843, $\mathrm{C}\triangleq600{\cdot}001$ grains of brass $+78{\cdot}832$ grains of platinum in air ($t=24{\cdot}23$, $b=756{\cdot}78$). C in water ($t=18{\cdot}1$) $\triangleq-9{\cdot}707$ grains of platinum. Hence C in air ($t=24{\cdot}23$, $b=756{\cdot}78$) $\triangleq$ C in water ($t=18{\cdot}1$) $+600{\cdot}001$ grains of brass $+88{\cdot}539$ grains of platinum in air ($t=24{\cdot}23$, $b=756{\cdot}78$). The weights displace 0·092 grain of air; C displaces 0·810 grain of air. Hence C displaces 689·258 grains of water at 18·1, and the volume of C at 0° is equal to the volume of 689·562 grains of water at its maximum density. Hence $v\mathrm{O}=683{\cdot}66$, $v\mathrm{M}=688{\cdot}96$. By weighing in air and in water it was found that $v\mathrm{V}=706{\cdot}34$. The large differences between these numbers show that the volume of the lost standard cannot be inferred with any high degree of probability from a comparison of the volumes of the three remaining pounds.

The only resource now remaining was indicated by Professor SCHUMACHER's remarks on the figure of the lost troy pound:—"As soon as the Imperial standard troy pound was brought to Somerset House, Captain NEHUS's first care was to make an accurate drawing of its shape and marks, measuring all its dimensions with the greatest care. The annexed drawing represents this pound in its actual dimensions, and is now, since the original has been destroyed by the calamitous fire that consumed the two Houses of Parliament in 1834, the only thing remaining which can preserve an idea of it." An application was made to Professor SCHUMACHER for the original drawing, if still in existence, or for any information that would show how far the accuracy of the wood engraving might be depended upon. In his reply, dated October 3, 1843, he wrote as follows:—"The dimensions of the lost standard were only taken with a bow-legged compass in order to give an accurate *drawing* of the standard pound, and in this respect I called them in my papers accurate, but they certainly are not sufficient to give a near approximation of its volume. I find that he (Captain v. NEHUS) has immediately transferred the taken dimensions to paper. This paper, with the original drawing, has served to give the woodcut in the Philosophical Transactions, but to my best recollection Mr. BAILY

U. Fig. 6.



has returned it, though I cannot find it amongst my papers. Even if you had NEHUS's original drawing you would not be able to find the volume, the only height he has *measured* being that of the whole pound. The heights of the points *a*, *b*, *c* are only found by holding a scale in a vertical position near the pound. The diameters on the contrary are measured."

By a comparison of the figure of U with a profile of V traced mechanically, and with careful measurements of its axis and diameters, the axis and the extreme diameters of the knob and cylindrical portion of U appear to be a very little greater than the corresponding dimensions of V, the differences in other parts being exactly where we might expect the drawing to be inaccurate from the manner in which it was made. (In the figures of the weights V, O, M, B, the dotted line is the profile of U.) The diameters and axis of U being measured with a bow-legged compass, were more likely to err in excess than in defect. Making every allowance for this, it did not seem possible, on looking at the profiles of U and V, to suppose that the volume of U was less than that of V. But the volume of O, as well as that of M, being less than that of V, it appeared that of the three weights V, O, M, V approximated most nearly to U in volume. As the existing data were utterly insufficient to determine how much, if at all, U exceeded V in volume, it appeared safest to assume the volumes of U and V to have been equal. This course was recommended also by Professor SCHUMACHER in his letter of October 3, 1843.

Long after this resolution had been taken and acted upon, and the new standard constructed in accordance therewith, the troy pound O came by purchase into the hands of the Committee. The surface of O was studded with numerous small pores, showing it to be an extremely bad casting. It was only after repeatedly boiling the water in which it was suspended that the air-bubbles which attached themselves to the pores ceased to appear. It was weighed in water April 2, and then left to dry till April 28, when on being weighed in air it appeared to be about 16 grains too heavy. By heating it to above the boiling-point, the water that had been retained in the cavities was expelled, and the weight reduced to 5759·83 grains. Afterwards, by placing it in a jar containing water, under the receiver of an air-pump, and alternately exhausting the receiver and boiling the water, the cavities communicating with the surface were found capable of containing 21·37 grains of water. This explains the seeming paradox, that although the linear dimensions of O are hardly less than those of V, and sensibly greater than those of M and B, its specific gravity is considerably greater than that of V, and slightly exceeds that of either M or B.

Of the weights used in the following weighings, those marked (100), (200), (400) ..., of nearly 100, 200, 400 ... grains respectively, are of bronze, for which log $\Delta=0·92260$. The smaller weights are of platinum. By a mean of four observations, April 30, 1853,

$O \triangleq (3200)+(1600)+(800)+(100)+(32)+(16)+(8)+(4)-0·1360$ grain in air ($D=12$, $C=12·2$, $F=756·4$, $E=11·8$).

$O \triangleq 5699·9704$ grains of bronze $+59·8607$ grains of platinum in air ($t=12·1$, $b=755·4$).

By one observation, April 2, 1853, O in water (B=10·4) ≏ (3200)+(1600)+(200) (bronze)+(64)+(16)+(2)+(1)−(8)−0·075 grain (platinum) in air (D=10·9, C=11, F=754·7, E=10·8).

O in water (t=10·16) ≏ 4999·9640 grains of bronze + 74·7759 grains of platinum in air (t=10·96, b=753·82),

Hence vO=685·665, ΔO=8·40036, O=5759·8333 grains.

The weight of O is given more accurately by the following observations :—

July 15, 1853. D=17·55, C=17·6, F=747·55, E=17·6. 100 parts=0·3916 grain.

Left-hand. gr.	Right-hand.	Scale.	Left-hand.	Right-hand. gr.	Scale.
O+0·1113+X	T+Y	27·15	T+Y	O+0·1113+X	16·66
O+0·13 +X	T+Y	21·06	T+Y	O+0·13 +X	20·50
O+0·13 +X	T+Y	21·15	T+Y	O+0·13 +X	20·97
O+0·13 +Y	T+X	21·70	T+X	O+0·13 +Y	19·79
O+0·13 +Y	T+X	22·04	T+X	O+0·13 +Y	20·45
O+0·13 +Y	T+X	21·41	T+X	O+0·13 +Y	20·59

O ≏ T−0·13196 grain in air (t=17·61, b=745·87).

O displaces 0·8146 grain, T−0·132 grain displaces 0·3231 grain of air (t=17·61, b=745·87). Hence O=T+0·3595 grain.

In air (t=18·7, b=755·64), O ≏ T−0·1363 gr., and T ≏ U−0·0074 gr. Therefore O ≏ U−0·1437 gr.

By the good offices of Sir J. F. W. Herschel, at that time Master of the Mint, permission was obtained from the Treasury to weigh M in air, and to repeat the observations with the stereometer for finding its volume with more care and more leisurely than on the former occasion, when they were made in Mr. Bingley's house.

By three series of comparisons made with the stereometer,

May 11, 12, 1855 . . . vM=vO+4·33=690·00

September 16, 1843 . . $\begin{cases} v\text{M}=v\text{C}-0{\cdot}6 = 688{\cdot}96 \\ v\text{M}=v\text{O}+5{\cdot}3 = 690{\cdot}97 \end{cases}$

According to a mean of all three results, the volume of M at 0° is equal to the volume of 689·98 grains of water at its maximum density.

Comparison of M with T, July 14, 1853.

D=18·2, C=18·3, F=743·2, E=17·7. 100 parts=0·3916 grain.

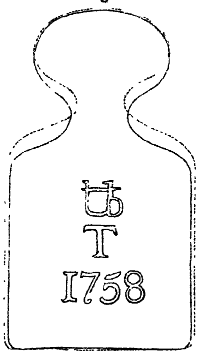
Left-hand. gr.	Right-hand.	Scale.	Left-hand.	Right-hand. gr.	Scale.
M+0·06+Y	T+X	15·30	T+X	M+0·06+Y	24·55
M+0·04+Y	T+X	20·02	T+X	M+0·04+Y	18·80
M+0·04+Y	T+X	20·31	T+X	M+0·04+Y	18·97
M+0·04+X	T+Y	19·05	T+Y	M+0·04+X	17·59
M+0·04+X	T+Y	18·12	T+Y	M+0·04+X	18·44
M+0·04+X	T+Y	18·94	T+Y	M+0·04+X	19·01

M ≏ T−0·0415 grain in air (t=18·28, b=741·52).

M displaces 0·81284 grain, T−0·0415 grain displaces 0·32045 grain of air ($t=18·28$; $b=741·52$). Hence $M=T+0·4509$ grain. $\Delta M=8·3491$.

In air ($t=18·7$, $b=755·64$), $M \bumpeq T-0·04012$ gr. But $T \bumpeq U-0·0074$ gr. Therefore $M \bumpeq U-0·0475$ gr.

B. Fig. 7.



In a letter from William Miller, Esq., of the Bank of England, dated August 22, 1855, I was apprised of the existence at the Bank of a fourth troy pound of 1758, and soon afterwards received it from him with permission to weigh it in air and also in water. This weight (B) is a bad casting, though much better than O. The upright stroke of the 5 is left incomplete. The under surface is slightly concave, the depth of the concavity being about 0·01 inch.

By one comparison, October 6, 1855, $B \bumpeq (3200)+(1600)+(800)+(100)$ (bronze) $+(32)+(16)+(8)+(4)+0·271$ grain (platinum) in air ($C=16·0$, $F=746·3$, $E=16·4$).

$B \bumpeq 5699·9704$ grains of bronze $+$ 60·2678 grains of platinum in air ($t=16·03$, $b=744·77$).

By observations made October 2, 1855, B in water ($K=17·67$) $\bumpeq (3200)+(1600)+(200)$ (bronze)$+(64)+(4)+0·2505$ grain (platinum) in air ($C=17·0$, $F=754·55$, $E=17·4$).

B in water ($t=17·66$) $\bumpeq$ 4999·9640 grains of bronze $+$68·3459 grains of platinum in air ($t=17·05$, $b=752·88$).

Hence $vB=692·815$, $\Delta B=8·3175$, $B=U+0·2487$ grain. $B \bumpeq U+0·2653$ grain in air ($t=18·7$, $b=755·64$).

The magnitude of the differences between these weights is very remarkable, considering that O, M, B are in a state of very excellent preservation, V alone exhibiting traces of rough usage.

Linear dimensions of the different weights in inches.

	U.	V.	O.	M.	B.
Axis	2·576	2·57	2·568	2·538	2·564
Diameter of knob...	1·012	0·98	0·973	1·020	1·024
Diameter of neck...	0·359	——	0·383	0·391	——
Diameter at d	——	1·43	1·416	1·391	1·387
Diameter at e	1·445	1·43	1·424	1·389	——
Diameter at f	——	——	1·434	1·385	1·377

The above comparison shows that the linear dimensions of U were very sensibly larger than those of M or B, and so far justifies the assumption that the volume of U was larger than that of M or B, probably not less than that of V; for it cannot be supposed that the weight selected as the standard was a bad and porous casting like O, the linear dimensions of which are nearly equal to those of V, though its volume is considerably less in consequence of the numerous cavities that exist in it communicating with its surface.

If U, the lost standard, be supposed to have the same density as V, the volume of U at 0° C. will be equal to the volume of 706·676 grains of water of maximum density.

$$Sp \bumpeq U-0{\cdot}00857 \text{ grain in air } (t=18{\cdot}68,\ b=754{\cdot}93).$$
$$RS \bumpeq U-0{\cdot}00205 \text{ grain in air } (t=18{\cdot}74,\ b=757{\cdot}06).$$

Sp displaces 0·32544 grain, U displaces 0·84646 grain of air. Therefore

$$Sp=U-0{\cdot}52956 \text{ grain}.$$

RS displaces 0·32629 grain, U displaces 0·84865 grain of air. Therefore

$$RS=U-0{\cdot}52441 \text{ grain}.$$

The density of V is nearly the average density of brass or bronze weights, as appears by the following list, which includes all the determinations of densities of such weights I have been able to find. The specific gravities of the Russian weights and of some others marked (K), taken from the second volume of KUPFFER's work, entitled 'Travaux de la Commission pour fixer les Mesures et les Poids de l'Empire de Russie,' are reduced from 13°·33 R. to the specific gravity as defined by BESSEL, by the addition of 0·002. Those marked (M) are from my own observations.

Russian lb. No. 8 (K)	7·872
Russian lb. No. 9 (K)	7·872
Russian lb. No. 7 (K)	7·882
Russian lb. No. 6 (K)	7·932
Prussian divided pound (K)	7·952
Russian lb. (K)	7·972
G lb. No. 21 (M)	7·974
Russian lb. No. 5 (K)	7·992
SCHUMACHER's troy pound K	7·994
59 lbs. No. 9, 12 to 71 (K), all nearly	8·0
Kilogramme, Modena Trans. t. xxv.	8·025
KLAPROTH's kilogramme, Berlin Trans. 1825.	8·055
G lb. No. 5 (M)	8·061
G lb. No. 16 (M)	8·073
Russian troy pound (K)	8·092
G lb. No. 25 (M)	8·101
G lb. No. 17 a (M)	8·117
G lb. No. 7 (M)	8·122
G lb. No. 28 (M)	8·126
Kew lb. marked 7000 (M)	8·128
Swedish pound (K)	8·132
Kilogramme, electro-gilt (M)	8·133
G lb. No. 24 (M)	8·142
Russian lb. No. 1 (K)	8·142
STEINHEIL's divided kilogramme, Munich Trans.	8·150
G lb. No. 23 (M)	8·151
Mr. VANDOME's troy pound V, 1758 (M)	8·151
G lb. No. 30 (M)	8·153
G lb. No. 26 (M)	8·153
G lb. No. 27 (M)	8·162
Russian lb. No. (2) (K)	8·162
G lb. No. 8 (M)	8·163
G lb. No. 29 (M)	8·186
Russian lb. No. 3 (K)	8·192
G lb. No. 22 (M)	8·198
Kew Standard lb., electro-gilt (M)	8·204
SCHUMACHER's troy pound Sb	8·228
Russian lb. No. 4 (K)	8·272
G lb. No. 10 (M)	8·283
G lb. No. 6 (M)	8·287
G lb. No. 18 (M)	8·303
G lb. No. 3 (M)	8·304
Mr. BARROW's lb., electro-gilt (M)	8·310
Bank troy pound B, 1758 (M)	8·317
G lb. No. 12 (M)	8·319
Kilogramme (M)	8·320
G lb. not numbered (M)	8·320
Exchequer kilogramme (M)	8·328
Russian divided lb. D (K)	8·332
G lb. No. 19 (M)	8·340
G lb. No. 2 (M)	8·341
Mint troy pound M, 1758 (M)	8·349
G lb. No. 14 (M)	8·349
Exchequer 10 lb., electro-gilt (M)	8·354
G lb. No. 15 (M)	8·361
G lb. No. 1 (M)	8·361
G lb. No. 11 (M)	8·363
G lb. No. 4 (M)	8·365
Weight of 6400 grains (M)	8·368
Russian lb. M (K)	8·373
ALCHORNE's troy pound O, 1758 (M)	8·403
Mint 10 troy ounces, electro-gilt (M)	8·414
G lb. No. 13 (M)	8·413
Exchequer 10 troy ounces, electro-gilt (M)	8·460
Electro-gilt lb. No. 35 (M)	8·470
Electro-gilt lb. No. 32 (M)	8·470
Electro-gilt lb. No. 33 (M)	8·479
An electro-gilt lb. (M)	8·481
Electro-gilt lb. No. 36 (M)	8·496
An electro-gilt lb. (M)	8·507
Russian lb. N (K)	8·508
Electro-gilt lb. No. 34 (M)	8·512
Electro-gilt lb. No. 31 (M)	8·514
G lb. No. 20 (M)	8·558
G lb. No. 17 b (M)	8·558

The Commissioners for the Restoration of the Standards of Weight and Measure, in their Report dated December 21, 1841, recommended that the avoirdupois pound of 7000 grains be adopted instead of the troy pound of 5760 grains, as the new Parliamentary Standard of weight, and that the new standard and four copies of it be constructed of platinum. In accordance with this recommendation, five platinum weights were made by Mr. BARROW, a little in excess of 7000 grains. The form of these pounds is that of a cylinder nearly 1·35 inch in height and 1·15 inch in diameter, with a groove round it, the middle of which is about 0·34 inch below the top of the cylinder, for insertion of the prongs of a forked lifter of ivory. They are marked PS 1844 1 lb.; PC No. 1 1844 1 lb.; PC No. 2 1844 1 lb.; PC No. 3 1844 1 lb.; PC No. 4 1844 1 lb., respectively.

The weight of 7000 grains might have been formed from one of 5760 grains, by the use of either a decimal or a binary system of weights. In either case, however, the number of the weights to be compared with one or the other or both of the weights of 7000 grains and 5760 grains would have been large, and the errors of the comparisons between themselves might by their accumulation sensibly affect the resulting weight of 7000 grains. Moreover, the repeated comparison of weights made up of the sum of several others, was a very troublesome process previous to the use of the method described in page 764, which had not been thought of at the time the weights were ordered. These two evils were in a great measure avoided by the use of a platinum weight T of about 5760 grains, or, more correctly, very nearly equal to Sp or RS, and of the following series of auxiliary weights, also of platinum, and all constructed by Mr. BARROW: A, B, C, D each of 1240 grains; F of 800 grains; G of 440 grains; H of 360 grains; K, L, M, N each of 80 grains; R, S each of 40 grains, nearly. The platinum of which the five lbs., T and the auxiliary weights were made, was prepared by Messrs. JOHNSON and COCK.

The numbers of the weights of each denomination, and their values, are given by the quotients and divisors obtained in the conversion of $\frac{7000}{5760}$ into a continued fraction. The errors of these weights are found by the following comparisons:—Sp and RS with T; T with A+B+C+D+F; each of the weights A, B, C, D with F+G; F with G+H; G with each of the weights H+K, H+L, H+M, H+N; H with K+L+M+N+R and K+L+M+N+S; each of the weights K, L, M, N with R+S. Sp and RS, instead of being true troy pounds, and consequently equal to U in a vacuum, had been adjusted so as to appear as heavy as U nearly, when weighed in air of ordinary density, and are therefore lighter than U by about 0·53 grain, the weight of the air contained in a space equal to the difference between the volume of U and that of Sp or RS. A space equal to the difference between the volume of 7000 grains of metal of the density of U and 7000 grains of platinum, contains about 0·645 grain of air. Calling this Q, PS may be compared with each of the weights T+A+Q, T+B+Q, T+C+Q, T+D+Q. In order to determine the error of the weight of 0·645 grain with the greatest precision, ten weights Q of 0·645 grain each, so accurately

adjusted that no sensible difference could be detected between them, a weight V of 6·451 grains, and a weight W of 12·901 grains, all of platinum, were obtained from Mr. BARROW. Then, Y and Z being platinum weights of 20 grains each belonging to the two ROBINSON'S balances, the following comparisons became possible:—each of the weights R and S with Y+Z; each of the weights Y and Z with W+V+ each of the weights Q; W with V+ sum of ten weights Q; V with the sum of the ten weights Q. In comparing PS with each of the weights T+A+Q, T+B+Q, T+C+Q, T+D+Q, the weight Q was changed at the end of every four comparisons, and thus each of the ten weights Q used in turn in a series of forty comparisons.

The following comparisons of the auxiliary weights with Sp and RS, and with each other, were made for the purpose of finding their errors preparatory to a more accurate adjustment, and in order to obtain a series of weights to be used in finding the densities of T and of the five platinum lbs. of 7000 grains each, from June 4, 1844, to the end of the year. In the comparisons of A+B+C+D+F with Sp and RS, X and Y denote the weights of the two detached pans. The counterpoise is placed in the left-hand pan. The numbers in each column are the readings of the scale in the position of equilibrium of the balance, when the weight at the head of the column is in the right-hand pan. In these, and all the other comparisons of weights in air, the results of the alternate weighings are arranged in separate columns.

June 18, 1844. 100 parts =0·2208 grain.

A+B+C+D+F+Y.	Sp+X.	A+B+C+D+F+X.	Sp+Y.
27·80	25·30	22·90	22·50
27·00	26·10	22·90	21·90
25·20	24·05	21·40	22·05
25·30	24·60	26·45	26·50
24·60	24·40	29·50	27·15
24·40	23·50	28·65	28·50
23·55	22·75	27·30	28·50
23·00	23·00	29·30	28·90
22·55	22·20	30·10	29·65
22·40	21·90	30·40	29·75
245·80	237·80	268·90	265·40

10(A+B+C+D+F+Y)≏10(Sp+X)+8·0 parts. 10(A+B+C+D+F+X)≏10(Sp+Y)+3·5 parts.
A+B+C+D+F≏Sp+0·00127 grain.

June 12, 1844. 100 parts =0·2293 grain.

A+B+C+D+F+Y.	RS+X.	A+B+C+D+F+X.	RS+Y.
21·20	20·70	18·60	19·60
19·00	21·10	18·10	20·20
19·30	21·60	18·70	20·00
19·50	20·80	18·50	20·00
19·40	21·30	19·10	19·35
18·70	20·80	18·00	19·85
117·10	126·30	111·00	119·00

6(A+B+C+D+F+Y)≏6(RS+X)−9·2 parts. 6(A+B+C+D+F+X)≏6(RS+Y)−8·0 parts.
A+B+C+D+F≏RS−0·00329 grain.

For the platinum of which T and the auxiliary weights are made $\log \Delta = 1{\cdot}32566$ by SCHUMACHER's Tables. For Sp and RS $\log \Delta = 1{\cdot}32608$, by the same tables. The space vU$-v$Sp contains 0·53 grain of air nearly. Therefore the weight of Sp or RS is nearly 5759·47 grains. The weight of A+B+C+D+F is very nearly the same. Hence v(A+B+C+D+F)=272·09, vSp=271·84, vRS=271·84, the unit of volume being the volume of a grain of water at its maximum density. Therefore

v(A+B+C+D+F) is larger than vSp or vRS by the volume of 0·25 grain of water, or 0·00030 grain of air of ordinary density.

Hence A+B+C+D+F=Sp+0·00157 gr. But Sp=U−0·52956 gr. Therefore A+B+C+D+F=U−0·52799 gr. Also A+B+C+D+F=RS−0·00299 gr. But RS=U−0·52441 gr. Therefore A+B+C+D+F=U−0·52740 gr. Giving the former value twice the weight of the latter,

A+B+C+D+F=U−0·52779=5759·47221 grains.

In the following comparisons of the auxiliary weights with each other the $10\frac{1}{2}$-inch balance by ROBINSON was used.

June 6, 1844. 100 parts =0·3593 grain.

A.	F+G.	B.	F+G.
1·80	1·75	2·70	2·75
2·50	2·35	2·65	3·05
2·50	2·85	2·55	2·90
2·90	3·25	2·60	3·05
3·20	3·15	2·80	2·70
3·00	3·05	2·50	3·25
3·25	3·15	2·70	3·05
3·15	3·05	2·50	3·25
2·90	2·85	2·80	3·20
3·00	2·90	2·95	3·30
2·75	2·55	3·00	3·35
2·85	2·85	2·75	3·30
33·80	33·75	32·50	37·15

12A=12(F+G)−0·05 part. 12B=12(F+G)+4·65 parts.

A=F+G−0·00001 grain. B=F+G+0·00138 grain.

June 7, 1844. 100 parts =0·3593 grain.

C.	F+G.	D.	F+G+0·004 gr.
3·10	3·00	3·10	3·20
2·75	3·10	3·15	3·15
3·10	2·85	3·20	3·00
2·80	3·00	3·20	3·15
2·75	2·90	2·90	3·10
3·65	3·65	2·95	2·95
3·65	3·65	3·00	3·05
3·40	2·90	3·30	3·55
2·90	2·90	3·00	3·30
2·60	3·10	2·80	2·90
2·75	2·95	3·30	3·20
2·90	3·00	3·00	3·15
36·35	37·00	36·90	37·70

12C=12(F+G)+0·65 part. 12D=12(F+G+0·004 gr.)+0·8 part.

C=F+G+0·00020 grain. D=F+G+0·00424 grain.

June 8, 1844. 100 parts = 0·2855 grain.

gr.	Scale.		Scale.
F+0·010	4·55	G+H	2·45
F+0·020	1·15	G+H	2·60
F+0·010	5·00	G+H	2·45
F+0·020	1·00	G+H	2·60
F+0·010	4·80	G+H	2·95
F+0·020	1·60	G+H	3·00
F+0·010	5·30	G+H	2·65
F+0·020	1·10	G+H	2·70
F+0·010	4·80	G+H	3·00
F+0·020	1·45	G+H	2·75
F+0·015	3·10	G+H	2·80
F+0·015	3·50	G+H	2·95
12F+0·18	37·35	12(G+H)	32·90

12F+0·18 gr. = 12(G+H) − 4·45 parts.

F = G+H − 0·01606 grain.

June 8, 1844. 100 parts = 0·2708 grain.

	Scale.	gr.	Scale.		Scale.	gr.	Scale.
G	3·20	H+K+0·005	2·97	G	3·60	H+L+0·004	3·55
G	3·10	H+K+0·004	3·15	G	3·45	H+L+0·004	3·60
G	3·20	H+K+0·004	3·30	G	3·25	H+L+0·004	4·05
G	3·15	H+K+0·004	3·25	G	3·40	H+L+0·004	3·75
G	3·45	H+K+0·004	3·25	G	3·50	H+L+0·004	4·00
G	3·15	H+K+0·004	3·00	G	3·30	H+L+0·004	4·10
G	3·20	H+K+0·004	3·15	G	3·40	H+L+0·004	4·15
G	3·35	H+K+0·004	3·30	G	3·60	H+L+0·006	3·35
G	3·20	H+K+0·004	3·45	G	3·55	H+L+0·006	3·25
G	3·30	H+K+0·004	3·00	G	3·40	H+L+0·006	3·05
G	3·10	H+K+0·004	3·30	G	3·25	H+L+0·006	2·95
G	3·20	H+K+0·004	3·60	G	3·00	H+L+0·006	2·85
12G	38·60	12(H+K)+0·049	38·72	12G	40·70	12(H+L)+0·058	42·65

12G = 12(H+K) + 0·049 gr. + 0·12 pt.

G = H+K+0·00411 grain.

12G = (H+L)12 + 0·058 gr. + 1·95 pt.

G = H+L+0·00527 grain.

June 10, 1844. 100 parts = 0·2708 grain.

	Scale.	gr.	Scale.		Scale.	gr.	Scale.
G	2·85	H+M+0·003	3·35	G	2·65	H+N+0·003	2·65
G	2·90	H+M+0·006	2·30	G	2·70	H+N+0·003	3·25
G	2·85	H+M	4·70	G	3·20	H+N+0·003	3·25
G	3·00	H+M+0·003	3·45	G	3·30	H+N+0·003	3·20
G	2·95	H+M+0·003	3·35	G	2·85	H+N+0·003	3·20
G	2·70	H+M+0·003	3·10	G	3·50	H+N+0·003	3·00
G	2·60	H+M+0·004	2·90	G	3·30	H+N+0·003	3·65
G	3·15	H+M+0·004	3·10	G	3·55	H+N+0·003	3·65
G	3·00	H+M+0·004	3·25	G	3·10	H+N+0·003	3·25
G	3·10	H+M+0·004	3·05	G	3·60	H+N+0·003	3·60
G	3·05	H+M+0·004	2·95	G	3·30	H+N+0·003	3·20
G	3·05	H+M+0·004	3·15	G	3·30	H+N+0·003	3·30
12G	35·20	12(H+M)+0·042	38·65	12G	38·35	12(H+N)+0·036	39·20

12G = 12(H+M) + 0·042 gr. + 3·45 pt.

G = H+M+0·00428 grain.

12G = 12(H+N) + 0·036 gr. + 0·85 pt.

G = H+N+0·00319 grain.

June 11, 1844. 100 parts =0·2535 grain.

K..R=K+L+M+N+R.

	Scale.	gr.	Scale.
H	2·90	K..R	3·30
H	2·70	K..R	3·30
H	2·70	K..R	3·45
H	2·90	R..R	3·30
H	2·60	K..R	3·30
H	2·75	K..R	3·25
H	2·65	K..R	3·10
H	2·50	K..R	3·10
H	2·60	K..R +0·003	2·20
H	2·70	K..R +0·003	2·20
H	2·70	K..R +0·003	2·35
H	2·70	K..R +0·003	2·30
12H	32·40	12(K..R)+0·012	35·15

12H=12(K..R)+0·012 gr.+2·75 pt.
H=K+L+M+N+R+0·00158 grain.

K..S=K+L+M+N+S.

	Scale.	gr.	Scale.
H	2·70	K..S +0·003	3·00
H	2·90	K..S +0·005	2·00
H	2·85	K..S +0·004	2·40
H	2·80	K..S	4·05
H	2·75	K..S	3·90
H	2·35	K..S +0·006	1·50
H	2·30	K..S +0·003	2·50
H	2·70	K..S +0·003	2·60
H	2·80	K..S +0·003	3·00
H	3·05	K..S +0·003	2·70
H	3·00	K..S +0·003	2·80
H	2·95	K..S +0·003	2·60
12H	33·15	12(K..S)+0·036	33·05

12H=12K..S+0·036 gr.−0·10 pt.
H=K+L+M+N+S+0·00298 grain.

June 19, 1844. 100 parts =0·1198 grain.

gr.	Scale.		Scale.
K	3·20	R+S	2·15
K	3·80	R+S	2·80
K+0·002	2·45	R+S	2·90
K+0·002	2·15	R+S	2·60
K	3·90	R+S	2·90
K+0·002	2·00	R+S	2·60
K	3·85	R+S	3·10
K+0·002	2·45	R+S	2·85
K	3·80	R+S	3·00
K+0·002	2·35	R+S	2·70
K	4·20	R+S	3·05
K+0·002	2·45	R+S	2·80
12K+0·012	36·60	12(R+S)	33·45

12K+0·012 gr.=12(R+S)−3·15 pt.
K=R+S−0·00131 grain.

gr.	Scale.		Scale.
L	5·70	R+S	3·10
L+0·003	3·00	R+S	3·15
L+	5·20	R+S	2·65
L+0·002	2·85	R+S	2·20
L+0·002	3·20	R+S	2·80
L	4·70	R+S	2·70
L+0·004	1·40	R+S	2·40
L	4·60	R+S	2·30
L+0·002	3·00	R+S	2·60
L+0·004	1·45	R+S	2·20
L+0·004	2·50	R+S	3·55
L+0·004	2·50	R+S	3·30
12L+0·025	40·10	12(R+S)	32·95

12L+0·025 gr.=12(R+S)−7·15 pt.
L=R+S−0·00280 grain.

June 19, 1844. 100 parts=0·1198 grain.

gr.	Scale.		Scale.
M	4·55	R+S	3·65
M+0·002	2·75	R+S	3·60
M	4·65	R+S	4·00
M+0·002	2·60	R+S	2·90
M	4·00	R+S	3·20
M+0·002	2·40	R+S	2·90
M	4·10	R+S	3·10
M+0·003	1·60	R+S	2·95
M	4·00	R+S	2·90
M+0·003	1·40	R+S	2·85
M	4·00	R+S	2·90
M+0·003	1·60	R+S	2·70
12M+0·015	37·65	12(R+S)	37·65

12M+0·015 gr.=12(R+S)
M=R+S−0·00125 grain.

	Scale.		Scale.
N	3·15	R+S	3·00
N	3·30	R+S	3·00
N	3·50	R+S	3·10
N	3·10	R+S	3·10
N	3·55	R+S	3·20
N	3·20	R+S	2·80
N	3·40	R+S	3·30
N	2·80	R+S	3·20
N	3·30	R+S	2·80
N	2·95	R+S	2·95
N	2·90	R+S	2·85
N	3·10	R+S	2·80
12N	38·25	12(R+S)	36·10

12N=12(R+S)+2·15 pt.
N=R+S+0·00022 grain.

				gr.
T	=			5759·48815
A+B+C+D+F	=			5759·47221
A	=	F+G	−	0·00001
B	=	F+G	+	0·00138
C	=	F+G	+	0·00020
D	=	F+G	+	0·00424
F	=	G+H	−	0·01606
G	=	H+K	+	0·00411
G	=	H+L	+	0·00527
G	=	H+M	+	0·00428
G	=	H+N	+	0·00319
H	=	K+L+M+N+R	+	0·00158
H	=	K+L+M+N+S	+	0·00298
K	=	R+S	−	0·00131
L	=	R+S	−	0·00280
M	=	R+S	−	0·00125
N	=	R+S	+	0·00022

A+B+C+D	=	4F+4G	+	0·00581
4F	=	4G+4H	−	0·06424
4G	=	4H+K+L+M+N	+	0·01685
2H	=	2(K+L+M+N)+R+S	+	0·00456
K+L+M+N	=	4(R+S)	−	0·00514
4(K+L+M+N)	=	16(R+S)	−	0·02056
4H	=	4(K+L+M+N)+2(R+S)	+	0·00912
4G	=	4H+K+L+M+N	+	0·01685
4F	=	4G+4H	−	0·06424
5760 grains	=	A+B+C+D+F	+	0·52779
4H	=	18(R+S)	−	0·01144
4G	=	22(R+S)	+	0·00027
4F	=	40(R+S)	−	0·07541
4F+4G	=	62(R+S)	−	0·07514
A+B+C+D	=	62(R+S)	−	0·06933
A+B+C+D+F	=	72(R+S)	−	0·08818

72(R+S)	=	5759·56039
18(R+S)	=	1439·89010
40(R+S)	=	3199·75577
10(R+S)	=	799·93894
62(R+S)	=	4959·62143
22(R+S)	=	1759·86567
A+B+C+D	=	4959·55210
4F+4G	=	4959·54629
F+G	=	1239·88657

Values of the auxiliary weights from June 4, 1844, to the end of the year.

		gr.			gr.
A	=	1239·88656	K	=	79·99258
B	=	1239·88795	L	=	79·99109
C	=	1239·88677	M	=	79·99264
D	=	1239·89081	N	=	79·99411
F	=	799·92009	R	=	39·99624
G	=	439·96648	S	=	39·99764
H	=	359·96966			

In January 1845 the auxiliary weights were adjusted so as to reduce still further the differences between the sums of the weights compared with each other.

Observations for finding the density of T.

T weighed in air.

June 22, 1844. 100 parts $=0{\cdot}19917$ grain.

Sp+0·02 gr.+Y.	T+X.	Sp.+0·02 gr.+X.	T+Y.
14·20	13·35	20·20	19·55
15·85	14·70	21·40	21·40
16·25	14·35	22·00	22·10
16·10	14·40	23·00	22·30
15·50	13·80	23·50	22·40
15·30	13·00	23·10	20·80
14·10	12·90	21·85	21·60
14·55	12·65	22·35	21·00
13·90	12·00	21·95	21·30
14·10	12·60	22·10	21·90
13·70	11·90	22·70	21·95
163·55	145·65	244·15	236·30

11(Sp+0·02 gr.+Y)≏11(T+X)+17·9 parts. 11(Sp+0·02 gr.+X)≏11(T+Y)+7·85 parts.

T≏Sp+0·01767 grain.

May 1, 1844. 100 parts $=0{\cdot}1456$ grain.

T.	RS+0·01 gr.
22·10	22·20
23·10	24·85
26·00	24·55
29·45	26·70
29·20	28·10
28·90	27·70
30·10	26·90
28·60	27·80
29·70	27·65
30·20	29·07
277·35	265·52

10T≏10(RS+0·01 gr.)+11·83 parts.

T≏RS+0·01172 grain.

For T, by SCHUMACHER's Tables, $\log\Delta=1{\cdot}32566$. For Sp and RS, $\log\Delta=1{\cdot}32608$ by the same Tables. The space vU$-v$Sp contains 0·53 grain of air nearly. Therefore the weight of Sp is nearly 5759·47 grains, and that of T nearly 5759·49 grains. Hence vT$=271{\cdot}63$, vSp$=271{\cdot}37$, vRS$=271{\cdot}37$. Therefore vT is larger than vSp or vRS, by the volume of 0·26 grain of water, or 0·00031 grain of air of ordinary density.

Hence	T =Sp +0·01798. (gr.)
But	Sp =U −0·52956.
Therefore	T =U −0·51158.
Also	T =RS+0·01203.
But	RS=U −0·52441.
Therefore	T =U −0·51238.

Giving the former value twice the weight of the latter,

T=U−0·51185 grain=5759·48815 grains.

In the observations for finding the apparent weight of T in water, the same pro-

cess was followed as in finding the density of V. The thermometers H, P were suspended in the water with their bulbs in a horizontal plane through the middle of T. t denotes the temperature of the air in centesimal degrees; b the height of the mercury in the barometer in millimètres, reduced to 0° C.; 100 parts =0·220 grain.

July 13, 1854. T and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
18·30	19·40	0·00	20·45
18·20	19·30	0·00	20·20
18·20	19·10	0·00	20·50
18·10	19·05	0·00	18·80
18·10	19·05	0·00	22·50
18·10	19·00	0·00	20·15
18·10	19·00	0·00	19·60
18·10	19·00	0·00	19·70
18·05	19·00	0·00	19·30
18·05	19·00	0·00	20·95
18·13	19·09	0·00	20·21

Hook in water. In right-hand pan.

	gr.	Scale.
	A+B+C+D+G+K+(8)+0·44	20·25
	+0·44	20·25
Air.	+0·48	48·40
t=18·2, b=760·21.	+0·40	12·20
	+0·40	10·05
	+0·40	3·00
	+0·4267	19·1

T in water (t=18·27)≏A+B+C+D+G+K+(8)+0·4291 grain in air (t=18·2, b=760·21).

July 15, 1844. T and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
17·90	18·65	0·04	21·10
18·00	18·90	0·04	24·90
18·02	18·95	0·04	25·15
18·00	19·00	0·03	23·55
18·05	19·00	0·02	22·25
18·00	19·00	0·02	22·00
18·15	19·10	0·01	19·80
18·20	19·10	0·01	19·90
18·20	19·10	0·01	19·95
18·06	18·98	0·244	22·07

Hook in water. In right-hand pan.

	gr.	Scale.
	A+B+C+D+G+K+(8)+0·42	15·30
	+0·43	16·65
	+0·44	18·70
	+0·44	18·85
Air.	+0·46	21·90
t=18·4, b=757·77.	+0·44	17·07
	+0·46	28·60
	+0·46	26·70
	+0·46	26·70
	+0·46	29·95
	+0·447	22·04

T in water (t=18·18)≏A+B+C+D+G+K+(8)+0·4227 grain in air (t=18·4, b=757·77).

July 29, 1844. T and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
19·00	19·95	0·86	19·5
		0·84	17·0
		0·88	23·0
19·00	19·95	0·84	16·5
		0·88	22·4
19·00	19·90	0·88	23·3
		0·84	15·5
		0·88	25·0
		0·84	16·2
		0·88	23·0
19·00	19·90	0·84	17·8
		0·88	24·1
19·00	19·92	0·8617	20·27

Hook in water. In right-hand pan.

	gr.	Scale.
A+B+C+D+G+K+(8)+	1·32	19·6
	1·32	20·2
Air.	1·32	21·5
t=20·4, b=761·17.	1·32	21·4
	1·32	21·0
	1·32	21·0
	1·32	20·78

T in water (t=19·11)≏A+B+C+D+G+K+(8)+0·4572 grain in air (t=20·4, b=761·17).

August 1, 1844. T and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
		0·86	20·8
		0·85	18·5
		0·86	19·5
		0·86	21·4
		0·85	20·4
18·00	18·80	0·85	17·3
		0·86	21·0
		0·85	15·1
		0·86	21·7
		0·86	22·6
		0·86	22·8
17·95	18·75	0·85	20·8
17·98	18·78	0·8567	20·16

Hook in water. In right-hand pan.

	gr.	Scale.
A+B+C+D+G+K+(8)+	1·28	19·5
	1·28	19·8
Air.	1·28	20·0
t=18·4, b=754·70.	1·28	20·5
	1·28	20·6
	1·28	19·6
	1·28	19·5
	1·28	19·93

T in water (t=18·04)≏A+B+C+D+G+K+8+0·4238 grain in air (t=18·4, b=754·70).

Apparent weight of T in water, weighed with platinum weights.

Water. t.	Apparent weight of T. gr.	Air. t.	b.
18·27	5487·9398	18·2	760·21
18·18	5487·9334	18·4	757·77
19·11	5487·9679	20·4	761·17
18·04	5487·9345	18·4	754·70

The resulting values of ΔT and log ΔT are:—

	ΔT.	log ΔT.
	21·1661	1·325642
	21·1661	1·325641
	21·1656	1·325631
	21·1667	1·325654
Mean ……	21·1661	1·325642
10−log ΔT		8·674358
T (reduced)	5759·471	3·760383
vT ………..	272·108	2·434741

Hence the volume of T at 0° is equal to the volume of 272·108 grains of water at its maximum density.

The resulting values of log ΔT by SCHUMACHER's Tables are:—

		log ΔT.
		1·325653
		1·325651
		1·325642
		1·325664
Mean log ΔT		1·325664
10 − log ΔT		8·674336
T (reduced)	5759·471	3·760383
vT	272·094	2·434719

Hence, using SCHUMACHER's Tables, the volume of T at 0° is equal to the volume of 272·094 grains of water at its maximum density.

Comparison of T *with* Sp *and* RS.

In December 1844 T was reduced till its weight was very nearly equal to that of Sp, and afterwards compared with Sp and RS. The weights of the two scale-pans are denoted by X and Y. The thermometers C, D were suspended in the balance case with their bulbs in a horizontal plane passing through the middle of the weights. F is the reading of ERNST's barometer, E that of the attached thermometer. The comparisons were made in a cellar under the north side of the Mineralogical Museum in Cambridge. The balance was placed upon a thick stone slab forming the sill of a recess containing a window so situated, that the lowest part of the window was considerably higher than the top of the balance case.

January 15, 1845. 100 parts =0·16949 grain.

T+X.	Sp+Y.		T+Y.	Sp+X.
17·14	15·95	D= 9·93	18·50	17·70
18·27	16·50	C= 9·90	19·15	18·31
18·82	17·97	F=754·8	19·00	18·50
19·44	17·56	E= 7·7	18·53	18·66
20·17	18·01		18·45	18·31
93·84	85·99		93·63	91·48

5(T+X)≏5(Sp+Y)+7·85 parts. 5(T+Y)≏5(Sp+X)+2·15 parts.

10T≏10Sp+0·01695 grain in air (t=9·91, b=754·30).

February 21, 1845. 100 parts $=0{\cdot}17396$ grain.

Sp+X, T+Y.	T+Y, Sp+X.	Sp+Y, T+X.	T+X, Sp+Y.
24·76	25·00	23·77	21·20
25·05	25·21	22·87	20·22
25·32	25·26	22·61	18·76
24·28	22·67	18·37	18·15
24·60	23·76	20·40	17·82
21·35	20·68	17·37	17·40
19·43	22·91	19·55	18·06
21·52	20·17	21·90	19·68
20·72	22·40	20·70	19·53
20·35	19·80	21·38	20·95
227·38	227·86	208·92	191·77

D= 6·95
C= 7·0
F=759·5
E= 9·3

$20(\mathrm{T}+\mathrm{Y})\bumpeq 20(\mathrm{Sp}+\mathrm{X})-0{\cdot}48$ part. $20(\mathrm{T}+\mathrm{X})\bumpeq 20(\mathrm{Sp}+\mathrm{Y})+17{\cdot}15$ parts.

$40\mathrm{T}\bumpeq 40\mathrm{Sp}+0{\cdot}02900$ grain in air ($t=6{\cdot}94$, $b=758{\cdot}80$).

February 26, 1845. 100 parts $=0{\cdot}17156$ grain.

T+X.	Sp+Y.	T+Y.	Sp+X.
21·48	20·46	22·62	22·62
21·97	21·83	22·83	23·45
22·51	23·20	23·18	23·52
23·00	21·98	22·50	22·15
23·22	22·62	22·71	22·95
23·30	23·63	22·87	22·67
24·21	24·62	22·45	22·46
24·92	24·48	22·92	22·68
184·61	182·82	182·08	182·50

D= 6·30
C= 6·35
b=752·41

$8(\mathrm{T}+\mathrm{X})\bumpeq 8(\mathrm{Sp}+\mathrm{Y})+1{\cdot}79$ part. $8(\mathrm{T}+\mathrm{Y})\bumpeq 8(\mathrm{Sp}+\mathrm{X})-0{\cdot}42$ part.

$16\mathrm{T}\bumpeq 16\mathrm{Sp}+0{\cdot}00235$ grain in air ($t=6{\cdot}28$, $b=752{\cdot}41$).

July 26, 1845. 100 parts $=0{\cdot}33860$ grain.

T+Y.	Sp+X.	T+X.	Sp+Y.
19·88	18·16	18·25	18·41
19·32	18·97	18·32	18·56
19·27	19·06	18·31	17·90
19·70	18·55	18·26	18·48
19·51	19·16	18·30	17·75
19·63	18·88	17·70	18·10
19·36	20·03	17·75	17·96
19·00	19·31	18·38	18·26
19·56	18·75	17·91	18·15
19·12	18·72	17·17	18·24
18·90	18·67	17·08	17·43
19·00	18·24	17·33	17·16
232·25	226·50	214·76	216·40

D= 17·5
C= 17·5
F=761·5
E= 18

$12(\mathrm{T}+\mathrm{Y})\bumpeq 12(\mathrm{Sp}+\mathrm{X})+5{\cdot}75$ parts. $12(\mathrm{T}+\mathrm{X})\bumpeq 12(\mathrm{Sp}+\mathrm{Y})-1{\cdot}64$ part.

$24\mathrm{T}\bumpeq 24\mathrm{Sp}+0{\cdot}01392$ grain in air ($t=17{\cdot}53$, $b=759{\cdot}79$).

August 16, 1845. 100 parts = 0·27581 grain.

Sp+Y, T+X.	T+X, Sp+Y.		Sp+X, T+Y.	T+Y, Sp+X.
19·22	20·30		17·87	19·50
19·16	20·15		20·02	18·36
20·67	20·52		20·04	17·82
21·27	22·02		20·35	17·14
19·11	20·51		20·30	17·51
21·20	20·54	D= 15·9	20·29	17·32
21·32	21·60	C= 15·9	19·34	17·77
20·27	19·69	F=760·2	19·65	17·61
18·31	20·31	E= 15·9	19·77	17·29
20·07	20·84		20·77	19·98
21·49	20·75		18·77	16·79
19·69	19·84		17·79	18·54
19·82	19·82		19·80	17·14
17·45	19·36		19·77	19·47
279·05	286·25		274·53	252·24

28(T+X)≏28(Sp+Y)−7·20 parts. 28(T+Y)≏28(Sp+X)+22·29 parts.

56T≏56Sp+0·04162 grain in air (t=15·91, b=758·70).

August 18, 1845. 100 parts = 0·27959 grain.

Sp+X, T+Y.	T+Y, Sp+X.		Sp+Y, T+X.	T+X, Sp+Y.
18·85	18·34		19·94	19·42
19·90	20·49		18·16	19·39
20·05	18·17		19·17	21·21
20·27	20·67	D= 15·92	19·86	18·82
20·32	18·66	C= 15·9	18·99	19·36
20·35	20·12	F=756·5	18·50	20·49
20·61	19·95	E= 16	20·02	18·56
21·02	19·24		18·76	18·61
21·39	20·99		19·01	18·37
20·96	20·00		17·81	17·92
203·72	196·63		190·22	192·15

20(T+Y)≏20(Sp+X)+7·09 parts. 20(T+X)≏20(Sp+Y)−1·93 part.

40T≏40Sp+0·01443 grain in air (t=15·93, b=754·99).

August 19, 1845. 100 parts = 0·27204 grain.

Sp+Y, T+X.	T+X, Sp+Y.		Sp+X, T+Y.	T+Y, Sp+X.
19·56	21·19		18·19	18·54
18·91	20·79	D= 15·55	20·95	19·95
20·93	20·25	C= 15·6	20·83	18·15
18·78	21·25	F=740·7	20·68	17·86
19·12	21·45	E= 15·5	20·60	18·50
97·30	104·93		101·25	93·00

10(T+X)≏10(Sp+Y)−7·63 parts. 10(T+Y)≏10(Sp+X)+8·25 parts.

20T≏20Sp+0·00169 grain in air (t=15·6, b=739·29).

August 27, 1845. 100 parts = 0·26861 grain.

Sp+Y, T+X.	T+X, Sp+Y.		Sp+X, T+Y.	T+Y, Sp+X.
24·20	22·27		22·24	23·45
24·34	21·01		22·10	22·47
24·66	22·40		22·06	23·00
24·22	22·32	D= 14·93	21·96	22·37
24·15	22·32	C= 14·93	21·67	22·61
23·86	22·17	F=766·1	22·27	22·30
25·50	21·07	E= 15	22·57	22·82
23·40	21·67		22·02	22·40
23·21	21·32		21·69	23·12
23·51	21·16		22·17	21·94
241·05	217·71		220·75	226·48

20(T+X)≏20(Sp+Y)+23·34 parts. 20(T+Y)≏20(Sp+X)−5·73 parts.

40T≏40Sp+0·04730 grain in air (t=14·95, b=764·69).

August 29, 1845. 100 parts $=0{\cdot}26700$ grain.

Sp+X, T+Y.	T+Y, Sp+X.		Sp+Y, T+X.	T+X, Sp+Y.
24·67	25·17		24·85	24·67
25·64	24·77		26·95	24·15
25·24	25·62		24·85	24·82
25·15	24·76	D= 14·55	26·62	23·97
25·19	24·80	C= 14·57	25·27	23·76
25·34	26·70	F=771·95	25·31	23·70
25·74	24·74	E= 15	23·99	23·51
25·25	26·29		25·86	22·35
25·35	25·70		24·27	23·52
25·10	24·54		25·44	23·35
252·67	253·09		253·41	237·80

$20(T+Y)\bumpeq 20(Sp+X)-0{\cdot}42$ part. $20(T+X)\bumpeq 20(Sp+Y)+15{\cdot}61$ parts.

$40T\bumpeq 40Sp+0{\cdot}04056$ grain in air ($t=14{\cdot}58$, $b=770{\cdot}53$).

January 16, 1845. 100 parts $=0{\cdot}16949$ grain.

T+Y.	RS+X.		T+X.	RS+Y.
16·47	17·70		18·05	20·75
17·90	20·06	D= 9·03	18·46	20·62
17·31	21·00	C= 9·10	18·90	20·51
18·54	21·47	F=764·0	18·55	21·95
18·35	22·67	E= 8	19·41	21·99
88·57	102·90		93·37	105·82

$5(T+Y)\bumpeq 5(RS+X)-14{\cdot}33$ parts. $5(T+X)\bumpeq 5(RS+Y)-12{\cdot}45$ parts.

$10T\bumpeq 10RS-0{\cdot}04539$ grain in air ($t=9{\cdot}05$, $b=763{\cdot}46$).

July 28, 1845. 100 parts $=0{\cdot}33720$ grain.

T+X.	RS+Y.		T+Y.	RS+X.
19·37	21·07		18·66	19·03
19·58	21·90		18·95	19·59
20·25	22·20		18·46	19·78
19·95	22·22		18·84	19·70
20·10	21·50	D= 16·95	18·67	19·65
19·66	22·05	C= 17·0	18·47	19·78
18·77	21·27	F=755·60	18·97	19·46
18·85	20·68	E= 17·2	18·20	18·70
19·08	19·51		18·90	19·20
18·71	20·52		18·55	19·82
18·92	20·52		18·33	19·60
19·26	20·40		18·72	19·60
232·50	253·84		223·72	233·91

$12(T+X)\bumpeq 12(RS+Y)-21{\cdot}34$ parts. $12(T+Y)\bumpeq 12(RS+X)-10{\cdot}19$ parts.

$24T\bumpeq 24RS-0{\cdot}10632$ grain in air ($t=17{\cdot}01$, $b=753{\cdot}95$).

August 14, 1845. 100 parts $=0{\cdot}29755$ grain.

RS+X, T+Y.	T+Y, RS+X.		RS+Y, T+X.	T+X, RS+Y.
20·51	22·76		19·25	23·77
20·50	23·11		19·90	23·99
19·95	21·97		18·75	23·57
20·70	21·46		18·56	22·80
20·57	22·55	D= 15·95	18·50	22·27
19·70	21·47	C= 15·95	20·84	24·35
21·67	23·47	F=760·85	18·77	22·21
20·54	23·25	E= 15·9	18·17	23·31
20·95	23·74		17·80	22·99
21·10	22·84		18·37	22·71
21·34	23·75		17·91	23·35
227·53	250·37		206·82	255·32

$22(T+Y)\bumpeq 22(RS+X)-22{\cdot}84$ parts. $22(T+X)\bumpeq 22(RS+Y)-48{\cdot}5$ parts.

$44T\bumpeq 44RS-0{\cdot}21227$ grain in air ($t=15{\cdot}96$, $b=759{\cdot}34$).

August 15, 1845. 100 parts $=0{\cdot}27921$ grain.

RS+X, T+Y.	T+Y, RS+X.		RS+Y, T+X.	T+X, RS+Y.
18·05	20·30		15·70	18·89
18·64	21·87		14·89	20·05
18·67	21·75		16·06	19·79
16·41	19·47		16·39	20·05
16·96	20·75	D= 16·25	16·66	19·85
16·59	20·59	C= 16·30	17·00	19·72
16·46	20·04	F=756·9	18·81	21·60
17·17	20·10	E= 16	16·48	21·62
17·79	21·00		17·60	20·47
17·75	19·79		17·94	20·85
18·21	20·53		15·22	17·54
192·70	226·19		182·75	220·43

$22(T+Y) \triangleq 22(RS+X)-33{\cdot}49$ parts. $22(T+X) \triangleq 22(RS+Y)-37{\cdot}68$ parts.

$44T \triangleq 44RS-0{\cdot}19871$ grain in air ($t=16{\cdot}29$, $b=755{\cdot}39$).

1845.	t.	b.	gr.
January 15.	9·91	754·30	10T≏ 10Sp +0·01695
February 21.	6·94	758·80	40T≏ 40Sp +0·02900
February 26.	6·28	752·41	16T≏ 16Sp +0·00235
July 26.	17·53	759·79	24T≏ 24Sp +0·01392
August 16.	15·91	758·70	56T≏ 56Sp +0·04162
August 18.	15·93	754·99	40T≏ 40Sp +0·01443
August 19.	15·6	739·29	20T≏ 20Sp +0·00169
August 27.	14·95	764·69	40T≏ 40Sp +0·04730
August 29.	14·58	770·53	40T≏ 40Sp +0·04056
			286T≏286Sp +0·20782
	In air ($t=13{\cdot}74$, $b=758{\cdot}91$)		T≏ Sp +0·00073

	t.	b.	gr.
January 16.	9·05	763·46	10T≏ 10RS−0·04539
July 28.	17·01	753·95	24T≏ 24RS−0·10632
August 14.	15·96	759·34	44T≏ 44RS−0·21227
August 15.	16·29	755·39	44T≏ 44RS−0·19871
			122T≏122RS−0·56269
	In air ($t=15{\cdot}72$, $b=757{\cdot}19$)		T≏ RS−0·00461

Since ΔSp was computed by means of SCHUMACHER's Tables, we must employ the value of ΔT obtained by means of the same Tables in reducing the comparisons of T with Sp and RS. By these Tables $\log \Delta=1{\cdot}32566$ for T, and $\log \Delta=1{\cdot}32608$ for Sp and RS.

$$T \triangleq Sp+0{\cdot}00073 \text{ grain in air } (t=13{\cdot}74,\ b=758{\cdot}91).$$

T displaces 0·33356 grain of air, Sp displaces 0·33324 grain of air. Therefore

$$T=Sp+0{\cdot}00105 \text{ grain.}$$

$$T \triangleq RS-0{\cdot}00461 \text{ grain in air } (t=15{\cdot}72,\ b=757{\cdot}19).$$

T displaces 0·33037 grain of air, RS displaces 0·33005 grain of air. Therefore

$$T=RS-0{\cdot}00429 \text{ grain.}$$

If we suppose U to have had the same density as V, $Sp=U-0{\cdot}52956$ grain, and $RS=U-0{\cdot}52441$ grain. The former gives $T=U-0{\cdot}52851$ grain, the latter $T=U-0{\cdot}52870$. Mean, giving to the former twice the weight of the latter, because

the number of comparisons of Sp with U, and of T with Sp is about twice as large as the number of comparisons of RS with U, and T with RS,

$$T=U-0{\cdot}52857 \text{ grain} =5759{\cdot}47143 \text{ grains}.$$

During the comparisons of Sp and RS with U in Somerset House, the mean value of t was 18·7, and that of b was 755·64.

T displaced 0·32605 grain of air, U displaced 0·84717 grain of air.

Hence $T \triangleq U-0{\cdot}00745$ grain in air ($t=18{\cdot}7$, $b=755{\cdot}64$) in Somerset House.

Comparisons of the auxiliary weights in January and February 1845.

The comparisons of the auxiliary weights among each other, after their second adjustment, were made with my 10½-inch ROBINSON. The comparisons of A+B+C+D+F with T were made with BARROW's balance.

February 22 and 24, 1845. 100 parts =0·17274 grain.

S=A+B+C+D+F.

T+X.	S+Y.	T+Y.	S+X.
22·00	20·06	19·16	20·65
21·05	29·37	21·52	21·50
22·16	20·50	23·17	22·73
21·46	20·52	23·86	21·00
22·73	21·21	20·88	20·47
22·71	21·31	21·60	20·67
23·21	21·32	20·93	21·10
23·53	21·68	21·20	21·67
23·68	21·50	22·50	21·40
24·05	22·00	21·61	21·77
23·27	21·23	21·65	21·26
249·85	231·70	238·08	234·22

11(T+X)=11(S+Y)+18·15 parts. 11(T+Y)=11(S+X)+3·86 parts.

T=A+B+C+D+F+0·00173 grain.

Jan. 29, Feb. 5 and 6, 1845. 100 parts =0·3063 grain.

A, F+G.	F+G, A.	B, F+G.	F+G, B.
1·80	2·29	3·35	3·15
1·95	2·28	3·46	3·01
2·45	2·57	3·20	2·65
2·75	3·75	3·35	2·85
3·00	3·59	3·90	3·05
3·39	3·35	4·14	2·85
2·97	2·50	3·66	1·69
2·05	2·30	2·73	1·80
2·65	2·50	3·41	1·76
1·94	2·51	2·50	2·13
2·21	1·93	2·60	1·28
2·29	2·86	3·00	2·09
2·20	2·69	2·88	1·95
2·55	2·73	3·00	1·75
2·71	3·44	3·18	2·80
2·71	3·56	3·08	2·15
2·66	2·86	3·16	2·65
42·28	47·71	54·60	39·61

A=F+G−0·00049 grain. B=F+G+0·00135 grain.

C, F+G.	F+G, C.	D, F+G.	F+G, D.
2·65	1·70	2·45	1·90
3·90	2·95	3·36	2·76
3·52	2·38	3·17	3·04
3·45	2·20	3·55	2·41
4·01	2·91	4·15	2·90
4·40	2·85	3·99	2·50
3·75	1·65	3·23	1·88
3·00	1·96	2·58	2·40
3·54	1·69	3·08	1·79
2·73	1·60	2·38	1·95
2·80	1·23	2·40	1·66
3·50	1·56	2·80	2·00
3·08	1·69	2·59	2·25
3·05	2·33	2·80	2·75
3·35	2·51	3·03	2·70
3·41	2·21	3·00	2·88
3·18	2·44	2·90	2·40
57·32	35·86	51·46	40·17

C=F+G+0·00193 grain. D=F+G+0·00102 grain.

Jan. 27 and 29, 1845. 100 parts =0·27082 grain.

F, G+H.	G+H, F.
2·64	2·85
2·69	2·97
2·30	2·86
2·55	2·82
2·36	2·91
2·51	3·07
2·27	2·77
2·16	2·59
2·72	2·75
2·52	3·46
2·41	3·39
2·59	2·87
29·72	35·31

F=G+H−0·00063 grain.

Jan. 25, 27. 100 parts =0·25016 grain.

G, H+K.	H+K, G.	G, H+L.	H+L, G.
4·65	3·00	5·10	2·60
4·67	3·34	5·22	2·67
3·30	1·94	3·72	1·34
3·85	2·76	4·40	2·17
3·87	2·85	4·52	2·32
3·99	3·01	4·66	2·44
3·47	2·15	3·85	1·82
3·51	2·45	3·89	1·77
3·35	2·05	3·89	1·66
3·49	2·45	3·87	1·86
38·15	26·00	43·12	20·65

G=H+K+0·00152 grain. G=H+L+0·00281 grain.

G, H+M.	H+M, G.	G, H+N.	H+N, G.
4·70	3·20	4·35	3·65
4·45	3·22	4·29	3·80
2·95	2·02	2·57	2·27
3·67	2·70	3·32	3·00
3·77	2·80	3·40	3·06
3·77	3·00	3·47	3·15
3·40	2·30	3·00	2·56
3·45	2·35	3·10	2·55
3·32	2·12	3·07	2·47
3·54	2·45	3·15	2·60
37·02	26·16	33·72	29·11

G=H+M+0·00136 grain. G=H+N+0·00058 grain.

Jan. 25, Feb. 7. 100 parts =0·251256 grain.

H, K+L+M+N+R.	K+L+M+N+R, H.	H, K+L+M+N+S.	K+L+M+N+S, H.
3·55	3·85	3·70	3·95
3·40	3·80	3·55	3·70
3·80	2·70	3·65	3·80
3·50	3·80	3·50	3·70
3·60	3·70	3·65	3·80
3·80	3·70	3·60	3·70
3·76	3·31	3·77	3·30
4·32	3·61	4·00	3·37
3·95	3·52	3·60	3·32
3·75	3·29	3·65	3·16
4·02	3·42	4·02	3·31
4·10	3·62	4·01	3·71
3·90	3·42	3·86	3·15
3·60	3·10	3·52	3·25
3·74	3·50	3·74	3·51
3·79	3·12	3·72	3·05
2·71	2·27	2·62	2·26
63·29	57·73	62·16	58·02

H=K+L+M+N+R+0·00034 grain. H=K+L+M+N+S+0·00030 grain.

January 25, and February 7. 100 parts =0·26560 grain.

K, R+S.	R+S, K.	L, R+S.	R+S, L.
3·45	3·80	2·80	4·00
3·25	3·30	2·70	3·70
3·20	3·30	2·75	3·80
3·10	2·90	2·70	3·30
3·00	3·30	2·75	3·70
3·20	3·15	2·70	3·70
2·00	2·05	1·45	2·62
2·75	2·95	2·30	3·37
3·22	3·20	2·55	3·81
2·90	3·00	2·32	3·60
2·99	3·17	2·37	3·65
2·99	3·22	2·40	3·77
36·05	37·34	29·79	43·02

K=R+S−0·00014 grain. L=R+S−0·00146 grain.

M, R+S.	R+S, M.	N, R+S.	R+S, N.
3·50	3·25	3·70	2·95
3·30	3·20	3·50	2·95
3·20	3·20	3·50	2·80
3·10	2·95	2·90	2·70
3·20	3·40	3·50	2·90
3·30	3·15	3·50	2·90
2·07	2·05	2·31	1·75
2·96	2·69	3·17	2·64
3·20	3·35	3·47	2·90
2·92	3·05	3·26	2·64
3·07	3·10	3·41	2·66
2·96	3·12	3·36	2·77
36·78	36·51	39·58	32·56

M=R+S+0·00003 grain. N=R+S+0·00078 grain.

T =A+B+C+D+F	+0·00173
A =F+G	−0·00049
B =F+G	+0·00135
C =F+G	+0·00193
D=F+G	+0·00102
F =G+H	−0·00063
G =H+K	+0·00152
G =H+L	+0·00281
G =H+M	+0·00136
G =H+N	+0·00058
H=K+L+M+N+R	+0·00034
H=K+L+M+N+S	+0·00030
K=R+S	−0·00014
L =R+S	−0·00146
M=R+S	+0·00003
N=R+S	+0·00078

	gr.
A+B+C+D=4F+4G	+0·00381
4G=4H+K+L+M+N	+0·00627
2H=2(K+L+M+N)+R+S	+0·00064
K+L+M+N=4(R+S)	−0·00079
4(K+L+M+N)=16(R+S)	−0·00316
4H=4(K+L+M+N)+2(R+S)	+0·00128
4G=4H+K+L+M+N	+0·00627
4F=4G+4H	−0·00252
A+B+C+D=4F+4G	+0·00381
T=A+B+C+D+F	+0·00173
4H=18(R+S)	−0·00188
4G=22(R+S)	+0·00360
4F=40(R+S)	−0·00080
4F+4G=62(R+S)	+0·00280
A+B+C+D=62(R+S)	+0·00661
T=72(R+S)	+0·00814

		grains.
T	=	5759·47143
72(R+S)	=	5759·46329
18(R+S)	=	1439·86582
40(R+S)	=	3199·70183
10(R+S)	=	799·92546
62(R+S)	=	4959·53783
22(R+S)	=	1759·83601
A+B+C+D	=	4959·54444
4F+4G	=	4959·54063
F+G	=	1239·88516

		grains.
A	=	1239·88467
B	=	1239·88651
C	=	1239·88709
D	=	1239·88618
F	=	799·92526
G	=	499·95990
H	=	359·96598
K	=	79·99241
L	=	79·99109
M	=	79·99258
N	=	79·99333
R	=	39·99625
S	=	39·99629

In the summer of 1845, after the adjustment and comparison of PS 200 times with T+0·645 grain together with each of the weights A, B, C, D in succession, the comparisons of the auxiliary weights with each other and with T presented some unaccountable discordances. By a most troublesome repetition of the weighings with different combinations of the weights, it became evident that A, C, F were subject to a very sensible fluctuation. This was at last found to be due to the circumstance that the platinum of which they were made had been very badly prepared, and contained cavities filled with a substance which attracted moisture from the air. In order to remove this injurious matter, they were digested in boiling water and then placed in a platinum capsule over a spirit-lamp, the heat of which caused a brown liquid to escape from the openings in their surfaces. After repeating this process several times till the coloured liquid ceased to appear, and till it was supposed that the whole of the deliquescent substance was removed, it was found that A, C and F had lost 0·04 grain, 0·031 grain, and 0·04 grain respectively. It now became obvious that all the weighings into which either A, B or F entered, must be repeated. This involved the rejection of the observations for determining the weight of PS, and the comparison of PS with the kilogramme, as well as those for comparing the auxiliary weights themselves. The weights lost by A, C, F were made up by the addition of bits of wire. In the following comparisons A denotes the weight marked A+0·04 grain, C the weight marked C+0·031 grain, F the weight marked F+0·04 grain.

June 26, 1846. 100 parts =0·26694 grain.

S=A+B+C+D+F.

S+Y, T+X.	T+X, S+Y.	S+X, T+Y.	T+Y, S+X.
19·97	19·80	18·00	18·24
20·04	19·40	17·44	18·30
20·57	18·66	17·60	18·54
20·41	18·89	17·95	18·61
19·75	18·65	17·90	17·96
19·96	19·55	17·82	18·60
19·99	18·80	17·27	18·16
18·92	18·52	17·17	18·41
18·41	18·59	18·27	18·14
18·94	17·22	17·60	18·29
196·96	188·08	177·02	183·25

$20(T+X)=20(S+Y)+8·88$ parts $\qquad$ $20(T+Y)=20(S+X)-6·23$ parts.

$$40T=40(A+B+C+D+F)+0·00708 \text{ grain.}$$

July 1. 100 parts =0·26965 grain.

S+Y, T+X.	T+X, S+Y.	S+X, T+Y.	T+Y, S+X.
17·61	18·09	15·67	16·70
17·67	17·87	16·17	16·99
16·95	17·81	16·01	15·85
17·09	17·47	15·55	16·96
17·04	17·49	16·19	16·89
17·06	17·59	16·15	16·07
16·82	17·36	15·52	16·00
16·79	17·70	15·47	16·46
17·15	17·19	15·40	16·80
16·81	17·01	15·90	15·99
170·99	175·58	158·03	164·71

20(T+X)=20(S+Y)−4·59 parts. 20(T+Y)=20(S+X)−6·68 parts.

40T=40(A+B+C+D+F)−0·03039 grain.

T=A+B+C+D+F−0·00029 grain.

June 25, July 3, 4, 6, 1846. 100 parts =0·49505 grain.

A, F+G.	F+G, A.	B, F+G.	F+G, B.
3·52	3·95	3·34	3·92
3·12	3·77	3·07	3·60
2·67	2·70	2·72	2·92
2·67	3·44	2·75	3·26
2·92	3·22	2·97	2·71
2·70	3·44	2·57	3·47
3·25	3·74	2·96	3·57
2·97	3·44	3·07	3·46
2·84	3·67	2·79	3·52
2·79	3·39	2·79	3·41
2·87	3·37	2·95	3·44
2·61	3·19	2·47	3·22
2·50	2·89	2·26	3·27
2·46	3·00	2·29	3·00
2·54	2·98	2·47	3·15
2·82	3·32	2·61	3·39
2·95	3·61	2·86	3·67
2·75	3·42	3·22	4·00
2·74	4·16	2·96	4·09
2·74	3·67	3·06	4·12
2·99	3·80	2·66	4·04
2·79	4·00	2·69	3·75
62·21	76·17	61·53	76·98

A=F+G−0·00157 grain. B=F+G−0·00174 grain.

June 25, July 3, 4, 6, 1846. 100 parts =0·49505 grain.

C, F+G.	F+G, C.	D, F+G.	F+G, D.
3·21	3·94	3·05	3·77
3·05	3·87	3·02	3·56
2·70	2·90	2·72	2·89
2·80	2·97	2·69	2·92
3·12	3·29	3·10	3·21
2·52	3·46	2·85	3·51
2·89	3·70	3·02	3·97
2·92	3·20	2·91	3·16
2·55	3·44	2·75	3·12
2·70	3·42	2·90	3·47
2·82	3·60	2·84	3·57
2·35	3·59	2·10	3·31
2·67	3·05	2·27	3·35
2·16	3·24	2·65	3·26
2·50	3·12	2·32	3·35
2·50	3·11	2·54	3·55
2·77	3·61	2·67	4·01
2·97	3·84	2·62	3·75
2·62	3·70	2·35	3·70
2·51	3·67	2·56	3·91
2·86	3·80	2·46	3·71
2·86	3·75	2·45	3·49
60·05	76·27	58·84	76·54

C=F+G−0·00182 grain. D=F+G−0·00199 grain.

June 25, July 3, 4, 5, 6, 1846. 100 parts =0·42553 grain.

F, G+H.	G+H, F.
4·72	4·07
4·60	4·02
5·05	4·19
4·05	3·96
3·95	3·32
3·99	3·37
3·82	3·45
3·20	3·17
3·55	3·32
3·45	3·04
4·01	3·60
4·10	3·57
4·14	3·55
4·17	3·39
4·17	3·77
4·46	3·56
4·24	3·52
4·19	3·29
4·15	3·60
4·07	3·44
4·09	3·31
4·37	3·46
4·44	3·67
4·49	3·31
4·40	3·02
4·30	3·80
4·40	3·50
4·45	3·34
4·11	3·64
4·54	3·32
4·17	3·12
129·84	108·69

F=G+H+0·00145 grain.

The relations between G, H, K, L, M, N, R, S are given by the observations of 1845.

	gr.
T = A + B + C + D + F	−0·00029
A = F + G	−0·00157
B = F + G	−0·00174
C = F + G	−0·00182
D = F + G	−0·00199
F = G + H	+0·00145
G = H + K	+0·00152
G = H + L	+0·00281
G = H + M	+0·00136
G = H + N	+0·00058
H = K + L + M + N + R	+0·00034
H = K + L + M + N + S	+0·00030
K = R + S	−0·00014
L = R + S	−0·00146
M = R + S	+0·00003
N = R + S	+0·00078

A + B + C + D	= 4F + 4G	−0·00712
4G	= 4H + K + L + M + N	+0·00627
2H	= 2(K + L + M + N) + R + S	+0·00064
K + L + M + N	= 4R + 4S	−0·00079
4(K + L + M + N)	=	16(R + S) − 0·00316
4H	= 4(K + L + M + N) +	2(R + S) + 0·00128
4H	=	18(R + S) − 0·00188
K + L + M + N	=	4(R + S) − 0·00079
4G	= 4H + K + L + M + N	+0·00627
4G	=	22(R + S) + 0·00360
4H	=	18(R + S) − 0·00188
4G + 4H	=	40(R + S) + 0·00172
4F	= 4G + 4H	+0·00580
4F	=	40(R + S) + 0·00752
4G	=	22(R + S) + 0·00360
4F + 4G	=	62(R + S) + 0·01112
A + B + C + D	=	62(R + S) + 0·00400
F	=	10(R + S) + 0·00188
A + B + C + D + F	=	72(R + S) + 0·00588
T	=	72(R + S) + 0·00559

		grains.
T	=	5759·47143
72(R + S)	=	5759·46584
18(R + S)	=	1439·86646
40(R + S)	=	3199·70324
10(R + S)	=	799·92581
62(R + S)	=	4959·54003
22(R + S)	=	1759·83678
A + B + C + D	=	4959·54403
4F + 4G	=	4959·55115
F + G	=	1239·88779
A	=	1239·88622
B	=	1239·88605
C	=	1239·88597
D	=	1239·88580
F	=	799·92769
G	=	439·96009
H	=	359·96614
K	=	79·99244
L	=	79·99112
M	=	79·99261
N	=	79·99336
R	=	39·99627
S	=	39·99631

In the observations for finding specific gravities made after June 1846, A, C, F were used without the bits of wire added to make up their loss of weight. Their values were, therefore,—

	grains.
A	=1239·84622
C	=1239·85497
F	= 799·88769

Y, Z are weights of 19·998 grains each, W a weight of 12·901 grains, V a weight of 6·451 grains, Q the mean of the ten weights of 0·645 grain nearly.

August 22, 1845, January 17, 1846. 100 parts =0·4065 grain.

Y+Z, R.	R, Y+Z.	Y+Z, S.	S, Y+Z.
3·69	3·49	3·61	3·54
3·50	3·40	3·47	3·45
3·57	3·42	3·50	3·37
3·50	3·44	3·52	3·39
3·56	3·42	3·56	3·40
3·55	3·34	3·47	3·36
21·37	20·51	21·13	20·51

Y+Z=R+0·00029 grain. Y+Z=S+0·00021 grain.

10(R+S)=799·92581 grains. Y+Z=39·99654 grains.

In the following comparisons of W+V+Q with Y and Z, the Q was changed at the end of each weighing, so that each Q was weighed eight times with W+V.

August 22, 1845, January 17, 1846. 100 parts =0·3992 grain.

W+V+Q, Y.	Y, W+V+Q.	W+V+Q, Z.	Z, W+V+Q.
3·02	3·91	3·17	3·85
3·10	3·92	3·32	3·70
3·10	3·96	3·27	3·76
3·11	3·94	3·35	3·75
3·16	4·07	3·35	3·67
3·05	4·05	3·31	3·79
3·20	4·01	3·31	3·62
3·10	4·06	3·22	3·67
3·17	4·00	2·82	3·57
3·20	3·97	3·25	3·67
3·01	3·80	3·19	3·71
2·88	3·86	3·22	3·69
2·91	3·85	3·23	3·62
2·97	3·81	3·20	3·59
3·01	3·81	3·25	3·61
3·08	3·87	3·20	3·56
2·97	3·84	3·22	3·67
2·98	3·86	3·23	3·64
3·04	3·83	3·27	3·59
3·07	3·80	3·28	3·70
61·13	78·22	64·66	73·43

W+V+Q=Y−0·001705 grain. W+V+Q=Z−0·000875 grain.

August 22, 1845, Jan. 17, 1846. 100 parts = 0·4096 grain.

W, V+10Q.	V+10Q, W.	V, 10Q.	10Q, V.
3·29	3·65	3·45	3·62
3·30	3·56	3·42	3·47
3·29	3·56	3·37	3·41
3·32	3·61	3·44	3·38
3·25	3·60	3·45	3·46
3·30	3·59	3·37	3·50
19·75	21·57	20·50	20·84

2V+20Q=2W+0·00124 grain. 40Q=4V+0·00046 grain.

		gr.
Y+Z	=	39·99654
2W+2V+2Q=Y+Z−		0·00258
2V+20Q	=2W +	0·00124
40Q	=4V +	0·00046
62Q	=	39·99566
Q	=	0·64509

Let (64), (32), (16), (8), (4), (2), (1) denote platinum weights belonging to Barrow's balance, 2J the sum of two platinum grain weights belonging to Robinson's balances, G a brass grain weight.

April 11, 1845. 100 parts = 0·29261 grain.

K,	(64)+(16)	2·47	(64)+(16), K	4·82
L,	(64)+(16)	1·88	(64)+(16), L	5·45
M,	(64)+(16)	2·44	(64)+(16), M	4·80
N,	(64)+(16)	2·70	(64)+(16), N	4·35
R+S,	(64)+(16)	2·32	(64)+(16), R+S	4·85
K,	(64)+(16)	2·30	(64)+(16), K	4·65
L,	(64)+(16)	1·80	(64)+(16), L	5·32
M,	(64)+(16)	2·42	(64)+(16), M	4·60
N,	(64)+(16)	2·65	(64)+(16), N	4·29
R+S,	(64)+(16)	2·35	(64)+(16), R+S	4·61
		23·33		47·74

		gr.
5[(64)+(16)]	=K+L+M+N+R+S+	0·01786
K+L+M+N	=4(R+S) −	0·00079
K+L+M+N+R+S=5(R+S)	−	0·00079
5(R+S)	=	399·96290
K+L+M+N+R+S=		399·96211
5[(64)+(16)]	=	399·97997
(64)+(16)	=	79·99599

March 19, 1845. 100 parts = 0·29485 grain.

	Scale.		Scale.
R, (32)+(8)	2·310	(32)+(8), R	3·675
S, (32)+(8)	2·640	(32)+(8), S	3·650
R, (32)+(8)	2·550	(32)+(8), R	3·675
S, (32)+(8)	2·675	(32)+(8), S	3·650
R, (32)+(8)	2·775	(32)+(8), R	3·725
S, (32)+(8)	2·860	(32)+(8), S	3·875
R, (32)+(8)	2·660	(32)+(8), R	3·790
S, (32)+(8)	2·825	(32)+(8), S	3·790
	21·295		29·830

16[(32)+(8)]=8(R+S)+0·02517 grain.

January 19, 1846. 100 parts $=0{\cdot}40140$ grain.

	Scale.		Scale.
R, (32)+(8)	3·300	(32)+(8), R	3·975
S, (32)+(8)	3·362	(32)+(8), S	3·875
R, (32)+(8)	3·300	(32)+(8), R	3·950
S, (32)+(8)	3·362	(32)+(8), S	3·875
R, (32)+(8)	3·200	(32)+(8), R	3·925
S, (32)+(8)	3·287	(32)+(8), S	3·825
R, (32)+(8)	3·175	(32)+(8), R	3·900
S, (32)+(8)	3·275	(32)+(8), S	3·862
R, (32)+(8)	3·212	(32)+(8), R	3·812
S, (32)+(8)	3·375	(32)+(8), S	3·800
	32·848		38·799

$$
\begin{aligned}
36[(32)+(8)] &= 18(R+S)+\overset{\text{gr.}}{0{\cdot}04906}\\
2[(32)+(8)] &= R+S+0{\cdot}00273\\
R+S &= 79{\cdot}99258\\
2[(32)+(8)] &= 79{\cdot}99531\\
(32)+(8) &= 39{\cdot}99765
\end{aligned}
$$

January 19, 1846. 100 parts $=0{\cdot}40140$ grain.

(64), (32)+(16)+(8)+(4)+(2)+2J.	(32)+(16)+(8)+(4)+(2)+2J, (64).
3·487	3·375
3·512	3·462
3·750	3·525
3·837	3·487
3·800	3·562
18·386	17·411

$(32)+(16)+(8)+(4)+(2)+2J=(64)-0{\cdot}00038$ grain.

(32), (16)+(8)+(4)+(2)+2J.	(16)+(8)+(4)+(2)+2J, (32).
3·325	3·450
3·437	3·537
3·512	3·537
3·625	3·550
3·587	3·637
17·486	17·711

$(16)+(8)+(4)+(2)+2J=(32)+0{\cdot}00009$ grain.

(16), (8)+(4)+(2)+2J.	(8)+(4)+(2)+2J, (16).	(8), (4)+(2)+2J.	(4)+(2)+2J, (8).
3·525	3·225	3·437	3·275
3·512	3·375	3·500	3·300
3·662	3·350	3·587	3·425
3·625	3·350	3·612	3·475
3·687	3·437	3·625	3·500
18·011	16·737	17·761	16·975

$(8)+(4)+(2)+2J=(16)-0{\cdot}00051$ grain. $(4)+(2)+2J=(8)-0{\cdot}00032$ grain.

January 16, 1845. 100 parts $=0{\cdot}4096$ grain.

(4), (2)+2J.	(2)+2J, (4).	(2), 2J.	2J, (2).
3·312	3·212	3·287	3·387
3·412	3·425	3·337	3·450
3·612	3·425	3·325	3·487
3·587	3·412	3·312	3·537
3·562	3·500	3·387	3·512
17·485	16·974	16·648	17·373

$(2)+2J=(4)-0{\cdot}00021$ grain. $2J=(2)+0{\cdot}00030$ grain.

March 20, 1845. 100 parts $=0{\cdot}29485$ grain.

(2), (1)+G.	(1)+G, (2).	(1), G.	G, (1).
2·98	3·46	2·80	3·65
3·10	3·57	2·75	3·72
3·12	3·67	3·02	3·72
9·20	10·70	8·57	11·09

$(1)+G=(2)+0{\cdot}00074$ grain. $G=(1)+0{\cdot}00124$ grain.

Hence $2(1)=(2)-0{\cdot}00050$ grain.

		gr.
2J	= (2)	+ 0·00030
2J+(2)	= (4)	− 0·00021
2J+(2)+(4)	= (8)	− 0·00032
2J+(2)+(4)+(8)	=(16)	− 0·00051
2J+(2)+(4)+(8)+(16)	=(32)	+ 0·00009
2J+(2)+(4)+(8)+(16)+(32)	=(64)	− 0·00038

Hence

2J	= (2)	+ 0·00030
4J	= (4)	+ 0·00009
8J	= (8)	+ 0·00007
16J	=(16)	− 0·00005
32J	=(32)	+ 0·00050
64J	=(64)	+ 0·00053
80J	=(64)+(16)+0·00048=79·99647	
40J	=(32)+(8)+0·00057 =39·99822	

Mean.........80J=79·99646

64J=63·99717	(64)	=63·99664
32J=31·99858	(32)	=31·99808
16J=15·99929	(16)	=15·99934
8J= 7·99965	(8)	= 7·99958
4J= 3·99982	(4)	= 3·99973
2J= 1·99991	(2)	= 1·99961
J= 0·99996	(1)	= 0·99956

Observations for finding the density of PC No. 1.

Weighing of PC No. 1 in air.

August 12, 1844. 100 parts $=0{\cdot}2564$ grain.

PC.	T+B+0·67 grain.
19·00	20·80
21·65	21·00
21·30	21·50
20·70	20·50
20·50	20·80
21·35	20·95
21·95	20·40
20·35	18·90
19·80	19·00
19·50	18·60
18·95	17·60
18·20	17·70
18·10	17·60
18·00	16·85
17·50	16·95
17·60	16·90
17·45	16·50
17·40	16·85
17·15	17·00
18·10	17·30
384·55	373·70

PC No. 1 $\triangleq$ T+B+0·6714 gr. T=5759·4881 gr., B=1239·8880 gr. Therefore PC No. 1 $\triangleq$ 7000·0475 grains.

For PC No. 1, $\log\Delta=1\cdot32566$. For T+B+0·6714, $\log\Delta=1\cdot32560$.

Hence vPC No. 1=330·70, v(T+B+0·6714 gr.)=330·72. vPC No. 1 is less than v(T+B+0·6714) by the volume of 0·02 grain of water, or of 0·00002 grain of air of ordinary density. Hence

PC No. 1=7000·0475 grains.

July 18, 1844. 100 parts =0·26 grain.

PC No. 1 and hook in water.		In right-hand pan.	
H.	P.	gr.	Scale.
16·95	17·65	0·12	19·4
16·9	17·6	0·12	19·9
16·9	17·65	0·12	20·1
16·9	17·6	0·12	20·8
16·8	17·55	0·12	21·2
16·8	17·5	0·12	21·5
16·8	17·45	0·12	19·1
16·8	17·5	0·12	19·0
16·75	17·45	0·12	19·4
16·75	17·45	0·12	20·0
16·835	17·53	0·120	20·04

Hook in water.	In right-hand pan.	
	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·68	27·4
	0·66	20·4
	0·66	20·4
	0·66	19·3
	0·66	16·0
Air.	0·66	14·0
t=17·0, b=754·76.	0·68	22·0
	0·68	22·5
	0·66	19·2
	0·66	19·2
	0·66	16·5
	0·68	23·0
	0·68	26·2
	0·6677	20·47

PC No. 1 in water (t=16·95) $\triangleq$ T+F+K+(16)+(8)+(4)+(2)+0·5466 gr. in air (t=17·0, b=754·76).

July 22, 1844. 100 parts =0·26 grain.

PC No. 1 and hook in water.		In right-hand pan.	
H.	P.	gr.	Scale.
		0·04	22·0
		0·04	22·0
		0·08	41·6
		0·00	15·7
		0·04	22·5
19·4	20·4	0·00	13·0
		0·00	7·9
		0·00	15·5
		0·04	32·8
		0·04	21·5
		0·00	12·5
		0·00	12·5
19·4	20·4	0·0233	19·96

Hook in water. In right-hand pan.

	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·72	28·5
	0·68	7·5
	0·70	15·0
	0·71	21·6
Air.	0·70	18·0
t=20·0, b=765·83.	0·72	26·3
	0·68	13·5
	0·72	23·0
	0·72	33·0
	0·68	9·0
	0·703	19·54

PC No. 1 in water (t=19·54)≏T+F+K+(16)+(8)+(4)+(2)+0·6811 gr. in air (t=20·0, b=765·83).

July 25, 1844. 100 parts =0·26 grain.

PC No. 1 and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
21·03	22·15	0·20	36·5
21·0	22·10	0·16	24·8
		0·12	11·0
		0·16	22·5
		0·14	18·5
21·0	22·15	0·16	24·4
21·0	22·1	0·12	9·0
		0·14	18·0
21·0	22·1	0·12	13·0
		0·16	22·5
21·0	22·1	0·15	20·4
21·0	22·05	0·15	19·5
20·95	22·0	0·15	19·7
20·9	22·0	0·15	20·1
20·99	22·08	0·1486	19·99

Hook in water. In right-hand pan.

	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·98	28·3
	0·94	15·7
	0·96	18·0
	0·98	25·6
	0·94	15·5
	0·96	20·4
	0·98	23·4
	0·94	13·2
	0·98	25·5
	0·94	13·4
	0·96	19·5
	0·98	28·5
	0·94	13·1
	0·96	18·8
	0·960	99·99

PC No. 1 in water (t=21·15)≏T+F+K+(16)+(8)+(4)+(2)+0·8114 gr. in air (t=21·6, b=761·37).

August 2, 1844. **100 parts = 0·26 grain.**

PC No. 1 and hook in water.		In right-hand pan.	
H.	P.	gr.	Scale.
		0·33	19·2
17·9	18·6	0·35	20·0
		0·35	20·2
17·8	18·5	0·35	20·5
		0·33	19·0
17·6	18·4	0·35	20·1
17·6	18·4	0·35	18·2
		0·37	20·6
17·5	18·35	0·37	21·0
		0·37	25·6
17·5	18·25	0·37	24·4
17·4	18·2	0·33	18·1
		0·33	14·0
		0·33	10·8
		0·33	8·2
		0·37	24·5
17·61	18·39	0·3487	19·02

Hook in water.	In right-hand pan.	
	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·94	23·1
	0·90	11·0
	0·94	16·5
	0·96	22·4
Air.	0·94	22·6
t=18·5, b=746·12.	0·94	21·2
	0·96	25·0
	0·92	17·5
	0·96	20·4
	0·96	20·0
	0·942	19·97

PC No. 1 in water (t=17·68)≏T+F+K+(16)+(8)+(4)+(2)+0·5908 gr. in air (t=18·5, b=746·12).

Apparent weight of PC No. 1 in water weighed with platinum weights.

Water. t.	Apparent weight of PC No. 1. gr.	Air. t.	Air. b.
16·95	6669·9457	17·0	754·76
19·54	6670·0802	20·0	765·83
21·15	6670·2105	21·6	761·37
17·68	6669·9899	18·5	746·12

The resulting values of ΔPC No. 1 and log ΔPC No. 1 are,—

	ΔPC No. 1.	log ΔPC No. 1.
	21·1664	1·325647
	21·1662	1·325644
	21·1687	1·325695
	21·1673	1·325666
Mean	21·16715	1·325663
10 − log Δ............		8·674337
PC No. 1 (reduced)	7000·004	3·845098
vPC No. 1	330·701	2·519435

Observations for finding the density of PC No. 2.

Weighing of PC No. 2 in air.

August 13, 1844. 100 parts $=0{\cdot}2432$ grain.

PC.	T+B+0·653 gr.
20·20	19·90
21·30	20·80
21·40	21·50
22·30	22·40
22·80	24·40
22·85	22·45
22·95	22·85
23·80	23·15
24·00	23·10
24·35	24·35
21·47	19·85
21·37	20·65
21·57	20·95
21·52	20·90
21·00	20·55
20·32	20·10
20·22	19·87
20·62	19·72
20·52	19·62
19·95	19·77
434·51	424·88

PC No. $2 \triangleq \mathrm{T}+\mathrm{B}+0{\cdot}65417$ gr. $\mathrm{T}=5759{\cdot}4881$ gr. $\mathrm{B}=1239{\cdot}8880$ gr. Therefore PC No. $2 \triangleq 7000{\cdot}0303$ grains.

For PC No. 2, $\log \Delta=1{\cdot}32560$. For $\mathrm{T}+\mathrm{B}+0{\cdot}6542$, $\log \Delta=1{\cdot}32564$.

Hence vPC No. $2=330{\cdot}75$, $v(\mathrm{T}+\mathrm{B}+0{\cdot}6542)=330{\cdot}72$. vPC No. 2 is greater than $v(\mathrm{T}+\mathrm{B}+0{\cdot}6542)$ by the volume of 0·03 grain of water, or of 0·00004 grain of air of ordinary density. Hence

PC No. $2=7000{\cdot}0303$ grains.

PC No. 2.

July 18, 1844. 100 parts $=0{\cdot}26$ grain.

PC No. 2 and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
		0·20	20·6
		0·20	21·2
		0·20	21·1
		0·20	21·3
		0·20	21·5
		0·20	23·6
		0·18	23·2
		0·16	13·0
16·5	17·3	0·16	15·7
		0·18	22·0
		0·20	25·4
		0·24	38·5
		0·16	15·3
		0·14	12·5
		0·20	23·5
		0·16	16·1
		0·18	18·9
		0·20	21·3
16·5	17·3	0·1867	20·82

Hook in water.	In right-hand pan.	
	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·60	11·9
	0·68	27·4
	0·64	13·2
	0·66	20·9
Air.	0·70	44·5
t=17·0, b=753·94.	0·64	17·6
	0·60	12·2
	0·68	25·1
	0·60	9·4
	0·68	25·5
	0·648	20·77

PC No. 2 in water (t=16·58)≏T+F+K+(16)+(8)+(4)+(2)+0·4612 grain in air (t=17·0, b=753·94).

July 23, 1844. 100 parts =0·26 grain.

PC No. 2 and hook in water.		In right-hand pan.	
H.	P.	gr.	Scale.
19·8	20·7	0·00	14·1
		0·04	22·1
19·9	20·85	0·04	30·0
		0·00	14·5
		0·04	34·0
19·95	20·95	0·00	15·2
		0·04	24·8
20·0	20·95	0·00	16·0
20·0	21·0	0·04	27·0
20·0	21·0	0·00	16·5
20·0	21·0	0·04	30·5
20·05	21·05	0·00	15·2
19·95	20·92	0·020	21·66

Hook in water.	In right-hand pan.	
	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·68	23·0
	0·64	14·5
Air.	0·68	23·5
t=21·5, b=761·72.	0·64	15·0
	0·68	26·5
	0·64	12·0
	0·660	19·08

PC No. 2 in water (t=20·07)≏T+F+K+(16)+(8)+(4)+(2)+0·6467 gr. in air (t=21·5, b=761·72).

July 27, 1844. 100 parts =0·26 grain.

PC No. 2 and hook in water.		In right-hand pan.	
H.	P.	gr.	Scale.
20·5	21·6	0·23	15·5
		0·26	21·8
		0·26	23·4
		0·23	14·6
		0·26	23·5
20·5	21·5	0·26	23·7
		0·23	18·5
20·5	21·55	0·23	14·8
		0·26	21·6
		0·26	23·5
20·5	21·55	0·248	20·09

Hook in water.	In right-hand pan.	
	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·98	27·0
	0·98	25·5
	0·94	13·7
	0·98	24·8
	0·94	11·6
Air.	0·94	10·8
t=22·3, b=765·63.	0·98	22·9
	0·94	12·1
	0·98	22·5
	0·98	32·0
	0·94	12·8
	0·98	22·1
	0·98	24·5
	0·9646	20·18

PC No. 2 in water (t=20·65)≏T+F+K+(16)+(8)+(4)+(2)+0·7164 grain in air (t=22·3, b=765·63).

August 3, 1844. 100 parts =0·26 grain.

PC No. 2 and hook in water.		In right-hand pan.	
H.	P.	gr.	Scale.
18·0	18·9	0·41	21·5
17·95	18·8	0·40	23·0
		0·40	21·5
17·5	18·4	0·40	21·5
17·4	18·2	0·39	18·7
		0·40	11·5
17·1	17·95	0·40	17·4
16·95	17·6	0·44	20·5
		0·44	22·6
16·6	17·5	0·44	24·0
17·36	18·19	0·412	20·22

Hook in water.	In right-hand pan.	
	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·94	28·8
	0·94	28·0
	0·90	11·8
Air.	0·94	22·0
t=17·5, b=744·15.	0·90	12·0
	0·94	22·6
	0·94	25·0
	0·90	12·7
	0·94	23·0
	0·9267	20·31

PC No. 2 in water (t=17·5)≏T+F+K+(16)+(8)+(4)+(2)+0·5144 grain in air (t=17·5, b=744·15).

Apparent weight of PC No. 2 in water, weighed with platinum weights.

Water.	Apparent weight of PC No. 2.	Air.	
t.	gr.	t.	b.
16·58	6669·8603	17·0	753·94
20·07	6670·0458	21·5	761·72
20·65	6670·1155	22·3	765·63
17·50	6669·9135	17·5	744·15

Resulting values of ΔPC No. 2 and log ΔPC No. 2.

	ΔPC No. 2.	log ΔPC No 2.
	21·1656	1·325631
	21·1640	1·325599
	21·1631	1·325579
	21·1633	1·325584
Mean	21·1640	1·325598
10− log Δ		8·674402
PC No. 2 (reduced)	7000·002	3·845098
*v*PC No. 2	330·750	2·519500

Observations for finding the density of PS.

Apparent weight in water of the hook and wire by which PS, PC No. 3 and PC No. 4 were suspended in water.

The weight of 123·6 inches of the platinum wire by which the weights were suspended was 38·815 grains. Hence one inch displaces 0·01365 grain of water. 100 parts of the micrometer scale correspond to a displacement of 0·01365 grain of water. With 110 grains in each pan, 100 parts =0·2559 grain. With the hook suspended in water, 100 parts =0·2559 grain +0·01365 grain =0·27 grain. The upper hook by which the wire was attached to the scale-pan was changed twice during the weighings.

March 27, 1845. Hook in water.

In right-hand pan.

gr.	Scale.
0·00	18·2
0·02	20·2
0·00	18·9
0·02	20·6
0·00	18·9
0·02	20·5
0·00	20·5
0·00	18·5
0·02	19·5
0·00	18·9
0·00	20·0
0·0073	19·52

Counterpoise balances hook in water + 0·0086 gr. in air.

In right-hand pan.

gr.	Scale.
(64)+(32)+(8)+(4)+(1)+0·88	21·75
+0·87	18·82
+0·88	22·12
+0·87	18·55
0·875	20·31

C.poise bal. (64)+(32)+(8)+(4)+(1)+0·8742 gr.

H. in w. ≏ (64)+(32)+(8)+(4)+(1)+0·8656 gr. in a.

April 1, 1845. Hook in water.

In right-hand pan.

gr.	Scale.
2·30	19·9
2·30	20·0
2·30	19·0
2·30	19·2
2·32	21·2
2·30	20·8
2·30	19·5
2·3029	19·94

Counterpoise balances hook in water + 2·3031 gr. in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·18	19·65
+0·18	19·72
0·180	19·68

Counterpoise balances K+(32)+0·1808 grain.

Hook in water ≏ K+(32)−2·1223 grains in air.

April 5. Hook in water.

In right-hand pan.

gr.	Scale.
2·30	20·2
2·30	20·4
2·28	20·5
2·24	12·0
2·36	25·5
2·24	7·0
2·34	26·0
2·30	21·0
2·30	22·3
2·30	22·5
2·296	19·5

Counterpoise balances hook in water + 2·2972 gr. in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·18	17·15
0·19	20·55
0·20	23·87
0·190	20·64

Counterpoise balances K+(32)+0·1884 grain.
Hook in water ≏K+(32)−2·1088 grains in air.

April 8. Hook in water.

In right-hand pan.

gr.	Scale.
2·30	18·0
2·30	20·5
2·30	18·8
2·30	19·3
2·30	19·5
2·30	18·5
2·30	23·1
2·30	18·9
2·30	20·2
2·30	19·2
2·300	19·6

Counterpoise balances hook in water+2·3011 gr. in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·18	18·05
0·19	21·7
0·18	18·0
0·1883	19·25

Counterpoise balances K+(32)+0·1902 grain.
Hook in water ≏K+(32)−2·1109 grains in air.

April 16. Hook in water.

In right-hand pan.

gr.	Scale.
2·30	18·9
2·30	19·0
2·30	19·0
2·30	19·0
2·30	19·5
2·30	20·0
2·30	20·7
2·30	20·0
2·300	19·51

Counterpoise balances hook in water+2·3013 gr. in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·18	17·65
0·18	18·55
0·19	19·95
0·19	21·65
0·19	21·8
0·18	16·8
0·185	19·4

Counterpoise balances K+(32)+0·1865 grain.
Hook in water ≏K+(32)−2·1143 grains in air.

Hook in water.

In right-hand pan.

gr.	Scale.
2·30	17·1
2·30	18·2
2·30	18·5
2·30	18·9
2·30	18·9
2·30	19·9
2·30	21·5
2·30	19·0
2·30	18·5
2·300	18·94

Counterpoise balances hook in water+2·3002 gr. in air.

April 17. In right-hand pan.

gr.	Scale.
K+(32)+0·18	15·9
0·19	17·6
0·22	29·05
0·18	15·3
0·19	15·7
0·20	21·05
0·1933	19·1

Counterpoise balances K+(32)+0·1956 grain.
Hook in water ≏K+(32)−2·1046 grains in air.

Hook in water.

In right-hand pan.

gr.	Scale.
2·30	31·0
2·24	12·1
2·28	25·0
2·27	22·9
2·26	19·7
2·26	19·0
2·28	25·0
2·26	18·2
2·27	20·5
2·26	18·0
2·28	21·0
2·26	17·0
2·28	20·9
2·26	19·5
2·28	21·0
2·30	26·5
2·26	15·6
2·28	18·0
2·30	24·8
2·28	15·0
2·30	24·5
2·28	19·0
2·2745	20·65

Counterpoise balances hook in water+2·2727 gr. in air.

April 30. In right-hand pan.

gr.	Scale.
K+(32)+0·20	24·0
0·18	14·8
0·19	18·2
0·20	22·7

0·20	20·8
0·18	14·95
0·20	20·6
0·20	20·05
0·20	20·0
0·1944	19·57

Counterpoise balances K+(32)+0·1955 grain.

Hook in water ≏K+(32)−2·0772 grains in air.

May 5. Hook in water.

In right-hand pan.

gr.	Scale.
2·27	21·1
2·27	21·0
2·26	22·9
2·25	16·5
2·26	19·2
2·27	22·5
2·25	18·5
2·26	20·6
2·26	20·9
2·25	15·2
2·26	11·2
2·29	20·0
2·29	20·6
2·29	20·5
2·2664	19·33

Counterpoise balances h. in w. +2·2682 grains in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·20	19·0
0·21	20·0
0·21	22·3
0·21	23·9
0·20	16·7
0·20	18·5
0·21	19·5
0·21	19·5
0·21	19·0
0·21	19·1
0·207	19·75

Counterpoise balances K+(32)+0·2076 grain.

Hook in water ≏K+(32)−2·0606 grains in air.

May 7. Hook in water.

In right-hand pan.

gr.	Scale.
2·26	11·2
2·29	20·0
2·29	20·6
2·29	20·5
2·29	25·5
2·26	15·6
2·26	15·0
2·28	20·5
2·28	20·7
2·28	19·5
2·28	19·9
2·28	20·8
2·28	22·4
2·28	22·5
2·24	15·0
2·24	15·0
2·28	25·0
2·28	26·0
2·26	20·0
2·26	20·6
2·26	19·8
2·26	19·5
2·2718	19·8

Counterpoise balances h. in w. +2·2723 grains in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·20	20·8
0·20	20·2
0·20	21·3
0·20	24·0
0·20	23·4
0·18	19·2
0·18	20·9
0·18	17·1
0·18	17·2
0·1911	20·45

Counterpoise balances K+(32)+0·1899 grain.

Hook in water ≏K+(32)−2·0824 grains in air.

Hook in water.

In right-hand pan.

gr.	Scale.
2·26	18·9
2·27	21·9
2·26	19·8
2·27	22·8
2·26	20·0
2·26	19·0
2·27	22·1
2·26	20·0
2·26	19·3
2·26	20·0
2·26	20·1
2·26	20·4
2·26	19·9
2·26	20·0
2·26	20·0
2·25	17·4
2·27	24·3
2·2618	20·35

Counterpoise balances h. in w. +2·2609 grains in air.

May 10. In right-hand pan.

gr.	Scale.
K+(32)+0·20	27·3
0·17	15·95
0·18	19·85
0·18	19·6
0·18	19·2
0·18	18·8
0·1817	20·1

Counterpoise balances K+(32)+0·1814 grain.

Hook in water ≏K+(32)−2·0795 grains in air.

May 22. Hook in water.

In right-hand pan.

gr.	Scale.
2·26	23·7
2·24	16·5
2·25	20·1
2·25	20·0
2·25	20·2
2·25	19·4
2·25	19·25
2·25	19·3
2·26	23·2
2·25	19·0
2·25	19·0
2·26	23·45
2·2517	20·26

Counterpoise balances h. in w. +2·2510 grains in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·20	27·6
0·16	12·4
0·18	20·05
0·18	19·95
0·18	20·0
0·18	19·5
0·180	19·92

Counterpoise balances K+(32)+0·1802 grain.
Hook in water ≏K+(32)−2·0708 grains in air.

Hook in water.

In right-hand pan.

gr.	Scale.
0·12	21·2
0·12	21·0
0·11	17·1
0·12	20·9
0·11	16·9
0·12	20·6
0·12	21·8
0·12	27·0
0·10	13·15
0·12	16·0
0·12	23·9
0·10	18·3
0·12	24·0
0·10	17·8
0·12	24·0
0·10	18·0
0·12	21·4
0·10	17·0
0·12	22·3
0·1137	20·12

Counterpoise balances h. in w. +0·1134 grain in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·30	28·6
0·28	20·4
0·27	20·7
0·26	10·3
0·28	20·0
0·30	27·65
0·28	19·45
0·26	16·25
0·28	17·3
0·30	25·8
0·28	19·4
0·281	20·53

Counterpoise balances K+(32)+0·2796 grain.
Hook in water ≏K+(32)+0·1662 grain in air.

May 31. Hook in water.

In right-hand pan.

gr.	Scale.
0·12	23·7
0·10	16·7
0·11	20·5
0·11	18·2
0·12	10·5
0·14	19·5
0·14	31·7
0·11	17·5
0·12	21·4
0·10	14·2
0·11	18·0
0·12	21·5
0·11	18·3
0·12	22·0
0·11	18·5
0·12	21·9
0·11	18·45
0·1159	19·57

Counterpoise balances h. in w. +0·1170 grain in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·29	24·2
0·28	20·8
0·27	16·8
0·28	20·0
0·28	19·7
0·280	20·3

Counterpoise balances K+(32)+0·2792 grain.
Hook in water ≏K+(32)+0·1622 grain in air.

Apparent weight of the hook in water.

	gr.	Means.
March 27	109·8592	
April 1	109·8686	
April 5	109·8821	109·875
April 8	109·8800	
April 16	109·8761	
April 17	109·8863	
April 30	109·9137	
May 5	109·9303	
May 7	109·9085	109·917
May 10	109·9114	
May 22	109·9201	
May 30	112·1567	112·155
May 31	112·1527	

March 27 ... April 17. Hook in water $\bumpeq$ 109·875 grains of platinum in air.

April 30 ... May 22. Hook in water $\bumpeq$ 109·917 grains of platinum in air.

May 30 ... 31. Hook in water $\bumpeq$ 112·155 grains of platinum in air.

With 6680 grains in the right-hand pan,—March 27...April 8, 100 parts = 0·197 grain. April 29...May 3, 100 parts = 0·32 grain, liable to some uncertainty on account of the state of the knife-edge. May 9...May 30, 100 parts = 0·207 grain. The dimensions of the wire by which the lbs. were suspended was such that a portion of the wire corresponding to 100 parts of the scale displaced 0·01365 grain of water. Hence, when a platinum lb. is suspended in water from the right-hand pan,—

March 27...April 8	100 parts = 0·211 grain.
April 29...May 3	100 parts = 0·33 grain.
May 9...May 30	100 parts = 0·221 grain.

The thermometer B was suspended in the water with its bulb in a horizontal plane through the middle of the lb.; the thermometer C was placed in the balance case. F denotes the reading of Ernst's barometer, E that of its attached thermometer.

The counterpoise in the left-hand pan is supposed to be in equilibrium with the weights in or suspended from the right-hand pan, when the reading of the scale is 20.

Weighing of PS in air.

June 4, 1845. 100 parts = 0·2649 grain.

PS+X.	T+C+0·68 gr.+Y.		PS+Y.	T+C+0·68 gr.+X.
19·70	18·60		15·05	19·40
19·50	18·95		16·10	18·00
18·70	18·05		15·10	17·50
18·40	17·10	D= 13·3	15·70	18·00
17·00	18·00	C= 13·4	14·90	16·90
16·20	18·10	F=750·7	PS+Y.	T+C+0·67 gr.+X.
17·20	18·60	E= 14	16·40	14·30
17·80	17·75		14·65	15·80
18·10	19·05		16·25	13·80
16·90	18·70		14·05	13·70
179·50	183·70		15·50	12·75
			153·70	160·15

10(PS+X) $\bumpeq$ 10(T+C+0·68 gr.+Y)−4·2 parts. 10(PS+Y) $\bumpeq$ 10(T+C+0·675 gr.+X)−6·45 parts.

20PS=20(T+C)+13·52179 grains.

100 parts $=0.3108$ grain.

PS+X.	T+C+0·68 gr.+Y.		PS+Y.	T+C+0·67 gr.+X.
15·50	16·55		21·10	19·30
15·90	15·20	D= 13·5	18·70	16·40
13·70	14·30	C= 13·55	17·20	16·00
14·95	14·10	F=752·4	16·50	18·90
18·00	15·90	E= 14·1	17·15	14·80
13·90	14·90		17·15	15·30
91·95	90·95		107·70	100·70

$6(PS+X) \triangleq 6(T+C+0{\cdot}68 \text{ gr.}+Y)+1$ part. $6(PS+Y) \triangleq 6(T+C+0{\cdot}67 \text{ gr.}+X)+7$ parts.

$12PS \triangleq 12(T+C)+8{\cdot}12486$ grains.

$32PS \triangleq 32(T+C)+21{\cdot}64665$ grains.

$PS \triangleq T+C+0{\cdot}67646$ grain in air ($t=13{\cdot}45$, $b=750{\cdot}09$).

$T=5759{\cdot}47143$ grains. $C=1239{\cdot}88708$ grains. $PS \triangleq 7000{\cdot}03495$ grains in air.

For PS, $\log \Delta=1{\cdot}32545$. For $T+C+0{\cdot}676$ grain, $\log \Delta=1{\cdot}32566$. Hence $vPS=330{\cdot}86$, $v(T+C+0{\cdot}676 \text{ gr.})=330{\cdot}70$. Therefore vPS is larger than $v(T+C+0{\cdot}676 \text{ gr.})$ by the volume of 0·16 grain of water, or of 0·00019 grain of air. Hence

$$PS=7000{\cdot}03516 \text{ grains.}$$

Weighing of PS in water.

April 1, 1845. PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
9·05	7·85	769·8	8·1	1·00	14·5
				1·08	20·0
9				1·08	22·0
8·95	8·33	769·65	8·5	1·00	13·1
				1·08	24·0
8·9				1·04	19·5
8·87	8·4			1·08	27·2
				1·00	12·0
8·85				1·04	19·0
				1·04	18·5
8·94	8·19	769·72	8·3	1·044	18·98

Counterpoise in air ($C=8{\cdot}19$, $F=769{\cdot}4$, $E=8{\cdot}3$) balances PS and hook in water ($B=8{\cdot}94$) + 1·0462 grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+N+R+(16)+(4)+	1·10	20·67
+	1·10	20·7
	1·10	20·68

Counterpoise balances $T+F+K+N+R+(16)+(4)+1{\cdot}0987$ grain in air.

PS and h. in w. ($B=8{\cdot}94$) $\triangleq T+F+K+N+R+(16)+(4)+0{\cdot}0525$ grain in air ($C=8{\cdot}19$, $F=769{\cdot}72$, $E=8{\cdot}3$).

PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
13·05	8·85	762·25	9·6	1·00	27·4
				0·90	9·5
				0·98	21·9
12·85	8·93			0·96	19·0
12·5				0·94	18·0

				0·96	18·3
12·4		762·13	9·65	0·98	19·4
12·25				0·98	20·0
12·01	9·05			0·98	18·5
12				1·00	20·0
11·95				1·00	20·0
11·65	9·35	762·0	9·5	1·00	21·0
11·53				1·00	19·2
11·50				1·00	19·5
11·43				1·00	17·0
11·40	9·4			1·00	18·6
12·18	9·18	762·06	9·57	0·98	19·2

Counterpoise in air (C=9·18, F=762·06, E=9·57)≏PS and hook in water (B=12·18)+0·9817 grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+N+R+(16)+(4)	+1·10	20·02
	+1·10	20·25
	+1·10	21·2
	1·10	20·49

Counterpoise ≏T+F+K+N+R+(16)+(4)+1·0990 grain in air.

PS and h. in w. (B=12·18)≏T+F+K+N+R+(16)+(4)+0·1173 gr. in air (C=9·18, F=762·06, E=9·57).

April 5. PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
7·5	8	764·3	8·4	1·00	10·0
				1·04	18·2
				1·06	19·1
7·5				1·08	21·8
				1·07	21·5
				1·04	19·4
7·55				1·04	20·7
				1·00	8·0
				1·10	28·5
				1·04	21·5
				1·02	16·5
				1·04	21·3
7·6				1·04	21·55
	8·5	763·9	8·95	1·02	20·6
7·65				1·00	4·5
				1·00	7·9
				1·10	23·5
				1·00	7·9
7·65				1·10	23·5
				1·10	25·4
				1·10	27·5
				1·10	29·4
	8·95			1·10	33·0
7·65				1·00	9·0
				1·04	18·0
7·65				1·04	21·0
				1·04	21·6
				1·04	21·4
				1·04	22·0
7·6	8·48	764·1	8·67	1·0479	19·47

Counterpoise in air (C=8·48, F=764·1, E=8·67)≏PS and hook in water (B=7·6)+1·0490 grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+N+R+(16)+(4)	+1·10	20·7
	+1·10	21·0
	1·100	20·85

Counterpoise ≏ T+F+K+N+R+(16)+(4)+1·0983 grain in air.

PS and h. in w. (B=7·6) ≏ T+F+K+N+R+(16)+(4)+0·0493 gr. in air (C=8·48, F=764·1, E=8·67).

May 3. PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
11·85				1·06	21·1
				1·06	22·3
11·7				1·06	20·0
				1·04	14·0
	11·5	759·4	12·4	1·06	19·5
				1·02	12·4
				1·06	23·0
				1·10	28·9
11·65				1·02	12·7
				1·06	23·7
11·6	12·2	759·1	12·2	1·02	15·4
				1·06	23·5
				1·04	21·6
				1·03	16·9
				1·04	20·5
				1·03	20·3
				1·00	12·5
				1·10	27·6
11·6				1·00	17·0
				1·10	29·4
				1·04	20·8
				1·02	16·8
				1·04	21·0
				1·03	18·2
11·5				1·04	20·5
				1·04	20·9
11·67	11·85	759·25	12·3	1·045	20·02

Counterpoise in air (C=11·85, F=759·25, E=12·3) ≏ PS and hook in water (B=11·67)+1·045 grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)	+1·18	26·9
	+1·14	15·0
	+1·16	20·3
	+1·16	21·8
	+1·16	17·5
	1·160	20·3

Counterpoise ≏ T+F+K+L+R+(16)+(4)+1·159 grain in air.

PS and h. in w. (B=11·67) ≏ T+F+K+L+R+(16)+(4)+0·114 gr. in air (C=11·85, F=759·25, E=12·3).

May 5. PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
10·05	10·4	759·1	11	1·04	18·0
				1·08	24·5
				1·04	19·4
				1·06	22·5
				1·05	21·5
				1·04	15·5
				1·04	16·7

				1·06	20·1
				1·06	20·4
				1·06	21·8
10·05				1·06	22·0
10·07	11·3	758·7	11·5	1·06	21·7
				1·04	17·7
				1·05	20·2
10·08				1·05	20·3
				1·05	20·05
				1·05	20·1
				1·05	21·1
10·09				1·06	21·15
				1·04	15·9
				1·04	16·8
				1·06	23·5
10·07	10·85	758·9	11·25	1·0518	20·04

Counterpoise in air (C=10·85, F=758·9, E=11·25)≏PS and hook in water (B=10·07)+1·0517 grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·16	17·0
	1·17	15·7
	1·18	19·8
	1·18	17·0
	1·18	18·2
	1·21	22·1
	1·21	23·17
	1·20	23·4
	1·20	24·5
	1·1878	20·1

Counterpoise ≏T+F+K+L+R+(16)+(4)+1·1875 grain in air.

PS and h. in w.(B=10·07)≏T+F+K+L+R+(16)+(4)+0·1358 gr. in a.(C=10·85, F=758·9, E=11·25).

May 28.

		PS and hook in water.		In right-hand pan.	
B.	C.	F.	E.	gr.	Scale.
				0·21	24·0
		759·4	11	0·21	22·7
10·18				0·20	20·6
				0·20	22·5
				0·18	18·4
				0·20	21·4
10·2	11·1			0·18	20·3
				0·18	20·4
				0·16	9·0
				0·18	13·0
				0·20	22·5
10·4	11·6	758·9	11·4	0·18	20·1
				0·18	20·6
10·45				0·18	20·1
				0·17	19·4
				0·16	15·9
				0·20	25·9
10·31	11·35	759·15	11·2	0·1865	19·81

C.poise in air (C=11·35, F=759·15, E=11·2) balances PS and hook in water (B=10·31)+0·1869 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	2·52	18·6
	2·53	20·55
	2·54	29·5
	2·52	21·0
	2·54	30·7
	2·50	8·6
	2·52	18·9
	2·52	18·55
	2·53	21·65
	2·5244	20·89

Counterpoise balances T+F+K+L+R+(16)+(4)+(2)+0·5224 grain in air.
PS & h. in w. (B=10·31) ≏ T+F+K+L+R+(16)+(4)+(2)+0·3355 gr. (C=11·35, F=759·15, E=11·2).

B.	C.	F.	E.	gr.	Scale.
	PS and hook in water.			In right-hand pan.	
	10·5	756·0	11	0·18	22·0
				0·17	17·9
				0·18	20·9
				0·17	20·8
				0·18	24·8
10·3				0·17	21·2
10·4	10·6	755·6	11	0·15	14·1
				0·18	26·0
				0·15	15·2
				0·18	27·4
				0·15	12·8
				0·18	26·8
				0·15	12·6
10·35	10·55	755·8	11	0·1685	20·2

Counterpoise in air (C=10·55, F=755·8, E=11) balances PS and hook in water (B=10·35)+0·1681 gr. in air.
Counterpoise balances T+F+K+L+R+(16)+(4)+(2)+0·5211 grain in air. (Mean, May 28, 30.)
PS & h. in w. (B=10·35) ≏ T+F+K+L+R+(16)+(4)+(2)+0·3530 gr. in a. (C=10·55, F=755·8, E=11).

B.	C.	F.	E.	gr.	Scale.
	PS and hook in water.			In right-hand pan.	
10·12	10·65	758·85	11	0·18	26·0
				0·15	12·7
				0·18	27·5
10·2				0·16	16·1
				0·17	23·1
10·25				0·16	14·5
				0·17	20·5
				0·17	21·0
10·3		759·4	12	0·16	14·5
				0·17	20·5
10·55	12·1	760·05	12·1	0·18	27·8
				0·14	8·1
				0·16	20·0
				0·18	27·7
				0·16	19·5
				0·16	19·55
				0·14	9·6
10·6				0·18	31·5
				0·16	19·0
				0·16	20·7
				0·16	18·8
10·6				0·16	15·4
10·4	11·37	759·58	11·8	0·1641	19·73

Counterpoise in air (C=11·37, F=759·58, E=11·8) balances PS and hook in water (B=10·4)+0·1647 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	2·52	21·3
	2·52	22·4
	2·52	18·7
	2·50	9·5
	2·54	29·1
	2·52	19·7
	2·52	19·6
	2·520	20·04

Counterpoise balances T+F+K+L+R+(16)+(4)+(2)+0·5199 grain in air.

PS and h. in w. (B=10·4)≏T+F+K+L+R+(16)+(4)+(2)+0·3552 gr. (C=11·37, F=759·58, E=11·8).

Apparent weight of PS in water, weighed with platinum weights.

Water. *t.*	Apparent weight of PS. gr.	Air. *t.*	*b.*
8·95	6669·555	8·18	769·13
12·15	6669·620	9·17	761·32
7·62	6669·552	8·47	763·47
11·63	6669·573	11·83	758·19
10·05	6669·595	10·84	757·97
10·29	6669·556	11·34	758·22
10·33	6669·573	10·54	754·90
10·38	6669·575	11·36	758·58

Resulting values of ΔPS and log ΔPS.

	ΔPS.	log ΔPS.
	21·1570	1·325455
	21·1572	1·325457
	21·1581	1·325477
	21·1553	1·325420
	21·1590	1·325496
	21·1563	1·325440
	21·1573	1·325461
	21·1574	1·325463
Mean	21·1572	1·325459
10− log Δ		8·674541
PS (reduced)	7000·0009	3·845098
*v*PS	330·856	2·519639

Observations for finding the density of PC No. 3.

Weighing of PC No. 3 in air.

June 10, 1845. 100 parts =0·273 grain.

PC+X.	T+C+0·67 gr.+Y.		PC+Y.	T+C+0·67 gr.+X.
16·75	18·25		24·2	22·4
16·6	17·8		24·6	22·4
16·7	13·4		PC+Y.	T+C+0·68 gr.+X.
16·9	14·7		20·5	21·0
15·9	14·4		19·55	21·1
15·5	15·55	D= 14·1	20·2	20·6
17·5	15·65	C= 14·3	20·4	20·7
17·2	14·4	F=771·6	19·5	20·9
17·1	14·1	E= 14·9	20·0	20·5
16·3	15·0		19·2	20·7
15·25	15·6		19·1	21·7
15·1	14·4		19·85	21·4
14·8	13·9		18·65	21·4
81·6	67·2		18·9	20·5
			215·85	230·5

13(PC+X)≏13(T+C)+8·71 gr.+14·4 pt. 13(PC+Y)≏13(T+C)+8·82 gr.−10·65 pt.

26PC≏26(T+C)+17·54024 grains.

June 11. 100 parts $=0{\cdot}29$ grain.

PC+Y.	T+C+0·68 gr.+X.		PC+X.	T+C+0·68 gr.+Y.
26·15	26·3	D= 14	22·3	24·0
23·45	25·0	C= 14	22·1	23·2
23·1	25·6	F=769·5	23·9	23·8
24·5	25·0	E= 15	22·62	24·15
97·2	101·9		90·95	95·15

$4(PC+Y) \triangleq 4(T+C+0{\cdot}68\text{ gr.}+X)-4{\cdot}7$ pt. $\qquad 4(PC+X) \triangleq 4(T+C+0{\cdot}68\text{ gr.}+Y)-4{\cdot}2$ pt.

$$8PC \triangleq 8(T+C)+5{\cdot}41864 \text{ grains.}$$

$$34PC \triangleq 34(T+C)+22{\cdot}95888 \text{ grains.}$$

$$PC \triangleq T+C+0{\cdot}67526 \text{ grain in air } (t=14{\cdot}07,\ b=769{\cdot}66).$$

$$\text{PC No. } 3 \triangleq 7000{\cdot}03375 \text{ grains of platinum in air } (t=14{\cdot}07,\ b=769{\cdot}66).$$

For PC No. 3, $\log \Delta=1{\cdot}32555$. For T+C+0·675 gr., $\log \Delta=1{\cdot}32566$. Hence vPC No. 3 $=330{\cdot}79$, (vT+C+0·675). Therefore vPC No. 3 is greater than v(T+C+0·675 gr.) by the volume of 0·09 grain of water, or of 0·00011 grain of air. Hence

$$\text{PC No. } 3=7000{\cdot}03386 \text{ grains.}$$

Weighing of PC No. 3 in water.

March 31, 1845. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
9·18				1·00	27·8
				0·90	8·8
9·1				0·97	18·5
				0·98	20·0
9				0·98	21·55
		768·75	8·8	0·97	20·0
8·95	8·65			1·00	22·8
		768·83	9	0·97	20·6
				0·96	19·4
				0·96	19·3
8·93	8·84			0·97	19·7
				0·97	18·5
8·9		768·8	9	0·97	20·4
				0·97	21·0
9·01	8·74	768·79	8·93	0·9693	19·88

Counterpoise in air (C=8·74, F=768·79, E=8·93) balances PC and hook in water (B=9·01)+0·9696 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+(1)+	0·10	15·9
	0·11	22·75
	0·10	17·3
	0·10	17·9
	0·11	25·2
	0·104	19·81

Counterpoise balances T+F+K+L+R+(16)+(4)+(1)+0·1044 grain.

PC and h. in w. (B=9·01) $\triangleq$ T+F+K+L+R+(16)+(4)+0·1348 gr. in air (C=8·74, F=768·79, E=8·93).

	PC and hook in water.				In right-hand pan.	
	B.	C.	F.	E.	gr.	Scale.
April 7.					1·00	19·0
					1·00	29·5
					1·00	26·5
	8·45	8·4	759·8	8·9	1·00	20·2
					1·00	24·5
	8·43				0·96	13·6
					0·92	14·5
					0·92	9·5
					1·04	34·0
April 8.	7·4		749·2	8·4	1·00	19·0
					0·96	15·0
	7·37				1·00	20·0
					1·00	25·5
		8·4			0·96	14·5
					1·00	25·0
					0·96	17·0
					1·00	20·0
					1·00	27·5
					0·96	18·0
					1·00	21·6
					0·96	9·5
					1·00	21·7
	7·4				0·99	21·5
					0·95	14·5
		8·6	748·7	9·0	0·99	21·2
					0·95	7·5
					0·99	20·0
					0·99	21·6
					1·00	27·0
					1·00	24·6
					0·96	11·6
					1·00	23·7
	7·74	8·47	752·57	8·77	0·9831	20·27

Counterpoise in air (C=8·47, F=752·57, E=8·77) balances PC and h. in w. (B=7·74)+0·9825 gr. in air.

	In right-hand pan.	
	gr.	Scale.
T+F+K+N+R+(16)+(4)+	1·10	25·75
	1·08	14·45
	1·09	19·52
	1·09	19·91

Counterpoise balances T+F+K+N+R+(16)+(4)+1·0902 grain.

PC and h. in w. (B=7·74)≏T+F+K+N+R+(16)+(4)+0·1077 gr. in air (C=8·47, F=752·57, E=8·77).

	PC and hook in water.				In right-hand pan.	
	B.	C.	F.	E.	gr.	Scale.
April 17	7·65	8·15	770·87	8·8	1·00	19·1
					1·04	33·5
					1·00	18·5
					1·02	20·7
	7·66				1·00	20·5
					1·02	20·5
					0·96	18·6
					1·00	19·1
	7·66				1·04	22·8
					1·00	19·5
					0·96	18
					1·00	19
					1·04	22·4

7·85	9·3	770·7	9·4	0·96	17·0
				1·00	19·3
				1·04	22·5
				1·00	18·6
				1·04	23·4
				0·96	18·9
				1·00	20·5
				0·96	18·0
				1·00	20·4
7·73	8·7	770·78	9·1	1·0018	20·49

Counterpoise in air (C=8·7, F=770·78, E=9·1) balances PC and h. in water (B=7·73)+1·0008 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·12	23·7
	1·10	12·15
	1·11	17·93

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1141 grain.

PC and h. in w. (B=7·73)≏T+F+K+L+R+(16)+(4)+0·1133 gr. in air (C=8·7, F=770·78, E=9·1).

May 9. PC and hook in water. In right-hand pan.

B.	C.	F.	F.	gr.	Scale.
10·4	10·3	750·5	10·6	0·98	23·5
				0·94	7·5
				0·98	23·5
				0·97	17·3
				0·98	21·5
10·35				0·97	16·0
				0·98	20·4
				0·98	22·0
				0·98	23·5
				0·97	18·3
10·33	10·7	750·4	10·9	0·98	24·0
				0·97	18·4
				0·98	21·2
10·25				0·97	12·5
				0·98	18·6
				0·99	31·5
				0·98	25·2
				0·97	23·5
10 25	10·9			0·97	11·0
10·3	10·63	750·45	10·75	0·9747	19·97

Counterpoise in air (C=10·63, F=750·45, E=10·75) balances PC and h. in w. (B=10·3)+0·9748 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·13	20·65
	1·13	20·55
	1·13	20·45
	1·13	20·55

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1289 grain.

PC & h. in w. (B=10·3)≏T+F+K+L+R+(16)+(4)+0·1541 gr. in air (C=10·63, F=750·45, E=10·75).

May 10.

		PC and hook in water.		In right-hand pan.	
B.	C.	F.	E.	gr.	Scale.
				0·97	12·4
9·5	9·8	749·6	10·4	1·01	39·4
				0·98	17·6
				0·99	22·2
				0·98	23·0
				0·97	13·2
				0·98	24·4
				0·97	12·9
9·5				0·98	19·4
				0·98	19·4
				0·98	19·5
				0·99	23·0
9·55	10·8	749·6	10·9	0·97	19·8
				0·97	19·0
				0·98	21·5
9·55				0·98	21·5
				0·97	18·7
				0·98	23·1
				0·97	21·0
9·6				0·98	25·0
				0·96	17·1
9·6				0·96	16·9
9·54	10·3	749·6	10·65	0·9773	20·45

Counterpoise in air (C=10·3, F=749·6, E=10·65) balances PC and hook in water (B=9·54)+0·9764 gr. in air.

	In right-hand pan. gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·13	21·6
	1·13	22·3
	1·12	16·8
	1·13	22·5
	1·13	20·8
	1·128	20·8

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1264 grain.

PC and h. in w. (B=9·54) $\simeq$ T+F+K+L+R+(16)+(4)+0·1500 gr. in air (C=10·3, F=749·6, E=10·65).

Apparent weight of PC No. 3 in water, weighed with platinum weights.

Water. t.	Apparent weight of PC No. 3. gr.	Air. t.	b.
9·02	6669·635	8·73	768·13
7·76	6669·610	8·46	751·95
7·75	6669·614	8·69	770·09
10·28	6669·613	10·64	749·59
9·52	6669·609	10·3	748·76

Resulting values of ΔPC No. 3 and log ΔPC No. 3.

	ΔPC No. 3.	log ΔPC No. 3.
	21·1623	1·325563
	21·1622	1·325561
	21·1618	1·325555
	21·1602	1·325519
	21·1609	1·325534
	21·1615	1·325546
10−log Δ..........		8·674454
PC No. 3 (reduced)	6999·999	3·845098
vPC No. 3	330·790	2·519552

Observations for finding the density of PC No. 4.

Weighing of PC No. 4 in air.

June 2, 3, 1845. 100 parts =0·26 grain.

PC+Y.	T+C+0·68 gr.+X.		PC+X.	T+C+068 gr.+Y.
25·25	25·25		24·9	25·8
25·65	26·75		25·5	26·2
25·8	26·6		25·3	26·0
24·9	25·7		27·1	27·1
24·5	25·5		28·8	26·5
24·3	25·0		27·8	26·8
23·8	23·7		28·3	27·9
23·0	23·35	D= 12·5	28·25	27·55
22·4	22·9	C= 12·48	20·25	20·7
21·5	22·8	F=752·93	21·5	19·5
20·9	21·3	E= 13·97	20·8	18·7
20·75	21·0		19·3	20·7
19·9	20·6		21·35	19·6
22·0	24·8		20·7	19·6
21·8	22·25		25·5	23·2
22·1	23·3		25·1	23·5
22·35	22·7		24·0	23·55
22·0	23·6		23·5	22·45
21·7	23·0		23·65	21·9
435·6	450·1		461·6	447·25

19(PC+Y)≏19(T+C+0·68 gr.+X)−14·5 parts. 19(PC+X)≏19(T+C+0·68 gr.+Y)+14·13 parts.

PC No. 4≏T+C+0·67999 grain in air (t=12·51, b=751·68).

PC No. 4≏7000·03848 grains of platinum in air (t=12·51, b=751·68).

For PC No. 4, log Δ=1·32543. For T+C+0·680 grain, log Δ=1·32566. vPC No. 4=330·88, v(T+C+0·68)=330·70. Therefore vPC No. 4 is greater than v(T+C+0·680 grain) by the volume of 0·18 grain of water, or of 0·00022 grain of air. Hence

PC No. 4=7000·03870 grains.

Weighing of PC No. 4 in water.

March 26, 1845. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
8·93	7·5	759·3	7·9	3·00	21·65
				2·96	12·5
8·92		795·15	8·0	2·99	18·5
				3·00	21·7
8·85				3·00	22·0
8·5	7·85	759·5	8·0	3·00	21·1
				2·98	19·5
				2·96	17·2
				3·00	21·8
8·7	7·67	759·36	7·97	2·9878	19·55

Counterpoise in air (C=7·67, F=759·36, E=7·97) balances PC and hook in water (B=8·7)+2·9887 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+N+K+S+(16)+(4)+	3·00	13·6
	3·04	35·8
	3·00	15·4
	3·04	35·6
	3·02	25·1

Counterpoise balances T+F+N+K+S+(16)+(4)+3·0103 grains.

PC and h. in w. (B=8·7) ≏ T+F+N+K+S+(16)+(4)+0·0216 gr. in air (C=7·67, F=759·36, E=7·97).

March 27.	PC and hook in water.			In right-hand pan.	
B.	C.	F.	E.	gr.	Scale.
6·8	7·4	757·9	7·9	1·10	20·1
				1·10	20·8
				1·10	24·0
				1·10	23·6
				1·06	17·0
6·85				1·04	9·5
				1·10	20·0
		757·9	8	1·10	21·0
6·95				1·07	17·7
				1·08	18·2
6·97	8	757·9	8	1·10	19·1
				1·10	20·0
				1·10	21·2
6·89	7·7	757·9	7·97	1·0885	19·4

Counterpoise in air (C=7·7, F=757·9, E=7·97) balances PC and hook in water (B=6·89)+1·0898 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+N+S+(16)+(4)+	1·100	23·92
	1·080	14·45
	1·00	24·3
	1·09	20·89

Counterpoise balances T+F+K+N+S+(16)+(4)+1·0882 grain.

PC and h. in w. (B=6·89) ≏ T+F+K+N+S+(16)+(4)−0·0003 gr. in air (C=7·7, F=757·9, E=7·97).

April 16.	PC and hook in water.			In right-hand pan.	
B.	C.	F.	E.	gr.	Scale.
7·87	8	771·3	8·5	1·10	20·9
				1·08	18·4
7·87				1·10	21·6
				1·08	19·8
7·85				1·09	19·8
				1·12	25·5
				1·04	16·0
7·85	8·35			1·08	18·6
				1·12	22·5
				1·08	18·5
		770·9	8·8	1·12	31·0
				1·08	19·0
				1·08	18·7
7·85	8·8	770·75	8·9	1·08	18·3
				1·12	23·0
				1·08	18·6
				1·08	19·5
				1·12	22·0
				1·08	19·7
				1·08	19·0

				1·12	21·0
				1·08	20·5
7·87	8·8			1·08	19·0
7·86	8·44	770·92	8·77	1·0909	20·47

Counterpoise in air (C=8·44, F=770·93, E=8·77) balances PC and h. in w. (B=7·86)+1·0899 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·10	15·95
	1·11	22·65
	1·10	16·55
	1·11	23·5
	1·105	19·66

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1057 grain.
PC and h. in w. (B=7·86)≏T+F+K+L+R+(16)+(4)+0·0158 gr. in air (C=8·44, F=770·92, E=8·77).

April 29. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
11·1	10·5	761·53	11	1·10	27·0
				1·06	14·7
				1·10	27·5
				1·06	17·9
				1·08	20·4
				1·08	25·0
				1·08	20·5
11·05				1·06	17·1
				1·08	22·0
				1·06	21·3
				1·06	21·3
11·03				1·04	19·0
11·01		761·57	11·5	1·06	22·6
				1·04	20·5
				1·00	12·3
				1·04	20·5
				1·02	16·7
11	11·55			1·04	20·0
11·04	10·84	761·55	11·2	1·0589	20·35

Counterpoise in air (C=10·84, F=761·55, E=11·25) balances PC and h. in w. (B=11·04)+1·0577 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·18	23·7
	1·16	20·7
	1·16	17·0
	1·16	15·0
	1·18	24·4
	1·168	20·18

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1674 grain.
PC and h. in w. (B=11·04)≏T+F+K+L+R+(16)+(4)+0·1097 gr. in a. (C=10·85, F=761·55, E=11·25).

April 30. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
10	10·7	763·2	11·1	1·08	25·0
				1·04	17·5
				1·08	23·5
				1·04	13·0
				1·08	22·9
10·05				1·04	12·5
				1·07	21·5

				1·04	11·4
				1·07	19·0
				1·08	20·5
10·07				1·07	19·5
				1·08	22·1
10·1	11·7	763·1	11·9	1·08	21·6
				1·07	20·5
10·13				1·06	18·0
				1·07	19·7
				1·06	18·4
10·2				1·07	21·4
				1·06	19·2
				1·07	20·4
				1·06	19·1
				1·07	21·6
10·3	11·9	762·7	11·9	1·07	22·0
10·12	11·25	763·05	11·63	1·0657	19·58

Counterpoise in air (C=11·25, F=763·05, E=11·63) balances PC and hook in water (B=10·12)+1·0671 gr.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·18	20·3
	1·18	23·6
	1·18	22·5
	1·17	17·6
	1·1775	21

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1745 grain.

PC and h. in w. (B=10·12) $\bumpeq$ T+F+K+L+R+(16)+(4)+0·1072 gr. in air (C=11·25, F=763·05, E=11·63).

May 20.	B.	C.	F.	E.	gr.	Scale.
	10·5	9·9	759·3	10·4	1·02	14·2
					1·04	29·7
					1·04	19·0
					1·04	25·0
					1·04	26·6
					1·00	5·3
					1·04	30·1
					1·00	8·7
May 21.					1·04	17·8
					1·04	19·8
					1·04	21·5
					1·04	23·0
	9·35	10	754·5	10·2	1·03	17·2
					1·03	16·9
					1·04	21·2
					1·04	21·2
					1·03	17·4
			753·25	10·5	1·03	18·5
					1·04	20·4
					1·04	21·2
					1·03	18·1
					1·04	21·5
					1·03	19·0
	9·5	10·6			1·04	21·5
					1·03	19·8
					1·04	22·2
					1·03	19·7
	9·78	10·17	755·68	10·37	1·0333	19·5

(PC and hook in water: B. In right-hand pan: C., F., E., gr., Scale.)

Counterpoise in air (C=10·17, F=755·68, E=10·37) balances PC and h. in w. (B=9·78)+1·0344 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·11	21·1
	1·11	19·6
	1·11	17·8
	1·12	23·4
	1·11	18·1
	1·11	17·6
	1·11	17·0
	1·1114	19·23

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1130 grain.

PC & h. in w. (B=9·78) ≏ T+F+K+L+R+(16)+(4)+0·0786 gr. in air (C=10·17, F=755·68, E=10·37).

May 22. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
9·4	9·7	756·5	10·5	1·03	16·3
				1·04	22·5
				1·03	17·0
				1·04	21·9
9·45				1·03	18·0
				1·04	22·5
				1·03	18·4
				1·04	22·8
				1·03	18·5
9·45				1·04	22·7
9·5	9·8	756·8	10·9	1·03	20·4
				1·03	21·1
				1·02	18·05
				1·03	21·05
				1·02	17·55
9·55				1·03	21·0
				1·02	19·6
				1·03	22·8
				1·02	18·7
9·6				1·03	21·55
9·5	9·75	756·65	10·7	1·0305	20·12

Counterpoise in air (C=9·75, F=756·65, E=10·7) balances PC and hook in water (B=9·5)+1·0302 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·12	24·95
	1·11	14·9
	1·12	24·9
	1·10	14·6
	1·12	24·1
	1·11	14·4
	1·10	19·64

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1108 grain.

PC and h. in w. (B=9·5) ≏ T+F+K+L+R+(16)+(4)+0·0806 gr. in air (C=9·75, F=756·65, E=10·7).

Apparent weight of PC No. 4 in water, weighed with platinum weights.

Water. t.	Apparent weight of PC No. 4. gr.	Air. t.	Air. b.
8·71	6669·524	7·65	758·82
6·92	6669·503	7·68	757·36
7·88	6669·516	8·43	770·27
11·01	6669·568	10·85	760·61
10·11	6669·566	11·26	762·06
9·76	6669·537	10·17	754·85
9·49	6669·539	9·75	755·79

Resulting values of ΔPC No. 4 and log ΔPC No. 4.

	ΔPC No. 4.	log ΔPC No. 4.
	21·1553	1·325420
	21·1552	1·325417
	21·1551	1·325416
	21·1556	1·325426
	21·1567	1·325449
	21·1554	1·325422
	21·1558	1·325430
	21·1556	1·325426
10 − log Δ		8·674574
PC No. 4 (reduced)	6999·998	3·845098
*v*PC No. 4............	330·881	2·519672

Comparison of PS *with each of the weights* T+Q+A, T+Q+B, T+Q+C, T+Q+D.

The weights Q were changed, so that each of the ten weights Q was used the same number of times in each series of weighings. The cistern of the barometer was 305 mm. above the weights. Therefore 0·03 mm. must be added to F.

June 13, 1846. 100 parts = 0·31041 grain.

PS+X, T+Q+A+Y.	T+Q+A+Y, PS+X.		PS+Y, T+Q+A+X.	T+Q+A+X, PS+Y.
18·60	15·56		17·97	15·91
17·15	15·29		17·42	15·99
18·35	15·74		17·44	15·84
17·75	15·31	D= 19·65	17·91	16·22
17·50	15·55	C= 19·6	16·77	15·92
17·86	14·96	F=767·6	17·75	15·65
17·21	15·04	E= 20·9	17·41	16·01
17·54	15·55		18·19	15·80
18·00	15·39		17·34	16·00
17·50	14·81		18·02	15·87
177·46	153·20		176·22	159·21

20(PS+X) ≏ 20(T+Q+A+Y) − 24·26 parts. 20(PS+Y) ≏ 20(T+Q+A+X) − 17·01 parts.

40PS ≏ 40(T+Q+A) − 0·12811 grain in air (t=19·68, b=765·49).

June 24. 100 parts = 0·3006 grain.

PS+Y, T+Q+A+X.	T+Q+A+X, PS+Y.		PS+X, T+Q+A+Y.	T+Q+A+Y, PS+X.
15·71	14·25		14·49	13·35
15·87	14·06	D= 19·65	18·44	16·90
14·92	14·17	C= 19·65	18·72	16·74
15·29	13·86	F=748·5	18·34	16·92
14·71	13·70	E= 20·3	17·74	16·85
76·50	70·04		87·73	80·76

10(PS+Y) ≏ 10(T+Q+A+X) − 6·46 parts. 10(PS+X) ≏ 10(T+Q+A+Y) − 6·97 parts.

20PS ≏ 20(T+Q+A) − 0·04038 grain in air (t ≏ 19·70, b=746·53).

June 29. 100 parts = 0·29539 grain.

PS+Y, T+Q+A+X.	T+Q+A+X, PS+Y.		PS+X, T+Q+A+Y.	T+Q+A+Y, PS+X.
16·95	15·70		19·02	15·49
16·92	15·82	D= 18·8	18·85	15·90
16·41	14·25	C= 18·8	18·39	15·86
16·35	14·17	F=758·0	18·25	15·30
16·46	14·52	E= 20	18·55	16·65
83·09	74·46		93·06	79·20

10(PS+Y) ≏ 10(T+Q+A+X) − 8·63 parts. 10(PS+X) ≏ 10(T+Q+A+Y) − 13·86 parts.

20PS ≏ 20(T+Q+A) − 0·06643 grain in air (t=18·83, b=756·03).

June 12. 100 parts=0·30735 grain.

PS+Y, T+Q+B+X.	T+Q+B+X, PS+Y.		PS+X, T+Q+B+Y.	T+Q+B+Y, PS+X.
16·74	18·17		17·99	15·26
17·00	17·52		17·56	15·09
17·12	17·11		17·60	14·94
17·27	17·96	D= 19·05	17·65	15·32
17·00	17·10	C= 19·1	17·91	14·44
16·80	17·80	F=769·4	16·92	15·06
17·20	17·61	E= 20·4	16·97	14·67
16·86	17·82		17·10	14·87
16·66	17·21		17·77	15·44
16·16	16·97		17·64	14·75
168·81	175·27		175·11	149·84

$20(PS+Y) \bumpeq 20(T+Q+B+X)+6{\cdot}46$ parts. $20(PS+X) \bumpeq 20(T+Q+B+Y)-25{\cdot}27$ parts.

$40PS \bumpeq 40(T+Q+B)-0{\cdot}05781$ grain in air ($t=19{\cdot}12$, $b=767{\cdot}35$).

June 24. 100 parts=0·30065 grain.

PS+X, T+Q+B+Y.	T+Q+B+Y, PS+X.		PS+Y, T+Q+B+X.	T+Q+B+X, PS+Y.
18·46	17·21		18·06	15·52
17·94	17·27	D= 19·65	17·36	14·62
17·97	16·99	C= 19·68	17·42	15·06
18·46	16·97	F=748·5	17·95	15·29
17·60	17·02	E= 20·3	17·45	15·20
90·43	85·46		88·24	75·69

$10(PS+X) \bumpeq 10(T+Q+B+Y)-4{\cdot}97$ parts. $10(PS+Y) \bumpeq 10(T+Q+B+X)-12{\cdot}55$ parts.

$20PS \bumpeq 20(T+Q+B)-0{\cdot}05267$ grain in air ($t=19{\cdot}7$, $b=746{\cdot}53$).

June 29. 100 parts=0·29539 grain.

PS+X, T+Q+B+Y.	T+Q+B+Y, PS+X.		PS+Y, T+Q+B+X.	T+Q+B+X, PS+Y.
18·84	17·50		17·37	16·65
18·35	17·09	D= 18·8	17·36	17·19
18·31	16·96	C= 18·8	18·16	17·05
17·47	16·44	F =758·0	17·17	16·91
17·85	16·46	E= 20	17·54	16·69
90·82	84·45		87·60	84·49

$10(PS+X) \bumpeq 10(T+Q+B+Y)-6{\cdot}37$ parts. $10(PS+Y) \bumpeq 10(T+Q+B+X)-3{\cdot}11$ parts.

$20PS \bumpeq 20(T+Q+B)-0{\cdot}02800$ grain in air ($t=18{\cdot}83$, $b=756{\cdot}03$).

June 25. 100 parts=0·29161 grain.

PS+Y, T+Q+C+X.	T+Q+C+X, PS+Y.		PS+X, T+Q+C+Y.	T+Q+C+Y, PS+X.
18·00	17·97		18·47	17·44
17·70	18·51	D= 19·1	18·57	17·07
17·69	18·35	C= 19·1	18·42	16·99
17·70	17·97	F=756·85	18·31	17·47
17·72	18·44	E= 19·75	18·01	17·52
88·81	91·24		91·78	86·49

$10(PS+Y) \bumpeq 10(T+Q+C+X)+2{\cdot}43$ parts. $10(PS+X) \bumpeq 10(T+Q+C+Y)-5{\cdot}29$ parts.

$20PS \bumpeq 20(T+Q+C)-0{\cdot}00834$ grain in air ($t=19{\cdot}15$, $b=754{\cdot}92$).

June 27. 100 parts=0·29438 grain.

PS+X, T+Q+C+Y.	T+Q+C+Y, PS+X.		PS+Y, T+Q+C+X.	T+Q+C+X, PS+Y.
19·94	18·05		17·40	15·52
19·57	18·30		17·90	16·57
19·32	17·91		16·84	16·40
18·59	17·70	D= 18·6	17·42	16·17
18·17	17·21	C= 18·6	16·94	15·92
17·77	16·49	F=755·7	16·80	14·70
18·07	16·71	E= 19·1	16·74	16·01
17·42	16·44		16·37	15·84
17·91	16·00		16·70	15·50
17·60	16·99		16·27	15·30
184·36	171·80		169·38	157·93

$20(PS+X) \bumpeq 20(T+Q+C+Y)-12{\cdot}56$ parts. $20(PS+Y) \bumpeq 20(T+Q+C+X)-11{\cdot}45$ parts.

$40PS \bumpeq 40(T+Q+C)-0{\cdot}07068$ grain in air ($t=18{\cdot}65$, $b=753{\cdot}85$).

June 30. 100 parts=0·29982 grain.

PS+Y, T+Q+C+X.	T+Q+C+X, PS+Y.		PS+X, T+Q+C+Y.	T+Q+C+Y, PS+X.
20·09	18·96		18·64	16·27
19·51	18·75	D= 18·8	18·62	16·39
19·60	18·67	C= 18·85	18·30	15·62
19·06	17·85	F=761·2	17·69	15·55
18·89	18·44	E= 19·9	18·36	15·36
97·15	92·67		91·61	79·19

$10(PS+Y) \bumpeq 10(T+Q+C+X)-4{\cdot}48$ parts. $10(PS+X) \bumpeq 10(T+Q+C+Y)-12{\cdot}42$ parts.

$20PS \bumpeq 20(T+Q+C)-0{\cdot}05067$ grain in air ($t=18{\cdot}87$, $b=759{\cdot}23$).

June 5 and 6. 100 parts=0·29998 grain.

PS+Y, T+Q+D+X.	T+Q+D+X, PS+Y.		PS+X, T+Q+D+Y.	T+Q+D+Y, PS+X.
15·24	14·16		21·59	20·85
14·37	14·25		21·17	20·04
13·86	12·47		20·70	19·89
13·21	12·17	D= 18·3	20·56	19·57
12·65	12·27	C= 18·27	20·51	19·16
11·60	12·56	F=766·8	18·30	18·77
18·90	19·90	E= 19·65	18·36	18·60
18·91	19·70		18·21	18·51
19·26	19·49		18·20	18·17
18·52	19·50		18·44	18·61
156·52	156·47		196·04	192·17

$20(PS+Y) \bumpeq 20(T+Q+D+X)-0{\cdot}05$ part. $20(PS+X) \bumpeq 20(T+Q+D+Y)-3{\cdot}87$ parts.

$40PS \bumpeq 40(T+Q+D)-0{\cdot}01176$ grain in air ($t=18{\cdot}32$, $b=764{\cdot}85$).

June 20. 100 parts=0·31320 grain.

PS+X, T+Q+D+Y.	T+Q+D+Y, PS+X.		PS+Y, T+Q+D+X.	T+Q+D+X, PS+Y.
18·92	17·32		19·55	18·34
18·52	19·31		19·76	18·42
18·82	18·80		19·47	18·11
18·36	19·01	D= 21·25	18·89	17·61
18·42	18·42	C= 21·25	18·97	17·94
17·95	18·77	F=768·9	18·70	17·81
18·82	18·46	E= 22·4	18·44	17·95
18·76	18·30		18·96	17·85
20·22	19·59		19·16	18·11
19·71	18·84		19·09	17·85
188·50	186·82		190·99	179·99

$20(PS+X) \bumpeq 20(T+Q+D+Y)-1{\cdot}68$ part. $20(PS+Y) \bumpeq 20(T+Q+D+X)-11{\cdot}0$ parts.

$40PS \bumpeq 40(T+Q+D)-0{\cdot}03971$ grain in air ($t=21{\cdot}31$, $b=766{\cdot}60$).

June 30. 100 parts=0·29981 grain.

PS+X, T+Q+D+Y.	T+Q+D+Y, PS+X.		PS+Y, T+Q+D+X.	T+Q+D+X, PS+Y.
18·70	16·22		16·16	16·74
17·95	16·32	D= 18·8	16·96	16·92
17·75	16·00	C= 18·85	16·24	16·55
17·24	15·56	F=761·20	17·32	16·84
17·61	15·90	E= 19·9	16·34	16·56
89·25	80·00		83·02	83·61

10(PS+X)≏10(T+Q+D+Y)−9·25 parts. 10(PS+Y)≏10(T+Q+D+X)+0·59 part.

20PS≏20(T+Q+D)−0·02596 grain in air (t=18·87, b=759·23).

	gr.	t.	b.
40PS≏40(T+Q+A)	−0·12811	19·68	765·49
20PS≏20(T+Q+A)	−0·04038	19·70	746·53
20PS≏20(T+Q+A)	−0·06643	18·83	756·03
40PS≏40(T+Q+B)	−0·05781	19·12	767·35
20PS≏20(T+Q+B)	−0·05267	19·7	746·53
20PS≏20(T+Q+B)	−0·02800	18·83	756·03
20PS≏20(T+Q+C)	−0·00834	19·15	750·60
40PS≏40(T+Q+C)	−0·07068	18·65	753·85
20PS≏20(T+Q+C)	−0·05067	18·87	759·23
40PS≏40(T+Q+D)	−0·01176	18·32	764·85
40PS≏40(T+Q+D)	−0·03971	21·31	766·60
20PS≏20(T+Q+D)	−0·02596	18·87	759·23

Means.

	gr.	t.	b.
PS≏T+Q+A	−0·002936	19·47	758·38
PS≏T+Q+B	−0·001731	19·19	759·31
PS≏T+Q+C	−0·001621	18·83	754·38
PS≏T+Q+D	−0·000774	19·63	764·43

Mean of all.

	gr.	t.	b.
PS≏T+Q+¼(A+B+C+D)	−0·00177	19·28	759·12

Or, since A+B+C+D=4F+4G−0·00712 grain,

PS≏T+Q+F+G−0·00355 grain in air (t=19·28, b=759·12).

T+Q+F+G−0·00355 grain =7000·00073 grains.

PS displaces 0·39744 grain of air, T+Q+F+G−0·00355 gr. displaces 0·39727 gr. of air. Hence

PS=T+Q+F+G−0·00338 grain.

Hence, supposing U to have had the same density as V, PS=7000·00093 grains, of which U contained 5760.

Apparent weight of the Commercial Standard.

During the comparison of U with Sp and RS in Somerset House, the mean temperature was 18°·7 C., and the mean height of the barometer, reduced to 0° C., was 755·64 mm. PS displaced 0·38035 grain of air, T+Q+F+G displaced 0·38019 grain of air, when t=18·7, b=755·64. Hence PS≏T+Q+F+G−0·00354 grain in air (t=18·7, b=755·64) in Somerset House.

Let W be a weight of 7000 grains of the same density as U. In air ($t=18{\cdot}7$, $b=755{\cdot}64$) $U \triangleq T+0{\cdot}00745$ grain. Therefore $W \triangleq \frac{7000}{5760}(T+0{\cdot}00745 \text{ grain})$.

$$\frac{7000}{5760}(T+0{\cdot}00745 \text{ gr.})=T+0{\cdot}00745 \text{ gr.}+\frac{124}{576}(T+0{\cdot}00745 \text{ gr.}).$$

$$T=72(R+S)+0{\cdot}00559 \text{ grain, and } 4F+4G=62(R+S)+0{\cdot}01112 \text{ grain.}$$

$$\frac{124}{576}(T+0{\cdot}00745 \text{ gr.})=\frac{62}{4{\cdot}72}[72(R+S)+0{\cdot}01303 \text{ gr.}]=\frac{62}{4}(R+S)+0{\cdot}00281 \text{ gr.}$$
$$=F+G+0{\cdot}00003 \text{ gr.}$$

Hence $$\frac{7000}{5760}[T+0{\cdot}00745 \text{ grain}]=T+F+G+0{\cdot}00748 \text{ grain.}$$

Therefore $$W \triangleq T+F+G+0{\cdot}00748 \text{ grain.}$$

Hence in Somerset House, $PS \triangleq W+Q-0{\cdot}01102$ gr. in air ($t=18{\cdot}7$, $b=755{\cdot}64$).

$Q-0{\cdot}01102$ grain $=0{\cdot}63407$ grain. The weight of 0·63407 grain was adjusted by PS. This, in air ($t=18{\cdot}7$, $b=755{\cdot}64$), is equivalent to a weight of 0·63413 grain adjusted by W.

The commercial standard lb. is a brass weight which in air ($t=18{\cdot}7$, $b=755{\cdot}64$), or at the temperature 65°·66 FAHRENHEIT, under the pressure of 29·750 inches of mercury at 32° FAHRENHEIT, in Somerset House, for which $\log \Delta=7{\cdot}078324-10$, appears to weigh as much as W, or as much as $PS-Q+0{\cdot}01102$ grain. For in air having the above-mentioned temperature and pressure, the apparent weight of such a lb. would be $\frac{7000}{5760}$ of that of the lost standard. This result is not affected by the uncertainty of the value of the specific gravity of the lost standard.

In the following comparisons of the platinum copies of the pound with PS, I, K, L, M, N will be used to denote PS, PC No. 1, PC No. 2, PC No. 3, PC No. 4 respectively. The zero of the scale of the barometer was 305 millimètres above the centre of gravity of the weights.

Comparison of PC No. 1 *with* PS.

February 16, 1846.	100 parts=0·26222 grain.		February 18.	100 parts=0·27683 grain.
K+Y, I+X.	I+X, K+Y.		K+X, I+Y.	I+Y, K+X.
20·14	20·30		15·32	17·76
20·44	20·61		15·29	17·40
21·25	20·94		15·51	17·41
20·64	19·61	D= 16·05	15·80	17·67
20·89	20·94	C= 16·1	16·96	17·75
20·01	20·85	F=766·8	16·46	17·24
20·57	19·46	E= 16·42	15·35	17·52
20·04	19·82		16·24	16·62
20·70	19·75		16·60	16·46
19·65	19·81		16·27	16·94
204·33	202·09		159·80	172·77

$20(K+Y) \triangleq 20(I+X)-2{\cdot}24$ parts. $\qquad$ $20(K+X) \triangleq 20(I+Y)+12{\cdot}97$ parts.

$40K \triangleq 40I+0{\cdot}03031$ grain in air ($t=16{\cdot}09$, $b=765{\cdot}25$).

February 20. 100 parts = 0·22624 grain.

K+Y, I+X.	I+X, K+Y.		K+X, I+Y.	I+Y, K+X.
21·70	23·17		18·09	19·79
23·27	22·27		18·55	19·02
22·22	22·86		19·11	20·25
22·29	23·07	D = 9·3	19·70	20·40
21·66	21·20	C = 9·5	18·54	20·74
21·26	20·77	F = 766·8	19·16	20·39
20·95	21·24	E = 16·8	19·12	20·35
20·50	20·26		19·02	19·72
19·62	20·36		18·99	20·07
20·55	19·65		19·50	20·59
214·02	214·85		189·78	201·32

$20(K+Y) \bumpeq 20(I+X) + 0·83$ part. $20(K+X) \bumpeq 20(I+Y) + 11·54$ parts.

$40K \bumpeq 40I + 0·02799$ grain in air ($t = 9·4$, $b = 765·30$).

February 25. 100 parts = 0·29063 grain.

K+X, I+Y.	I+Y, K+X.		K+Y, I+X.	I+X, K+Y.
20·35	21·56		17·10	17·15
19·85	21·67		16·05	16·66
19·36	21·00		17·45	17·51
18·84	20·69	D = 18·05	16·81	17·70
18·62	19·92	C = 18·1	16·57	16·45
18·67	19·75	F = 753·05	17·19	17·51
18·90	19·12	E = 18·5	16·69	17·77
18·62	18·97		16·50	17·42
18·37	19·06		16·92	16·42
18·21	18·22		16·76	16·49
189·79	199·96		168·04	171·08

$20(K+X) \bumpeq 20(I+Y) + 10·17$ parts. $20(K+Y) \bumpeq 20(I+X) + 3·04$ parts.

$40K \bumpeq 40I + 0·03839$ grain in air ($t = 18·08$, $b = 751·28$).

March 6. 100 parts = 0·29781 grain.

K+Y, I+X.	I+X, K+Y.		K+X, I+Y.	I+Y, K+X.
18·00	19·00		15·22	15·36
17·34	17·31		12·85	13·22
17·41	18·06		20·41	20·36
16·32	17·07	D = 17·85	20·05	19·50
16·07	17·57	C = 17·85	20·12	19·37
16·35	16·65	F = 755·3	19·05	19·17
16·10	16·24	E = 18·35	18·07	18·42
15·80	16·37		17·47	17·42
15·54	16·46		16·54	16·55
16·04	15·97		15·80	15·62
164·97	170·70		175·58	174·99

$20(K+Y) \bumpeq 20(I+X) + 5·73$ parts. $20(K+X) \bumpeq 20(I+Y) - 0·59$ part.

$40K \bumpeq 40I + 0·01531$ grain in air ($t = 17·88$, $b = 753·54$).

April 20. 100 parts=0·24107 grain.

K+X, I+Y.	I+Y, K+X.		K+Y, I+X.	I+X, K+Y.
20·09	21·27		18·71	19·11
20·65	21·30		18·42	18·47
20·62	21·25		18·52	18·75
20·71	21·17	D= 12·25	18·46	19·01
20·52	21·44	C= 12·3	17·65	18·89
19·99	20·00	F=766·2	17·76	18·00
19·34	20·79	E= 13	18·27	18·35
19·57	20·50		18·00	18·50
19·27	19·71		18·55	18·97
19·20	20·00		18·45	19·09
199·97	207·43		182·79	187·14

$20(K+X) \triangleq 20(I+Y)+7·47$ parts. $20(K+Y) \triangleq 20(I+X)+4·35$ parts.

$40K \triangleq 40I+0·02849$ grain in air (t=12·3, b=765·07).

gr.	t.	b.
40K≏ 40I+0·03003	16·09	765·25
40K≏ 40I+0·02799	9·4	765·30
40K≏ 40I+0·03839	18·08	751·28
40K≏ 40I+0·01531	17·88	753·54
40K≏ 40I+0·02849	12·3	765·07
200K≏200I+0·14021		

PC No. 1≏PS+0·00070 grain in air (t=14·75, b=760·09).

PC No. 1 displaces 0·40452 grain of air, PS displaces 0·40471 grain. Therefore

PC No. 1=PS+0·00051 grain.

Comparison of PC No. 2 *with* PS.

February 11, 12. 100 parts=0·25720 grain.

L+X, I+Y.	I+Y, L+X.		L+Y, I+X.	I+X, L+Y.
16·36	15·89		15·61	14·81
17·69	16·14		14·66	15·44
17·10	15·60		15·64	15·46
16·74	15·59	D= 14·95	14·41	13·54
16·94	15·04	C= 15	13·56	13·17
15·19	14·00	F=762·52	17·04	17·06
15·30	13·41	E= 15·42	16·26	17·01
14·95	13·37		15·61	16·16
14·61	13·10		15·87	15·77
14·81	13·24		14·75	15·77
159·69	145·38		153·41	154·19

$20(L+X) \triangleq 20(I+Y)-14·31$ parts. $20(L+Y) \triangleq 20(I+X)+0·78$ part.

$40L \triangleq 40I-0·03480$ grain in air (t=15·0, b=761·1).

March 13, 14. 100 parts=0·29530 grain.

L+X, I+Y.	I+Y, L+X.		L+Y, I+X.	I+X, L+Y.
23·25	24·55		22·30	20·62
23·81	24·06		22·59	21·10
23·34	23·21		21·85	20·50
22·35	22·66	D= 17·55	21·49	20·00
23·16	22·99	C= 17·55	20·55	19·55
21·87	23·16	F=773·5	19·72	19·47
22·91	23·00	E= 18	21·14	19·54
22·11	21·62		20·89	18·91
22·36	23·81		19·67	18·32
22·22	23·72		21·04	19·66
21·86	22·85		20·85	19·17
249·24	255·63		232·09	216·84

22(L+X)≏22(I+Y)+6·39 parts. 22(L+Y)≏22(I+X)−15·25 parts.

44L≏44I−0·02616 grain in air (t=17·58, b=771·73).

March 17. 100 parts=0·26284 grain.

L+X, I+Y.	I+Y, L+X.		L+Y, I+X.	I+X, L+Y.
21·35	22·07		20·76	18·81
20·37	19·92		20·89	19·02
20·60	19·82		19·66	18·92
18·56	19·12	D= 16·22	20·44	18·47
19·42	19·26	C= 16·25	19·89	17·75
19·00	18·85	F=750·05	20·41	19·00
17·85	19·19	E= 17·1	21·19	19·36
18·02	18·74		20·42	18·36
17·34	17·94		20·51	19·84
16·50	16·86		20·62	19·84
14 59	15·55		20·52	19·40
203·60	207·31		225·31	208·77

22(L+X)≏22(I+Y)+3·71 parts. 22(L+Y)≏22(I+X)−17·54 parts.

44L≏44I−0·03635 grain in air (t=16·25, b=748·46).

April 1. 100 parts=0·27933 grain.

L+Y, I+X.	I+X, L+Y.		L+X, I+Y.	I+Y, L+X.
19·40	20·75		17·44	14·86
19·39	20·47		17·12	14·47
18·62	20·92		16·49	14·60
18·02	20·16	D= 16·2	16·54	14·96
17·64	19·16	C= 16·3	16·02	14·17
17·84	18·87	F=750·7	16·49	14·92
17·29	18·80	E= 18·6	16·52	14·65
17·37	18·51		16·72	15·29
16·56	17·41		16·42	15·15
16·29	17·64		16·62	16·00
16·04	17·36		16·72	15·42
194·49	210·05		183·10	164·49

22(L+Y)≏22(I+X)+15·56 parts. 22(L+X)≏22(I+Y)−18·61 parts.

44L≏44I−0·00852 grain in air (t=16·27, b=748·93).

April 8. 100 parts=0·24979 grain.

L+X, I+Y.	I+Y, L+X.		L+Y, I+X.	I+X, L+Y.
24·39	24·05		19·64	19·72
24·72	23·51		20·55	19·70
25·07	24·69		19·49	19·51
24·80	24·59	D= 15·02	19·84	19·04
24·52	21·70	C= 15·02	19·67	18·71
24·60	22·90	F=745·2	20·17	19·80
24·42	22·37	E= 15·9	19·94	19·46
24·25	22·19		20·47	19·17
24·12	22·55		20·79	19·45
23·46	22·84		20·16	19·42
24·00	21·16		20·24	19·34
268·35	252·55		220·96	213·32

22(L+X)≏22(I+Y)−15·80 parts. 22(L+Y)≏22(I+X)−7·64 parts.

44L≏44I−0·05855 grain in air (t=15·04, b=743·77).

gr.	t.	b.
40L≏ 40I−0·03480	15·0	761·1
44L≏ 44I−0·02616	17·58	771·73
44L≏ 44I−0·03635	16·25	748·46
44L≏ 44I−0·00852	16·27	748·93
44L≏ 44I−0·05855	15·04	743·77
216L≏216I−0·16438		

PC No. 2≏PS−0·00076 grain in air (t=16·12, b=754·68).

PC No. 2 displaces 0·39966 grain of air, PS displaces 0·39979 grain. Hence

PC No. 2=PS−0·00089 grain.

Comparison of PC No. 3 *with* PS.

February 9, 10. 100 parts=0·26146 grain.

M+Y, I+X.	I+X, M+Y.		M+X, I+Y.	I+Y, M+X.
17·65	17·54		17·64	16·26
17·15	17·52		17·86	16·07
16·31	16·42	D= 15·15	17·02	14·90
16·44	16·25	C= 15·25	17·85	15·22
16·04	15·56	F=771·1	17·79	15·79
15·47	15·46	E= 15·5	17·06	14·96
16·52	15·09		16·69	14·77
15·49	16·00		15·80	14·05
14·99	16·15		16·29	14·09
16·17	15·95		15·42	14·17
162·23	161·94		169·42	150·28

20(M+Y)≏20(I+X)−0·29 part. 20(M+X)≏20(I+Y)−19·14 parts.

40M≏40I−0·05080 grain in air (t=15·22, b=769·64).

February 13. 100 parts $=0{\cdot}26281$ grain.

M+X, I+Y.	I+Y, M+X.
23·49	21·00
23·55	21·11
22·10	20·77
22·65	20·82
23·57	20·27
21·99	20·12
22·25	20·55
22·41	20·54
22·01	21·27
21·59	21·30
225·61	207·75

D= 15·73
C= 15·82
F=767·55
E= 16·05

February 14. 100 parts $=0{\cdot}26683$ grain.

M+Y, I+X.	I+X, M+Y.
19·89	19·01
19·99	19·40
19·27	18·14
19·26	17·47
19·55	18·34
18·85	17·65
18·12	17·70
17·95	16·41
18·12	17·24
17·91	17·14
188·91	178·50

$20(M+X) \triangleq 20(I+Y)-17{\cdot}86$ parts. $20(M+Y) \triangleq 20(I+X)-10{\cdot}41$ parts.

$40M \triangleq 40I-0{\cdot}07472$ grain in air ($t=15{\cdot}8$, $b=766{\cdot}04$).

March 18. 100 parts $=0{\cdot}24265$ grain.

M+Y, I+X.	I+X, M+Y.		M+X, I+Y.	I+Y, M+X.
16·80	16·40		18·99	17·24
17·71	14·65		19·06	17·82
17·92	15·94		19·44	17·76
17·54	14·40	D= 15·13	19·39	17·96
16·66	15·16	C= 15·22	19·06	16·76
16·07	14·40	F=752·4	18·25	17·00
16·39	14·00	E= 15·9	18·24	17·05
15·52	13·36		18·64	16·85
14·92	13·42		17·25	17·01
15·35	13·75		17·57	16·15
164·88	145·48		185·89	171·60

$20(M+Y) \triangleq 20(I+X)-19{\cdot}4$ parts. $20(M+X) \triangleq 20(I+Y)-14{\cdot}29$ parts.

$40M \triangleq 40I-0{\cdot}08176$ grain in air ($t=15{\cdot}19$, $b=750{\cdot}95$).

March 19. 100 parts $=0{\cdot}24331$ grain.

M+X, I+Y.	I+Y, M+X.		M+Y, I+X.	I+X, M+Y.
22·79	21·66		17·54	17·11
23·35	21·60		17·27	15·17
23·04	22·27		17·45	15·54
22·50	21·76	D= 14·0	16·77	14·84
23·12	21·65	C= 14·08	15·97	14·09
22·82	21·57	F=753·2	15·75	13·97
22·30	21·51	E= 14·45	15·74	14·64
22·21	21·42		15·92	13·57
21·40	20·65		15·24	13·40
20·15	20·01		15·64	14·50
223·68	214·10		163·29	146·83

$20(M+X) \triangleq 20(I+Y)-9{\cdot}58$ parts. $20(M+Y) \triangleq 20(I+X)-16{\cdot}46$ parts.

$40M \triangleq 40I-0{\cdot}06336$ grain in air ($t=14{\cdot}06$, $b=751{\cdot}92$).

April 11. 100 parts =0·23525 grain.

M+Y, I+X.	I+X, M+Y.		M+X, I+Y.	I+Y, M+X.
23·40	21·36		17·26	14·67
23·45	21·81		16·90	15·66
23·41	22·11		16·82	15·60
22·27	20·89		17·37	15·69
18·04	16·45	D= 14·0	17·47	15·57
18·21	16·52	C= 14·0	16·47	15·65
17·80	16·40	F=750·2	15·89	15·75
17·10	15·25	E= 14·7	15·02	15·42
17·20	14·94		16·79	15·62
17·47	16·00		16·41	15·12
17·62	15·20		17·55	15·99
215·97	196·93		183·95	170·74

22(M+Y)≏22(I+X)−19·04 parts. 22(M+X)≏22(I+Y)−13·21 parts.

44M≏44I−0·07587 grain in air (t=14·02, b=748·89).

gr.	t.	b.
40M≏ 40I−0·05080	15·22	769·64
40M≏ 40I−0·07472	15·80	766·04
40M≏ 40I−0·08176	15·19	750·95
40M≏ 40I−0·06336	14·06	751·92
44M≏ 44I−0·07587	14·02	748·89
204M≏204I−0·34651		

PC No. 3≏PS−0·00170 grain in air (t=14·84, b=757·32).

PC No. 3 displaces 0·40302 grain of air, PS displaces 0·40310 grain. Therefore

PC No. 3=PS−0·00178 grain.

Comparison of PC No. 4 *with* PS.

February 7. 100 parts =0·29952 grain.

N+X, I+Y.	I+Y, N+X.		N+Y, I+X.	I+X, N+Y.
22·42	19·56		19·34	19·35
22·40	19·12		19·85	20·02
21·76	18·30		20·22	19·57
20·85	17·77	D= 17·4	19·67	20·04
21·14	17·10	C= 17·45	20·79	19·35
19·69	17·06	F=757·4	20·01	19·45
19·46	16·55	E= 17·65	20·20	18·89
20·20	16·97		19·84	19·04
20·00	15·72		20·09	19·24
19·89	16·01		20·25	19·85
207·81	174·16		200·26	194·80

20(N+X)≏20(I+Y)−33·65 parts. 20(N+Y)≏20(I+X)−5·46 parts.

40N≏40I−0·11715 grain in air (t=17·46, b=755·72).

March 23. · 100 parts =0·22348 grain.

N+Y, I+X.	I+X, N+Y.		N+X, I+Y.	I+Y, N+X.
20·49	17·81		16·41	13·50
20·61	17·35		17·21	14·20
21·05	18·07		17·49	14·31
21·17	18·19	D= 9·25	17·59	13·75
20·07	18·05	C= 9·4	17·47	13·31
20·04	17·49	F=743·0	17·72	14·10
19·15	16·44	E= 14·6	17·27	14·66
18·92	17·37		17·64	13·97
18·45	16·62		17·72	13·74
18·30	16·26		17·86	13·81
198·25	173·65		174·38	139·35

20(N+Y)≏20(I+X)−24·6 parts. 20(N+X)≏20(I+Y)−35·03 parts.

40N≏40I−0·13326 grain in air (t=9·32, b=741·72).

March 30. 100 parts =0·27112 grain.

N+X, I+Y.	I+Y, N+X.		N+Y, I+X.	I+X, N+Y.
20·04	16·10		20·01	18·74
17·45	16·05		19·81	18·07
17·89	15·70		19·11	18·57
17·87	15·05	D= 16	19·95	18·56
22·11	18·50	C= 16	19·60	17·95
21·50	18·30	F=755·0	19·69	17·94
21·30	19·29	E= 17·9	19·70	18·45
20·31	18·49		18·75	18·35
20·91	18·66		17·80	16·31
20·80	17·50		17·20	15·86
200·18	173·64		191·62	178·80

20(N+X)≏20(I+Y)−26·54 parts. 20(N+Y)≏20(I+X)−12·82 parts.

40N≏40I−0·10671 grain in air (t=16·01, b=753·3).

April 13. 100 parts =0·24070 grain.

N+X, I+Y.	I+Y, N+X.		N+Y, I+X.	I+X, N+Y.
18·27	15·11		16·69	13·36
17·77	15·90		16·79	13·12
19·05	16·07		16·57	12·60
18·59	15·52		16·09	12·35
18·46	15·89	D= 14·2	25·15	20·67
17·27	15·19	C= 14·25	20·99	17·45
16·65	14·22	F=753·45	21·47	18·71
16·85	15·17	E= 15·3	20·92	18·45
16·76	14·45		20·77	19·35
17·26	14·91		21·51	19·46
16·56	14·00		21·34	19·49
193·49	166·43		218·29	185·01

22(N+X)≏22(I+Y)−27·06 parts. 22(N+Y)≏22(I+X)−33·28 parts.

44N≏44S−0·14524 grain in air (t=14·24, b=752·06).

April 22. 100 parts $=0.24294$ grain.

N+X, I+Y.	I+Y, N+X.		N+Y, I+X.	I+X, N+Y.
19·40	17·39		17·29	14·55
19·92	16·57		17·71	14·10
19·80	16·95		17·34	14·81
19·80	16·50	D= 12·75	17·67	14·41
19·75	17·01	C= 12·8	17·66	14·59
18·56	16·09	F=760·9	17·86	14·22
18·40	15·22	E= 13·95	17·60	13·70
18·29	15·69		17·34	14·81
18·42	15·26		17·64	15·81
17·36	14·80		17·64	13·71
189·70	161·48		175·75	144·71

$20(N+X) \triangleq 20(I+Y)-28{\cdot}22$ parts. $20(N+Y) \triangleq 20(I+X)-31{\cdot}04$ parts.

$40N \triangleq 40I-0{\cdot}14397$ grain in air ($t=12{\cdot}8$, $b=759{\cdot}67$).

gr.	*t.*	*b.*
$40N \triangleq 40I-0{\cdot}11715$	17·46	755·72
$40N \triangleq 40I-0{\cdot}13326$	9·32	741·72
$40N \triangleq 40I-0{\cdot}10671$	16·01	753·30
$44N \triangleq 44I-0{\cdot}14524$	14·24	752·06
$40N \triangleq 40I-0{\cdot}14397$	12·80	759·67
$204N \triangleq 204I-0{\cdot}64633$		

PC No. 4 $\triangleq$ PS$-0{\cdot}00317$ grain in air ($t=13{\cdot}97$, $b=752{\cdot}49$).

PC No. 4 displaces 0·40184 grain of air, PS displaces 0·40181 grain. Therefore

PC No. 4=PS−0·00314 grain.

Determination of the volume of Professor SCHUMACHER's *pound* Sp+V.

The troy pounds Sp, Sb, K were returned to Professor SCHUMACHER, accompanied by a platinum weight V of about 1240·53 grains which with Sp makes a pound of 7000 grains.

Weighing of V in air.

January 28. 100 parts $=0.26743$ grain. $Z=F+H+R+S+Q$.

Z+0·06 gr. +X, V+Y.	V+Y, Z+X.	Z+0·06 gr. +Y, V+X.	V+X, Z+Y.
18·87	20·05	20·02	20·57
19·55	20·87	20·20	21·37
19·56	20·75	20·70	21·22
19·55	21·25	20·15	21·20
19·77	21·27	20·21	21·10
97·30	104·19	101·28	105·46

$10(V+Y) \triangleq 10(Z+0{\cdot}03\text{ gr.}+X)-6{\cdot}89$ parts. $10(V+X) \triangleq 10(Z+0{\cdot}03\text{ gr.}+Y)-4{\cdot}18$ parts.

$20V \triangleq 20Z+0{\cdot}6\text{ gr.}-0{\cdot}02960$ gr.

$V \triangleq 1240{\cdot}530$ grains of platinum in air.

$vV=59{\cdot}34$, $vZ=58{\cdot}61$, $vV-vZ=0{\cdot}73=$ volume of 0·0009 grain of air. Hence

V=1240·5309 grains.

Weighing of V in water.

The wire by which V was suspended in water was so fine, that the effect of the displacement of the water by it, on the sensibility of the balance, may be neglected. The thermometer B was suspended in the water, with its bulb in a horizontal plane through V. The counterpoise in the left-hand pan is supposed to be in equilibrium with the weights suspended from the right-hand end of the beam, when the reading of the scale is 19.

January 24, 26, 1846. 100 parts = 0·2626 grain.

		V and hook in water.		In right-hand pan.	
B.	C.	F.	E.	gr.	Scale.
12·16				0·64	23·20
				0·62	14·80
				0·63	19·35
	11·1			0·63	18·50
				0·63	18·90
12·15				0·63	19·40
12·05				0·64	20·57
				0·63	18·15
		752·47	15·95	0·64	20·60
				0·63	18·30
12·0	11·35			0·63	18·20
				0·64	20·65
12·0				0·63	19·85
	11·37			0·63	19·50
				0·63	19·57
				0·63	20·10
				0·63	20·05
11·1				0·63	19·90
				0·63	19·85
				0·63	19·85
	11·35	742·2	16	0·63	19·65
11·1				0·63	19·52
11·76	11·29	747·33	16	0·6314	19·52

Counterpoise in air (C=11·29, F=747·33, E=16) balances V and hook in water (B=11·76)+0·630 gr. in air.

In right-hand pan.	
gr.	Scale.
F+H+R+0·06	18·72
+0·06	18·82
+0·06	18·86
+0·06	18·92
+0·06	18·61
0·06	18·79

Counterpoise balances F+H+R+0·0606 grain.

V and hook in water (B=11·76) $\triangleq$ F+H+R−0·5694 grain in air (C=11·29, F=747·33, E=16).

January 26. 100 parts = 0·292 grain.

Hook in water. In right-hand pan.	
gr.	Scale.
1·61	20·00
1·61	20·05
1·61	20·20
1·61	19·40
1·61	20·70
1·61	21·45

1·61	23·05
1·61	22·55
1·61	22·25
1·61	22·75
1·61	21·24

Counterpoise balances hook in water +1·6045 grain in air.

In right-hand pan.

	gr.	Scale.
(16)+(2)+(1)+0·7 gr.	+0·00	20·92
	0·00	20·87
	0·00	20·95
	0·00	21·47
	0·00	21·05

Counterpoise balances (16)+(2)+(1)+0·694 grain in air.

Hook in water ≏(16)+(2)+(1)−0·9105 grain in air.

Hook in water ≏18·0880 grains.

V in water (t=11·67)≏1181·2028 grains of platinum in air (t=11·30, b=745·84).

Hence ΔV=20·8834, log ΔV=1·319797, vV=59·403.

Comparison of Sp+V *with* PS.

The zero of the scale of the barometer was 305 millimètres above the centre of gravity of the weights.

April 15, 1846. 100 parts =0·24256 grain.

PS+Y. Sp+V+X.	Sp+V+X, PS+Y.		PS+X, Sp+V+Y.	Sp+V+Y. PS+X.
19·35	19·92		18·72	19·61
20·30	20·21		18·90	19·75
19·85	20·51		18·35	19·69
20·10	20·32	D= 13·7	18·30	19·40
19·40	20·42	C= 13·8	19·05	19·67
19·35	19·40	F=756·4	18·20	19·74
19·44	19·67	E= 14·1	18·86	19·46
19·30	20·29		18·54	19·61
19·35	20·02		18·84	18·99
19·35	19·20		18·82	19·82
195·79	199·96		186·58	195·74

20(Sp+V+X)≏20(PS+Y)−4·17 parts. 20(Sp+V+Y)≏20(PS+X)−9·16 parts.

40(Sp+V)≏40PS−0·03233 grain in air (t=13·78, b=755·15).

April 17. 100 parts =0·24555 grain.

PS+Y, Sp+V+X.	Sp+V+X, PS+Y.		PS+X, Sp+V+Y.	Sp+V+Y, PS+X.
21·29	19·27		18·74	19·65
20·59	18·94		19·42	20·74
21·09	21·15		19·96	20·50
21·17	20·89	D= 13·55	19·66	20·17
20·35	20·57	C= 13·56	19·84	20·32
20·15	20·15	F=760·65	19·90	20·67
20·06	21·26	E= 14	20·09	20·37
20·35	19·20		20·00	20·60
20·17	18·91		20·05	20·34
19·87	18·89		19·81	20·19
205·09	199·23		197·47	203·55

20(Sp+V+X)≏20(PS+Y)+5·86 parts. 20(Sp+V+Y)≏20(PS+X)−6·08 parts.

40(Sp+V)≏40PS−0·00054 grain in air (t=13·59, b=759·41).

April 18. 100 parts = 0·24021 grain.

PS+Y, Sp+V+X.	Sp+V+X, PS+Y.		PS+X, Sp+V+Y.	Sp+V+Y, PS+X.
24·30	24·69		16·20	17·36
24·76	25·35		16·25	17·30
18·77	19·39		17·47	17·50
19·10	19·74	D= 13·05	17·21	18·14
18·82	19·46	C= 13·1	18·52	19·60
19·10	18·96	F=761·8	19·35	20·62
18·20	19·36	E= 13·4	19·17	19·74
18·06	18·82		19·10	19·52
18·41	19·31		19·42	19·27
18·19	17·97		18·96	19·16
197·71	203·05		181·65	188·21

20(Sp+V+X)≏20(PS+Y)−5·34 parts. 20(Sp+V+Y)≏20(PS+X)−6·56 parts.

40(Sp+V)≏40PS−0·02858 grain in air (t=13·1, b=760·63).

April 21. 100 parts = 0·24160 grain.

PS+X, Sp+V+Y.	Sp+V+Y, PS+X.		PS+Y, Sp+V+X.	Sp+V+X, PS+Y.
21·57	20·41		14·97	14·65
20·97	20·02		14·60	15·47
20·21	19·12		14·40	14·84
19·66	18·70	D= 12·6	14·40	15·37
18·89	18·07	C= 12·63	14·02	15·86
19·29	17·60	F=764·03	14·52	15·39
19·02	17·52	E= 13·9	14·10	16·15
18·75	17·22		14·86	14·97
17·97	16·86		15·05	15·34
17·95	16·77		14·82	15·60
194·28	182·29		145·74	153·64

20(Sp+V+Y)≏20(PS+X)−11·99 parts. 20(Sp+V+X)≏20(PS+Y)−7·90 parts.

40(Sp+V)≏40PS−0·04805 grain in air (t=12·64, b=762·79).

April 23. 100 parts = 0·23834 grain.

PS+X, Sp+V+Y	Sp+V+Y, PS+X.		PS+Y, Sp+V+X.	Sp+V+X, PS+Y.
21·50	21·50		19·66	19·64
20·85	21·80		19·72	20·14
21·14	20·90		19·12	20·04
20·92	21·96	D= 12·3	19·19	20·06
20·11	21·36	C= 12·35	19·39	20·56
20·24	21·11	F=759·0	19·52	20·36
20·12	21·12	E= 12·9	19·90	19·62
19·74	20·76		19·67	19·96
20·11	20·67		19·81	20·21
19·32	20·40		18·91	20·20
204·05	211·58		194·89	200·79

20(Sp+V+Y)≏20(PS+X)−7·53 parts. 20(Sp+V+X)≏20(PS+Y)−5·90 parts.

40(Sp+V)≏40PS−0·03200 grain in air (t=12·35, b=757·43).

gr.	t.	b.
40(Sp+V)≏40PS−0·03233	13·78	755·15
40(Sp+V)≏40PS−0·00054	13·59	759·41
40(Sp+V)≏40PS−0·02858	13·10	760·63
40(Sp+V)≏40PS−0·04805	12·64	762·79
40(Sp+V)≏40PS−0·03200	12·35	757·43

Mean ...... Sp+V≏PS−0·00071 grain in air (t=13·09, b=759·08).

$vSp=271\cdot832$ as computed by means of SCHUMACHER's tables, but vT is $0\cdot014$ less as computed by SCHUMACHER's tables than as computed by the tables used in calculating vV and vPS; therefore probably vSp would be $271\cdot846$ if computed by the latter tables, $vV=59\cdot403$, $vPS=330\cdot856$. Hence $v(Sp+V)=331\cdot249$, $v(Sp+V)-vPS=0\cdot393$.

$v(Sp+V)-vPS=$ volume of $0\cdot00048$ grain of air. Therefore

$$Sp+V=PS-0\cdot00023 \text{ grain.}$$

Comparison of the Pound with the Kilogramme.

The necessity of a new comparison of the kilogramme with English weights appears by the following extract from a paper by Professor MOLL in the Journal of the Royal Institution, No. 4, 1831. "Each of the governments who sent Commissioners to the Committee of Weights and Measures in Paris were presented with a kilogramme and a mètre by the French government. A similar present was made to each of the Commissioners. These weights and measures were fabricated under the eyes of the united Commission and marked with their particular stamp, after being previously examined, it is said, with great care. The kilogramme and the mètre of the late M. VAN SWINDEN are now in my hands. I have compared VAN SWINDEN's kilogramme and several standards of the same weight with the English troy weights of ROBINSON, and I am very sorry to say that the result has left me in an entire darkness as to the real value of the kilogramme.

VAN SWINDEN's kilogramme	15432·295
A kilogramme modèle made by FORTIN and belonging to Government	15432·752
A kilogramme modèle made by GANDOLFI, Balancier de la Monnaye de Paris, and sent during the French occupation of this country by the Parisian Mint to the Mint at Utrecht	15432·730
A kilogramme by the same artist, also belonging to the Mint at Utrecht	15432·752
A kilogramme by NAGEL of Amsterdam, and serving as a standard at the Mint	15432·920
Another, made as a standard for the Royal Institute of Holland . .	15432·985
Another by the same, in my possession	15433·420
Another, also said to be a standard	15434·91
Annuaire du Bureau des Longitudes	15438·355
KELLY	15433."
According to HASSLER (Comparison of Weights and Measures reported to the Senate of the United States, 1832), the weight of an original brass kilogramme presented to M. TRALLES by the French Committee of Weights and Measures, in grains of which the United States Mint troy pound adjusted by Captain KATER, contains 5760	15433·159
Result of the comparison of a troy pound sent to Paris, according to a statement of the French Minister of the Interior, in a letter to the English Commissioners of Weights and Measures, dated Paris, February 28, 1821	15432·719

By the good offices of M. ARAGO, permission was obtained from the French Government to compare the pound with the standard kilogramme of platinum deposited in the Archives on the 22nd of June, 1799, known as the 'kilogramme des Archives.' The weights selected for comparison with the standard kilogramme, which henceforward will be designated by the letter 𝔄, were PC No. 1 and PC No. 2, together with the auxiliary weight Ḃ and a platinum weight V of about 192·4 grains, making altogether about 15432·35 grains. In order to obtain a second comparison perfectly independent of the former, a kilogramme of bronze was constructed by Mr. BARROW, with which, after comparison with 𝔄, it was my intention to compare PS together with each of the four platinum copies of the pound in turn, and other platinum weights sufficient to make up a kilogramme.

By some most unaccountable oversight 𝔄 had never been weighed in water previous to its final adjustment. Afterwards, on account of its legal importance, it was considered hazardous to immerse it in water, especially as, from the method of preparing platinum at that time in use (fusion with arsenious acid and subsequent ignition of the arsenide of platinum under a muffle till the arsenic was burnt away), there is reason to suppose that it is not entirely free from an admixture of arsenic which, on being wetted, might oxidize and then dissolve, and thus produce a very sensible alteration of weight. Its form is that of a cylinder of about 39·4 millimètres in diameter and 39·7 millimètres high, having its edges rounded by a surface 0·75 millimètre broad, and having a radius of about 3 millimètres. An approximate value of its density was obtained by Professor SCHUMACHER and OLUFSEN in the following manner. In August and November 1831, the density of a kilogramme 𝔖 in Professor SCHUMACHER's possession was found to be 21·212, by weighing it in water and in air. In March 1832, by measuring pairs of diameters at right angles to each other in planes cutting the axis in eight different points, and the distances between nine corresponding pairs of points in the ends of the cylinder, the volume of 𝔖 at 0° appeared to be 47114·4 cubic millimètres. In the autumn of 1834, Professor OLUFSEN measured two diameters at right angles to each other at the middle and at each end of 𝔄, and the distances between eight pairs of corresponding points in the circular ends. The volume of 𝔄 at 0°, deduced from these linear dimensions appeared to be 48615·4 cubic millimètres, and consequently its density 20·644*. These measurements, though made with the utmost care, appeared to be too few, and confined to too small a number of points, to determine the density of 𝔄 in this manner with sufficient accuracy. I therefore resolved to compare its volume by means of the stereometer, with that of a brass cylinder of nearly the same dimensions, the volume of which might afterwards be found by weighing in air and in water.

The representations of M. ARAGO procured for me the privilege of forwarding the balance and stereometer unexamined from Havre to the Douane in Paris, where I received them without being obliged to unpack the cases in which they were con-

* SCHUMACHER's Jahrbuch für 1836, p. 237.

tained. From M. LETRONNE and M. LALLEMAND, Officers of the Archives, to the latter of whom the custody of the standards was confided, I received every possible assistance. The balance was mounted on a strong and heavy carpenter's bench in a room paved with brick, on the ground floor of the Archives. On unpacking the stereometer, the graduated tube was found to be broken, in consequence, as M. BUNTEN affirmed, of mere contact with a slender iron wire used in cleaning the tube with cotton wool, and left in it in ignorance of the peculiar action of iron wire on the interior of a glass tube, and not from any violent shock. M. BUNTEN replaced the broken tube, which had been divided into inches, by a tube divided into centimètres, and traced upon the slip of ivory a scale of 10 millimètres divided to every 0·2 of a millimètre. I procured one of ERNST's cistern barometers, which, after hanging all night by the side of the standard barometer of the observatory, was compared with it on the following day by one of the Assistants. M. GAMBEY was commissioned to construct a brass cylinder, either solid or, if hollow, air-tight, nearly of the dimensions of 𝔄, a cylindrical cup to receive the kilogramme or the model, fitting into the cup of the stereometer, and a second cylinder closed at both ends, to fill up as much as possible of the space left vacant in the cup of the stereometer. While with M. GAMBEY, I ascertained that he had some platinum kilogrammes finished, with the exception of the final reduction. This appeared to be a favourable opportunity for commencing the formation of a collection of accurate copies of foreign standards, which had been recommended in Art. 33 of the Report of the Committee, dated December 21, 1841. Also the comparison of 𝔄 with a copy having nearly the same density and expansion, and unalterable by mere exposure to the atmosphere, promised to be much more serviceable in finding the relation between the French and English standards of weight, than its comparison with a copy expanding nearly twice as much by heat, and having nearly three times its volume, and liable to become considerably heavier by oxidation in the course of a few years. I therefore applied to the Astronomer Royal for authority to purchase a platinum kilogramme for the use of the Committee. While waiting for his reply, I occupied myself with the comparison of 𝔄 with PC No. 1+PC No. 2+B+V.

The platinum kilogramme being a cylinder without a knob, does not admit of being lifted with a fork, consequently, in putting it into the scale-pan or taking it out, it must be held in the hand, a glove or a piece of silk being of course interposed between the fingers and the weight. The insertion of the hand into the balance case, and the communication of its warmth to the weight itself, are so prejudicial to the accuracy of a weighing, that it became necessary to seek for some method of obviating this source of error. Such a method was found in the employment of the detached scale-pans described in page 764, and answered so well, that I afterwards continued to use it in comparing weights of the usual form.

On the 16th of September, the Astronomer Royal having approved of the purchase of a platinum kilogramme for the use of the Committee, I procured one from

M. Gambey. In form it resembles 𝔄, except that it is not quite so large, in consequence of the greater density of the metal of which it is composed. The cylindrical surface and the ends are very accurately worked. The metal of which it is made is greatly superior to that of which the standard lb. and its copies are constructed, as not the slightest indication of any defective place can be observed on its surface. This kilogramme will be designated by the letter ℭ. The comparisons of 𝔄 with PC No. 1+PC No. 2+B+V were now discontinued, being of secondary importance after the acquisition of ℭ, and serving mainly to control the comparison of ℭ with 𝔄.

A considerable number of observations were made with the stereometer on the 25th of September, for the purpose of finding the volume of 𝔄 by comparing it with that of a hollow brass cylinder M of nearly the same dimensions. Towards the end of the series, M being in the cup, the mercury in the graduated tube was observed to descend perceptibly though very slowly. This was caused apparently by the passage of a small quantity of air into the upper part of the graduated tube, where the pressure was about half that of the external air. At first it was supposed that the air found an entrance either between the rim of the cup and the glass plate which closed it, or through the screw joint by which the cup was connected with the collar into which the graduated tube was cemented, or, lastly, through the cemented joint itself. The cement was then varnished with shell-lac dissolved in alcohol, and the screw and glass plate carefully smeared with lard, so as to render the passage of air through the joints all but impossible. The mercury in the graduated tube still continued to descend when M was in the cup, but not otherwise. It appeared therefore probable that the soldering of M had given way under the changes of pressure to which it had been exposed, so as to allow the air enclosed within it to escape slowly, when the pressure of the air in the cup was diminished. This conjecture was verified, on attempting to weigh M in water on the 22nd and 23rd of April, 1845, when M was found to have increased in weight after being left all night in the water in which it had been weighed; and on placing it in the receiver of an air-pump and partially exhausting the air, drops of water made their appearance at the junction of the plane and cylindrical surfaces at one end of the cylinder. It was evidently useless to continue the observations with M, on account of its presumed leakage; a second series of stereometer observations was therefore made on the 5th of October, in which the volume of 𝔄 was compared with that of ℭ.

Stereometer observations for finding the volume of 𝔄.

At 16° the mercury contained in the graduated tube between 163·2 mm. and 108·9 mm. weighed 1812·35 grains, and the mercury contained between 108·9 mm. and 0·0 mm. weighed 3614·99 grains. Hence the mercury contained between 0 mm. and 136 mm. weighed 4532·87 grains, and each mm. of the tube near 136 mm. contained 33·38 grains of mercury. The volume of PM is expressed in terms of the

volume of a grain of mercury at 16°. The temperature of the mercury in the stereometer was determined by a thermometer placed in the jar which received the mercury that escaped on opening the stopcock at the lower end of the tube. Let t denote this temperature. The barometer was suspended near the stereometer, and was presumed to have the same temperature. The height of the column of mercury in it, corrected for its constant error, but not reduced to 0°, will be denoted by h.

𝕮 in cup.

$t=16{\cdot}92$, $h=750{\cdot}24$.

PM.	PC.
136·43	518·10
136·60	518·80
136·40	519·10
136·40	519·20
136·35	518·85
136·40	519·10
136·25	518·20
136·45	518·85
136·41	518·77

CM= 382·36
h—CM= 376·89
Vol. PM=4546·56
Vol. cup—vol. 𝕮=4481·4

𝕬 in cup.

$t=17{\cdot}1$, $h=759{\cdot}07$.

PM.	PC.
136·35	529·55
136·15	530·50
136·00	531·00
136·20	530·65
136·45	531·50
136·30	531·30
136·00	530·50
135·90	530·15
136·10	530·70
136·20	531·10
136·65	531·95
136·40	531·70
136·22	530·88

CM= 394·66
h—CM= 364·41
Vol. PM=4540·2
Vol. cup—Vol. 𝕬=4192·2

𝕮 in cup.

$t=17{\cdot}3$, $h=758{\cdot}65$.

PM.	PC.
136·10	517·90
136·20	518·55
136·10	518·40
136·05	517·90
136·45	518·90
136·05	517·40
136·25	517·75
136·35	518·60
136·20	518·15
139·65	519·30
136·23	518·28

CM= 382·05
h—CM= 376·60
Vol. PM=4540·7
Vol. cup—Vol. 𝕮=4475·9

Hence, at 17°·1, vol. cup — vol. 𝕮 = volume of 4478·65 grains of mercury at 16°, and vol. cup — vol. 𝕬 = volume of 4192·2 grains of mercury at 16°. Therefore, at 17°·1, vol. 𝕬 — vol. 𝕮 = volume of 286·45 grains of mercury at 16°.

v𝕬—v𝕮 in vol. of a grain of mercury at 16° ...	286·45	2·45705
Δ mercury at 0° : Δ mercury at 16°	1·00287	0·00124
		2·45829
Δ mercury at 0° : Δ water at 4°.....................	13·596	1·13341
v𝕬—v𝕮 at 17°·1		1·32488
Δ platinum at 0° : Δ platinum at 17°·1		0·00020
v𝕬—v𝕮 at 0°..	21·119	1·32468

Weighing of 𝕮 in water.

The kilogramme, after having been washed with alcohol, was suspended from the right-hand pan of the balance, by a loop of platinum wire wound round it, so as to hang in the middle of the jar destined to receive the water in which it was to be weighed. In order to prevent the formation of air-bubbles, the water was poured through a funnel having a very small opening inserted into the upper end of a

glass tube the lower end of which reached to the bottom of the jar. The left-hand pan contained the counterpoise consisting of 14003 grains of bronze, for which $\log \Delta = 0{\cdot}92260$. The thermometers C, D were suspended in the water with their bulbs in a horizontal plane through the centre of gravity of ℭ. The counterpoise is supposed to be in equilibrium with the weights in or suspended from the right-hand pan, when the reading of the scale is 20.

Let K, L denote PC No. 1, PC No. 2 respectively, and P, Q, R the bronze weights marked 100, 200, 400. ℭ and the loop of wire being suspended in water, and the weight in the first column placed in right-hand pan, the second column gives the corresponding reading of the scale.

September 27, 1844. 100 parts $=0{\cdot}500$ grain.

C.	D.	F.	E.	gr.	Scale.
14·97	15·0			0·0	20·1
				0·0	20·15
				0·0	20·15
				0·0	20·2
		764·65	16·1	0·0	18·9
				0·0	18·9
				0·0	19·9
				0·0	20·0
				0·0	20·05
				0·0	19·9
14·97	15·0			0·0	20·25
				0·0	20·3
		764·65	16·1	0·0	20·3
				0·0	20·35
				0·0	20·4
				0·0	20·4
14·98	15·0			0·0	18·8
				0·0	18·8
		764·55	16·2	0·0	18·8
				0·0	20·5
				0·0	20·45
				0·0	20·45
				0·0	20·55
				0·0	20·45
				0·0	20·45
14·97	15·0	764·62	16·13	0·0	19·98

Counterpoise in air ($t=16{\cdot}05$, $b=763{\cdot}08$) balances ℭ and wire loop in water ($t=15{\cdot}0$) $+0{\cdot}0001$ grain in air.

The wire loop alone being suspended in the water contained in the jar, to which a volume of water equal to that of ℭ had been added, and the weights $K+L+(2)+(1)+P+Q+R$, together with the weight given in the first column, being placed in the right-hand pan, the second column gives the corresponding reading of the scale.

D.	C.	F.	E.	gr.	Scale.
				0·60	45·00
				0·50	22·80
				0·50	22·35
				0·44	11·85
				0·48	15·00
				0·48	15·80
				0·50	20·80

				0·50	20·80
				0·40	4·25
				0·40	4·20
16·05	16·05	764·23	16·2	0·50	20·80
				0·50	20·80
				0·50	20·85
				0·50	23·20
				0·50	23·20
				0·50	21·00
				0·50	21·00
				0·50	21·10
				0·50	21·15
				0·48	20·25
				0·48	20·40
				0·46	18·45
				0·46	18·40
				0·48	20·15
				0·48	20·40
16·4	16·4			0·48	20·45
				0·48	20·50
				0·48	20·50
				0·48	20·50
16·22	16·22	764·23	6·2	0·4848	19·86

In a.(t=16·24, b=762·68) counterpoise balances the wire loop in w.+K+L+(2)+(1)+0·4855 gr.+P+Q+R.

A correction must be applied for the change of density of the air displaced by the counterpoise after 𝕮 was taken out of the water. When 𝕮 and the wire loop were in the water, t=15·07, b=763·08. When the wire loop alone was in the water and weights in the right-hand pan, t=16·24, b=762·68. The counterpoise displaces 2·0453 grs. of air (t=16·07, b=76·308), and 2·0429 grs. of air (t=16·24, b=762·68). 14003−2·0453 grs. balance 𝕮 and wire loop in water (t=15·0)+0·0010 grain in air. 14003−2·0429 grains balance the wire loop in water and K+L+(2)+(1)+0·4855 grain+P+Q+R in air (t=16·22, b=762·68).

Hence 𝕮 in water (t=15·0) ≏ K+L+(2)+(1)+0·4830 grain +P+Q+R in air (t=16·24, b=762·68).

Comparison of P+Q+R with the sum of the auxiliary platinum weights G, K, L, M, (16), (4).

March 5, 1845. 100 parts =0·244 grain.

	pt.
P+Q+R ≏ G+K+L+M+(16)+(4)	+0·70
	+0·95
	+0·82
	+1·50

P+Q+R ≏ G+K+L+M+(16)+(4)+0·00242 grain in air (t=4·88, b=763·86).

P+Q+R=700·003 grains nearly.

G+K+L+M+(16)+(4)+0·00242=699·93747 grains.

The weights of the air displaced by the bronze and platinum weights are 0·10668 gr. and 0·04217 gr. respectively. Hence P+Q+R=700·00197 grains.

The weight of air displaced by the platinum and bronze weights used in weighing 𝕮 in water =0·40355 gr.+0·40361 g.+0·00020 gr.+0·10212 gr.=0·90948 grain.

K+L+(2)+(1)+0·4830 grain	14003·48359	
P+Q+R	700·00197	
Apparent weight of $\mathfrak{C}$ in water....................	14703·48556	
Weight of air displaced by the weights	0·90948	
Weight of $\mathfrak{C}$ in water	14702·57608	
Weight of $\mathfrak{C}$	15432·32653	
Weight of water at 15·0 displaced by $\mathfrak{C}$	729·75045	2·8631744
Max. density of water: density of water at 15·0		0·0003708
Δ pt. at 15·0 : Δ pt. at 0°..............................		9·9998241
$v\mathfrak{C}$	730·078	2·8633693
$\mathfrak{C}$..	15432·32653	4·1884314
$\Delta\mathfrak{C}$	21·13791	1·3250621

The stereometer observations gave $v\mathfrak{A}-v\mathfrak{C}=21·119$. Hence $v\mathfrak{A}=751·197$.

In 1845 the Committee received from Professor SCHUMACHER an account in manuscript of fourteen newer and more accurate determinations of the density of $\mathfrak{S}$, and of several hundred measurements of its linear dimensions, together with a copy of Professor STEINHEIL's paper entitled "Ueber das Bergkrystall-Kilogramm*," containing numerous determinations of the linear dimensions of $\mathfrak{A}$ by himself and GAMBEY in May 1837, in addition to those made by OLUFSEN in 1834. The linear dimensions of $\mathfrak{S}$ and $\mathfrak{A}$ were all measured with the same instrument, which was made by GAMBEY.

Assuming the linear expansion of platinum to be 0·0000085655 for 1° C., $\Delta\mathfrak{S}=21·2037$, by fourteen observations made February 3...24, 1837. (The value $\Delta\mathfrak{S}=21·2047$ given in STEINHEIL's memoir is the mean of four sets of observations of rather doubtful accuracy made in September 1831.) By the comparisons of $\mathfrak{S}$ with $\mathfrak{A}$, April 9...14, 1835, $\mathfrak{S}=999·9993$ grammes. Previous to February 9, 1837, $\mathfrak{S}$ had lost about 1·6 milligramme, in consequence, as was supposed, of being washed with alcohol. In May 1837, therefore, its weight may be taken $=999·998$ grammes nearly. The volumes of $\mathfrak{S}$ and $\mathfrak{A}$ at 0°, deduced from their linear dimensions, appeared to be 47147 and 48650 cubic millimètres respectively.

$\Delta\mathfrak{S}$..........................	21·2037	1·3264117
$v\mathfrak{S}$(in cubic millimètres)	47147	4·6734541
$\mathfrak{A}:\mathfrak{S}$		0·0000009
		5·9998667
$v\mathfrak{A}$ (in cubic millimètres)	48650	4·6870828
$\Delta\mathfrak{A}$..........................	20·5487	1·3127839
$\mathfrak{A}$ (in grains nearly)......	15432·35	4·1884321
$v\mathfrak{A}$	751·014	2·8756482

Hence the volume of $\mathfrak{A}$ at 0° is equal to the volume of 751·014 grains, or 48·665 grammes of water at its maximum density.

The observations by which this value of $v\mathfrak{A}$ was obtained are so numerous, and made with such extreme care, that I have no hesitation in adopting it in preference to the value 751·197 resulting from the observations made with the stereometer.

The value of $v\mathfrak{A}$ just found depends upon $\Delta\mathfrak{S}$; therefore, in computing $v\mathfrak{A}-v\mathfrak{C}$, for reducing the weighings by which $\mathfrak{C}$ is compared with $\mathfrak{A}$, the density of $\mathfrak{C}$ must

* Abhandlungen der K. Akademie der Wissenschaften zu München, B. 4, S. 165.

be calculated with the same data that were used in calculating the density of $\mathfrak{S}$, subtracting 0·00018 from log β on account of the difference between the force of gravity in Altona and in Paris. The same expansions of platinum and bronze are to be used in calculating the weight of the air displaced by the weights used in weighing $\mathfrak{C}$ in water, as were used in calculating their volumes. 16°·24 C.=61°·23 F., 762·68 mm. =30·027 in., 15°·0 C.=59°·0 F.

Weight of air displaced by the weights used in weighing $\mathfrak{C}$ in water.

30·027	1·47751	30·027	1·47751
61·23	5·61139	61·23	5·61139
vK at 61·23	2·51963	vL at 61·23	2·51969
0·40600	9·60853	0·40606	9·60859
30·027	1·47751	30·027	1·47751
61·23	5·61139	61·23	5·61139
v[(2)+(1)+0·483 gr.] at 61·23	9·21641	v[P+Q+R] at 61·23	1·92286
0·00020	6·30531	0·10275	9·01176

Weight of air displaced by the weights =0·40600 gr.+0·40606 gr.+0·00020 gr.+0·10275 gr.=0·91501 gr.

Calculation of the volume and density of $\mathfrak{C}$.

Apparent weight of $\mathfrak{C}$ in water	14703·48556	
Weight of air displaced by the weights	0·91501	
Weight of $\mathfrak{C}$ in water	14702·57055	
Weight of $\mathfrak{C}$	15432·32653	
Weight of water (t=59·0) displaced	729·75598	2·8631777
Max. density of water : Δ water at 59°·0 F.		0·0003606
Δ pt. at 59°·0 F. : Δ pt. at 38° F.		9·9998326
$v\mathfrak{C}$	730·081	2·8633709
$\mathfrak{C}$	15432·327	4·1884315
$\Delta\mathfrak{C}$	21·13783	1·3250606

$v\mathfrak{A}$=751·014. Hence the volume of $\mathfrak{A}$—the volume of $\mathfrak{C}$, at the freezing-point, is equal to the volume of 20·933 grains of water at its maximum density.

Comparison of $\mathfrak{C}$ with $\mathfrak{A}$.

The zero of the scale of the barometer was 180 mm. below the middle of the weights, consequently 0·02 mm. must be subtracted from the height of the mercury in the barometer. The left-hand pan contained a counterpoise, and $\mathfrak{A}$ and $\mathfrak{C}$ in the detached scale-pans, the weights of which are denoted by X and Y, were suspended alternately from the agate-plane which rested on the right-hand knife-edge.

September 28, 1844. 100 parts=0·49579 grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
22·71	21·00	C= 17·0	21·13	23·22	C= 17·25
23·36	22·40	F=761·67	23·37	23·29	F=760·3
25·20	23·63	E= 16·55	23·58	23·21	E= 17·0
25·62	23·96		23·67	24·18	
24·77	24·24		24·10	23·66	
24·76	24·77		23·25	23·17	
24·51	24·02		23·50	24·88	
24·46	24·46		24·61	24·17	

24·90	24·04		23·98	23·67	
24·68	25·35		23·49	23·37	
23·82	24·45		24·07	23·73	
24·92	23·93		23·16	22·81	
24·97	24·48		22·50	24·21	
23·87	24·10		22·75	22·90	
24·76	24·71		24·05	24·00	
24·24	24·59		23·46	24·32	
24·62	24·35		23·48	23·22	
24·26	24·09	C= 17·25	23·27	24·22	C= 17·4
23·97	23·91	F=760·2	23·93	24·30	F=759·5
23·50	23·84	E= 17·0	24·48	23·99	E= 17·15
480·32	487·90		472·83	474·52	

$20(\mathfrak{C}+Y) \bumpeq 20(\mathfrak{A}+X)+7·58$ parts. $20(\mathfrak{C}+X) \bumpeq 20(\mathfrak{A}+Y)-1·69$ part.

$\mathfrak{C} \bumpeq \mathfrak{A}+0·0007251$ grain in air ($t=17·27$, $b=758·79$).

October 1. 100 parts=0·47393 grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
19·55	19·50	C= 14·9	21·54	21·15	C= 15·8
19·87	19·06	F=767·44	20·39	21·49	F=764·82
20·11	19·44	E= 15·05	20·54	20·42	E= 15·1
20·07	18·71		20·82	22·15	
19·40	19·44		21·49	21·02	
20·62	19·78		22·97	22·20	
19·54	18·78		23·35	22·72	
19·31	18·52		22·19	22·70	
19·55	18·25		23·47	23·06	
19·87	19·53		22·66	22·24	
20·09	18·58		22·70	22·66	
19·57	18·48		22·25	23·01	
19·06	18·90		22·05	22·09	
18·80	18·76		22·71	22·67	
19·00	17·84		22·87	22·93	
18·84	19·09		22·37	22·37	
18·79	17·45		22·51	22·82	
18·50	18·46	C= 15·8	24·03	23·42	C= 15·75
18·46	18·52	F=764·82	24·00	23·27	F=763·85
18·25	18·15	E= 15·1	23·85	24·01	E= 15·2
387·25	375·24		448·76	448·40	

$20(\mathfrak{C}+Y) \bumpeq 20(\mathfrak{A}+X)+12·01$ parts. $20(\mathfrak{C}+X) \bumpeq 20(\mathfrak{A}+Y)+0·36$ part.

$\mathfrak{C} \bumpeq \mathfrak{A}+0·001466$ grain in air ($t=15·59$, $b=763·79$).

October 3. 100 parts =0·48591 grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
21·07	19·62	C= 15·65	17·40	18·34	C= 16·4
20·56	20·42	F=761·57	18·57	17·83	F=760·73
20·86	20·27	E= 16·0	18·50	18·59	E= 16·05
20·51	20·49		18·75	19·22	
19·60	19·96		18·89	18·95	
19·98	19·12		18·96	18·80	
20·34	20·01		18·55	18·79	
20·01	18·99		18·51	18·47	
19·61	19·27		18·30	18·46	
19·92	19·60		18·06	17·75	
19·55	19·91		18·01	17·62	
20·03	19·69		18·38	17·40	
19·59	19·98		18·27	18·36	
19·55	19·29		18·12	18·05	
20·11	19·01		18·19	18·06	
19·59	19·37		18·39	18·45	
19·14	19·62		18·54	18·16	

19·64	18·70	C= 16·4	18·37	17·45	C= 16·42
18·99	19·32	F=760·73	18·26	17·99	F=760·5
17·79	19·27	E= 16·05	18·09	17·36	E= 16·0
398·44	391·91		367·11	364·10	

20($\mathfrak{C}$+Y)≏20($\mathfrak{A}$+X)+6·53 parts. 20($\mathfrak{C}$+X)≏20($\mathfrak{A}$+Y)+3·01 parts.

$\mathfrak{C}$≏$\mathfrak{A}$+0·001159 grain in air (t=16·25, b=759·31).

October 8. 100 parts =0·48077 grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
21·10	20·59	C= 14·7	18·62	18·69	C= 15·65
21·57	20·95	F=760·9	18·80	18·40	F=759·1
21·97	20·50	E= 14·6	19·30	19·05	E= 14·8
22·37	20·94		18·96	18·33	
22·14	21·97		19·19	19·09	
22·35	21·87		18·95	18·51	
21·61	20·54		18·81	18·62	
21·10	20·90		20·24	19·68	
20·97	20·42		19·06	18·75	
21·55	21·20		19·64	18·64	
21·19	21·05		19·37	18·61	
21·82	20·59		19·40	18·10	
20·57	20·54		18·96	19·03	
21·74	20·54		19·30	18·50	
20·84	20·12		19·10	18·95	
20·56	19·96		19·59	18·54	
21·02	20·90		18·80	18·60	
20·54	19·57	C= 15·65	18·86	18·84	C= 15·35
20·35	19·60	F=759·1	19·47	18·87	F=756·6
19·52	20·07	E= 14·8	19·10	19·47	E= 14·5
424·88	412·82		383·52	375·27	

20($\mathfrak{C}$+Y)≏20($\mathfrak{A}$+X)+12·06 parts. 20($\mathfrak{C}$+X)≏20($\mathfrak{A}$+Y)+8·25 parts.

$\mathfrak{C}$≏$\mathfrak{A}$+0·002441 grain in air (t=15·37, b=757·80).

October 9. 100 parts =0·48757 grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
20·97	20·67	C= 14·35	19·26	19·24	C= 15·45
21·85	21·64	F=746·68	19·48	19·41	F=745·15
22·06	22·22	E= 14·1	19·46	19·47	E= 14·6
22·19	22·24		19·56	19·40	
22·05	22·58		19·24	19·75	
22·35	22·22		19·47	20·25	
22·57	22·51		19·34	19·37	
22·22	22·17		19·88	19·49	
21·55	22·22		19·65	19·58	
22·09	21·64		19·50	19·91	
21·81	21·53		19·19	19·52	
22·04	21·42		19·62	19·49	
21·79	21·57		19·70	19·89	
21·34	20·87		19·57	19·57	
21·05	21·11		19·67	19·54	
21·00	20·66		20·24	19·46	
20·75	20·51		19·66	19·50	
20·31	19·86	C= 15·45	20·12	19·60	C= 15·47
20·05	19·65	F=745·15	19·42	19·16	F=743·9
20·29	19·84	E= 14·6	19·39	19·45	E= 15·05
430·33	427·13		391·42	391·05	

20($\mathfrak{C}$+Y)≏20($\mathfrak{A}$+X)+3·2 parts. 20($\mathfrak{C}$+X)≏20($\mathfrak{A}$+Y)+0·37 part.

$\mathfrak{C}$≏$\mathfrak{A}$+0·000435 grain in air (t=15·21, b=743·89).

	gr.	t.	b.
September 28.	$\mathfrak{C} \bumpeq \mathfrak{A}+0\cdot000725$	17·27	758·79
October 1.	$\mathfrak{C} \bumpeq \mathfrak{A}+0\cdot001466$	15·59	763·79
October 3.	$\mathfrak{C} \bumpeq \mathfrak{A}+0\cdot001159$	16·25	759·31
October 8.	$\mathfrak{C} \bumpeq \mathfrak{A}+0\cdot002441$	15·37	757·8
October 9.	$\mathfrak{C} \bumpeq \mathfrak{A}+0\cdot000435$	15·21	743·89

If $v\mathfrak{A}-v\mathfrak{C}=20\cdot933$, as given by the observations of SCHUMACHER combined with those of OLUFSEN, STEINHEIL and GAMBEY, the true weight of $\mathfrak{A}$ compared with $\mathfrak{C}$ will be larger than its apparent weight by the weight of air displaced by a mass of platinum the volume of which at 0° is equal to the volume of 20·933 grains of water at its maximum density.

Weight of air displaced by $\mathfrak{A}$— weight of air displaced by $\mathfrak{C}$.

September 28.	0·025316	October 8.	0·025462
October 1.	0·025643	October 9.	0·025007
October 3.	0·025430		

Therefore

	gr.
September 28.	$\mathfrak{C}=\mathfrak{A}-0\cdot02459$
October 1.	$\mathfrak{C}=\mathfrak{A}-0\cdot02417$
October 3.	$\mathfrak{C}=\mathfrak{A}-0\cdot02427$
October 8.	$\mathfrak{C}=\mathfrak{A}-0\cdot02302$
October 9.	$\mathfrak{C}=\mathfrak{A}-0\cdot02457$
Mean	$\mathfrak{C}=\mathfrak{A}-0\cdot02412$ grain.

If $v\mathfrak{A}-v\mathfrak{C}=21\cdot119$, as given by the stereometer observations, the differences of the weights of air displaced by $\mathfrak{A}$ and $\mathfrak{C}$ will be

September 28.	0·025541	October 8.	0·025688
October 1.	0·025871	October 9.	0·025230
October 3.	0·025655		

Therefore

	gr.
September 28.	$\mathfrak{C}=\mathfrak{A}-0\cdot02482$
October 1.	$\mathfrak{C}=\mathfrak{A}-0\cdot02439$
October 3.	$\mathfrak{C}=\mathfrak{A}-0\cdot02450$
October 8.	$\mathfrak{C}=\mathfrak{A}-0\cdot02325$
October 9.	$\mathfrak{C}=\mathfrak{A}-0\cdot02480$
Mean	$\mathfrak{C}=\mathfrak{A}-0\cdot02435$ grain.

Comparison of $\mathfrak{C}$ *with each of the weights* I+K+A, I+L+B, I+M+Γ, I+N+Δ.

The use of auxiliary weights corresponding to the divisors obtained in the process of expressing in a continued fraction the ratio of one to the other of the two weights to be compared, answered so well in deducing the pound of 7000 grains from the pound troy of 5760 grains, that I resolved to employ it in comparing the kilogramme with the pound. The weight of $\mathfrak{C}$ is nearly 15432·325 grains, and that of each of the weights I, K, L, M, N, the new standard lb. and its four copies respectively, is 7000 grains. Then, each of the weights A, B, Γ, Δ being 1432·324 grains, Z being 1270·707 grains, and Θ being 161·629 grains, the following comparisons became possible:—

$\mathfrak{C}$ with each of the weights I+K+A, I+L+B, I+M+Γ, I+N+Δ; each of the weights K, L, M, N with I; I with A+B+Γ+Δ+Z; each of the weights A, B, Γ, Δ with Z+Θ. The value of Θ in terms of I was known from comparisons made for a different purpose. The weights A, Γ, Δ consisted of the auxiliary platinum weights A, C, D, used in forming the lb. of 7000 grains, with the addition to each of a three rouble platinum coin and a little more than 33 grains of platinum wire weights; B of the auxiliary platinum weight B together with the platinum weight V+0·03 gr. which had been used with K+L in weighing the kilogramme des Archives. Z consisted of the platinum weights F, G and 30·85 grains of platinum wire weights. Θ was made up of the platinum weights L and M of nearly 80 grains each, one of the 1-grain weights of the two ROBINSON's balances, used by turns, and one of the ten weights Q of 0·64509 grains each, also used by turns.

The zero of the scale of the barometer was 305 mm. above the middle of the weights, 0·03 mm. must therefore be added to the observed value of F.

Let $\mathfrak{K}$=I+K+A, $\mathfrak{L}$=I+L+B, $\mathfrak{M}$=I+M+Γ, $\mathfrak{N}$=I+N+Δ.

February 27, 1846. 100 parts=0·65509 gr.

$\mathfrak{K}$+Y, $\mathfrak{C}$+X.	$\mathfrak{C}$+X, $\mathfrak{K}$+Y.	
19·26	18·80	
19·05	19·04	
19·00	18·50	
18·01	17·96	
18·00	18·15	
18·82	17·02	
17·57	17·21	
17·67	17·02	D= 18·9
17·17	16·60	C= 18·9
16·82	16·54	F=755·9
17·30	16·25	E= 19·0
17·07	16·69	
17·11	16·69	
17·26	16·86	
17·51	17·46	
17·72	17·40	
17·55	17·34	
302·89	295·53	

34($\mathfrak{C}$+X)≏30($\mathfrak{K}$+Y)+7·36 parts.

March 2. 100 parts=0·67362 grain.

$\mathfrak{K}$+X, $\mathfrak{C}$+Y.	$\mathfrak{C}$+Y, $\mathfrak{K}$+X.	
16·46	15·25	
16·94	16·84	
17·22	17·29	
16·39	17·15	
15·89	15·80	
16·15	16·35	
16·77	16·26	
16·04	16·60	D= 19·6
16·24	15·96	C= 19·6
15·91	15·75	F=763·2
15·99	14·76	E= 22·0
15·97	14·66	
15·65	14·56	
15·29	14·32	
16·20	14·79	
15·69	14·75	
15·56	13·95	
274·36	265·04	

34($\mathfrak{C}$+Y)≏34($\mathfrak{K}$+X)+9·32 parts.

68$\mathfrak{C}$≏68$\mathfrak{K}$+0·11100 grain in air (t=19·3, b=757·52).

January 31. 100 parts=0·56899 grain.

$\mathfrak{L}$+X, $\mathfrak{C}$+Y.	$\mathfrak{C}$+Y, $\mathfrak{L}$+X.	
18·90	17·64	
18·82	18·61	
18·79	18·14	
19·00	17·72	D= 11
18·51	17·85	C= 11·05
18·81	17·82	F=760·4
17·84	16·92	E= 17·5
18·36	17·52	
18·72	18·17	
18·64	17·30	
186·39	177·69	

20($\mathfrak{C}$+Y)≏20($\mathfrak{L}$+X)+8·7 parts.

February 2. 100 parts=0·61069 grain.

$\mathfrak{L}$+Y, $\mathfrak{C}$+X.	$\mathfrak{C}$+X, $\mathfrak{L}$+Y.	
18·01	17·70	
16·51	16·84	
16·51	17·06	
16·04	16·07	D= 15·7
15·75	16·22	C= 15·75
15·74	16·02	F=759·0
15·46	16·15	E= 16·0
16·06	16·64	
16·12	16·07	
15·97	16·54	
162·17	165·31	

20($\mathfrak{C}$+X)≏20($\mathfrak{L}$+Y)−3·14 parts.

40$\mathfrak{C}$≏40$\mathfrak{L}$+0·03032 grain in air (t=13·37, b=758·12).

March 24. 100 parts=0·57337 grain.

$\mathfrak{L}+X$, $\mathfrak{C}+Y$.	$\mathfrak{C}+Y$, $\mathfrak{L}+X$.	
17·57	17·45	
16·34	15·97	
16·02	15·54	
16·54	15·04	D= 9·55
16·60	16·34	C= 9·7
16·02	15·05	F=746·3
15·92	15·45	E= 11·6
16·04	15·25	
15·50	15·25	
15·49	15·05	
162·04	156·39	

$\mathfrak{L}+Y$, $\mathfrak{C}+X$.	$\mathfrak{C}+X$, $\mathfrak{L}+Y$.	
16·56	15·92	
17·35	16·30	
16·62	16·54	
16·20	16·07	D= 9·55
16·22	16·12	C= 9·7
15·67	15·46	F=746·3
16·45	16·05	E= 11·6
16·35	15·40	
16·10	16·36	
16·11	16·24	
163·63	160·46	

$20(\mathfrak{C}+Y)\bumpeq 20(\mathfrak{L}+X)+5{\cdot}65$ parts. $20(\mathfrak{C}+X)\bumpeq 20(\mathfrak{L}+Y)+3{\cdot}17$ parts.

$40\mathfrak{C}\bumpeq 40\mathfrak{L}+0{\cdot}05057$ grain in air ($t=9{\cdot}62$, $b=745{\cdot}38$).

February 3. 100 parts=0·64809 grain.

$\mathfrak{M}+Y$, $\mathfrak{C}+X$.	$\mathfrak{C}+X$, $\mathfrak{M}+Y$.	
18·67	18·06	
18·29	18·17	
17·90	17·70	
17·64	16·97	D= 16·05
17·61	17·26	C= 16·15
17·00	16·16	F=759·1
16·35	16·24	E= 16·6
16·97	16·19	
16·61	16·37	
16·72	16·25	
173·76	169·37	

February 4. 100 parts=0·64288 grain.

$\mathfrak{M}+X$, $\mathfrak{C}+Y$.	$\mathfrak{C}+Y$, $\mathfrak{M}+X$.	
18·92	17·37	
18·37	17·39	
16·84	17·45	
18·57	17·26	D= 16·35
17·87	16·71	C= 16·4
17·24	15·82	F=765·2
17·01	15·00	E= 17·1
16·87	14·45	
17·40	14·71	
16·65	14·37	
175·74	160·53	

$20(\mathfrak{C}+X)\bumpeq 20(\mathfrak{M}+Y)+4{\cdot}39$ parts. $20(\mathfrak{C}+Y)\bumpeq 20(\mathfrak{M}+X)+15{\cdot}21$ parts.

$40\mathfrak{C}\bumpeq 40\mathfrak{M}+0{\cdot}12623$ grain in air ($t=16{\cdot}24$, $b=760{\cdot}56$).

March 25. 100 parts=0·57820 grain.

$\mathfrak{M}+Y$, $\mathfrak{C}+X$.	$\mathfrak{C}+X$, $\mathfrak{M}+Y$.	
19·22	17·11	
19·01	17·34	
18·90	17·52	
18·80	17·10	D= 9·5
18·55	17·72	C= 9·5
18·30	16·57	F=747·3
17·84	17·02	E= 15·7
17·57	16·37	
17·85	16·02	
17·62	15·95	
183·66	168·72	

$\mathfrak{M}+X$, $\mathfrak{C}+Y$.	$\mathfrak{C}+Y$, $\mathfrak{M}+X$.	
16·56	15·82	
16·62	14·89	
17·37	15·62	
17·17	15·16	D= 9·5
17·67	14·71	C= 9·5
17·07	15·27	F=747·3
17·54	15·52	E= 15·7
17·06	15·99	
17·66	14·77	
17·46	14·95	
172·18	152·70	

$20(\mathfrak{C}+X)\bumpeq 20(\mathfrak{M}+Y)+14{\cdot}94$ parts. $20(\mathfrak{C}+Y)\bumpeq 20(\mathfrak{M}+X)+19{\cdot}48$ parts.

$40\mathfrak{C}\bumpeq 40\mathfrak{M}+0{\cdot}19902$ grain in air ($t=9{\cdot}55$, $b=745{\cdot}88$).

February 5. 100 parts $=$ 0·63391 grain.

$\mathfrak{N}$+X, $\mathfrak{C}$+Y.	$\mathfrak{C}$+Y, $\mathfrak{N}$+X.	
17·50	16·19	
16·92	15·71	
17·09	17·00	
16·70	16·66	D $=$ 16·45
16·80	15·76	C $=$ 16·45
15·79	15·32	F $=$ 759·65
15·97	14·71	E $=$ 16·9
16·00	14·74	
15·69	14·49	
15·80	14·89	
164·26	155·47	

$20(\mathfrak{C}+\mathrm{Y}) \bumpeq 20(\mathfrak{N}+\mathrm{X})+8{\cdot}79$ parts.

February 6. 100 parts $=$ 0·64246 grain.

$\mathfrak{N}$+Y, $\mathfrak{C}$+X.	$\mathfrak{C}$+X, $\mathfrak{N}$+Y.	
15·74	16·17	
14·97	15·79	
13·44	13·62	
18·19	17·74	D $=$ 17·5
16·87	17·70	C $=$ 17·5
16·52	17·07	F $=$ 763·8
16·02	17·20	E $=$ 18·6
16·54	17·69	
17·79	18·37	
17·55	18·57	
163·63	169·92	

$20(\mathfrak{C}+\mathrm{X}) \bumpeq 20(\mathfrak{N}+\mathrm{Y})-6{\cdot}29$ parts.

$40\mathfrak{C} \bumpeq 40\mathfrak{N}+0{\cdot}01531$ grain in air ($t=17{\cdot}0$, $b=760{\cdot}02$).

March 26. 100 parts $=$ 0·60994 grain.

$\mathfrak{N}$+X, $\mathfrak{C}$+Y.	$\mathfrak{C}$+Y, $\mathfrak{N}$+X.	
18·02	17·90	
18·00	17·84	
17·09	18·06	
18·37	17·66	D $=$ 15·6
17·94	18·45	C $=$ 15·65
17·41	17·81	F $=$ 752·05
17·72	18·19	E $=$ 17·4
17·21	17·46	
16·84	17·91	
17·29	17·51	
16·79	16·65	
192·68	195·44	

$22(\mathfrak{C}+\mathrm{Y}) \bumpeq 22(\mathfrak{N}+\mathrm{X})-2{\cdot}76$ parts.

$\mathfrak{N}$+Y, $\mathfrak{C}$+X.	$\mathfrak{C}$+X, $\mathfrak{N}$+Y.	
14·89	15·54	
15·94	15·45	
16·14	15·91	
16·26	15·75	D $=$ 16·4
15·54	15·97	C $=$ 16·4
15·46	15·12	F $=$ 752·05
15·61	15·35	E $=$ 17·4
15·56	15·81	
15·01	15·24	
15·82	15·66	
15·25	15·40	
171·48	171·20	

$22(\mathfrak{C}+\mathrm{X}) \bumpeq 22(\mathfrak{N}+\mathrm{Y})+0{\cdot}28$ part.

$44\mathfrak{C} \bumpeq 44\mathfrak{N}-0{\cdot}01513$ grain in air ($t=16{\cdot}03$, $b=750{\cdot}42$).

	gr.	t.	b.	No. of observations.
$\mathfrak{C} \bumpeq \mathfrak{K}$	$+0{\cdot}00163$	19·3	757·52	68
$\mathfrak{C} \bumpeq \mathfrak{L}$	$+0{\cdot}00101$	11·5	756·25	80
$\mathfrak{C} \bumpeq \mathfrak{M}$	$+0{\cdot}00407$	12·9	753·22	80
$\mathfrak{C} \bumpeq \mathfrak{N}$	$+0{\cdot}00000$	16·51	754·99	84

$v\mathrm{I}=330{\cdot}856$, $v\mathrm{K}=330{\cdot}701$, $v\mathrm{L}=330{\cdot}750$, $v\mathrm{M}=330{\cdot}790$, $v\mathrm{N}=330{\cdot}881$.

For the weights A, B, Γ, Δ of 1432·324 grains each, $\log \Delta=1{\cdot}325642$. Therefore the volume of either of them at 0° will be 67·6704. Hence $v\mathfrak{K}=729{\cdot}227$, $v\mathfrak{L}=729{\cdot}276$, $v\mathfrak{M}=729{\cdot}316$, $v\mathfrak{N}=729{\cdot}407$. $v\mathfrak{C}=730{\cdot}078$, and, therefore, $v\mathfrak{C}-v\mathfrak{K}=0{\cdot}851$, $v\mathfrak{C}-v\mathfrak{L}=0{\cdot}802$, $v\mathfrak{C}-v\mathfrak{M}=0{\cdot}762$, $v\mathfrak{C}-v\mathfrak{N}=0{\cdot}671$.

The true weight of $\mathfrak{C}$ compared with $\mathfrak{K}$, $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$ will be larger than its apparent weights, by the weights of air displaced by masses of platinum the volumes of which at 0° are respectively equal to the volumes of 0·851 gr., 0·802 gr., 0·762 gr., 0·671 gr. of water at its maximum density. These are 0·00102 gr., 0·00099 gr., 0·00093 gr., 0·00081 gr. respectively.

Hence

$$\begin{aligned}\mathfrak{C}&=I+K+A+0{\cdot}00265 \text{ gr.}\\ \mathfrak{C}&=I+L+B+0{\cdot}00200\\ \mathfrak{C}&=I+M+\Gamma+0{\cdot}00500\\ \mathfrak{C}&=I+N+\Delta+0{\cdot}00081\end{aligned}$$

March 29. 100 parts=0·24964 grain.

H=A+B+Γ+Δ+Z.

H+Y, I+X.	I+X, H+Y.		H+X, I+Y.	I+Y, H+X.	
26·62	26·31		17·00	17·99	
26·76	26·52		17·57	17·90	
21·99	22·50		17·14	17·75	
21·11	21·99		16·61	18·75	
20·12	21·90	D= 14·8	16·96	17·97	D= 15·1
20·80	21·26	C= 14·8	16·57	17·52	C= 15·2
19·46	19·44	F=766·9	17·57	18·74	F=766·9
19·65	20·74	E= 16·4	17·46	18·80	E= 16·4
19·77	20·24		17·00	17·42	
19·49	20·06		17·46	17·60	
18·94	19·42		17·32	18·50	
18·32	19·49		17·05	18·54	
253·03	259·87		205·71	217·48	

24(I+X)≏24(H+Y)−6·84 parts. 24(I+Y)≏24(H+X)−11·77 parts.

I≏H−0·00097 grain in air (t=15·0, b=765·35).

Log ΔH=1·325642, therefore vH=330·717, vI=330·856. Hence vI−vH=0·139. The true weight of I is greater than its apparent weight when compared with H, by 0·00017 grain, the weight of air displaced by a mass of platinum the volume of which at 0° is equal to the volume of 0·139 gr. of water of maximum density. Hence

$$I=A+B+\Gamma+\Delta+Z-0{\cdot}00080 \text{ grain.}$$

March 27. 100 parts=0·25617 grain.

Z+Θ+X, A+0·01 gr.+Y.	A+0·01 gr.+Y, Z+Θ+X.	Z+Θ+Y, A+0·01 gr.+X.	A+0·01 gr.+X, Z+Θ+Y.
16·49	18·69	15·44	17·70
15·92	18·27	15·40	18·24
16·37	18·24	15·10	17·61
15·69	18·15	15·17	17·89
15·51	18·57	15·34	16·86
79·98	91·92	76·45	88·30

10(A+0·01 gr.+Y)=10(Z+Θ+X)−11·94 parts. 10(A+0·01 gr.+X)=10(Z+Θ+Y)−11·85 parts.

A=Z+Θ−0·01305 grain.

Z+Θ+Y, B+0·01 gr.+X.	B+0·01 gr.+X, Z+Θ+Y.	Z+Θ+X, B+0·01 gr.+Y.	B+0·01 gr.+Y, Z+Θ+X.
18·77	19·85	16·64	18·47
18·35	20·47	16·82	18·70
17·47	19·92	16·81	18·40
17·25	20·14	16·60	18·35
17·37	19·79	16·35	17·85
89·21	100·17	83·22	91·77

10(B+0·01 gr.+X)=10(Z+Θ+Y)−10·96 parts. 10(B+0·01 gr.+Y)=10(Z+Θ+X).

B=Z+Θ−0·01250 grain.

$Z+\Theta+Y$, $\Gamma+0{\cdot}02$ gr.$+X$.	$\Gamma+0{\cdot}01$ gr.$+X$, $Z+\Theta+Y$.	$Z+\Theta+X$, $\Gamma+0{\cdot}02$ gr.$+Y$.	$\Gamma+0{\cdot}01$ gr.$+Y$, $Z+\Theta+X$.
18·27	18·47	17·92	18·76
18·67	18·86	18·27	18·45
17·44	18·62	21·92	23·45
18·22	18·41	22·20	23·57
17·90	18·35	21·92	22·99
90·50	92·71	102·23	107·22

$10(\Gamma+0{\cdot}015\text{ gr.}+X)=10(Z+\Theta+Y)-2{\cdot}21$ parts. $10(\Gamma+0{\cdot}015\text{ gr.}+Y)=10(Z+\Theta+X)-4{\cdot}99$ parts.

$$\Gamma=Z+\Theta-0{\cdot}01592\text{ grain.}$$

$Z+\Theta+X$, $\Delta+0{\cdot}01$ gr.$+Y$.	$\Delta+0{\cdot}01$ gr.$+Y$, $Z+\Theta+X$.	$Z+\Theta+Y$, $\Delta+0{\cdot}01$ gr.$+X$.	$\Delta+0{\cdot}01$ gr.$+X$, $Z+\Theta+Y$.
19·51	20·30	18·57	19·11
20·04	20·22	17·76	18·80
20·05	19·57	18·61	18·75
19·40	19·15	18·64	18·00
18·81	19·14	17·69	18·21
97·81	98·38	91·27	92·87

$10(\Delta+0{\cdot}01\text{ gr.}+Y)=10(Z+\Theta+X)-0{\cdot}57$ part. $10(\Delta+0{\cdot}01\text{ gr.}+X)=10(Z+\Theta+Y)-1{\cdot}60$ part.

$$\Delta=Z+\Theta-0{\cdot}01028\text{ grain.}$$

$$\begin{aligned} & \quad\text{gr.}\\ L &= 79{\cdot}99112\\ M &= 79{\cdot}99261\\ J &= 0{\cdot}99996\\ Q &= 0{\cdot}64509\\ \Theta=L+M+J+Q &= 161{\cdot}62878 \end{aligned}$$

$$\begin{array}{lll} \text{gr.} & \text{gr.} & \text{gr.}\\ \mathfrak{C}=I+K+A+0{\cdot}00265 & K=I+0{\cdot}00051 & A=Z+\Theta-0{\cdot}01305\\ \mathfrak{C}=I+L+B+0{\cdot}00200 & L=I-0{\cdot}00089 & B=Z+\Theta-0{\cdot}01250\\ \mathfrak{C}=I+M+\Gamma+0{\cdot}00500 & M=I-0{\cdot}00178 & \Gamma=Z+\Theta-0{\cdot}01592\\ \mathfrak{C}=I+N+\Delta+0{\cdot}00081 & N=I-0{\cdot}00316 & \Delta=Z+\Theta-0{\cdot}01028 \end{array}$$

Whence

$$\begin{aligned} & \qquad\qquad\text{gr.}\\ \mathfrak{C}&=2I+Z+\Theta-0{\cdot}00989\\ \mathfrak{C}&=2I+Z+\Theta-0{\cdot}01139\\ \mathfrak{C}&=2I+Z+\Theta-0{\cdot}01270\\ \mathfrak{C}&=2I+Z+\Theta-0{\cdot}01263\\ \text{Mean......}\ \mathfrak{C}&=2I+Z+\Theta-0{\cdot}01165 \end{aligned}$$

$$\begin{array}{rl} & \text{gr.}\\ I=A+B+\Gamma+\Delta+Z & -0{\cdot}00080\\ A+B+\Gamma+\Delta=4Z+4\Theta & -0{\cdot}05175\\ I=5Z+4\Theta & -0{\cdot}05255\\ 5(Z+\Theta)=I+\Theta & +0{\cdot}05255 \end{array}$$

$I=7000{\cdot}00000$ grains of the new standard, $\Theta=161{\cdot}62878$ grains. Therefore $5(Z+\Theta)=7161{\cdot}68133$ grs., $Z+\Theta=1432{\cdot}33627$ grs. Therefore $\mathfrak{C}=15432{\cdot}32462$ grs.

According to the observations of SCHUMACHER, OLUFSEN, STEINHEIL and GAMBEY, $v\mathfrak{A}-v\mathfrak{C}=20{\cdot}933$, whence $\mathfrak{A}=\mathfrak{C}+0{\cdot}02412$ grain. Therefore the kilogramme des Archives $=15432{\cdot}34874$ grains, of which I, the new standard pound, contains $7000{\cdot}00000$. If we adopt for $v\mathfrak{A}-v\mathfrak{C}$ the value $21{\cdot}119$ given by the observations with the stereometer, $\mathfrak{A}=\mathfrak{C}+0{\cdot}02435$ grain, and therefore $\mathfrak{A}=15432{\cdot}34897$ grains. Of these two values of $\mathfrak{A}$ the former is probably the more accurate.

Comparison of $\mathfrak{A}$ *with* PC No. 1+PC No. 2+B+V+0·03 grain.

Let K, L denote PC No. 1, PC No. 2 respectively; $\mathfrak{P}$=K+L+B+V+0·03 grain. The zero of the scale of the barometer was 180 millimètres below the middle of the weights, consequently 0·02 mm. must be subtracted from the height of the mercury in the barometer.

September 16, 1844. 100 parts=0·565 grain.

$\mathfrak{A}$+X.	$\mathfrak{P}$+Y.		$\mathfrak{A}$+Y.	$\mathfrak{P}$+X.	
22·62	22·25	C= 20·4	20·46	20·69	C= 20·55
22·52	22·16	F=762·1	20·65	20·85	F=761·7
21·90	22·40	E= 20·1	20·51	21·16	E= 20·4
22·43	22·33		21·05	21·30	
22·65	22·53		20·71	20·71	
22·37	21·97		21·54	21·50	
21·48	21·81	C= 20·55	22·49	21·91	C= 20·55
21·23	21·19	F=761·7	21·80	21·55	F=761·35
21·25	20·86	E= 20·4	21·86	21·93	E= 20·55
198·45	197·50		191·07	191·60	

9($\mathfrak{A}$+X)≏9($\mathfrak{P}$+Y)+0·95 part. 9($\mathfrak{A}$+Y)≏9($\mathfrak{P}$+X)−0·53 part.

18$\mathfrak{A}$≏18$\mathfrak{P}$+0·00237 grain in air (t=20·61, b=759·64).

September 17. 100 parts=0·5746 grain.

$\mathfrak{A}$+X.	$\mathfrak{P}$+Y.		$\mathfrak{A}$+Y.	$\mathfrak{P}$+X.	
20·02	19·91	C= 19·8	19·05	19·37	C= 20·76
19·70	20·00	F=758·53	19·36	19·37	F=758·2
19·70	20·55	E= 20·4	19·51	19·24	E= 20·9
19·31	20·22		18·49	19·90	
19·57	20·32		19·45	19·56	
19·29	20·01		18·44	19·91	
20·09	20·47		19·39	19·37	
20·32	20·21	C= 20·76	18·82	19·80	C= 21
19·71	20·01	F=758·2	19·40	19·71	F=758·04
20·09	19·76	E= 20·9	19·03	19·15	E= 21
179·80	201·46		199·94	195·38	

10($\mathfrak{A}$+X)≏10($\mathfrak{P}$+Y)−3·66 parts. 10($\mathfrak{A}$+Y)≏10($\mathfrak{P}$+X)−4·44 parts.

20$\mathfrak{A}$≏20$\mathfrak{P}$−0·046543 grain in air (t=20·68, b=756·12).

September 19. 100 parts=0·5322 grain.

$\mathfrak{A}$+X.	$\mathfrak{P}$+Y.		$\mathfrak{A}$+Y.	$\mathfrak{P}$+X.	
25·73	25·59	C= 18·5	14·25	14·19	C= 18·8
25·32	26·00	F=758·23	18·94	19·61	F=758·12
24·73	26·46	E= 18	19·50	19·87	E= 17·6
24·52	25·41		19·42	20·00	
25·77	26·16		19·50	19·72	
25·66	25·44		19·89	20·05	
25·15	25·65		19·42	20·02	
25·93	25·59		20·10	20·19	
25·61	25·65	C= 18·8	19·49	20·04	C= 18·85
24·79	25·59	F=758·12	19·80	19·96	F=758·2
25·20	25·31	E= 17·6	19·23	19·22	E= 17·9
278·41	282·85		209·54	212·87	

11($\mathfrak{A}$+X)≏11($\mathfrak{P}$+Y)−4·44 parts. 11($\mathfrak{A}$+Y)≏11($\mathfrak{P}$+X)−3·33 parts.

22$\mathfrak{A}$≏22$\mathfrak{P}$−0·041356 grain in air (t=18·8, b=756·39).

By a mean of the 60 comparisons,

$$\mathfrak{A} \doteq \mathfrak{B} - 0{\cdot}00143 \text{ grain in air } (t=19{\cdot}97,\ b=757{\cdot}28).$$

The value of these observations is considerably diminished by the fact that B contained a small cavity filled with some hygroscopic substance, which renders its weight slightly variable.

The following comparisons of V+0·03 grain with the sum of the auxiliary weights M, N, (32) were made with ROBINSON'S $10\frac{1}{2}$-inch balance.

$$\Xi = M+N+(32)+0{\cdot}45 \text{ grain}.$$

March 29, 1845. 100 parts=0·290 grain.

V+0·03 gr., Ξ.	Ξ, V+0·03 gr.
4·27	3·05
4·42	3·05
4·57	3·20
4·55	3·25
17·81	12·55

$$8V+0{\cdot}03 \text{ gr.} = 8\Xi + 5{\cdot}26 \text{ parts}.$$

$$V+0{\cdot}03 \text{ gr.} = M+N+(32)+0{\cdot}45191 \text{ grain} = 192{\cdot}43590 \text{ grains}.$$

$v(B+V+0{\cdot}03 \text{ gr.})=67{\cdot}670$, $vK=330{\cdot}701$, $vL=330{\cdot}750$. Hence the volume of $\mathfrak{B}$ at 0° is equal to the volume of 729·121 grains of water of maximum density. But $v\mathfrak{A}=751{\cdot}014$, hence $v\mathfrak{A}-v\mathfrak{B}$= volume of 21·893 grains of water at its maximum density = volume of 0·02617 grain of air ($t=19{\cdot}97$, $b=757{\cdot}28$). Therefore $\mathfrak{A}=K+L+B+V+0{\cdot}03 \text{ gr.}+0{\cdot}02474$ gr. $K=7000{\cdot}00051$ grains, $L=6999{\cdot}99911$ grains, of which I contains 7000·00000. The best determination of the weight of B+V+0·03 gr. is that which was obtained in March 1846, in the process of comparing $\mathfrak{C}$ with I. This gave $B+V+0{\cdot}03 \text{ gr.}=Z+\Theta-0{\cdot}01250$ gr., $Z+\Theta=1432{\cdot}33627$ grs. Therefore $B+V+0{\cdot}03 \text{ gr.}=1432{\cdot}32377$ grains. Hence $\mathfrak{A}=15432{\cdot}34813$ grains.

A similar comparison made in November and December 1845, but afterwards rejected on account of the quantity of hygroscopic matter contained in the auxiliary weights A, C, F, gave,—$B+V+0{\cdot}03 \text{ gr.}=\Phi+\Theta+0{\cdot}01055$ gr., $\Phi+\Theta=1432{\cdot}31313$ grs. Therefore $B+V+0{\cdot}03 \text{ gr.}=1432{\cdot}32368$ grains. Whence $\mathfrak{A}=15432{\cdot}34804$ grains.

On the 29th of March, 1845, V+0·03 grain was compared eight times with M+N+(32)+0·45 grain, for the purpose of obtaining an approximate value of B+V+0·03 gr., to be used in the reduction of the weighings by which $\mathfrak{A}$ was compared with K+L+B+V+0·03 gr. These comparisons gave $V+0{\cdot}03 \text{ gr.}=M+N+(32)+0{\cdot}45191$ grain. Whence $V+0{\cdot}03 \text{ gr.}=192{\cdot}43590$ grains. By numerous comparisons of the auxiliary weights with each other and with T after its final reduction, in January and February 1845, $B=1239{\cdot}88650$ grains, of which T contains 5759·47141 and I contains 7000·00093. Hence $B=1239{\cdot}88634$ grains, of which T contains 5759·47067 and I contains 7000·00000. Therefore $B+V+0{\cdot}03 \text{ gr.}=1432{\cdot}32224$ grains, and $\mathfrak{A}=15432{\cdot}34660$ grains.

By a mean of 286 comparisons of T with Sp in January ... August, 1845, T≏Sp+0·00073 grain in air (t=13·74, b=758·91). vT−vSp=volume of 0·26 grain of water = volume of 0·00032 grain of air. Therefore T=Sp+0·00105 grain. If I=7000·00000 grains, T=5759·47067 grains, and therefore Sp=5759·46962 grains. According to the account in manuscript of Professor Schumacher's comparisons of his kilogramme 𝔖 with his troy pound Sp, by the observations of December 1831 and October 1835, 𝔖=15433·77226 grains, of which Sp contains 5760. After these comparisons 𝔖 lost a portion of its substance, and became 0·02557 grain lighter; for by a mean of forty comparisons by Petersen in March 1837, and thirty-six comparisons by Captain v. Nehus in February and March 1837, 𝔖=15433·74669 grains, of which Sp contains 5760. Hence, in 1837, 𝔖=15432·32555 grains, of which I contains 7000·00000. In Steinheil's Memoir, "Ueber das Bergkrystall-Kilogramm," (p. 168), the following passage occurs: "Conferenzrath Schumacher erhielt eine Platina-Copie des Archivkilogrammes, welche, zufolge sorgfältiger Abwägungen des Professor Olufsen aus Koppenhagen im April 1835 (siehe Jahrbuch für 1856 von Schumacher, p. 250) leichter war als das Archivkilogramm um 0·41 Milligrammen. Aus Unvorsichtigkeit eines Mechanikus wurde diese Copie später abgewaschen und hatte dadurch an Gewicht verloren. Um zu bestimmen, wie viel dieser Verlust betrug, wiederholte ich im Jahr 1837 die Vergleichung der Schumacher'schen Copie mit dem Normalkilogramm der Archive und es ergab sich, dass es nun 1·59 Milligrammen leichter war, also 1·09 Milligrammen an Gewicht durch die Operation des Abwaschens verloren hatte." The most probable explanation that can be offered of the discrepancy contained in these words, is that by an error of the press, 1·59 was substituted for 1·50. According to Kupffer, however (Travaux de la Commission pour fixer les mesures et les poids de l'Empire de Russie, t. ii. p. 413), the value of 𝔄−𝔖 is 1·592 milligramme. Supposing that in 1837, 𝔄−𝔖=1·50 milligramme =0·02315 gr., then 𝔄=15432·34870 grs. If 𝔄−𝔖=1·59 milligramme =0·02454 gr., 𝔄=15432·35009 grains.

By the observations of 1831 and 1835, 𝔖=15432·35112 grains, of which I contains 7000·00000. If in April 1835, 𝔄−𝔖=0·41 milligramme =0·00633 grain, 𝔄=15432·35745 grains. This difference of 0·00875 grain in the weight of 𝔄 between 1835 and 1837, is difficult to account for, unless due to an accumulation of dust since wiped off; for it cannot be supposed that a weight so carefully preserved as the kilogramme des Archives, could have lost nearly 0·01 grain of its substance in the course of two years.

All the other comparisons, in which the number of observations is large, yield results sufficiently accordant with the value of 𝔄 deduced from the comparisons of 𝔄 with 𝔈, and of 𝔈 with I+K+A, I+L+B, I+M+Γ, I+N+Δ, which gave,—

Kilogramme des Archives =15432·34874 grains, of which the new platinum standard pound contains 7000. Or, kilogramme des Archives=(2·20462125) standard platinum lb. Standard platinum lb. =453·5926525 grammes, of which the kilogramme des Archives contains 1000.

Weight of the Kilogramme type laiton.

This kilogramme of brass, which will be designated by the letter $\mathfrak{L}$, is deposited in the Ministère de l'Intérieure in Paris, and serves as a standard for the purpose of adjusting the kilogrammes used in commerce. In a paper in the 'Memorie di Matematica e di Fisica della Società Italiana delle Scienze residente in Modena,' t. xxv. p. 1, it is stated that in 1850, $\mathfrak{A} \triangleq L+89{\cdot}5$ milligrammes in air ($t=18{\cdot}90$, $b=763{\cdot}80$). A brass kilogramme marked No. IV., the volume of which is assumed to be equal to that of $\mathfrak{L}$ (the grounds for this assumption are not stated), was weighed in water at 16·97. The absolute weight of the water displaced by it was 124·5536 grammes.

Hence $v\mathfrak{L}$ assumed $= v$Kilo. No. IV.$=$ volume of 124·590 grammes $=$ volume of 1922·720 grains of water at its maximum density. $v\mathfrak{C}=730{\cdot}078$, $v\mathfrak{A}-v\mathfrak{C}=20{\cdot}933$. Therefore $v\mathfrak{A}=751{\cdot}011$. The volume of 89·5 milligrammes or 1·38120 grain of platinum $=$ volume of 0·065 grain of water. Therefore $v(\mathfrak{A}-89{\cdot}5 \text{ milligr.})=750{\cdot}946$. During the comparison, $\mathfrak{L}$ displaced 2·32768 grains of air, and $\mathfrak{A}$ displaced 0·90868 grain. Therefore $\mathfrak{L}=\mathfrak{A}+0{\cdot}03780$ grain. $\mathfrak{L}$ displaces 2·30493 grains, and $\mathfrak{A}$ displaces 0·89981 grain of air ($\log\Delta=7{\cdot}07832-10$). Therefore $\mathfrak{L} \triangleq \mathfrak{A}-1{\cdot}36732$ grain in air ($\log\Delta=7{\cdot}07832-10$).

The commercial standard lb. is a weight of 7000 grains, of the same density as the lost standard troy pound U. Denoting the commercial standard lb. by W, and the platinum standard by I, in air ($\log\Delta=7{\cdot}07832-10$) $W \triangleq I-0{\cdot}63407$ grain. Therefore $\frac{15432{\cdot}34874}{7000} W \triangleq \frac{15432{\cdot}34874}{7000} I-1{\cdot}39788$ grain. $v\mathfrak{A}-v\,\frac{15432{\cdot}349}{7000}\,I=$ volume of 29·599 grains of water of maximum density $=$ volume of 0·03547 grain of air. Therefore $\mathfrak{A}+0{\cdot}03547$ grain $\triangleq \frac{15432{\cdot}34874}{7000}$ I. Hence $\mathfrak{L} \triangleq \frac{15432{\cdot}34386}{7000} W$, in air, for which $\log\Delta=7{\cdot}07832-10$. $\mathfrak{L}$ appears to weigh 15432·344 grains, of which W contains 7000. W appears to weigh 453·59278 grammes, of which $\mathfrak{L}$ contains 1000.

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The Quartz Pound.

The hardness of quartz, its capability of receiving a high polish, the absence of any hygroscopic properties, and its indestructibility at the ordinary temperature of the atmosphere by any chemical agent except hydrofluoric acid, are such valuable qualities in a substance used for the construction of weights, that Professor Steinheil was induced to adopt it as the material for a copy of the kilogramme. The only objection to the use of a weight made of quartz is, that on account of the large amount of air displaced, the barometer and thermometer must be observed with extreme care during its comparison with a weight of any ordinary metal. Notwithstanding this disadvantage, it appeared desirable to the Committee that a weight of quartz should be constructed sufficiently near 7000 grains to admit of readily deducing the pound from it. They accordingly commissioned Mr. Barrow to construct a weight of

quartz of the form of a cube of about 2·2 inches, having its edges and angles rounded. Its apparent weight in air is intermediate between that of a lb. of platinum and a lb. of brass, approaching more nearly to the latter than to the former.

Weighing of the quartz weight in water.

The quartz weight, which will be designated by the letter Q, was suspended in water, from the right-hand pan of the balance, by a fine copper wire, one end of which was fastened to a thicker wire bent round Q in a plane through its centre, parallel to a pair of faces of the cube. 7·23 inches of the suspending wire weighed 0·96 grain. Hence a portion of the wire corresponding to 100 parts of the scale displaced 0·016 grain of water. When the counterpoise in the left-hand pan was in equilibrium with the weight A+B+D+G+K+M+S+(16)+(2)+0·1 grain in the right-hand pan, 100 parts =0·268 grain. Hence when Q is suspended in water 100 parts =0·284 grain. In the observations of July 15 the suspending wire alone immersed to the same depth as when Q was suspended in water ≏ 12·7029 grains of platinum in air. At the close of these observations the fine wire broke, and was replaced. In the observations which followed, the wire alone in water ≏ 12·5641 grains of platinum in air.

July 15, 1846. Q and wire in water. G.	L.	In right-hand pan. gr.	Scale.
118·05	62·85	3·04	15·30
		3·05	21·30
		3·04	21·45
		3·02	11·00
		3·02	17·00
		3·02	14·00
118·20	63·05	3·04	20·45
		3·04	20·40
		3·00	16·00
		3·00	13·60
118·25	63·30	3·03	20·45
		3·02	19·45
		3·02	15·50
		3·02	19·10
		3·02	19·40
		3·02	20·30
118·40	63·50	3·00	17·25
		3·00	15·70
		3·00	17·30
118·22	63·17	3·020	17·71

Counterpoise balances Q and wire in water (G=118·40, L=63·17)+3·0237 grains in air.

	In right-hand pan. gr.	Scale.
A+B+D+G+K+M+S+(16)+	2·68	24·6
	2·64	12·6
D=18·35, C=18·4, F=761·8, E=20.	2·67	23·2
	2·66	19·2
	2·6625	19·9

C.poise bal. A+B+D+G+K+M+S+(16)+2·6601 grs. in air (D=18·35, C=18·4, F=761·8, E=20).

Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
118·7	64·1	2·94	7·45
		2·94	11·90
		2·98	19·45
		2·98	18·35
119·0	64·5	2·98	21·50
		2·94	17·50
		2·96	21·70
118·85	64·3	2·9644	17·62

Counterpoise balances Q and wire in water (G=118·85, L=64·3)+2·9683 grains in air.
Q and wire in water (t=17·30)≏A+B+D+G+K+M+S+(16)−0·3359 gr. in air (t=18·41, b=759·79).

July 15, 1846. Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
		2·94	14·80
		2·96	19·70
119·25	64·90	2·96	22·90
		2·94	18·25
		2·94	20·75
		2·94	19·65
119·40	65·10	2·94	21·10
		2·94	21·15
119·32	65·00	2·945	19·79

Counterpoise balances Q and wire in water (G=119·32, L=65·0)+2·9428 grains in air.

	In right-hand pan.	
	gr.	Scale.
A+B+D+G+K+M+S+(16)	+2·66	17·60
	+2·67	19·85
D=18·5, C=18·5, F=761·8, E=20.	+2·67	19·97
	2·6667	19·14

C.poise bal. A+B+D+G+K+M+S+(16)+2·6663 grs. in air (D=18·5, C=18·5, F=761·8, E=20).

July 15, 1846. Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
119·60	65·50	2·94	26·10
		2·90	20·10
		2·90	12·10
		2·90	15·30
		2·94	21·50
		2·94	26·50
119·80	65·80	2·90	18·35
		2·90	16·25
119·7	65·65	2·915	19·52

Counterpoise balances Q and wire in water (G=119·7, L=65·15)+2·9135 grains in air.
Q and wire in water (t=17·44)≏A+B+D+G+K+M+S+(16)−0·2618 grain in air (t=18·54, b=759·79).

July 28. Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
	71·05	2·24	18·20
		2·24	18·00
		2·24	18·30
	71·10	2·24	17·50
		2·24	17·60

	2·24	17·10
	2·24	17·00
71·05	2·24	18·00
	2·24	17·30
	2·24	17·90
71·07	2·240	17·69

Counterpoise balances Q and wire in water (t=18·15)+2·2437 grains in air.

	In right-hand pan. gr.	Scale.
A+B+D+G+K+M+S+(16)	+2·10	23·80
	+2·08	16·27
D=18·45, C=18·5, F=768·9, E=19·7.	+2·09	20·00
	+2·08	15·15
	+2·09	19·30
	2·088	18·90

C.poise bal. A+B+D+G+K+M+S+(16)+2·0883 grs. in air (D=18·45, C=18·5, F=768·9, E=19·7).

Q and wire in water. G.	L.	In right-hand pan. gr.	Scale.
122·9	70·93	2·24	16·4
		2·28	28·0
		2·24	10·6
		2·24	17·0
		2·24	18·5
		2·24	18·0
		2·24	14·7
122·87	70·8	2·24	17·1
		2·28	23·2
		2·28	24·9
		2·24	14·0
		2·28	22·5
		2·28	30·5
		2·24	14·5
122·9	70·90	2·24	16·5
122·89	70·88	2·2533	19·09

Counterpoise balances Q and wire in water (t=18·13)+2·2530 grains in air.

Q and wire in water (t=18·14)≏A+B+D+G+K+M+S+(16)−0·1603 gr. in air (t=18·51, b=766·91).

July 29.

Q and wire in water. G.	L.	In right-hand pan. gr.	Scale.
119·10	64·60	2·54	17·0
		2·58	28·9
		2·50	10·5
		2·54	22·6
		2·50	13·1
		2·54	29·5
119·30	64·90	2·50	12·5
		2·50	9·9
		2·54	28·8
		2·52	23·1
119·45	65·13	2·50	20·0
		2·50	20·8
		2·48	13·5
		2·50	19·2
		2·50	23·5
		2·48	18·8
119·28	64·88	2·5137	19·48

Counterpoise balances Q and wire in water (G=119·28, L=64·88)+2·5123 grains in air.

In right-hand pan.

	gr.	Scale.
A+B+D+G+K+M+S+(16)	+2·08	20·6
	+2·07	16·3
D=18·4, C=18·4, F=766·1, E=20·7.	+2·08	19·6
	2·0767	18·83

C.poise balances A+B+D+G+K+M+S+(16)+2·0772 grs. in air (D=18·4, C=18·4, F=766·1, E=20·7).

Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
119·8	65·7	2·48	24·2
		2·44	7·5
		2·48	27·6
		2·44	11·8
		2·48	28·0
120·0	66·0	2·46	20·2
		2·44	8·5
		2·46	19·0
		2·46	21·8
		2·44	13·5
		2·46	22·2
120·15	66·3	2·44	18·5
		2·44	20·5
119·98	66·0	2·4554	18·71

Counterpoise balances Q and wire in water (G=119·98, L=66·0)+2·4562 grains in air.

Q and wire in water (t=17·46)≏A+B+D+G+K+M+S+(16)−0·4070 gr. in air (t=18·44, b=763·99).

July 29.

Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
120·30	66·57	2·44	18·9
		2·44	20·5
		2·44	24·5
		2·40	13·0
		2·44	26·9
120·45	66·90	2·40	13·6
		2·44	25·4
		2·40	13·7
		2·44	26·7
120·65	67·15	2·40	15·8
		2·44	27·5
		2·40	21·3
		2·40	20·5
120·47	66·87	2·421	20·64

Counterpoise balances Q and wire in water (G=120·47, L=66·8)+2·4163 grains in air.

In right-hand pan.

	gr.	Scale.
A+B+D+G+K+M+S+(16)	+2·08	20·30
	+2·08	18·30
D=18·5, C=18·6, F=765·4, E=20·4.	+2·08	19·02
	+2·08	18·52
	+2·04	2·95
	+2·12	33·57
	2·08	18·78

C.poise balances A+B+D+G+K+M+S+(16)+2·0806 grs. in air (D=18·5, C=18·6, F=765·4, E=20·4).

Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
121·05	67·80	2·38	22·9
		2·34	9·0
		2·38	25·6
		2·34	9·7
		2·38	26·7
121·25	68·05	2·34	13·9
		2·38	26·9
		2·34	15·5
		2·38	26·8
		2·34	17·3
		2·38	26·1
121·40	68·40	2·34	18·3
121·23	68·08	2·360	19·9

Counterpoise balances Q and wire in water (G=121·23, L=68·08)+2·3574 grains in air.

Q and wire in water (t=17·71)≏A+B+D+G+K+M+S+(16)−0·3062 gr. in air (t=18·59, b=763·33).

July 30.

Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
120·10	66·25	2·48	27·5
		2·44	20·1
		2·40	6·9
		2·40	5·0
		2·44	24·3
		2·40	8·0
120·25	66·50	2·43	21·9
		2·43	20·8
		2·43	21·2
120·40	66·70	2·42	19·5
120·25	66·48	2·427	17·52

Counterpoise balances Q and wire in water (G=120·25, L=66·48)+2·4312 grains in air.

	In right-hand pan.	
	gr.	Scale.
A+B+D+G+K+M+S+(16)	+2·08	18·25
	+2·08	18·45
D=18·6, C=18·6, F=764·2, E=22·3.	+2·08	18·97
	2·08	18·56

C.poise bal. A+B+D+G+K+M+S+(16)+2·0813 grs. in air (D=18·6, C=18·6, F=764·2, E=22·3).

Q and wire in water.		In right-hand pan.	
G.	L	gr.	Scale.
120·6	67·2	2·40	22·5
		2·37	12·5
		2·40	23·1
		2·38	18·1
120·85	67·5	2·38	18·2
		2·38	18·9
		2·38	21·0
121·0	67·8	2·38	21·7
		2·36	16·7
120·82	67·5	2·3811	19·19

Counterpoise balances Q and wire in water (G=120·82, L=67·5)+2·3806 grains in air.

Q and wire in water (t=17·65)≏A+B+D+G+K+M+S+(16)−0·3246 grain in air(t=18·64, b=761·90).

Apparent weight of Q in water, weighed with platinum weights.

Water.		Air.	
t.	gr.	t.	b.
17·3	Q≏4362·5201	18·41	759·79
17·44	Q≏4362·5942	18·54	759·79
18·14	Q≏4362·8345	18·51	766·91
17·46	Q≏4362·5878	18·44	763·99
17·71	Q≏4362·6886	18·59	763·33
17·65	Q≏4362·6702	18·64	761·90

Comparison of Q *with* PS.

During the comparison of Q with PS, the zero of the scale of the barometer was 305 millimètres above the centre of gravity of the weights, consequently 0·03 mm. must be added to the reading of the barometer. F denotes the reading of the barometer, E that of its attached thermometer. The thermometers C, D were suspended in the balance-case. PS will be designated by the letter I. The weight of 0·4 grain used with Q on the 3rd and 10th of August, was the 0·4 grain of Robinson's $5\frac{1}{2}$-inch balance; on the 5th it was the 0·4 grain of Barrow's balance; on the 6th it was the sum of the 0·3 grain and 0·1 grain of Barrow's balance; on the 7th it was the 0·4 grain of Robinson's $10\frac{1}{2}$-inch balance; on the 11th it was the sum of the 0·3 grain and 0·1 grain of Robinson's $5\frac{1}{2}$-inch balance. Z=0·4 gr.+0·3 gr.+0·1 gr. of Robinson's $5\frac{1}{2}$-inch balance+0·4 gr.+0·3 gr.+0·1 gr. of Barrow's balance+0·4 gr. of Robinson's $10\frac{1}{2}$-inch balance; (2) the 2-grain weight of Barrow's balance. The readings increase on placing a small weight in the left-hand pan.

100 parts=0·399 grain.

Z, (2).	(2), Z.
3·21	3·07
3·22	3·01
3·16	2·90
9·59	8·98

6Z =6(2)+0·00243 grain.

Z=(2)+0·000406 gr., (2)=1·999597 gr., Z=2·000003 grains, $\frac{1}{5}$Z=0·4000006 gr. Hence the mean value of the 0·4 grain used may be taken as exact.

August 3, 1846. 100 parts=0·31355 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, Q+0·4 gr.+X	21·16	Q+0·4 gr.+X, I+Y	18·57
				....	21·64	....	17·95
20·4	20·3			....	22·15	....	17·30
		760·0	21·5	....	22·15	....	16·57
20·6	20·5			....	22·42	....	17·51
20·65	20·6			I+X, Q+0·4 gr.+Y	23·12	Q+0·4 gr.+Y, I+X	16·41
20·65	20·6			I+0·02 gr.+X, Q+0·4 gr.+Y	17·09	Q+0·4 gr.+Y, I+0·02 gr.+X	22·51
		760·5	21·5	I+0·01 gr.+X, Q+0·4 gr.+Y	19·55	Q+0·4 gr.+Y, I+0·01 gr.+X	19·41
				....	20·19	....	19·49
				....	19·37	....	19·24
20·57	20·5	760·25	21·5	10I+0·05 gr., 10(Q+0·4 gr.)	208·84	10(Q+0·4 gr.), 10I+0·05 gr.	184·96

20(Q+0·4 gr.)≏20I+0·1 gr.+23·88 pts.

100 parts=0·31355 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, Q+0·4 gr.+Y	15·55	Q+0·4 gr.+Y, I+X	25·50
19·85	19·8	764·72	20·9	I+X, Q+0·42 gr.+Y	22·24	Q+0·42 gr.+Y, I+X	19·27
				I+X, Q+0·41 gr.+Y	18·59	Q+0·41 gr.+Y, I+X	22·29
				....	19·56	....	22·20
20	19·9			....	19·20	....	21·21
				I+Y, Q+0·41 gr.+X	19·94	Q+0·41 gr.+X, I+Y	20·75
20·05	19·95			....	18·79	....	20·47
		764·7	20·8	....	18·32	....	19·52
				....	18·96	....	20·29
20·15	20·05			....	18·65	....	19·50
20·01	19·92	764·71	20·85	10I, 10(Q+0·4 gr.)+0·1 gr.	189·80	10(Q+0·4 gr.)+0·1 gr., 10I	211·00

20(Q+0·4 gr.)≏20I−0·2 gr.−21·2 pts.

40(Q+0·4 gr.)≏40I−0·1 gr.+2·68 pts. in air (D=20·29, C=20·21, F=762·48, E=21·17).

Q+0·4 gr.≏I−0·00229 gr. in air (t=20·31, b=760·35).

August 5.

100 parts=0·31808 grain.

D.	C.	F.	E.		Scale.		Scale.
19·7	19·6			I+Y, Q+0·4 gr.+X	17·52	Q+0·4 gr.+X, I+Y	23·05
				....	18·07	....	22·82
				....	17·55	....	22·06
				....	18·31	....	22·44
20·0	19·87	762·2	20·4	....	19·44	....	22·47
				....	19·00	....	22·01
				....	19·47	....	22·06
				....	19·31	....	21·19
				....	19·35	....	20·11
20·2	20·1			....	19·81	....	19·65
19·97	19·86	762·2	20·4	10(I+Y), 10(Q+0·4gr.+X)	187·83	10(Q+0·4gr.+X), 10(I+Y)	217·86

20(Q+0·4 gr.+X)≏20(I+Y)−30·03 pts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, Q+0·4 gr.+Y	22·32	Q+0·4 gr.+Y, I+X	19·42
20·5	20·43			....	21·19	....	18·55
				....	22·05	....	18·31
				....	21·17	....	18·21
20·6	20·5	761·8	21	....	22·36	....	18·46
				....	22·11	....	19·97
				....	20·15	....	18·10
				....	20·01	....	17·60
20·65	20·6			....	20·04	....	17·71
				....	19·59	....	17·57
20·58	20·51	761·8	21	10(I+X), 10(Q+0·4 gr.+Y)	210·99	10(Q+0·4 gr.+Y), 10(I+X)	183·90

20(Q+0·4 gr.+Y)≏20(I+X)+27·09 pts.

40(Q+0·4 gr.)≏40I−2·94 pts. in air (D=20·27, C=20·18, F=762·0, E=20·7).

Q+0·4 gr.≏I−0·00023 grain in air (t=20·29, b=759·93).

August 6.

100 parts=0·31808 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, Q+0·4 gr.+Y	21·24	Q+0·4 gr.+Y, I+X	16·51
				....	21·17	....	16·71
20·7	20·6			....	22·05	....	18·66
				....	22·05	....	15·84
				I+0·01 gr.+X, Q+0·4 gr.+Y	18·57	Q+0·4 gr.+Y, I+0·01 gr.+X	20·01
		760·9	21·95	....	18·21	....	17·92
				....	18·92	....	17·54
21	20·93			....	18·55	....	17·57
				....	18·65	....	17·34
				....	19·59	....	17·00
20·85	20·76	760·9	21·95	10(I+X)+$\overset{\text{gr.}}{0{\cdot}06}$, 10(Q+$\overset{\text{gr.}}{0{\cdot}4}$+Y)	199·00	10(Q+$\overset{\text{gr.}}{0{\cdot}4}$+Y), 10(I+X)+$\overset{\text{gr.}}{0{\cdot}06}$	175·10

20(Q+0·4 gr.+Y)≏20(I+X)+0·12 gr.+23·9 pts.

D.	C.	F.	E.		Scale.		Scale.
				I+0·02 gr.+Y, Q+0·4 gr.+X	16·69	Q+0·4 gr.+X, I+0·02 gr.+Y	22·27
				....	16·37	....	20·55
21·1	21·06			....	16·32	....	21·22
				....	16·79	....	20·72
				I+0·01 gr.+Y, Q+0·4 gr.+X	20·10	Q+0·4 gr.+X, I+0·01 gr.+Y	18·49
		760·9	22	....	19·99	....	18·19
				....	20·52	....	18·34
21·2	21·15			....	19·75	....	17·44
				....	20·61	....	17·25
				....	19·56	....	18·77
21·15	21·1	760·9	22	10(I+Y)+0·14 gr., 10(Q+0·4 gr.+X)	186·70	10(Q+0·4 gr.+X), 10(I+Y)+0·14 gr.	193·24

20(Q+0·4 gr.+X)≏20(I+Y)+0·28 gr.−6·54 pts.

40(Q+0·4 gr.)≏40I+0·4 gr.+17·36 pts. in air (D=21, C=20·93, F=760·9, E=21·97).

Q+0·4 gr.≏I+0·01138 grain in air (t=21·03, b=758·68).

August 7. 100 parts=0·31299 grain.

D.	C.	F.	E.		Scale.		Scale.
20·2	20·1			I+Y, Q+0·4 gr.+X	25·40	Q+0·4 gr.+X, I+Y	16·72
20·35	20·3			I+0·01 gr.+Y, Q+0·4 gr.+X	22·02	Q+0·4 gr.+X, I+0·01 gr.+Y	18·61
		757·7	20·95	....	23·20	....	18·06
20·55	20·45			I+0·02 gr.+Y, Q+0·4 gr.+X	20·20	Q+0·4 gr.+X, I+0·02 gr.+Y	22·12
20·6	20·5			....	20·05	....	20·45
				....	20·49	....	21·22
20·7	20·6			....	20·17	....	19·90
20·85	20·7	757·6	21·2	....	20·17	....	20·89
20·95	20·9			....	20·91	....	19·72
				....	21·25	....	19·36
20·6	20·51	757·65	21·07	10(I+Y)+0·16 gr., 10(Q+0·4 gr.+X)	213·86	10(Q+0·4 gr.+X), 10(I+Y)+0·16 gr.	197·05

20(Q+0·4 gr.+X)≏20(I+Y)+0·32 gr.+16·81 pts.

D.	C.	F.	E.		Scale.		Scale.
20·92	20·85			I+0·02 gr.+X, Q+0·4 gr.+Y	20·82	Q+0·4 gr.+Y, I+0·02 gr.+X	19·54
		757·3	21·4	....	21·42	....	18·45
20·93	20·85			....	21·31	....	19·76
				....	21·76	....	18·67
20·97	20·9	757·2	21·5	....	22·11	....	19·42
				....	22·56	....	19·30
20·97	20·9			....	21·37	....	19·51
		756·83	21·5	....	21·67	....	20·60
20·95	20·9			....	23·00	....	19·46
				....	21·65	....	18·44
20·95	20·88	757·11	21·47	10(I+X)+0·2 gr., 10(Q+0·4 gr.+Y)	217·67	10(Q+0·4 gr.+Y), 10(I+X)+0·2 gr.	193·15

20(Q+0·4 gr.+Y)≏20(I+X)+0·4 gr.+24·52 pts.

40(Q+0·4 gr.)≏40I+0·72 gr.+41·33 pts. in air (D=20·77, C=20·7, F=757·38, E=21·27).

Q+0·4 gr.≏I+0·02123 grain in air (t=20·8, b=755·26).

August 11. 100 parts=0·31299 grain.

D.	C.	F.	E.		Scale.		Scale.
19·4	19·3			I+Y, Q+0·42 gr.+X	20·55	Q+0·42 gr.+X, I+Y	22·32
				....	20·06	....	20·12
19·6	19·5	765·52	20·6	....	19·97	....	19·37
				....	19·87	....	18·82
19·9	19·77			....	19·52	....	18·60
				....	19·05	....	16·22
19·98	19·85			....	20·61	....	16·50
		765·42	20·8	....	19·90	....	16·09
20	19·87			....	19·39	....	17·22
				....	20·19	....	16·50
19·78	19·66	765·47	20·7	10(I+Y), 10(Q+0·42 gr.+X)	199·11	10(Q+0·42 gr.+X), 10(I+Y)	181·76

20(Q+0·4 gr.+X)≏20(I+Y)−0·4 gr.+17·35 pts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, Q+0·42 gr.+Y	20·51	Q+0·42 gr.+Y, I+X	19·01
20·1	20	765·5	20·9	....	20·90	....	17·91
				....	20·49	....	17·27
20·15	20·03			....	20·81	....	17·40
		765·58	20·95	....	20·87	....	17·01
20·2	20·1			....	20·47	....	16·45
				....	20·31	....	16·95
20·25	20·1	765·43	20·9	....	20·95	....	16·31
				....	20·79	....	16·20
20·25	20·13			....	20·62	....	17·05
20·2	20·07	765·5	20·91	10(I+X), 10(Q+0·42 gr.+Y)	206·72	10(Q+0·42 gr.+Y), 10(I+Y)	171·56

20(Q+0·4 gr.+Y)≏20(I+Y)−0·4 gr.+35·16 pts.

40(Q+0·4 gr.)≏40I−0·8 gr.+52·51 pts. in air (D=19·99, C=19·86, F=765·48, E=20·8).

Q+0·4 gr.≏I−0·01589 grain in air (t=19·99, b=762·92).

gr.	t.	b.
Q≏I−0·40229	20·31	760·35
Q≏I−0·40023	20·29	759·93
Q≏I−0·38862	21·03	758·68
Q≏I−0·37877	20·80	755·26
Q≏I−0·41589	19·99	762·92

According to STEINHEIL, the density of quartz at 0° is 2·650962 of the maximum density of water, and its cubic expansion for 1° C. is 0·00003255.

The weighings of Q in water give

	Water at 18° displaced by Q.	Logarithms.
	2639·831	3·4215760
	2639·809	3·4215725
	2639·838	3·4215774
	2639·825	3·4215751
	2639·819	3·4215742
	2639·814	3·4215733
Mean	2639·823	3·4215748
Maximum density of water : Δ water at 18		0·0005867
vQ at 18	2643·391	3·4221615
Q	7002·368	3·8452449
ΔQ at 18 : maximum density of water	2·649009	0·4230834
ΔQ at 18 : Δ water at 18	2·652590	0·4236701

The density of STEINHEIL's kilogramme of quartz at 18° in terms of the density of water at 18° is 2·652908.

The logarithm of the volume of Q at $t°$, in volumes of a grain of water at its maximum density, will be 3·4221615+(t−18)×0·00001413. The comparisons of Q with I in air, reduced with this value of the volume of Q, give for the absolute weight of Q in a vacuum,

gr.	No. of comparisons.
Q=I+2·36801	40
Q=I+2·36871	40
Q=I+2·36817	40
Q=I+2·36782	40
Q=I+2·36715	40
Mean ......Q=I+2·36797	200

Secondary Standards.

The comparison of different brass weights with platinum weights, shows that, however carefully preserved, they are liable to gain from 0·01 grain to 0·02 grain or more in the course of a few years. Brass, therefore, unless very well protected by gilding, is quite unfit to be used in the construction of weights having that degree of accuracy which is required in secondary standards. Electro-gilding was tried in the first instance. It failed, however, to afford a sufficient protection to the metal underneath, and the weight of the pound when the gilding was completed, was so very uncertain as to render its adjustment an extremely troublesome process. It was afterwards discovered that these evils were due to the want of skill of the person employed as a gilder by Mr. Barrow, and not to any inherent defect in electro-gilding itself. Mr. Barrow then resorted to amalgam gilding. In order to adjust a weight, its under surface, which was slightly concave, was more thickly coated with gold than the rest of its surface. If too heavy, a little of the gold was removed by rubbing it with charcoal; if too light, more amalgam was added, and the mercury driven off by heat. Directions had been given that the weights should be made of the alloy used by Mr. Baily for the standard yard bars, consisting of thirty-two parts of copper, five of tin, and two of zinc. The densities, however, of the greater part of them indicate that these proportions have not been strictly observed. The lbs. numbered 31 up to 36, protected by electro-gilding, were constructed by Messrs. Ladd and Streathfield. The weighings were reduced with the expansions given in Table III.

The observations for finding the densities of the secondary standards, and for comparing them with the platinum standard, were made in a room in the basement of my own house in Cambridge, the brick floor of which afforded a perfectly firm foundation for a strong table on which the balance was mounted.

The weights employed in some of the weighings are of bronze, for which $\log \Delta = 0{\cdot}92250$.

Apparent weights of the secondary standards in water.

	Water.	Platinum.	Bronze.	Air.	
No. of lb.	*t*.	gr.	gr	*t*.	*b*.
1	15·11	6163·2354		17·03	755·10
2	18·41	6161·5534		19·86	760·32
3	17·75	6157·7809		19·94	753·04
4	15·35	6163·6422		15·62	767·02
5	14·69	6132·0586		16·45	755·40
6	14·31	6155·7369		16·34	758·92
7	14·24	6138·4869		15·17	760·01
8	17·48	6143·1594		19·10	760·86
9	16·31	6047·7747		17·01	763·14
10	14·71	6155·3347		16·88	758·12
11	17·80	6163·6244		18·49	750·29
12	15·28	6159·0015		16·09	767·20
13	10·79	6169·9472		12·20	734·36
14	14·42	6161·9845		15·54	755·94
15	10·23	6162·9240		11·01	751·19
16	17·43	6133·5979		17·93	754·30
17*a*	11·15	38·3615	6099·9740	12·42	758·02

17*b*	10·75	6182·2705		11·44	738·69
18	17·20	6157·6295		19·82	757·27
19	15·39	6161·0862		17·01	767·23
21	15·59	6122·6308		15·94	771·23
22	17·59	6146·8491		19·45	756·99
23	16·08	6141·7878		16·34	763·64
24	17·46	6140·9677		19·42	758·65
25	17·91	6136·6810		20·04	758·03
26	12·21	6141·5668		14·10	754·92
27	16·27	6142·9019		16·52	763·47
28	17·28	6139·2058		18·23	761·17
29	11·35	45·4802	6099·9740	13·35	758·40
30	16·18	6141·9389		17·06	763·07
31	14·53	78·7565	6099·9776	15·05	741·30
32	14·53	72·5862	6099·9776	15·11	741·50
33	14·63	75·3351	6099·9776	15·11	741·88
34	15·03	78·8455	6099·9776	15·01	752·51
35	15·03	74·5158	6099·9776	15·41	752·88
36	14·93	77·0155	6099·9776	15·31	753·07

Densities and volumes of the secondary standards at 0° C., the density being expressed in terms of the maximum density of water, and the volume in volumes of a grain of water at its maximum density.

No.	ΔG.	*v*G.	log *v*G.
1	8·36134	837·186	2·922822
2	8·34161	839·162	2·923846
3	8·30462	842·904	2·925778
4	8·36500	836·820	2 922632
5	8·06122	868·356	2·938698
6	8·28779	844·614	2·926658
7	8·12163	861·898	2·935456
8	8·16317	857·512	2·933240
9	7·37614	952·897	2·979046
10	8·28375	845·024	2·926869
11	8·36302	837·013	2·922732
12	8·31919	841·426	2·925016
13	8·43179	830·187	2·919176
14	8·34955	838·365	2·923433
15	8·36107	837·211	2·922835
16	8·07354	867·028	2·938033
17*a*	8·11718	862·365	2·935691
17*b*	8·55888	817·858	2·912678
18	8·30369	842·999	2·925827
19	8·33969	839·358	2·923947
21	7·97370	877·890	2·943440
22	8·19859	853·804	2·931358
23	8·15141	858·748	2·933866
24	8·14286	859·645	2·934319
25	8·10164	864·022	2·936525
26	8·15218	858·665	2·933824
27	8·16186	857·650	2·933310
28	8·12604	861·428	2·935219
29	8·18446	855·279	2·932108
30	8·15292	858·588	2·933785
31	8·51444	822·129	2·914940
32	8·47042	828·302	2·918189
33	8·47902	825·563	2·916750
34	8·51472	822·103	2·914926
35	8·47019	826·424	2·917203
36	8·49601	823·912	2·915881

Secondary standards weighed in air.

The small weights R were made by winding fine silvered brass wire, to which a weight was attached nearly as heavy as the wire could sustain, round a piece of steel pinion-wire. The pinion-wire, with the brass wire wound round it, and still stretched by the weight, was then clamped between two pieces of wood, and the rings separated by placing the back of a sharp penknife in one of the grooves of the pinion-wire, and pushing it on in the groove till all the coils of the fine wire were cut through. The weight of eighty-eight of the rings was found to be 0·99595 grain. Hence $R=0·011313$ grain. The weights S of 0·01136 grain each, used in two of the observations, were made in the same manner. $I \bumpeq T+Q+D-0·00077$ grain in air ($t=19·63$, $b=764·43$), $\log v(I-Q)=2·51960$, $\log v(T+D)=2·51942$. Hence $I=T+Q+D-0·00060$ gr. $T+D=I-0·64449$ gr. Let W be taken to denote the commercial lb. Then, in air, at 65°·66 F., bar. 29·75 inches ($t=18·7$, $b=755·64$), at Somerset House, or in air for which $\log \Delta=7·07832-10$, $W \bumpeq I-Q+0·01102$ gr. Also, since $T+Q+D \bumpeq I+0·00077$ gr., $W \bumpeq T+D+0·01025$ grain. $I-Q$ displaces 0·39641 grain, and $T+D$ displaces 0·39624 grain of air, for which $\log \Delta=7·07832-10$. M denotes the mean of the readings of the thermometers D, C.

G= lb. No. 1.

100 parts $+0·27297$ grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+Y, G+X	28·10	G+X, T+D+R+Y	18·87
				....	24·96	....	19·45
13·15	13·2			....	26·60	....	17·14
				....	27·77	....	18·81
				....	28·34	....	20·32
		752·4	13·85	T+D+2R+Y, G+X	21·12	G+X, T+D+2R+Y	22·47
				....	19·90	....	19·34
13·3	13·35			....	18·57	....	19·31
				....	19·47	....	17·77
				....	21·77	....	19·55
					236·60		193·03

$20(G+X) \bumpeq 20(T+D+Y)+30R+43·57$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+2R+X, G+Y	17·52	G+Y, T+D+2R+X	18·95
				....	19·25	....	20·47
13·5	13·5			....	20·00	....	22·09
				....	19·67	....	23·02
				....	20·06	....	22·66
		751·75	14	....	20·82	....	20·39
				....	21·55	....	22·15
13·55	13·55			....	20·50	....	22·75
				....	22·27	....	23·37
				....	23·22	....	22·55
					204·86		218·40

$20(G+Y) \bumpeq 20(T+D+X)+40R-13·54$ parts.

$40G \bumpeq 40(T+D)+70R+30·03$ parts, $M=13·39$, $F=752·07$, $E=13·92$.

$G \bumpeq T+D+0·02183$ grain in air ($t=13·42$, $b=750·83$).

$G=I-0·00732$ grain.

$G \bumpeq W+0·01956$ grain in air ($\log \Delta=7·07832-10$).

G= lb. No. 2.

100 parts =0·27039 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+R+Y	19·86	G+R+Y, T+D+X	19·05
				....	22·37	....	18·31
12·9	12·95			....	21·82	....	18·95
				....	19·64	....	17·31
				....	17·72	....	13·54
		748·5	13·4	....	18·00	....	13·36
				T+D+X, G+Y	13·06	G+Y, T+D+X	19·32
				....	13·27	....	18·47
13·1	13·13			....	13·96	....	19·71
				....	14·97	....	19·10
				....	15·60	....	18·87
					190·27		195·99

22(G+Y)+12R≏22(T+D+X)−5·72 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	18·00	G+X, T+D+Y	23·42
					20·61		22·85
13·45	13·48			....	18·47	....	21·16
				....	18·55	....	22·22
				....	18·84	....	22·49
		748·9	13·65	....	19·70	....	22·22
				....	17·44	....	20·54
				....	17·22	....	19·47
13·4	13·45			....	18·24	....	22·22
				....	18·17	....	21·35
				....	16·81	....	22·67
					202·05		240·61

22(G+X)≏22(T+D+Y)−38·56 parts.

44G≏44(T+D)−12R−44·28 parts, M=13·23, F=748·7, E=13·52.

G≏T+D−0·00587 grain in air (*t*=13·26, *b*=746·51).

G=I−0·03582 grain.

G≏W−0·01132 grain in air (log Δ=7·07832−10).

G= lb. No. 3.

100 parts=0·27397 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+Y, G+X	28·30	G+X, T+D+R+Y	23·15
				....	28·00	....	19·65
12·55	12·55			....	29·51	....	21·54
				....	27·62	....	20·90
				....	28·07	....	18·70
		755·8	13·3	T+D+2R+Y, G+X	23·04	G+X, T+D+2R+Y	24·64
				....	24·89	....	24·05
12·85	12·85			....	21·67	....	20·79
				....	22·80	....	20·29
				....	22·16	....	20·30
					256·06		214·01

20(G+X)≏20(T+D+Y)+30R+42·05 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+2R+X, G+Y	19·74	G+Y, T+D+2R+X	19·60
				....	22·09	....	22·70
·13	13·05			....	23·47	....	22·87
				....	20·30	....	21·32
				....	23·27	....	22·47
		756·3	13·6	....	22·25	....	21·30
				....	23·35	....	20·89
13·4	13·45			....	20·16	....	22·34
				....	20·62	....	21·24
				....	19·62	....	18·97
					214·87		213·70

$20(G+Y) \bumpeq 20(T+D+X)+40R+1{\cdot}17$ part.

$40G \bumpeq 40(T+D)+70R+43{\cdot}22$ parts, $M=12{\cdot}96$, $F=756{\cdot}05$, $E=13{\cdot}45$.

$G \bumpeq T+D+0{\cdot}02276$ grain in air ($t=12{\cdot}99$, $b=754{\cdot}86$).

$G=I+0{\cdot}00510$ grain.

$G \bumpeq W+0{\cdot}02512$ grain in air ($\log\Delta=7{\cdot}07832-10$).

G=lb. No. 4.

100 parts=0·27450 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+X, G+Y	30·12	G+Y, T+D+R+X	23·31
				....	22·02	....	17·34
12·75	12·78			....	25·09	....	16·42
				....	23·71	....	13·98
				....	21·66	....	15·15
		764·6	13·5	....	22·77	....	14·45
				T+D+2R+X, G+Y	15·36	G+Y, T+D+2R+X	14·84
				....	15·14	....	14·35
13·25	13·25			....	17·54	....	15·31
				....	15·86	....	16·81
				....	16·65	....	13·57
					225·92		175·53

$22(G+Y) \bumpeq 22(T+D+X)+32R+50{\cdot}32$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+2R+Y, G+X	17·09	G+X, T+D+2R+Y	16·57
				....	16·90	....	16·45
13·2	13·25			....	17·29	....	14·69
				....	15·44	....	15·50
				....	14·87	....	14·94
		762·8	13·8	....	16·12	....	13·61
				....	17·60	....	15·07
				....	14·97	....	14·47
13·4	13·4			....	15·95	....	14·94
				....	15·10	....	12·30
				....	14·45	....	14·72
					175·78		163·26

$22(G+X) \bumpeq 22(T+D+Y)+44R+12{\cdot}52$ parts.

$44G \bumpeq 44(T+D)+76R+62{\cdot}84$ parts, $M=13{\cdot}16$, $F=763{\cdot}7$, $E=13{\cdot}65$.

$G \bumpeq T+D+0{\cdot}02346$ grain in air ($t=13{\cdot}19$, $b=762{\cdot}47$).

$G=I+0{\cdot}00425$ grain.

$G \bumpeq W+0{\cdot}03157$ grain in air ($\log\Delta=7{\cdot}07832-10$).

G=lb. No. 5.

100 parts=0·27485 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+R+X	16·60	G+R+X, T+D+Y	16·00
				....	19·16	....	15·17
13	13·03			....	18·36	....	12·96
				....	15·47	....	10·62
				....	16·62	....	10·46
		767·55	13·8	T+D+Y, G+X	18·25	G+X, T+D+Y	17·74
				....	16·22	....	19·70
13·45	13·45			....	18·75	....	21·37
				....	18·15	....	21·35
				....	19·32	....	21·15
					176·90		166·52

20(G+X)≏20(T+D+Y)−10R+10·38 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	15·50	G+Y, T+D+X	20·17
				....	17·69	....	19·17
13·7	13·75			....	16·21	....	17·95
				....	15·30	....	18·06
				....	16·90	....	17·57
		767·5	14·1	....	16·35	....	19·62
				....	14·67	....	19·17
13·87	13·87			....	16·95	....	19·42
				....	16·60	....	19·15
				....	14·95	....	18·87
					161·12		189·15

20(G+Y)≏20(T+D+X)−28·03 parts.
40G≏40(T+D)−10R−17·65 parts, M=13·51, F=767·52, E=13·95.
G≏T+D−0·00404 grain in air (t=13·54, b=766·24).
G=I+0·01783 grain.
G≏W+0·00734 grain in air (log Δ=7·07832−10).

G=lb. No. 6.

100 parts=0·28770 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	17·49	G+Y, T+D+X	23·77
				....	20·07	....	23·05
14·35	14·35			....	18·75	....	20·84
				....	18·49	....	19·84
				....	18·56	....	22·10
		764·85	14·9	....	18·61	....	22·77
				....	19·65	....	23·31
14·6	14·6			T+D+X, G+R+Y	23·04	G+R+Y, T+D+X	18·54
				....	21·77	....	18·62
				....	21·16	....	17·37
					197·59		210·21

20(G+Y)≏20(T+D+X)−6R−12·62 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	17·65	G+X, T+D+Y	21·09
				....	18·20	....	21·02
15	15			....	19·61	....	21·05
				....	17·54	....	20·52
				....	17·60	....	19·56
		764·6	15·3	....	18·65	....	19·72
				....	18·62	....	20·19
15·1	15·3			....	15·72	....	19·12
				....	16·90	....	18·64
				....	16·50	....	19·12
					176·99		200·03

20(G+X)≏20(T+D+Y)−23·04 parts.

40G≏40(T+D)−6R−35·66 parts, M=14·77, F=764·72, E=15·1.

G≏T+D−0·00426 grain in air (t=14·79, b=763·30).

G=I−0·01714 grain.

G≏W+0·00083 grain in air (log Δ=7·07832−10).

G=lb. No. 7.

100 parts=0·26008 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+Y, G+X	18·15	G+X, T+D+R+Y	20·82
				....	21·24	....	22·49
14·2	14·2			....	20·66	....	22·74
				....	19·31	....	21·29
				....	18·46	....	19·34
		766·4	15·2	....	20·11	....	21·54
				....	21·06	....	20·34
14·6	14·6			....	20·77	....	21·36
				....	19·32	....	18·69
				....	20·36	....	19·67
					199·44		208·28

20(G+X)≏20(T+D+Y)+20R−8·84 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+X, G+Y	18·22	G+Y, T+D+R+X	19·10
				....	18·72	....	19·19
14·83	14·83			....	15·92	....	17·21
				....	17·34	....	17·72
				....	17·62	....	19·79
		765·9	15·2	....	18·66	....	20·21
				....	16·44	....	20·20
15	15			....	18·30	....	19·95
				....	16·19	....	19·72
				....	16·39	....	20·10
					173·80		193·19

20(G+Y)≏20(T+D+X)+20R−19·39 parts.

40G≏40(T+D)+40R−28·23 parts, M=14·66, F=766·15, E=15·2.

G≏T+D+0·00948 grain in air (t=14·68, b=764·72).

G=I+0·01933 grain.

G≏W+0·01658 grain in air (log Δ=7·07832−10).

G=lb. No. 8.

100 parts=0·28329 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+X, G+Y	30·32	G+Y, T+D+R+X	24·32
				....	20·42	....	12·95
14·8	14·8			....	25·95	....	19·30
				....	24·67	....	17·81
				....	23·59	....	16·55
		752·4	15·5	T+D+2R+X, G+Y	20·24	G+Y, T+D+2R+X	21·62
				....	21·17	....	20·57
15·25	15·25			....	19·71	....	20·37
				....	19·76	....	20·56
				....	20·34	....	19·32
					226·17		193·37

$20(G+Y) \triangleq 20(T+D+X)+30R+32{\cdot}8$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+2R+Y, G+X	18 22	G+X, T+D+2R+Y	15·90
				....	17·50	....	16·85
15·95	15·95			....	18·31	....	17·54
				....	16·31	....	15·22
				....	16·27	....	13·00
		753·0	16	....	14·76	....	14·10
				....	14·07	....	12·54
16·35	16·35			....	14·56	....	12·82
				....	16·40	....	12·46
				....	13·77	....	12·21
					160·17		142·64

$20(G+X) \triangleq 20(T+D+Y)+40R+17{\cdot}53$ parts.

$40G \triangleq 40(T+D)+70R+50{\cdot}33$ parts, $M=15{\cdot}59$, $F=752{\cdot}7$, $E=15{\cdot}75$.

$G \triangleq T+D+0{\cdot}02336$ grain in air ($t=15{\cdot}61$, $b=751{\cdot}24$).

$G=I+0{\cdot}01428$ grain.

$G \triangleq W+0{\cdot}01679$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 9.

100 parts=0·40919 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	24·34	G+Q+X, I+Y	21·67
				....	23·52	....	20·59
9·9	9·9	740·6	11	....	23·17	....	19·27
				....	22·55	....	18·57
				....	21·87	....	17·36
10·27	10·3	740·0	11·1	....	21·47	....	16·66
				....	20·94	....	16·31
				....	20·57	....	15·49
10·53	10·57	739·95	11·4	....	17·94	....	12·92
				....	17·34	....	12·49
					213·71		171·33

$20(G+Q+X) \triangleq 20(I+Y)+42{\cdot}38$ parts

D.	C.	F.	E.		Scale.		Scale.
				I+R+X, G+Q+Y	18·77	G+Q+Y, I+R+X	19·70
10·87	10·83	741·6	11·6	....	18·87	....	19·67
				....	17·32	....	19·02
				....	17·72	....	18·55
11·1	11·1			....	17·30	....	19·17
		742·0	11·6	....	16·82	....	18·09
				....	16·46	....	18·39
				....	16·55	....	18·24
11·15	11·2	742·9	11·7	....	16·05	....	17·36
				....	15·89	....	17·74
					171·75		185·93

$20(G+Q+Y) \bumpeq 20(I+X)+20R-14{\cdot}18$ parts.

$40(G+Q) \bumpeq 40I+20R+28{\cdot}2$ parts, $M=10{\cdot}64$, $F=741{\cdot}17$, $E=11{\cdot}42$.

$G+Q \bumpeq I+0{\cdot}00854$ grain in air ($t=10{\cdot}65$, $b=740{\cdot}25$).

$G=I+0{\cdot}11611$ grain.

$G \bumpeq W+0{\cdot}00426$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 10.

May 28, 1854. 100 parts = 0·28329 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+2R+X	21·55	G+2R+X, T+D+Y	19·56
14·6	14·6			....	23·96	....	21·32
				....	24·06	....	21·27
				....	22·36	....	19·46
				....	22·62	....	20·05
		756·2	15	T+D+Y, G+R+X	18·17	G+R+X, T+D+Y	24·45
				....	18·79	....	24·76
				....	17·95	....	24·06
				....	17·15	....	23·40
14·9	14·87			....	17·92	....	23·05
					204·53		221·38

$20(G+X) \bumpeq 20(T+D+Y)-30R-16{\cdot}85$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+2R+Y	19·82	G+2R+Y, T+D+X	17·85
15·05	15·05			....	20·87	....	18·61
				....	21·75	....	20·41
				....	22·16	....	20·15
				....	23·66	....	20·34
		756·3	15·4	....	22·85	....	20·20
				....	21·69	....	19·75
				....	19·79	....	17·37
15·2	15·2			....	20·71	....	17·49
				....	21·24	....	18·70
					214·54		190·87

$20(G+Y) \bumpeq 20(T+D+X)-40R+23{\cdot}67$ parts.

$40G \bumpeq 40(T+D)-70R+6{\cdot}82$ parts, $M=14{\cdot}94$, $F=756{\cdot}25$, $E=15{\cdot}2$.

$G \bumpeq T+D-0{\cdot}01931$ grain in air ($t=14{\cdot}96$, $b=754{\cdot}84$).

$G=I-0{\cdot}03910$ grain.

$G \bumpeq W-0{\cdot}02162$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 11.

January 6, 1853. 100 parts=0·41459 grain.

D.	C.	F.	E.	I+X, G+Q+2R+Y	Scale.	G+Q+2R+Y, I+X	Scale.
					25·89		24·55
10	10	749·9	10·8	....	25·85	....	23·57
				....	25·57	....	22·34
				....	24·59	....	21·65
				....	24·52	....	21·04
10·27	10·27	749·6	10·9	....	22·55	....	20·85
				....	23·19	....	19·86
				....	22·55	....	19·75
10·45	10·45	749·03	10·9	....	22·27	....	19·62
				....	22·51	....	19·32
					239·49		212·55

20(G+Q+Y)≏20(I+X)−40R+26·94 parts.

D.	C.	F.	E.	I+Y, G+Q+R+X	Scale.	G+Q+R+X, I+Y	Scale.
					18·57		20·96
10·53	10·6	748·8	11	....	19·01	....	21·40
				....	18·15	....	20·30
				....	17·57	....	20·86
				....	17·21	....	19·17
10·8	10·8	748·8	11·1	....	17·25	....	19·42
				....	16·37	....	18·12
				....	16·26	....	18·75
10·85	10·85	748·75	11·5	....	16·04	....	17·74
				....	15·35	....	17·32
					171·78		194·04

20(G+Q+X)≏20(I+Y)−20R−22·26 parts.
40(G+Q)≏40I−60R+4·68 parts, M=10·49, F=749·14, E=11·03.
G+Q≏I−0·01648 grain in air (t=10·49, b=748·25).
G=I−0·04208 grain.
G≏W−0·01499 grain in air (log Δ=7·07832−10).

G=lb. No. 12.

December 28, 1852. 100 parts=0·39337 grain.

D.	C.	F.	E.	I+Y, G+Q+X	Scale.	G+Q+X, I+Y	Scale.
					17·57		19·24
				....	16·66	....	17·70
10·4	10·4	752·4	11·4	....	17·92	....	19·00
				....	17·77	....	18·54
				....	20·52	....	21·50
10·8	10·8	752·5	11·5	....	19·99	....	20·17
				....	19·57	....	19·70
				....	19·79	....	19·59
10·73	10·73	752·5	11·5	....	19·40	....	18·62
				....	18·72	....	18·90
					187·91		192·96

20(G+Q+X)≏20(I+Y)−5·05 parts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+Y	16·96	G+Q+Y, I+X	18·99
				....	17·02	....	18·89
11·3	11·3	752·9	11·65	....	18·06	....	19·40
				....	18·61	....	20·12
				....	17·29	....	19·34
11·5	11·5	753·3	12	....	16·70	....	19·50
				....	19·31	....	18·71
				....	17·45	....	18·85
11·6	11·55	753·65	12	....	16·72	....	18·76
				....	16·77	....	17·91
					174·89		190·47

$20(G+Q+Y) \bumpeq 20(I+X)-15{\cdot}58$ parts.

$40(G+Q) \bumpeq 40I-20{\cdot}63$ parts, $M=11{\cdot}1$, $F=752{\cdot}88$, $E=11{\cdot}69$.

$G+Q \bumpeq I-0{\cdot}00203$ grain in air ($t=11{\cdot}11$, $b=751{\cdot}9$).

$G=I-0{\cdot}02060$ grain.

$G \bumpeq W+0{\cdot}00118$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 13.

December 29, 1852. 100 parts=0·38625 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+R+Y	23·04	G+Q+R+Y, I+X	20·54
				....	22·84	....	20·85
9·95	9·95	755·3	10·6	....	20·79	....	19·35
				....	20·90	....	18·17
				....	20·22	....	17·50
10·45	10·5	754·4	10·9	....	19·19	....	16·82
				....	19·45	....	15·65
				....	19·00	....	15·27
10·52	10·52	754·29	10·95	....	19·01	....	15·25
				....	19·82	....	14·71
					204·26		174·11

$20(G+Q+Y) \bumpeq 20(I+X)-20R+30{\cdot}15$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+R+X	19·26	G+Q+R+X, I+Y	15·19
				....	19·70	....	13·60
10·65	10·65	753·85	11·1	....	18·45	....	12·29
				....	18·30	....	12·64
				I+Y, G+Q+X	14·89	G+Q+X, I+Y	14·94
10·8	10·8	753·7	11·3	....	14·49	....	14·61
				I+R+Y, G+Q+X	11·41	G+Q+X, I+R+Y	18·45
				....	12·54	....	18·01
11	11	753·6	11·2	I+Y, G+Q+X	14·11	G+Q+X, I+Y	14·92
				....	15·19	....	14·41
					158·34		149·06

$20(G+Q+X) \bumpeq 20(I+Y)-4R+9{\cdot}28$ parts.

$40(G+Q) \bumpeq 40I-24R+39{\cdot}43$ parts, $M=10{\cdot}67$, $F=754{\cdot}00$, $E=11{\cdot}07$.

$G+Q \bumpeq I-0{\cdot}00298$ grain in air ($t=10{\cdot}68$, $b=753{\cdot}21$).

$G=I-0{\cdot}03331$ grain.

$G \bumpeq W+0{\cdot}00195$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 14.

January 5, 1853. 100 parts=0·42206 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	17·67	G+Q+X, I+Y	21·62
				....	17·20	....	21·20
10·43	10·47	753·85	11·3	....	15·97	....	19·99
				....	16·19	....	19·24
				....	14·34	....	18·17
10·75	10·8	754·1	11·6	....	14·39	....	17·70
				I+Y, G+Q+R+X	16·40	G+Q+R+X, I+Y	15·35
				....	16·40	....	14·50
11	11	754·0	11·8	....	16·10	....	14·31
				....	16·10	....	14·39
					160·76		176·47

$20(G+Q+X) \bumpeq 20(I+Y)-8R-15{\cdot}71$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+R+Y	21·02	G+Q+R+Y, I+X	19·56
				....	21·41	....	19·59
11·35	11·35	754·0	12	....	20·50	....	18·84
				....	20·71	....	19·49
				....	21·32	....	18·70
11·35	11·35	754·2	12	....	21·05	....	18·22
				....	20·59	....	17·52
				....	20·01	....	17·55
11·45	11·45	754·25	12	....	19·89	....	17·05
				....	19·90	....	16·74
					206·40		183·26

$20(G+Q+Y) \bumpeq 20(I+X)-20R+23{\cdot}14$ parts.

$40(G+Q) \bumpeq 40I-28R+7{\cdot}43$ parts, $M=11{\cdot}07$, $F=754{\cdot}06$, $E=11{\cdot}78$.

$G+Q \bumpeq I-0{\cdot}00713$ grain in air ($t=11{\cdot}08$, $b=753{\cdot}07$).

$G=I-0{\cdot}02844$ grain.

$G \bumpeq W-0{\cdot}00297$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 15.

January 30, 31, 1851. 100 parts=0·34349 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	18·80	G+Q+X, I+Y	18·90
				....	17·84	....	17·04
				....	17·26	....	16·46
				....	17·66	....	15·89
10·15	10·2	749·7	11	....	18·30	....	15·54
				....	18·22	....	15·00
				....	16·89	....	15·01
				....	17·49	....	14·70
				....	16·35	....	13·69
				....	16·07	....	13·07
					174·88		155·30

$20(G+Q+X) \bumpeq 20(I+Y)+19{\cdot}58$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+Y	22·60	G+Q+Y, I+X	16·50
				....	24·50	....	16·98
				....	23·96	....	17·96
				....	23·75	....	18·21
9·0	9·03	742·75	9·5	I+0·01 gr.+X, G+Q+Y	19·55	G+Q+Y, I+0·01 gr.+X	19·95
				....	19·79	....	20·50
				....	20·50	....	19·30
				....	20·82	....	19·74
				....	20·92	....	21·31
				....	21·75	....	20·45
					218·14		190·90

20(G+Q+Y)≏20(I+X)+0·12 gr.+27·24 parts.

40(G+Q)≏40I+0·12 gr.+46·82 parts, M=9·59, F=746·22, E=10·25.

G+Q≏I+0·00702 grain in air (t=9·59, b=745·43).

G=I−0·01864 grain.

G≏W+0·00820 grain in air (log Δ=7·07832−10).

G=lb. No. 15.

March 18, 1854. 100 parts=0·25957 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+2R+Y	20·12	G+2R+Y, T+D+X	15·65
				....	21·75	....	14·62
				....	22·95	....	15·50
				....	21·20	....	16·15
10·15	10·2			T+D+X, G+R+Y	17·20	G+R+Y, T+D+X	18·65
				....	17·42	....	20·20
				....	16·47	....	17·02
				....	18·50	....	17·95
				....	16·30	....	17·45
		764·0	10·9	....	15·55	....	18·17
					189·46		171·36

20(G+Y)≏20(T+D+X)−28R+18·10 parts.

D.	C.	F.	E.		Scale.		Scale.
		764·0	10·9	T+D+Y, G+R+X	18·80	G+R+X, T+D+Y	16·29
				....	18·39	....	14·27
				....	19·54	....	16·00
				....	21·42	....	16·25
10·48	10·53			T+D+Y, G+X	14·57	G+X, T+D+Y	20·87
				....	18·82	....	19·56
				....	16·06	....	20·95
				....	14·95	....	20·12
				....	16·15	....	19·50
				....	15·72	....	20·51
					170·42		184·32

20(G+X)≏20(T+D+Y)−8R−13·9 parts.

40G≏40(T+D)−36R+4·2 parts, M=10·34, F=764·0, E=10·9.

G≏T+D−0·00991 grain in air (t=10·34, b=763·1).

G=I−0·02180 grain.

G≏W+0·00504 grain in air (log Δ=7·07832−10).

Means of both series of comparisons:—

G=I−0·02022 grain.

G≏W+0·00662 grain in air (log Δ=7·07832−10).

G=lb. No. 16.

December 31, 1852.

100 parts=0·38541 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+5R+Y	24·06	G+Q+5R+Y, I+X	24·01
				....	22·67	....	21·71
10·4	10·4	766·2	11·1	....	22·04	....	21·29
				....	21·89	....	19·82
				....	21·70	....	18·44
10·75	10·75	765·87	11·5	....	17·86	....	16·67
				....	23·24	....	20·17
				....	21·85	....	19·97
11·07	11·07	765·7	11·7	....	21·62	....	18·71
				....	22·19	....	18·37
					219·12		199·16

$20(G+Q+Y) \bumpeq 20(I+X)-100R+19{\cdot}96$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+4R+X	19·42	G+Q+4R+X, I+Y	22·02
				....	17·55	....	22·55
11·35	11·3	765·6	11·8	....	17·96	....	19·89
				....	17·77	....	20·22
				....	18·82	....	19·19
11·4	11·4	765·5	11·8	....	18·84	....	21·27
				....	17·95	....	21·50
				....	17·07	....	20·57
11·5	11·5	765·35	11·9	....	17·15	....	19·65
				....	17·59	....	20·25
					180·12		207·11

$20(G+Q+X) \bumpeq 20(I+Y)-80R-26{\cdot}99$ parts.

$40(G+Q) \bumpeq 40I-180R-7{\cdot}03$ parts, $M=11{\cdot}07$, $F=765{\cdot}7$, $E=11{\cdot}61$.

$G+Q \bumpeq I-0{\cdot}05159$ grain in air ($t=11{\cdot}08$, $b=764{\cdot}71$).

$G=I-0{\cdot}02747$ grain.

$G \bumpeq W-0{\cdot}03636$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 17*a*.

April 8, 1854.

100 parts=0·30837 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+2R+X	14·39	G+2R+X, T+D+Y	24·65
				....	15·44	....	25·15
				....	16·15	....	24·29
				....	16·17	....	23·70
				....	15·65	....	24·47
14·2	14·2			T+D+Y, G+3R+X	18·64	G+3R+X, T+D+Y	19·80
				....	18·69	....	20·37
				....	18·47	....	20·04
				....	18·65	....	18·67
		767·1	14·7	....	17·37	....	16·20
					168·59		217·34

$20(G+X)+50R \bumpeq 20(T+D+Y)-47{\cdot}75$ parts.

D.	C.	F.	E.		Scale.		Scale.
		767·1	14·7	T+D+X, G+3R+Y	14·32	G+3R+Y, T+D+X	19·57
				....	15·20	....	20·25
				....	15·27	....	19·20
				....	15·15	....	20·15
				....	16·45	....	20·47
14·5	14·55			....	15·41	....	19·50
				....	15·42	....	18·80
				....	14·40	....	17·31
				....	13·90	....	19·25
				....	15·00	....	18·32
					150·52		192·82

20(G+Y)+60R≏20(T+D+X)−42·3 parts.

40G+110R≏40(T+D)−90·05 parts, M=14·36, F=767·1, E=14·7.

G≏T+D−0·03811 grain in air (t=14·38, b=765·72).

G=I−0·02614 grain.

G≏W−0·02944 grain in air (log Δ=7·07832−10).

G=lb. No. 17*b*.

100 parts=0·29133 grain.

	Scale.		Scale.
T+D+X, G+Y	16·97	G+Y, T+D+X	19·52
....	19·65	....	20·65
....	18·15	....	20·07
T+D+Y, G+X	20·82	G+X, T+D+Y	14·12
T+D+R+Y, G+X	17·10	G+X, T+D+R+Y	21·55
T+D+Y, G+X	21·15	G+X, T+D+R+Y	21·01
	113·84		116·92

12G≏12(T+D)+3R−3·08 parts, M=11·91. F=755·1, E=12·6.

G≏T+D+0·00208 grain in air (t=11·93, b=754·01).

G=I−0·04475 grain.

G≏W+0·00529 grain in air (log Δ=7·07832−10).

100 parts=0·27861 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	19·97	G+Y, T+D+X	13·44
				....	19·15	....	12·12
				....	18·30	....	13·85
				....	20·05	....	14·44
13·15	13·2			....	19·52	....	14·16
		749·1	13·65	T+D+R+X, G+Y	16·41	G+Y, T+D+R+X	16·87
				....	13·89	....	14·95
				....	16·11	....	15·26
				....	17·76	....	15·59
				....	14·71	....	17·81
					172·87		148·49

20(G+Y)≏20(T+D+X)+10R+24·38 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+Y, G+X	22·82	G+X, T+D+R+Y	23·71
				....	23·50	....	23·90
				....	23·74	....	25·10
				....	25·00	....	24·09
13·45	13·45			....	23·41	....	23·62
		749·3	13·8	....	21·66	....	21·80
				....	24·37	....	21·55
				....	22·85	....	23·91
				....	25·50	....	22·32
				....	26·22	....	24·00
					239·07		234·00

$20(G+X) \bumpeq 20(T+D+Y)+20R+5\cdot07$ parts.

$40G \bumpeq 40(T+D)+30R+29\cdot45$ parts, $M=13\cdot31$, $F=749\cdot2$, $E=13\cdot72$.

$G \bumpeq T+D+0\cdot01054$ grain in air ($t=13\cdot34$, $b=747\cdot99$).

$G=I-0\cdot04413$ grain.

$G \bumpeq W+0\cdot00591$ grain in air ($\log \Delta=7\cdot07832-10$).

Means ......$G=I-0\cdot04427$ grain.

$G \bumpeq W+0\cdot00577$ grain in air ($\log \Delta=7\cdot07832-10$).

G = lb. No. 18.

100 parts $=0\cdot41249$ grain.

D.	C.	F.	E.		Scale.		Scale.
				I+S+Y, G+Q+X	16·34	G+Q+X, I+S+Y	17·80
11·2	11·2	752·75	12	....	16·25	....	18·25
				....	16·59	....	18·02
11·37	11·37	752·8	11·9	....	16·35	....	17·54
				....	15·57	....	17·82
				....	14·77	....	15·14
11·5	11·5	752·8	11·95	....	15·06	....	15·84
				....	14·47	....	16·00
11·55	11·55	752·83	12	....	14·66	....	14·90
				....	14·47	....	15·82
					154·53		167·13

$20(G+Q+X) \bumpeq 20(I+Y)+20S+12\cdot6$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+S+X, G+Q+Y	20·34	G+Q+Y, I+S+X	18·12
11·65	11·65	753·6	12·1	....	21·20	....	16·65
				I+2S+X, G+Q+Y	17·55	G+Q+Y, I+2S+X	18·92
11·75	11·75	753·7	12·2	....	17·74	....	19·89
				....	17·07	....	18·21
				....	16·86	....	19·29
11·9	12	753·7	12·5	....	15·69	....	17·97
				....	14·52	....	17·40
12	12	753·7	12·5	....	15·90	....	17·84
				....	16·41	....	17·62
					173·28		181·91

$20(G+Q+Y) \bumpeq 20(I+X)+36S-8\cdot63$ parts.

$40(G+Q) \bumpeq 40I+56S+3\cdot97$ parts, $M=11\cdot61$, $F=753\cdot23$, $E=12\cdot13$.

$G+Q \bumpeq I+0\cdot01631$ grain in air ($t=11\cdot63$, $b=752\cdot2$).

$G=I-0\cdot00129$ grain.

$G \bumpeq W+0\cdot01862$ grain in air ($\log \Delta=7\cdot07832-10$).

G=lb. No. 19

100 parts=0·40766 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+S+Y, G+Q+X	19·66	G+Q+X, I+S+Y	18·52
				....	18·54	....	17·50
11·55	11·6	747·6	12·6	....	17·76	....	17·34
				....	18·75	....	17·57
				....	19·04	....	17·80
11·93	11·93	747·4	12·6	....	17·50	....	16·52
				....	16·96	....	15·60
				....	16·92	....	16·44
12·07	12·15	747·5	12·9	....	15·17	....	15·55
				....	16·16	....	14·74
					176·46		167·58

$20(G+Q+X) \bumpeq 20(I+Y)+20S+8{\cdot}88$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+2S+X, G+Q+Y	18·80	G+Q+Y, I+2S+X	17·69
12·8	12·8	736·9	13·35	....	18·72	....	17·54
				....	17·44	....	18·07
				....	17·50	....	16·12
				....	17·02	....	16·92
12·95	12·95	737·1		....	16·62	....	17·44
				....	16·01	....	17·27
				....	17·54	....	16·87
13	13·05	737·2	13·1	....	15·81	....	16·45
				....	17·87	....	16·27
					173·33		170·64

$20(G+Q+Y) \bumpeq 20(I+X)+40S+2{\cdot}69$ parts.

$40(G+Q) \bumpeq 40I+60S+11{\cdot}57$ parts, $M=12{\cdot}4$, $F=742{\cdot}28$, $E=12{\cdot}87$.

$G+Q \bumpeq I+0{\cdot}01822$ grain in air ($t=12{\cdot}42$, $b=741{\cdot}22$).

$G=I-0{\cdot}01473$ grain.

$G \bumpeq W+0{\cdot}00954$ grain in air ($\log\Delta=7{\cdot}07832-10$).

G=lb. No. 21.

100 parts=0·38653 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	22·66	G+Q+X, I+Y	18·64
				....	22·77	....	18·89
11·25	11·25	750·9	11·8	I+R+Y, G+Q+X	19·71	G+Q+X, I+R+Y	20·71
				....	19·69	....	21·15
				....	19·15	....	20·87
11·4	11·4	750·8	12	....	19·17	....	21·00
				....	18·80	....	20·56
				....	19·35	....	20·20
11·5	11·5	750·45	12	....	19·14	....	20·85
				....	19·12	....	20·04
					199·53		202·91

$20(G+Q+X) \bumpeq 20(I+Y)+16R-3{\cdot}38$ parts.

D.	C.	F.	E.	I+X, G+Q+Y	Scale.	G+Q+Y, I+X	Scale.
					18·21		17·90
				....	19·07	....	17·22
10·5	10·5	751·63	11·4	....	18·87	....	16·16
				....	18·09	....	15·85
				....	18·52	....	15·96
10·9	10·9	751·33	11·6	....	16·75	....	14·95
				....	17·06	....	14·10
				....	16·82	....	13·62
11·05	11·05	751·20	11·7	....	16·27	....	13·50
				....	16·36	....	13·25
					176·02		152·51

$20(G+Q+Y) \triangleq 20(I+X)+23 \cdot 51$ parts.

$40(G+Q) \triangleq 40I+16R+20 \cdot 13$ parts, $M=11 \cdot 1$, $F=751 \cdot 05$, $E=11 \cdot 76$.

$G+Q \triangleq I+0 \cdot 00647$ grain in air ($t=11 \cdot 2$, $b=760 \cdot 07$).

$G=I+0 \cdot 03971$ grain.

$G \triangleq W+0 \cdot 01779$ grain in air ($\log \Delta=7 \cdot 07832-10$).

G=lb. No. 22.

100 parts=0·38541 grain.

D.	C.	F.	E.	I+Y, G+Q+R+X	Scale.	G+Q+R+X, I+Y	Scale.
					23·66		24·26
10·87	10·87	756·65	11·8	....	20·81	....	22·32
				....	21·47	....	22·30
				....	20·87	....	21·62
11·35	11·35	756·85	11·8	....	19·40	....	20·37
				....	18·45	....	20·29
				....	19·39	....	19·45
11·45	11·47	757·20	11·9	....	19·45	....	19·62
				....	18·84	....	18·35
				....	20·21	....	18·92
					202·55		207·50

$20(G+Q+X)+20R \triangleq 20(I+Y)-4 \cdot 95$ parts.

D.	C.	F.	E.	I+X, G+Q+R+Y	Scale.	G+Q+R+Y, I+X	Scale.
					17·39		19·59
				....	18·72	....	19·37
11·55	11·57	758·63	12·1	....	17·41	....	17·49
				....	16·92	....	17·19
				....	16·65	....	16·36
11·7	11·7	758·65	12·2	....	17·96	....	16·99
				....	17·74	....	17·82
				....	18·02	....	18·12
11·85	11·8	758·73	12·3	....	17·96	....	18·12
				....	17·52	....	17·96
					176·29		179·01

$20(G+Q+Y)+20R \triangleq 20(I+X)-2 \cdot 72$ parts.

$40(G+Q)+40R \triangleq 40I-7 \cdot 67$ parts, $M=11 \cdot 46$, $F=757 \cdot 77$, $E=12 \cdot 06$.

$G+Q \triangleq I-0 \cdot 01205$ grain in air ($t=11 \cdot 47$, $b=756 \cdot 74$).

$G=I-0 \cdot 01214$ grain.

$G \triangleq W-0 \cdot 00519$ grain in air ($\log \Delta=7 \cdot 07832-10$).

G=lb. No. 23.

100 parts=0·38429 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	19·82	G+Q+X, I+Y	16·15
				....	18·96	....	15·99
10·53	10·6	759·15	11·2	....	18·55	....	15·10
				....	18·50	....	13·62
				....	15·56	....	12·00
11	11	759·39	11·6	....	14·80	....	11·57
				....	14·15	....	10·59
				....	14·67	....	10·63
11·2	11·2	759·7	11·6	....	20·57	....	16·59
				....	20·82	....	16·36
					173·40		138·62

$20(G+Q+X) \doteq 20(I+Y)+34·78$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+R+X, G+Q+Y	18·02	G+Q+Y, I+R+X	19·60
				....	17·87	....	20·19
11·25	11·25	759·9	11·4	....	17·65	....	20·15
				....	17·94	....	20·02
				....	17·72	....	20·25
11·4	11·4	760·3	11·5	....	17·21	....	19·49
				....	17·79	....	19·65
				....	17·27	....	19·55
11·5	11·5	760·65	11·6	....	17·01	....	18·97
				....	17·01	....	18·71
					175·49		196·58

$20(G+Q+Y) \doteq 20(I+R+X)-21·09$ parts.

$40(G+Q) \doteq 40I+20R+13·69$ parts, $M=11·15$, $F=759·85$, $E=11·48$.

$G+Q \doteq I+0·00697$ grain in air ($t=11·16$, $b=758·88$).

$G=I+0·01557$ grain.

$G \doteq W+0·01659$ grain in air ($\log \Delta=7·07832-10$).

G=lb. No. 24.

100 parts=0·40700 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+4R+Y	22·24	G+Q+4R+Y, I+X	22·84
9·6	9·65	747·9	10·6	....	22·27	....	22·74
				....	22·02	....	21·02
				....	20·24	....	21·97
				....	20·47	....	18·56
10	10	747·9	10·6	....	20·47	....	20·32
				....	19·75	....	19·41
				....	20·50	....	19·07
10·15	10·15	747·8	10·8	....	19·12	....	18·20
				....	19·25	....	18·17
					206·33		202·30

$20(G+Q+Y)+80R \doteq 20(I+X)+4·03$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+4R+X	20·41	G+Q+4R+X, I+Y	16·90
				....	20·36	....	16·67
10·4	10·4	747·8	11	....	19·90	....	16·42
				....	20·02	....	16·79
				I+Y, G+Q+3R+X	15·47	G+Q+3R+X, I+Y	16·36
10·55	10·6	747·7	11·1	....	14·56	....	16·55
				....	14·71	....	16·41
				....	14·72	....	16·05
10·65	10·7	747·7	11·2	....	14·24	....	15·69
				....	15·09	....	15·85
					169·48		163·69

$20(G+Q+X)+68R \bumpeq 20(I+Y)+5{\cdot}79$ parts.

$40(G+Q)+148R \bumpeq 40I+9{\cdot}82$ parts, $M=10{\cdot}23$, $F=747{\cdot}8$, $E=11$.

$G+Q \bumpeq I-0{\cdot}04086$ grain in air ($t=10{\cdot}23$, $b=746{\cdot}91$).

$G=I-0{\cdot}03932$ grain.

$G \bumpeq W-0{\cdot}03937$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 25.

100 parts=0·39825 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+2R+Y	17·65	G+Q+2R+Y, I+X	16·42
				....	17·00	....	15·80
10·52	10·55	768·7	11·5	....	15·40	....	14·55
				....	15·61	....	13·94
				....	17·21	....	16·42
10·85	10·85	769·4	11·65	....	16·11	....	15·86
				....	18·01	....	18·65
				....	17·84	....	18·59
11·1	11·1	769·7	11·9	....	16·86	....	18·07
				....	18·62	....	18·20
					170·31		166·50

$20(G+Q+2R) \bumpeq 20(I+X)+3{\cdot}81$ parts.

D.	C	F	E.		Scale.		Scale.
				I+Y, G+Q+2R+X	18·35	G+Q+2R+X, I+Y	18·09
				....	18·91	....	17·57
11·3	11·3	770·2	11·9	....	16·17	....	18·04
				....	18·06	....	18·01
				....	17·82	....	17·70
11·45	11·45	770·3	12	....	18·26	....	17·37
				....	17·22	....	16·91
				....	17·70	....	17·00
11·6	11·6	770·5	12·1	....	15·77	....	14·80
				....	14·51		15·64
					172·77		171·13

$20(G+Q+2R+X) \bumpeq 20(I+Y)+1{\cdot}64$ part.

$40G+Q+2R \bumpeq 40I+5{\cdot}45$ parts, $M=11{\cdot}13$, $F=769{\cdot}8$, $E=11{\cdot}84$.

$G+Q \bumpeq I-0{\cdot}02218$ grain in air ($t=11{\cdot}14$, $b=768{\cdot}77$).

$G=I+0{\cdot}00163$ grain.

$G \bumpeq W-0{\cdot}00368$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 25.

100 parts=0·39510 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+R+X	24·76	G+Q+R+X, I+Y	24·95
				....	24·41	....	24·65
9·35	9·35	751·3	10·2	....	24·32	....	23·77
				....	24·12	....	23·04
				....	23·01	....	21·67
9·55	9·6	750·9	10·5	....	22·61	....	20·61
				....	22·32	....	19·95
				....	21·91	....	19·86
9·9	9·9	749·8	10·6	....	21·30	....	18·89
				....	21·44	....	18·59
					230·20		215·98

20(G+Q+X)+20R≏20(I+Y)+14·22 parts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+Y	16·95	G+Q+Y, I+X	18·52
				....	16·72	....	18·64
10·2	10·2	748·5	10·6	....	15·85	....	18·66
				....	15·22	....	18·11
				....	15·51	....	17·77
10·35	10·35	748·0	10·65	....	16·04	....	18·56
				....	15·64	....	17·96
				....	16·24	....	17·95
10·4	10·4	747·55	10·65	....	17·52	....	18·37
				....	16·64	....	18·21
					162·33		182·75

20(G+Q+Y)≏20(I+X)−20·42 parts.

40(G+Q)+20R≏40I−6·2 parts, M=9·96, F=749·34, E=10·53.

G+Q≏I−0·00627 grain in air (t=9·96, b=748·51).

G=I+0·00265 grain.

G≏W−0·00266 grain in air (log Δ=7·07832−10).

G=lb. No. 25.

100 parts=0·26918 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+3R+Y	17·50	G+3R+Y, T+D+X	13·80
				T+D+X, G+R+Y	7·12	G+R+Y, T+D+X	19·19
				T+D+X, G+2R+Y	18·80	G+2R+Y, T+D+X	22·75
				....	19·09	....	24·15
8·55	8·6			....	22·35	....	25·40
				....	22·24	....	23·15
				....	22·10	....	22·65
				....	19·49	....	22·72
				T+D+X, G+3R+Y	23·37	G+3R+Y, T+D+X	17·81
		762·8	9·3	....	23·25	....	18·24
					194·31		208·84

20(G+Y)+44R≏20(T+D+X)−14·53 parts.

D.	C.	F.	E.		Scale.		Scale.
		762·8	9·3	T+D+Y, G+2R+X	19·27	G+2R+X, T+D+Y	20·91
				......	20·49	......	19·02
				......	21·62	......	20·00
				......	20·50	......	18·87
				......	20·65	......	18·45
8·9	8·93			......	20·70	......	19·07
				......	20·99	......	18·81
				......	20·49	......	17·77
				......	22·22	......	19·32
				......	22·02	......	18·80
					208·95		191·02

$20(G+X)+40R \bumpeq 20(T+D+Y)+17{\cdot}93$ parts.

$40G+84R \bumpeq 40(T+D)+3{\cdot}4$ parts, $M=8{\cdot}74$, $F=762{\cdot}8$, $E=9{\cdot}3$.

$G \bumpeq T+D-0{\cdot}02353$ grain in air ($t=8{\cdot}73$, $b=762{\cdot}1$).

$G=I+0{\cdot}00113$ grain.

$G \bumpeq W-0{\cdot}00418$ grain in air ($\log \Delta=7{\cdot}07832-10$).

Means of the three series of comparisons:—

$G=I+0{\cdot}00180$ grain.

$G \bumpeq W-0{\cdot}00351$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 26.

100 parts =0·28116 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	21·20	G+Y, T+D+X	19·67
14·3	14·35			......	20·32	......	18·80
				......	20·97	......	19·42
				......	22·27	......	19·46
				......	21·47	......	19·67
		753·9	14·8	......	18·75	......	17·71
				......	19·15	......	17·26
				......	20·20	......	18·81
				......	18·62	......	18·07
14·55	14·55			......	18·25	......	18·27
				......	19·91	......	17·40
					221·12		204·54

$22(G+Y) \bumpeq 22(T+D+X)+16{\cdot}58$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	17·16	G+X, T+D+Y	16·56
14·9	14·87			......	19·72	......	15·00
				......	19·97	......	15·16
				......	19·60	......	14·45
				......	18·54	......	17·47
		754·4	15·2	......	19·29	......	18·56
				......	20·31	......	16·69
				......	19·67	......	16·29
				......	18·86	......	13·37
15·3	15·3			......	16·87	......	13·92
				......	19·02	......	13·77
					209·01		171·24

$22(G+X) \bumpeq 22(T+D+Y)+37{\cdot}77$ parts.

$44G \bumpeq 44(T+D)+53{\cdot}35$ parts, $M=14{\cdot}76$, $F=754{\cdot}15$, $E=15$.

$G \bumpeq T+D+0{\cdot}00347$ grain in air ($t=14{\cdot}78$, $b=752{\cdot}77$).

$G=I-0{\cdot}00112$ grain.

$G \bumpeq W+0{\cdot}00002$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 27.

100 parts = 0·39809 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+2R+Y, G+Q+X	18·09	G+Q+X, I+2R+Y	17·52
				....	18·32	....	17·06
11·8	11·8	742·6	12	....	16·30	....	16·75
				....	16·15	....	16·41
				....	16·76	....	17·07
11·95	11·95	743·6	12·1	....	17·12	....	15·97
				....	16·36	....	15·24
				....	15·91	....	14·84
12	12	744·4	12·2	....	14·85	....	15·40
				....	14·65	....	15·34
					164·51		161·60

20(G+Q+X) ≏ 20(I+2R+Y)+2·91 parts.

D.	C.	F.	E.		Scale.		Scale.
				I+2R+X, G+Q+Y	18·46	G+Q+Y, I+2R+X	17·12
				....	17·15	....	16·05
11·05	11·1	739·83	12	....	14·47	....	12·70
				....	13·45	....	13·91
				....	19·89	....	18·46
11·47	11·5	740·7	12	....	18·46	....	19·22
				....	17·14	....	17·47
				....	18·30	....	18·02
11·6	11·63	740·97	12	....	17·11	....	17·25
				....	18·31	....	15·51
					172·74		167·71

20(G+Q+Y) ≏ 20(I+2R+X)+5·03 parts.

40(G+Q) ≏ 40(I+2R)+7·94 parts, M=11·65, F=742·01, E=12·05.

G+Q ≏ I+0·02342 grain in air (t=11·67, b=741·02).

G=I+0·01405 grain.

G ≏ W+0·01640 grain in air (log Δ=7·07832−10).

G = lb. No. 28.

100 parts = 0·30166 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+2R+Y	20·67	G+2R+Y, T+D+X	20·67
				....	20·21	....	22·60
				....	19·37	....	21·49
				....	20·55	....	19·46
				....	20·22	....	18·54
14·4	14·4			....	17·62	....	19·52
				....	18·75	....	18·80
				....	19·57	....	21·97
				....	17·46	....	21·19
		772·7	14·1	....	20·50	....	20·72
					194·92		204·96

20(G+Y)+40R ≏ 20(T+D+X)−10·04 parts.

D.	C.	F.	E.		Scale.		Scale.
		772·7	14·1	T+D+Y, G+2R+X	21·59	G+2R+X, T+D+Y	18·40
				....	21·65	....	17·57
				....	21·47	....	16·25
				....	21·46	....	18·16
				....	20·44	....	16·97
13·95	13·95			T+D+Y, G+R+X	17·70	G+R+X, T+D+Y	21·74
				....	17·30	....	19·05
				....	16·71	....	19·22
				....	15·39	....	19·66
				....	15·81	....	19·36
					189·52		186·38

$20(G+X)+30R \bumpeq 20(T+D+Y)+3{\cdot}14$ parts.

$40G+70R \bumpeq 40(T+D)-6{\cdot}9$ parts, $M=14{\cdot}17$, $F=772{\cdot}7$, $E=14{\cdot}1$.

$G \bumpeq T+D-0{\cdot}02032$ grain in air ($t=14{\cdot}19$, $b=771{\cdot}39$).

$G=I-0{\cdot}00416$ grain.

$G \bumpeq W-0{\cdot}00635$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 29.

100 parts=0·29084 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+R+Y	20·79	G+R+Y, T+D+X	22·32
				....	22·06	....	24·92
				....	21·35	....	25·32
				....	20·75	....	23·36
				....	19·75	....	21·85
13·3	13·3	768·8	13·85	....	22·22	....	24·47
				....	21·07	....	20·56
				....	21·25	....	23·87
				....	20·67	....	22·71
				....	21·07	....	23·31
				....	20·35	....	23·19
					231·33		255·88

$22(G+Y)+22R \bumpeq 22(T+D+X)-24{\cdot}55$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+R+X	22·19	G+R+X, T+D+Y	20·37
				....	23·92	....	19·62
				....	24·87	....	19·45
				....	25·59	....	18·32
				....	23·96	....	16·42
13·65	13·65	767·8	14	T+D+Y, G+X	19·92	G+X, T+D+Y	20·35
				....	20·27	....	20·97
				....	19·24	....	20·59
				....	19·24	....	23·22
				....	20·24	....	20·88
				....	19·95	....	20·20
					238·39		126·21

$22(G+X)+10R \bumpeq 22(T+D+Y)+18{\cdot}0$ parts.

$44G+32R \bumpeq 44(T+D)-6{\cdot}55$ parts, $M=13{\cdot}47$, $F=768{\cdot}3$, $E=13{\cdot}92$.

$G \bumpeq T+D-0{\cdot}00867$ grain in air ($t=13{\cdot}5$, $b=767{\cdot}04$).

$G=I-0{\cdot}00222$ grain.

$G \bumpeq W+0{\cdot}00296$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 30.

100 parts = 0·41249 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+Y	17·00	G+Q+Y, I+X	20·96
11·65	11·65	754·7	12·8	....	18·65	....	20·11
				....	17·57	....	20·35
11·93	11·93	755·0	12·9	....	16·87	....	19·82
				....	16·20	....	19·86
				....	16·10	....	18·70
12·10	12·10	755·0	12·9	....	15·36	....	18·09
				....	14·97	....	17·04
12·25	12·3	754·9	13	....	14·79	....	17·76
				....	14·37	....	17·84
					161·88		190·53

$20(G+Q+Y) \bumpeq 20(I+X) - 28·65$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	16·10	G+Q+X, I+Y	16·75
12·55	12·55	754·5	13·05	....	15·79	....	16·97
				....	14·90	....	16·50
12·55	12·6	754·25	13·1	....	14·77	....	16·42
				....	15·06	....	15·24
				....	15·05	....	15·71
12·65	12·7	754·1	13·1	....	15·32	....	15·51
				....	14·74	....	14·45
12·75	12·8	754·1	13·0	....	14·62	....	14·67
				....	14·87	....	14·99
					151·22		157·21

$20(G+Q+X) \bumpeq 20(I+Y) - 5·99$ parts.

$40(G+Q) \bumpeq 40I - 34·64$ parts, M = 12·31, F = 754·57, E = 12·98.

$G+Q \bumpeq I - 0·00357$ grain in air ($t = 12·33$, $b = 753·42$).

$G = I - 0·00269$ grain.

$G \bumpeq W - 0·00140$ grain in air ($\log \Delta = 7·07832 - 10$).

G = lb. No. 30.

100 parts = 0·24737 grain.

D.	C.	F.	E		Scale.		Scale.
				T+D+Y, G+2R+X	20·32	G+2R+X, T+D+Y	17·90
				....	21·12	....	16·85
				....	19·79	....	14·86
				....	19·31	....	15·82
				T+D+Y, G+R+X	15·86	G+R+X, T+D+Y	21·00
7·55	7·6			....	15·37	....	20·59
				....	14·62	....	21·20
				....	16·02	....	20·15
				....	16·67	....	21·00
		762·2	8·3	....	16·10	....	19·57
					175·18		188·94

$20(G+X) + 28R \bumpeq 20(T+D+Y) - 13·76$ parts.

D.	C.	F.	E.		Scale.		Scale.
		762·2	8·3	T+D+X, G+2R+Y	14·95	G+2R+Y, T+D+X	15·34
				....	14·74	....	15·55
				....	15·31	....	15·06
				....	16·15	....	16·41
7·9	7·93			....	16·26	....	18·32
				....	16·02	....	17·14
				....	17·05	....	16·45
				....	16·75	....	17·97
				....	15·60	....	16·45
				....	15·30	....	16·65
					158·12		165·34

$20(G+Y)+40R \doteq 20(T+D+X)-7{\cdot}22$ parts.

$40G+68R \doteq 40(T+D)-20{\cdot}98$ parts, $M=7{\cdot}74$, $F=762{\cdot}2$, $E=8{\cdot}3$.

$G \doteq T+D-0{\cdot}02053$ grain in air ($t=7{\cdot}72$, $b=761{\cdot}62$).

$G=I-0{\cdot}00072$ grain.

$G \doteq W+0{\cdot}00052$ grain in air ($\log \Delta=7{\cdot}07832-10$).

A mean of the two series of comparisons gives—

$G=I-0{\cdot}00170$ grain.

$G \doteq W-0{\cdot}00044$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 31.

100 parts $=0{\cdot}28531$ grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	18·60	G+Y, T+D+X	19·81
7·6	7·6			....	18·57	....	19·26
				....	19·04	....	19·11
		739·7	8·2	....	18·40	....	19·06
				....	17·55	....	18·46
7·9	7·85			....	17·44	....	18·52
				....	17·04	....	17·22
				....	17·86	....	17·39
8	8			....	17·67	....	16·47
		739·6	8·4	....	16·70	....	16·45
				....	16·60	....	16·92
				....	17·01	....	16·11
					212·48		214·78

$24(G+Y) \doteq 24(T+D+X)-2{\cdot}3$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	15·80	G+X, T+D+Y	15·45
8·13	8·1			....	16·16	....	15·16
				....	16·85	....	14·75
		739·65	8·6	....	14·84	....	14·41
8·2	8·2			....	16·70	....	14·90
				....	15·46	....	14·02
8·3	8·25			....	16·79	....	14·67
				....	15·02	....	14·02
		739·65	8·6	....	16·41	....	13·45
				....	15·21	....	14·70
8·35	8·3			....	15·57	....	13·59
				....	15·75	....	13·89
					190·56		173·01

$24(G+X) \doteq 24(T+D+Y)+17{\cdot}55$ parts.

$48G \doteq 48(T+D)+15{\cdot}25$ parts, $M=8{\cdot}01$, $F=739{\cdot}65$, $E=8{\cdot}45$.

$G \doteq T+D+0{\cdot}00091$ grain in air ($t=8{\cdot}0$, $b=739{\cdot}08$).

$G=I-0{\cdot}04410$ grain.

$G \doteq W+0{\cdot}00088$ grain in air ($\log\Delta=7{\cdot}07832-10$).

G=lb. No. 32.

100 parts=0·31852 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	17·60	G+X, T+D+Y	13·85
				T+D+R+Y, G+X	18·42	G+X, T+D+R+Y	20·95
15·9	15·8			....	18·60	....	21·72
				....	18·61	....	21·22
				T+D+Y, G+X	21·56	G+X, T+D+Y	18·04
				....	21·86	....	18·65
16·03	15·95			....	22·50	....	18·94
		757·8	16	T+D+R+Y, G+X	18·97	G+X, T+D+R+Y	20·12
				....	18·07	....	21·66
				....	17·35	....	21·96
16	15·9			....	17·92	....	20·42
				T+D+Y, G+X	22·29	G+X, T+D+Y	17·97
				....	20·90	....	17·05
					254·65		252·55

26(G+X)≏26(T+D+Y)+14R+2·1 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	24·36	G+Y, T+D+X	19·81
				....	24·32	....	21·04
15·85	15·8			....	23·51	....	21·35
				....	24·89	....	19·42
				....	23·80	....	21·05
16	15·95			....	23·69	....	19·55
		758·9	16·4	T+D+R+X, G+Y	19·82	G+Y, T+D+R+X	22·54
				....	20·35	....	21·74
16·15	16·05			....	19·42	....	22·47
				....	20·62	....	22·59
				....	20·11	....	22·49
				....	20·04	....	23·10
				....	19·01	....	23·31
					283·94		280·46

26(G+Y)≏26(T+D+X)+14R+3·48 parts.

52G≏52(T+D)+28R+5·58 parts, M=15·94, F=758·35, E=16·2.

G≏T+D+0·00643 grain in air (t=15·96, b=756·81).

G=I−0·03431.

G=lb. No. 32.

100 parts=0·31974 grain.

D	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	24·51	G+Y, T+D+X	26·27
				....	26·07	....	26·64
15·3	15·2			....	26·04	....	26·74
				....	26·12	....	26·10
15·4	15·4			....	26·41	....	26·61
				....	26·19	....	26·66
15·55	15·5			....	25·81	....	25·47
		764·1	16	....	24·31	....	24·99
15·7	15·65			....	25·00	....	25·22
				....	25·10	....	25·52
15·8	15·75			....	24·12	....	23·97
				....	24·04	....	24·22
				....	25·05	....	24·70
					328·77		333·11

26(G+Y)≏26(T+D+X)−4·34 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	25·15	G+X, T+D+Y	27·26
15·25	15·2			....	25·72	....	27·59
				....	26·32	....	27·80
15·5	15·4			....	24·44	....	26·40
				....	25·21	....	26·85
15·6	15·5	768·8	16	....	24·84	....	26·37
				....	24·55	....	25·45
15·75	15·7			....	25·40	....	26·56
				....	24·79	....	26·46
15·95	15·85			....	24·50	....	25·12
				....	24·76	....	25·26
16·03	15·98			....	24·05	....	24·75
				....	24·35	....	24·47
					325·08		340·34

$26(G+X) \bumpeq 26(T+D+Y)-15{\cdot}26$ parts.

$52G \bumpeq 52(T+D)-19{\cdot}60$ parts, $M=15{\cdot}58$, $F=766{\cdot}45$, $E=16$.

$G \bumpeq T+D-0{\cdot}00121$ grain in air ($t=15{\cdot}6$, $b=764{\cdot}92$). $G=I-0{\cdot}03465$ grain.

Mean ...$G=I-0{\cdot}03448$ grain. $G \bumpeq W+0{\cdot}00304$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 33.

100 parts=0·31374 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	17·00	G+X, T+D+Y	20·30
13·05	13·0			....	15·95	....	19·35
				....	16·09	....	20·39
				....	16·70	....	19·27
13·3	13·25			....	16·77	....	19·97
		761·1	14·1	....	16·45	....	19·89
				....	16·84	....	20·50
13·5	13·5			T+D+Y, G+R+X	20·40	G+R+X, T+D+Y	17·05
				....	19·79	....	16·65
		761·0	14·4	....	20·80	....	16·02
				....	20·52	....	16·27
13·7	13·65			....	19·39	....	14·15
				....	19·91	....	15·54
13·75	13·7			T+D+Y, G+X	16·95	G+X, T+D+Y	19·02
					253·56		254·37

$28(G+X)+12R \bumpeq 28(T+D+Y)-0{\cdot}81$ part.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	19·19	G+Y, T+D+X	23·41
12·85	12·8			....	19·85	....	24·49
				T+D+X, G+R+Y	23·47	G+R+Y, T+D+X	19·09
13·0	13·0			....	22·44	....	18·21
		759·7	14	....	22·20	....	18·79
13·25	13·2			....	22·37	....	18·51
				....	21·57	....	18·52
13·4	13·35			....	21·25	....	16·90
				T+D+X, G+Y	18·01	G+Y, T+D+X	20·46
13·5	13·45			....	16·72	....	19·65
		758·5	14·4	....	17·77	....	19·62
				....	18·57	....	20·75
				....	17·20	....	18·34
				....	17·16	....	18·29
					277·77		275·03

$28(G+Y)+12R \bumpeq 28(T+D+Y)+2{\cdot}74$ parts.

$56G+24R \bumpeq 56(T+D)+1{\cdot}93$ part, $M=13{\cdot}31$, $F=760{\cdot}07$, $E=14{\cdot}22$.

$G \bumpeq T+D-0{\cdot}00474$ grain in air ($t=13{\cdot}34$, $b=758{\cdot}77$).

$G=I-0{\cdot}04144$ grain. $G \bumpeq W-0{\cdot}00063$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 34.

100 parts=0·28057 grain.

D.	C.	F.	E.		Scale.		Scale.
6·85	6·8			T+D+X, G+R+Y	20·10	G+R+Y, T+D+X	28·05
				T+D+X, G+2R+Y	25·47	G+2R+Y, T+D+X	23·94
7·05	7·0			....	24·92	....	23·44
				....	24·96	....	23·62
7·25	7·25			....	22·87	....	20·95
		760·0	7·8	....	22·60	....	21·32
7·5	7·45			....	20·92	....	19·29
				....	20·72	....	19·47
7·1	7·1			....	20·15	....	19·86
				....	21·42	....	20·17
7·4	7·35			....	21·25	....	19·52
		759·8	8	....	20·64	....	20·05
				....	20·99	....	19·55
				....	21·50	....	18·44
					308·51		297·67

$28(G+Y)+52R \triangleq 28(T+D+X)+10{\cdot}84$ parts.

D.	C.	F.	E.		Scale.		Scale.
6·35	6·3			T+D+Y, G+2R+X	28·31	G+2R+X, T+D+Y	26·89
				....	28·52	....	26·29
6·55	6·5			....	20·42	....	16·22
				....	20·65	....	17·15
				....	18·99	....	17·17
6·65	6·6			....	18·47	....	15·84
6·65	6·6			T+D+Y, G+R+X	13·35	G+R+X, T+D+Y	19·14
		757·5	7·1	....	14·25	....	18·75
				....	13·16	....	17·32
6·85	6·8			....	15·27	....	18·69
				....	13·50	....	17·81
6·5	6·5			....	18·02	....	22·20
				....	17·92	....	23·11
6·7	6·65			....	19·35	....	23·32
					260·18		279·90

$28(G+X)+40R \triangleq 28(T+D+Y)-19{\cdot}72$ parts.

$56G+92R \triangleq 56(T+D)-8{\cdot}88$ parts, $M=6{\cdot}87$, $F=758{\cdot}7$, $E=7{\cdot}45$.

$G \triangleq T+D-0{\cdot}01904$ grain in air ($t=6{\cdot}84$, $b=758{\cdot}23$).

$G=I-0{\cdot}04584$ grain.

$G \triangleq W-0{\cdot}00089$ grain in air ($\log\Delta=7{\cdot}07832-10$).

G= lb. No. 35.

100 parts=0·30126 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	21·55	G+Y, T+D+X	27·80
14·8	14·7			T+D+X, G+R+Y	25·27	G+R+Y, T+D+X	25·90
				....	25·50	....	24·81
15	15			....	24·47	....	25·29
				....	23·45	....	23·77
15	14·95			....	24·11	....	22·99
		771·9	15·6	....	24·57	....	23·46
				....	23·92	....	24·04
15·05	15·05			....	24·82	....	23·22
				....	23·89	....	24·51
				....	23·81	....	24·10
14·95	14·9			....	24·40	....	23·66
				....	24·42	....	23·56
					315·18		317·11

$26(G+Y)+24R \triangleq 26(T+D+X)-1{\cdot}93$ part.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	21·72	G+X, T+D+Y	27·60
14·7	14·6			T+D+Y, G+R+X	25·81	G+R+X, T+D+Y	24·32
				....	25·66	....	25·32
				....	24·35	....	24·16
14·9	14·85			....	26·31	....	25·01
				....	24·80	....	24·60
		772	15·6	....	24·45	....	24·26
15·05	15			....	25·42	....	24·16
				....	24·02	....	23·64
				....	24·29	....	23·12
15·13	15·1			....	24·59	....	23·52
				....	25·40	....	24·25
				....	24·90	....	23·56
					321·72		317·52

26(G+X)+24R≏26(T+D+Y)+4·2 parts.

52G+48R≏52(T+D)+2·27 parts, M=14·92, F=771·95, E=15·6.

G≏T+D−0·01031 grain in air (t=14·95, b=770·45).

G=I−0·04019 grain.

G≏W−0·00041 grain in air (log Δ=7·07832−10).

G=lb. No. 36.

100 parts=0·307298 grain.

D.	C.	F.	E.		Scale.		Scale.
14·8	14·7			T+D+Y, G+R+X	24·50	G+R+X, T+D+Y	26·19
				....	25·12	....	25·70
				....	25·31	....	26·95
				....	25·17	....	26·07
15·15	15·1			....	23·89	....	26·46
				....	24·86	....	25·65
		772·4	16·2	....	25·05	....	26·60
15·35	15·25			....	25·61	....	24·61
				....	26·57	....	26·56
		772·4	16·4	....	24·52	....	24·94
15·35	15·25			....	25·25	....	26·05
				....	25·77	....	26·11
				....	25·75	....	24·92
					327·37		336·81

26(G+X)+26R≏26(T+D+Y)−9·44 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+R+Y	18·24	G+R+Y, T+D+X	22·09
14·55	14·5			....	17·56	....	20·46
				....	18·91	....	20·46
		776·3	15·5	....	16·92	....	18·44
				....	16·11	....	19·29
14·7	14·6			....	15·76	....	18·11
				T+D+X, G+2R+Y	19·71	G+2R+Y, T+D+X	15·59
				....	19·66	....	15·31
14·9	14·8			....	20·56	....	14·60
		776·1	15·5	T+D+X, G+R+Y	16·15	G+R+Y, T+D+X	18·60
				....	16·99	....	19·12
15	14·9			....	16·67	....	19·34
				....	17·05	....	18·40
					230·29		239·81

26(G+Y)+32R≏26(T+D+X)−9·52 parts.

52G+58R≏52(T+D)−18·96 parts, M=14·93, F=774·3, E=15·9.

G≏T+D−0·01374 grain in air (t=14·95, b=772·76).

G=I−0·04488 grain. G≏W−0·00209 grain in air (log Δ=7·07832−10).

G = lb. No. 36.

100 parts = 0·28057 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+R+X	23·57	G+R+X, T+D+Y	28·29
				....	19·09	....	23·11
7·4	7·35			....	18·51	....	22·09
		755·6	8	....	19·22	....	21·70
				....	17·69	....	22·11
7·55	7·5			....	17·90	....	21·05
					115·98		138·35

$12(G+X+R) \triangleq 12(T+D+X) - 22·37$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+R+Y	16·91	G+R+Y, T+D+X	20·04
7·65	7·6			....	16·97	....	20·06
				....	17·64	....	20·52
		755·6	8·2	....	17·76	....	20·50
				....	16·61	....	20·45
7·8	7·75			....	16·54	....	20·10
					102·43		121·67

$12(G+Y+R) \triangleq 12(T+D+X) - 19·24$ parts.

$24(G+R) \triangleq 24(T+D) - 41·61$ parts, $M=7·5$, $F=755·6$, $E=8·1$.

$G \triangleq T+D-0·01618$ grain in air ($t=7·48$, $b=755·05$).

$G = I - 0·04480$ grain.

$G \triangleq W - 0·00206$ grain in air ($\log \Delta = 7·07832-10$).

Mean...$G = I - 0·04484$ grain. $G \triangleq W - 0·00207$ grain in air ($\log \Delta = 7·07832-10$).

The 10-*lb. Weight.*

A weight of 10 lbs., or the weight of a gallon of water, protected by electro-gilding, was constructed by Mr. OERTLING. Let D denote the sum of the secondary standards, Nos. 9, 11, 12, 13, 14, 16, 17*a*, 17*b*, 29, 30; E the sum of Nos. 18, 19, 21, 22, 23, 24, 25, 27, 28, together with a weight Pd, for which $v\mathrm{Pd}=822·792$, $\mathrm{Pd} \triangleq W+0·00169$ grain in air ($\log \Delta = 7·07832-10$); F the sum of Nos. 1, 2, 3, 4, 5, 6, 7, 8, 10, 26; T the weight of 10 lbs. The balance employed, made by Mr. OERTLING and lent by him for the purpose of weighing T, has pans capable of being interchanged without taking out the weights contained in them. The weights of the pans will be denoted by X and Y. Each end of the beam carries a pointer, behind which is fixed an ivory scale with about 50 divisions to an inch. Every fifth division is marked by a longer line. The interval between two such consecutive lines is one part of the scale. The reading increases on placing a small weight in the left-hand pan. The index and scale were viewed through a telescope placed at a distance of about 12 feet from the scale. The absolute weight of water ($t=14·87$) displaced by T was 8378·8 grains. Hence $vT=8379·4$. $vD=8561·006$, $vE=8538·336$, $vF=8492·141$. In air for which $\log \Delta = 7·07832-10$, $D \triangleq 10W-0·06794$ grain, $E \triangleq 10W+0·02612$ grain, $F \triangleq 10W+0·08487$ grain.

1 part=0·1355 grain.

	Scale.		Scale.
T+Y, D+0·06 gr.+X	10·27	D+0·06 gr.+X, T+Y	9·65
....	9·75	....	9·54
....	9·84	....	9·14
....	9·45	....	9·28
....	9·65	....	9·12
T+X, D+0·06 gr.+Y	8·95	D+0·06 gr.+Y, T+X	9·56
....	9·21	....	9·60
....	8·60	....	9·74
....	8·71	....	9·35
....	8·78	....	9·43
T+Y, D+X	10·51	D+X, T+Y	7·37
....	9·58	....	5·62
....	9·72	....	8·16
....	11·11	....	6·12
....	10·82	....	7·75
....	10·79	....	7·95
T+X, D+Y	10·49	D+Y, T+X	8·47
....	10·71	....	8·01
T+X, D+0·1 gr.+Y	10·10	D+0·1 gr.+Y, T+X	8·87
....	9·71	....	8·60
T+X, D+0·2 gr.+Y	9·06	D+0·2 gr.+Y, T+X	9·04
....	9·54	....	8·72
T+Y, D+0·1 gr.+X	10·91	D+0·01 gr.+X, T+Y	8·70
....	11·01	....	8·06
T+Y, D+0·2 gr.+X	10·17	D+0·2 gr.+X, T+Y	9·22
....	10·51	....	9·36
....	10·10	....	8·91
....	9·97	....	9·06
T+X, D+0·2 gr.+Y	10·65	D+0·2 gr.+Y, T+X	9·95
....	10·94	....	10·17
....	10·46	....	9·65
....	10·67	....	10·05
....	10·89	....	10·10
....	10·15	....	10·24
	341·78		302·56

$68T \bumpeq 68D + 6{\cdot}8$ grain $+ 39{\cdot}22$ parts.

$T \bumpeq D + 0{\cdot}1781$ grain in air ($t = 16{\cdot}9$, $b = 759{\cdot}4$).

$T \bumpeq 10W + 0{\cdot}1080$ grain in air ($\log \Delta = 7{\cdot}07832 - 10$).

	Scale.		Scale.
T+Y, E+X	11·45	E+X, T+Y	9·55
....	11·21	....	9·65
....	11·27	....	9·42
....	10·76	....	9·70
....	10·91	....	9·46
T+X, E+Y	10·96	E+Y, T+X	9·47
....	10·44	....	10·62
....	10·85	....	9·90
....	11·96	....	10·09
....	12·92	....	10·00
	112·73		97·86

$20T \bumpeq 20E + 14{\cdot}87$ parts.

$T \bumpeq E + 0{\cdot}1007$ grain in air ($t = 17{\cdot}2$, $b = 755{\cdot}0$).

$T \bumpeq 10W + 0{\cdot}1262$ grain in air ($\log \Delta = 7{\cdot}07832 - 10$).

	Scale.		Scale.
T+X, F+Y	10·41	F+Y, T+X	10·36
....	10·52	....	11·76
....	11·30	....	11·84
....	10·89	....	10·41
....	10·86	....	11·67
T+Y, F+X	10·96	F+X, T+Y	10·15
....	10·00	....	9·91
....	10·30	....	9·62
....	10·94	....	9·35
....	10·40	....	9·45
	106·58		104·52

$20T \bumpeq 20F + 2{\cdot}06$ parts.

$T \bumpeq F + 0{\cdot}0140$ grain in air ($t = 17{\cdot}6$, $b = 754{\cdot}9$).

$T \bumpeq 10W + 0{\cdot}0986$ grain in air ($\log \Delta = 7{\cdot}07832 - 10$).

Mean......$T \bumpeq 10W + 0{\cdot}111$ grain in air ($\log \Delta = 7{\cdot}07832 - 10$).

Hence the 10-lb. weight appears to be 0·111 grain heavier than ten commercial pounds of the density of the lost standard, weighed in air, thermometer 65°·67 F., barometer 29·78 inches, the mercury being reduced to the freezing-point ($t=18{\cdot}7$, $b=755{\cdot}64$) at Somerset House.

Exchequer Kilogramme.

A kilogramme of gun-metal protected by electro-gilding, was constructed by Mr. Oertling for the use of the Exchequer. The absolute weight of the water ($t=17{\cdot}29$) displaced by it was 1852·212 grains. Hence, denoting this kilogramme by $\mathfrak{K}$, $v\mathfrak{K}=1852{\cdot}82$, $\log v\mathfrak{K}=3{\cdot}267834$, $\Delta\mathfrak{K}=8{\cdot}32910$. Let $\mathfrak{A}$ denote the Kilogramme des Archives, $\mathfrak{E}$ the English kilogramme of platinum. $\mathfrak{L}$ the kilogramme type laiton, and let $\mathfrak{O}$ denote $\mathfrak{K}$ together with a bit of platinum wire, the weight of which was 1·40002 grain, by a mean of six comparisons with the following weights belonging to Barrow's balance:—1 gr.+0·4 gr., and 1 gr.+0·3 gr.+0·1 gr.

100 parts=0·68159 grain.

$\mathfrak{O}$+X, $\mathfrak{E}$+Y.		$\mathfrak{E}$+X, $\mathfrak{O}$+Y.	$\mathfrak{O}$+Y, $\mathfrak{E}$+X.		$\mathfrak{E}$+X, $\mathfrak{O}$+Y.
13·76		14·07	15·72		18·26
19·24		16·94	17·17		18·42
18·70	C= 14·7, D=14·65	18·05	15·12	C= 15·3, D=15·3	18·17
18·05		17·50	16·69		18·24
18·61		19·25	17·96		17·72
17·94		19·29	15·27		18·15
17·64	F=762·6, E=15·5	19·19	17·00	F=762·6, E=15·6	17·99
18·10		18·05	16·07		16·31
18·01		17·70	14·51		15·47
17·17	C= 15·1, D=15·1	18·54	14·45	C= 15·55, D=15·57	18·21
16·50		18·45	15·87		16·19
17·99		18·36	15·89		17·00
211·71		215·39	191·72		210·13

$48\mathfrak{O} \bumpeq 48\mathfrak{E} + 22{\cdot}09$ parts.

$\mathfrak{K} + 1{\cdot}39687$grain $\bumpeq \mathfrak{E}$ in air ($t=15{\cdot}17$, $b=761{\cdot}13$).

$\mathfrak{K}$ displaces 2·26684 grs. of air; $\mathfrak{E}-1{\cdot}39687$ gr. of platinum displaces 0·89281 gr. of air. Therefore $\mathfrak{K}=\mathfrak{E}-0{\cdot}02284$ gr. But $\mathfrak{E}=\mathfrak{A}-0{\cdot}02412$ gr. Hence

$$\mathfrak{K}=\mathfrak{A}-0{\cdot}04696 \text{ grain.}$$

In air for which $\log \Delta = 7{\cdot}07832-10$, $\mathfrak{K}$ displaces 2·2211 grs., and $\mathfrak{A}-1{\cdot}4$ gr. of platinum displaces 0·89981 gr. Therefore $\mathfrak{K}+1{\cdot}3683$ gr. $\triangleq \mathfrak{A}$. But $\mathfrak{A} \triangleq \mathfrak{L}+1{\cdot}3673$ gr. Hence, when weighed in air of the above-mentioned density, $\mathfrak{K}$ appears to be 0·0010 gr. lighter than $\mathfrak{L}$.

Troy Ounce Weights.

A series of troy ounce weights from 500 ounces down to 0·001 ounce, of gun-metal protected by electro-gilding, was constructed by Messrs. Ladd and Streathfield for the use of the Exchequer. Each weight will be designated by the number expressing its approximate value in troy ounces, with a letter to distinguish it from other weights of nearly the same value. For the weights marked *s*, *t*, *z* $\log \Delta = 0{\cdot}92768$: for those marked *m*, *n* $\log \Delta = 0{\cdot}92501$. T denotes the platinum troy pound; U the lost standard troy pound, or 12 troy ounces. The comparisons of different combinations of these weights with T, and with one another, gave the following results:—

No. of Comp.		gr.	*t.*	*b.*
28	$4n+4t+2n+2t$	$\triangleq T-0{\cdot}02166$	18·6	754·6
24	$10n+\frac{1}{2}(2n+2t)$	$\triangleq T+0{\cdot}00078$	19·14	759·47
20	$10t+1s+1x$	$\triangleq T+0{\cdot}01370$	17·13	751·52
24	$5s+4t+1s+2t$	$\triangleq T+0{\cdot}00228$	17·78	760·1
48	$10s+1s+1x$	$\triangleq T+0{\cdot}00774$	17·9	755·68
32	$10m+2m$	$\triangleq T+0{\cdot}00060$	17·66	763·62
32	$4s+4m+2s+2m$	$\triangleq T-0{\cdot}00511$	16·5	763·44

In air for which $\log \Delta = 7{\cdot}07832-10$,

	gr.
$4n+4t+2n+2t$	$\triangleq T-0{\cdot}02133$
$10n+\frac{1}{2}(2n+2t)$	$\triangleq T+0{\cdot}00253$
$10t+1s+1x$	$\triangleq T+0{\cdot}01387$
$5s+4t+1s+2t$	$\triangleq T+0{\cdot}00688$
$10s+1s+1x$	$\triangleq T+0{\cdot}00925$
$10m+2m$	$\triangleq T+0{\cdot}00781$
$4s+4m+2s+2m$	$\triangleq T+0{\cdot}00639$
T	$\triangleq U-0{\cdot}00745$

Since these weights, the densities of which do not differ much from that of the lost standard U, are intended for use as commercial weights without applying any correction for the weight of the air displaced, $=$ may be substituted for $\triangleq$ in expressing their values in terms of U in air for which $\log \Delta = 7{\cdot}07832-10$.

	oz. gr.	
$4n+4t+2n+2t$	$=12-0{\cdot}02878$	1
$10n+\frac{1}{2}(2n+2t)$	$=12-0{\cdot}00492$	2
$10t+1s+1x$	$=12+0{\cdot}00642$	3
$5s+4t+1s+2t$	$=12-0{\cdot}00057$	4
$10s+1s+1x$	$=12+0{\cdot}00180$	5
$10m+2m$	$=12+0{\cdot}00036$	6
$4s+4m+2s+2m$	$=12-0{\cdot}00106$	7

No. of Comp.			gr.	
10	$4t$	$=2t+2n$	$+0{\cdot}01685$	8
10	$4n$	$=2t+2n$	$-0{\cdot}00090$	9
26	$2t$	$=2n$	$+0{\cdot}01121$	10
24	$10n$	$=5s+5m$	$+0{\cdot}00188$	11
16	$5s$	$=5m$	$+0{\cdot}00041$	12
10	$1s$	$=1m$	$+0{\cdot}00046$	13

50	1x	=1s	+0·00074	14
12	4t+1s	=5s	+0·00224	15
12	10t	=5s+4t+1s	+0·00270	16
12	4t	=2t+1s+1x	+0·00217	17
28	1s+1x	=2t	+0·00322	18
8	3s	=2t+1x	+0·00125	19
16	3m	=2m+1s	+0·00030	20
10	1s	=1z	+0·00017	21
20	4m	=2s+2m	+0·00059	22
20	4s	=2s+2m	+0·00059	23
36	2m	=2s	+0·00032	24
20	1s+1x	=2(0·5s+0·5m)	+0·00040	25
20	0·5s	=0·5m	+0·00001	26
6	0·4m+0·4s	=2(0·2n+0·2s)	+0·00013	27
10	1s	=0·4s+0·4m+0·2s	+0·00016	28
16	0·2s	=0·2n	+0·00049	29
10	0·4s	=0·4m	+0·00029	30
10	0·2s	=0·1s+0·1m	+0·00024	32
10	0·1m	=0·1s	+0·00070	33
6	0·3s+0·3m	=2×0·2s+0·1s+0·1m	+0·00045	34
6	0·1s+0·1m	=2(0·05s+0·05m)	+0·00019	35
10	0·05m	=0·05s	+0·00005	36
6	0·05s	=0·025s+0·025m	−0·00013	37
6	0·025m	=0·025s	+0·00018	38
4	0·1s	=0·04s+0·04m+½(0·02s+0·02m)	+0·00020	39
	0·02s	=0·02m	+0·00022	40
	0·04s	=0·02s+0·02m	+0·00017	41
	0·04m	=0·04s	+0·00010	42
4	0·02s	=0·01s+0·01m	−0·00051	43
4	0·01s	=0·01m	+0·00038	44
8	0·03s	=0·02s+0·01s	+0·00054	45
20	0·01s	=0·004s+0·004m+½(0·002s+0·002m)	+0·00030	46
10	0·004s	=0·002s+0·002m	−0·00041	47
10	0·004m	=0·002s+0·002m	−0·00032	48
6	0·002s	=0·001s+0·001n	−0·00004	49
6	0·001m	=0·001s	+0·00003	50
4	0·003s	=0·002s+½(0·001s+0·001n)	+0·00007	51
8	0·005s	=0·004s+½(0·001s+0·001n)	−0·00068	52

			oz.	gr.	
(3), (14), (15), (16), (17), (18)	2t	=	2	−0·00175	
(4), (14), (15), (17), (18)	2t	=	2	−0·00193	
(1), (8), (9), (10)	2t	=	2	−0·00185	
Mean	2t	=	2	−0·00184	53
(18), (53)	1s+1x	=	2	+0·00138	54
(5), (54)	10s	=	10	+0·00042	55
(14), (54)	1s	=	1	+0·00032	56
(7), (22), (23), (24)	2m	=	2	−0·00022	57
	2s	=	2	−0·00053	58
	4m	=	4	−0·00016	59
	4s	=	4	−0·00016	60
(6), (60)	10m	=	10	+0·00058	61
(10), (53)	2n	=	2	−0·01305	62
(53), (62), (2)	10n	=	10	+0·00253	63
(14), (53)	1x	=	1	+0·00106	64
(3), (54)	10t	=	10	+0·00504	65
(63), (11), (12)	5s	=	5	+0·00053	
(65), (15), (16)	5s	=	5	+0·00005	
(4), (15), (53)	5s	=	5	−0·00048	
Mean	5s	=	5	+0·00003	66
(12), (66)	5m	=	5	−0·00038	67
(20), (56), (57)	3m	=	3	+0·00040	68
(19), (53), (64)	3s	=	3	+0·00047	69
(21), (56)	1z	=	1	+0·00015	70
(25), (26), (54)	0·5s	=	0·5	+0·00013	71

(56), (27), (28), (29), (30)	0·4s	= 0·4	+0·00013	72
	0·2s	= 0·2	+0·00018	73
(32), (33), (73)	0·1m	= 0·1	+0·00032	74
	0·1s	= 0·1	−0·00038	75
(45), (73), (75)	0·3s	= 0·3	+0·00037	76
(74), (75), (35), (36)	0·05s	= 0·05	−0·00008	77
(77), (37), (38)	0·025s	= 0·025	−0·00007	78
(39), (75), (40), (41)	0·02s	= 0·02	−0·00066	79
	0·04s	= 0·04	+0·00015	80
(79), (43), (44)	0·01s	= 0·01	+0·00012	81
(45), (79), (81)	0·03s	= 0·03	+0·00000	82
(81), (46), (47), (48)	0·004s	= 0·004	−0·00019	83
	0·002s	= 0·002	+0·00018	84
(84), (49), (50)	0·001s	= 0·001	+0·00012	85
	0·001n	= 0·001	+0·00009	86
(51), (84), (85), (86)	0·003s	= 0·003	+0·00036	
(52), (83), (85), (86)	0·005s	= 0·005	−0·00076	

By a mean of 12 comparisons $20s = 10s + 10m - 0{\cdot}00386$ gr. $= 20$ ounces$-0{\cdot}00286$ gr.
By a mean of 12 comparisons $30s = 20s + 10m + 0{\cdot}00361$ gr. $= 30$ ounces$+0{\cdot}00133$ gr.

Let W denote a commercial lb. of the same density as the lost standard troy pound. In air for which $\log \Delta = 7{\cdot}07832 - 10$, the lb. Pd appears to be equal to W$+0{\cdot}00169$ gr. Let (1000), [1000] denote two brass weights of nearly 1000 grains each; (2000), (4000) two brass weights of nearly 2000 and 4000 grains each respectively. Then.

No. of Comp.			gr.
12	(4000)+(2000)+½{(1000)+[1000]}=Pd		−0·00582
12	(4000)	=(2000)+(1000)+[1000]	−0·00550
10	(2000)	=(1000)+[1000]	+0·00154
4	(1000)	=[1000]	+0·01360

Hence

		gr.
[1000]	=	999·99296
(1000)	=	1000·00656
(2000)	=	2000·00105
(4000)	=	3999·99506

The bronze weight (200) $= 199{\cdot}9971$ grains (in air).

By a mean of 28 comparisons, $40s =$ sum of secondary standard lbs. Nos. 9 and 11 $+(4000)+(1000)+(200)-0{\cdot}0171$ grain $= 40$ ounces$-0{\cdot}0298$ grain.

By a mean of 40 comparisons, $50s =$ sum of secondary standard lbs. Nos. 9, 11, 12 $+(2000)+(1000)+0{\cdot}0073$ grain $= 50$ ounces$+0{\cdot}0054$ grain.

By a mean of 60 comparisons, $100s =$ sum of secondary standard lbs. Nos. 9, 11, 12, 13, 14, 16$+(4000)+(2000)+0{\cdot}0701$ grain $= 100$ ounces$+0{\cdot}0193$ grain.

By a mean of 68 comparisons, $200s =$ sum of secondary standard lbs. Nos. 9, 11, 12, 13, 14, 16, 17*a*, 17*b*, 29, 30, 19, 21, 28$+(4000)+(1000)+0{\cdot}023$ grain $= 200$ ounces $-0{\cdot}022$ grain.

By a mean of 26 comparisons, $300s = 200s + 100s + 0{\cdot}0304$ gr. $= 300$ oz.$+0{\cdot}027$ gr.

By a mean of 20 comparisons, $400s = 300s + 100s - 0{\cdot}046$ gr. $= 400$ oz.$+0{\cdot}000$ gr.

By a mean of 20 comparisons, $500s = 300s + 200s - 0{\cdot}059$ gr. $= 500$ oz.$-0{\cdot}054$ gr.

The troy ounce weights marked *s* are deposited in the Exchequer.

The weights (12800), (6400), (3200), (1600), (800), (400), (200), (100), and a weight of 100 grains unmarked, which accompany BARROW's balance, are of bronze, for which $\log \Delta = 0{\cdot}92260$. Let them be denoted by W, V, U, T, S, R, Q, P, O respectively. It was found that

No. of Comp.		gr.
4	P = O	−0·0001
4	Q = P+O	−0·0037
4	R = Q+P+O	+0·0108
4	S = R+Q+P+O	−0·0009
4	T = S+R+Q+P+O	−0·0155
4	U = T+S+R+Q+P+O	−0·0207
4	V = U+T+S+R+Q+P+O	−0·0030
4	W = V+U+T+S+R+Q+P+O	−0·0516

Hence

	gr.
P =	O − 0·0001
Q =	2O − 0·0038
R =	4O + 0·0069
S =	8O + 0·0021
T =	16O − 0·0104
U =	32O − 0·0260
V =	64O − 0·0343
W =	128O − 0·1172

By a mean of 4 comparisons, the bronze weights U+T+S+P+platinum weights (32)+(16)+(8)+(4)+0·0133 gr. ≏ platinum troy pound T in air ($t=10{\cdot}45, b=758{\cdot}28$). Hence U+T+S+P=5699·9724 grains.

U+T+S+P=57O−0·0344 grain. Therefore 57O=5700·0068 grains. Hence

	gr.		gr.
(100) =	100·0000	(1600) =	1599·9915
(200) =	199·9964	(3200) =	3199·9778
(400) =	400·0074	(6400) =	6399·9733
(800) =	800·0031	(12800) =	12799·8981

The values assigned to these weights in computing the densities of some of the old troy pounds and secondary standard lbs., were obtained upon the supposition that the absolute weight of the platinum troy pound T was about 0·002 grain less than it afterwards appeared to be on reducing the observations with the weight of air as determined by REGNAULT. The error, which was not discovered till after going to press, is much too small to affect the last figure of the calculated densities.

Probable Errors of the Platinum Weights.

Probable errors of the comparisons of T, and of the auxiliary weights.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison. gr.	Probable error of n comparisons. gr.
815	T, Sp	168	0·000982	0·000076
818	T, RS	78	0·000727	0·000083
824	T, A+B+C+D+F	40	0·000540	0·000086
825	A, F+G	22	0·000287	0·000061
825	B, F+G	22	0·000560	0·000123
826	C, F+G	22	0·000521	0·000114
826	D, F+G	22	0·000669	0·000146
826	F, G+H	31	0·000458	0·000084
821	G, H+K	10	0·000170	0·000057
821	G, H+L	10	0·000154	0·000051

822	G, H+M	10	0·000159	0·000053
822	G, H+N	10	0·000111	0·000037
822	H, K+L+M+N+R	17	0·000298	0·000073
822	H, K+L+M+N+S	17	0·000241	0·000060
822	K, R+S	12	0·000134	0·000040
822	L, R+S	12	0·000178	0·000054
823	M, R+S	12	0·000139	0·000042
823	N, R+S	12	0·000122	0·000037
828	R, Y+Z	6	0·000072	0·000032
828	S, Y+Z	6	0·000063	0·000028
828	Y, W+V+Q	20	0·000097	0·000022
828	Z, W+V+Q	20	0·000046	0·000034
829	W, V+10Q	6	0·000053	0·000024
829	V, 10Q	6	0·000119	0·000053

According to the 300 comparisons of U, the lost troy pound, with Sp, and the 140 comparisons with RS, made in 1829, RS=Sp+0·00515 grain. According to the comparisons of T with Sp and RS in 1845, RS=Sp+0·00534 gr., a result which differs from the former by only 0·00019 gr. Giving to the comparisons of U and T with Sp twice the weight of their comparisons with RS, because the number of comparisons of U and T with Sp is about twice as great as the number of comparisons with RS, the probable error of T, in terms of Sp and RS, will be 0·000058 gr. The probable error of $\frac{1}{4}$(A+B+C+D) is 0·000026 gr., and that of Q is 0·000004 gr. Hence the probable error of T+Q+$\frac{1}{4}$(A+B+C+D) is 0·000064 gr.

Probable errors of the comparisons of I with T and the auxiliary weights.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison.	Probable error of n comparisons.
			gr.	gr.
858	I, T+Q+A	40	0·000569	0·000091
859	I, T+Q+B	40	0·000504	0·000081
859	I, T+Q+C	40	0·000648	0·000104
860	I, T+Q+D	50	0·000618	0·000083

Hence the probable error of the comparison of I with T+Q+$\frac{1}{4}$(A+B+C+D) is 0·000038 gr. But the probable error of T+Q+$\frac{1}{4}$(A+B+C+D) is 0·000064 gr. Therefore the probable error of I is 0·000074 grain. If we substitute for A, B, C, D their values in terms of F+G, the resulting values of I will be affected by the errors of the comparisons of A, B, C, D with F+G, combined with the errors of the comparisons of I with T+Q+A, T+Q+B, T+Q+C, T+Q+D. The differences of these values of I from the mean are −0·00097 grain, +0·00008 grain, +0·00011 grain, +0·00078 grain respectively.

Probable errors of the comparisons of I with K, L, M, N, the platinum lbs. Nos. 1, 2, 3, 4, and with Sp+V, Professor Schumacher's lb.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison.	Probable error of n comparisons.
			gr.	gr.
862	I, K	100	0·000514	0·000052
864	I, L	108	0·000659	0·000064
866	I, M	102	0·000485	0·000048
868	I, N	102	0·000555	0·000055
872	I, Sp+V	100	0·000495	0·000050

Probable errors of the weights (64), (32), (16), (8), (4), (2), J.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison. gr.	Probable error of n comparisons. gr.
829	(64)+16, $\frac{1}{3}$(K+L+M+N+R+S)	10	0·000150	0·000050
830	(64), (32)+(16)+(8)+(4)+(2)+2J	5	0·000144	0·000072
830	(32), (16)+(8)+(4)+(2)+2J	5	0·000095	0·000047
830	(16), (8)+(4)+(2)+2J	5	0·000082	0·000041
830	(8), (4)+(2)+2J	5	0·000036	0·000018
830	(4), (2)+2J	5	0·000031	0·000016
830	(2), 2J	5	0·000141	0·000070

The probable error of K+L+M+N+R+S is 0·000019 grain, and the probable error of J is 0·000015 grain.

Probable errors of the comparisons of 𝔄, the Kilogramme des Archives, with K+L+B, and with the platinum kilogramme 𝔈, and of the comparisons of 𝔈 with I+K+A, I+L+B, I+M+Γ, I+N+Δ.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison. gr.	Probable error of n comparisons. gr.
891	𝔄, K+L+B	60	0·001769	0·000230
882	𝔄, 𝔈	200	0·001594	0·000113
886	𝔈, I+K+A	34	0·001296	0·000226
886	𝔈, I+L+B	40	0·000790	0·000127
887	𝔈, I+M+Γ	40	0·001333	0·000213
888	𝔈, I+N+Δ	42	0·000987	0·000154

Probable errors of the auxiliary weights used in obtaining the value of 𝔈 in terms of I.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison. gr.	Probable error of n comparisons. gr.
889	I, A+B+Γ+Δ	24	0·000469	0·000097
889	A, Z+Θ	10	0·000373	0·000124
889	B, Z+Θ	10	0·000510	0·000170
890	Γ, Z+Θ	10	0·000372	0·000124
890	Δ, Z+Θ	10	0·000467	0·000146

Θ = sum of weights L, M of nearly 80 grains each, J, the mean of two grain weights, and Q, the mean of the ten weights of 0·64509 grain, the probable errors of which are 0·000083 grain, 0·000066 grain, 0·000015 grain, 0·000004 grain respectively. Therefore the probable error of Θ is 0·000107 grain. Hence the probable error of 𝔈 in terms of I is 0·000193 grain. But the probable error of the comparison of 𝔈 with 𝔄 is 0·000113 grain. Therefore the probable error of the value of 𝔄 in grains of which I contains 7000·00000, is 0·000224 grain.

The four different values of 𝔈 obtained by substituting for K, L, M, N their values in terms of I, and for A, B, Γ, Δ their values in terms of Z+Θ, are affected by the errors of the comparisons of 𝔈 with I+K+A, I+L+B, I+M+Γ, I+N+Δ, combined with the errors of the comparisons of A, B, Γ, Δ with Z+Θ. Their differences from the mean are +0·00176 grain, +0·00026 grain, −0·00105 grain, −0·00098 grain respectively.

The probable error of the value of 𝔄 in grains of which I contains 7000·00000, obtained by comparing it with K+L+B, is 0·000334 grain.

The differences between two or more series of comparisons of the same weights, though small, are larger than the probable error of each series would lead us to expect. Of the errors which affect the results of weighing, some partake too much of the nature of constant errors to be fairly estimated by the method of least squares. Of this kind is the error due to small differences of temperature of the weights. Whenever it was practicable, the weights to be compared were left in the balance-case during the night previous to the day on which they were compared. This precaution, however, was in some measure defeated, when a single weight was compared with the sum of several others; for the latter would be in advance of the former in following the changes of temperature during the time occupied by the comparisons. The effect of temperature on the apparent weight of any object appears to be due to currents of air ascending or descending, according as the weight is hotter or colder than the air in the balance-case. A brass kilogramme that had been left for several hours in the balance-case where the temperature was 5°·2 C., appeared to be about 5 milligrammes lighter after it had been heated up to 16°·4 C. The hygroscopic matter contained in some of the auxiliary weights, from which it was difficult to free them entirely by digestion in boiling water, may also have introduced a small error in one direction. In order to diminish, as much as possible, any inaccuracy resulting from this cause, all the more important weighings into which these weights entered, were made within the narrowest practicable limits of time. Many observations that could not be brought within such limits of time as were considered satisfactory, were rejected, the chance of a larger irregular error belonging to a small number of comparisons being considered less injurious to accuracy than the error in one direction to be apprehended in a larger number, extending over a considerable interval of time. That part of the weighing which depended upon the performance of the balances was most satisfactory. When the large balance, constructed by Mr. Barrow, was loaded with a pound in each pan, the probable error of a single comparison, by Gauss's method, was 0·00056 grain, or less than one-12 millionth of the weight in either pan; with a kilogramme in each pan, the probable error of a single comparison, by Borda's method, was 0·00162 grain, or less than one-9 millionth part of the weight in either pan; by Gauss's method it was 0·00112 grain, or one-14 millionth of the weight in either pan.

Legalization of the new Standards.

Legal authority has been given to the new Standard lb. and its four copies in platinum by an Act of Parliament, entitled "An Act for legalizing and preserving the restored Standards of Weights and Measures." The most important of those provisions of the Act which relate exclusively to the Standards of Weight, are contained in the following extracts:—

"Whereas by an Act of the Fifth Year of the Reign of King George the Fourth, Chapter Seventy-four, ... it was enacted ... that from and after the First Day of May

1825, the Standard Brass Weight of One Pound Troy Weight made in the Year 1758, then in the Custody of the Clerk of the House of Commons, should be and the same was thereby declared to be the original and genuine Standard Measure of Weight, and that such Brass Weight should be the 'Imperial Standard Troy Pound,' and should be and the same was declared to be the Unit or only Standard Measure of Weight from which all other Weights should be derived, computed, and ascertained, and that $\frac{1}{12}$ of the said Troy Pound should be an Ounce, and that $\frac{1}{20}$ of such Ounce should be a Pennyweight, and that $\frac{1}{24}$ of such Pennyweight should be a Grain, so that 5760 such Grains should be a Troy Pound, and that 7000 such Grains should be and they were thereby declared to be a Pound Avoirdupois: And whereas by the said Act Provision was made for restoring the said Imperial Troy Pound, in case of Loss, Destruction, Defacement, or other Injury, by Reference to the Weight of a Cubic Inch of Water: And whereas the said Standard Pound Troy (was) destroyed in the Fire at the Houses of Parliament: And whereas by the Researches of Scientific Men Doubts were thrown on the Accuracy of the Methods provided by the said Act for the Restoration of the said Standard: And whereas there exist Weights which had been accurately compared with the said Standard Pound Troy, which afforded sufficient Means for restoring such original Standard: And it having been deemed expedient that the Standard for Reference as a Measure of Weight should be a Pound Avoirdupois, there has been constructed a Pound Weight Avoirdupois equivalent to the Pound Avoirdupois of 7000 such Grains as are mentioned in the said recited Act, and Four accurate Copies of the said Pound Avoirdupois so constructed: And whereas the Standard Pound Avoirdupois so constructed as aforesaid, and the Copies thereof, are of Platinum, the Form being that of a Cylinder nearly 1·35 Inch in Height and 1·15 Inch in Diameter, with a Groove or Channel round it whose Middle is about 0·34 Inch below the Top of the Cylinder, for insertion of the Points of the Ivory Fork by which it is to be lifted; the Edges are carefully rounded off: And whereas the said Standard of Weight marked P.S. 1844, 1 lb. has been deposited in the Office of the Exchequer at *Westminster*, and One of the said Copies of the Standard of Weight marked No. 1. P.C. 1844, 1 lb. has been deposited at the Royal Mint; and One other of the said Copies of the Standard of Weight marked No. 2. P.C. 1844, 1 lb. has been delivered to the Royal Society of *London*; and One other of the said Copies of the Standard of Weight marked No. 3. P.C. 1844, 1 lb. has been deposited in the Royal Observatory of *Greenwich*; and the other of the said Copies of the Standard of Weight marked No. 4. P.C. 1844, 1 lb. has been immured in the Cill of the Recess on the East Side of the lower Waiting Hall in the New Palace at *Westminster*: And whereas it is expedient to legalize the Standards so constructed and to provide for the Preservation thereof: Be it therefore enacted ... as follows:—

I. So much of the said Act of the Fifth Year of King George the Fourth as relates to the Restoration of the Standard Troy Pound, in case of Loss, Destruction, Defacement, or other Injury, shall be repealed.

III. The said Weight of Platinum marked P.S. 1844, 1 lb., deposited in the Office of the Exchequer as aforesaid, shall be the legal and genuine Standard Measure of Weight, and shall be and be denominated the Imperial Standard Pound Avoirdupois, and shall be deemed to be the only Standard Measure of Weight from which all other Weights and other Measures having Reference to Weight shall be derived, computed, and ascertained, and One equal Seven Thousandth Part of such Pound Avoirdupois shall be a Grain, and Five Thousand seven hundred and sixty such Grains shall be and be deemed to be a Pound Troy.

VII. If at any Time hereafter the said Imperial Standard Pound Avoirdupois be lost, or in any Manner destroyed, defaced, or otherwise injured, the Commissioners of Her Majesty's Treasury may cause the same to be restored by Reference to or Adoption of any of the Copies so deposited as aforesaid, or such of them as may remain available for that Purpose."

Densities, Errors, and Distribution of the Standards of Weight.

The first column of the following Table contains the mark or designation of the weight; the second, its density at the temperature of melting snow (with one exception) in terms of the maximum density of water; the third, its error, in grains, when compared in a vacuum with PS, the new Imperial Standard lb.; the fourth, its error, in grains, when compared with W, the Commercial Standard lb., in air of the temperature 65°·55 FAHR., under the pressure of 29·750 inches of mercury at the temperature of melting snow ($t=18°{\cdot}7$ C., $b=\overset{\text{mm}}{755{\cdot}64}$), in Somerset House, or in air for which $\log\Delta=7{\cdot}07832-10$. The last column contains the name of the place of deposit, or of the country to which the weight has been sent.

Designation.	Density.	In a vacuum.	In air for which log Δ=7·07832−10.	Distribution.
		gr.	gr.	
P.S.	21·1572	0·00000	0·63407 too heavy.	Exchequer.
No. 1. P.C.	21·1671	0·00051 too heavy.	0·63477 too heavy.	Royal Mint.
No. 2. P.C.	21·1640	0·00089 too light.	0·63331 too heavy.	Royal Society.
No. 3. P.C.	21·1615	0·00178 too light.	0·63237 too heavy.	Royal Observatory, Greenwich.
No. 4. P.C.	21·1516	0·00314 too light.	0·63090 too heavy.	New Palace, Westminster.
Sp.+V.	21·1321	0·00023 too light.	0·63336 too heavy.	Observatory, Altona.
Troy Pd. T.	21·1661	0·52934 too light.	0·00745 too light.	Royal Observatory, Greenwich.
Gilt lb. No. 1.	8·36134	0·00732 too light.	0·01956 too heavy.	India
...... No. 2.	8·34161	0·03582 too light.	0·01132 too light.	Russia.
...... No. 3.	8·30462	0·00510 too heavy.	0·02512 too heavy.	Prussia.
...... No. 4.	8·36500	0·00425 too heavy.	0·03157 too heavy.	Bavaria.
...... No. 5.	8·06122	0·01783 too heavy.	0·00734 too heavy.	U.S. America.
...... No. 6.	8·28779	0·01714 too light.	0·00083 too heavy.	Edinburgh.
...... No. 7.	8·12163	0·01933 too heavy.	0·01658 too heavy.	Austria.
...... No. 8.	8·16317	0·01428 too heavy.	0·01679 too heavy.	Dublin.
...... No. 9.	7·37614	0·11611 too heavy.	0·00426 too heavy.	Canada.
...... No. 10.	8·28375	0·03910 too light.	0·02162 too light.	Cape of Good Hope.
...... No. 11.	8·36302	0·04208 too light.	0·01499 too light.	Sydney.
...... No. 12.	8·31919	0·02060 too light.	0·00118 too heavy.	Portugal.
...... No. 13.	8·43179	0·03331 too light.	0·00195 too heavy.	Spain.
...... No. 14.	8·34955	0·02844 too light.	0·00301 too light.	Holland.

Gilt lb. No. 15.	8·36107	0·02022 too light.	0·00667 too heavy.	France.
...... No. 16.	8·07354	0·02747 too light.	0·03640 too heavy.	Belgium.
...... No. 17*a*.	8·11718	0·02614 too light.	0·02948 too light.	Hanover.
...... No. 17*b*.	8·55888	0·04428 too light.	0·00542 too heavy.	Saxony.
...... No. 18.	8·30369	0·00129 too light.	0·01857 too heavy.	Sweden.
...... No. 19.	8·33969	0·01473 too light.	0·00950 too heavy.	Denmark.
...... No. 21.	7·97370	0·03971 too heavy.	0·01777 too heavy.	Switzerland.
...... No. 22.	8·19859	0·01214 too light.	0·00523 too light.	Sardinian States.
...... No. 23.	8·15141	0·01557 too heavy.	0·01655 too heavy.	Papal States.
...... No. 24.	8·14286	0·03932 too light.	0·03941 too light.	Naples.
...... No. 25.	8·10164	0·00180 too heavy.	0·00354 too light.	Turkey.
...... No. 26.	8·15218	0·00112 too light.	0·00001 too light.	Brazil.
...... No. 27.	8·16186	0·01405 too heavy.	0·01635 too heavy.	Buenos Ayres.
...... No. 28.	8·12604	0·00416 too light.	0·00638 too light.	Chili.
...... No. 29.	8·18446	0·00222 too light.	0·00293 too heavy.	Melbourne.
...... No. 30.	8·15292	0·00170 too light.	0·00050 too light.	Hobarton.
...... No. 31.	8·51444	0·04410 too light.	0·00080 too heavy.	
..... No. 32.	8·47042	0·03448 too light.	0·00304 too heavy.	Royal Observatory, Greenwich.
...... No. 33.	8·47902	0·04144 too light.	0·00063 too light.	Still disposable.
...... No. 34.	8·51472	0·04584 too light.	0·00089 too light.	
...... No. 35.	8·47019	0·04019 too light.	0·00041 too light.	Exchequer.
...... No. 36.	8·49601	0·04484 too light.	0·00207 too light.	R. Obs., Gr. Still disposable.
Quartz lb. (at 18° C.)	2·649009	2·36797 too heavy.	0·40147 too light.	Royal Observatory, Greenwich.
Platinum Kilogr. 𝕮.	21·13791	0·02412 too light.	....	Royal Observatory, Greenwich.
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oz.	0·00015	too heavy.	0·001	0·00012	too heavy.

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